ChatGPT

Reference Guide: Option Pricing Models and Implied Volatility Functions

Below is a structured reference for various plain-vanilla option types (calls and puts), mapping each to the appropriate pricing model and implied volatility calculation. For each category, we list the pricing model, implied volatility function (if applicable), expected formula inputs, and option type support. Function signatures are provided as a design guide for Python implementation.

1. European Options on Non-Dividend-Paying Stocks

This is the classic case of a European option on a stock with no dividends. The Black-Scholes model applies, providing a closed-form solution for call and put prices [1]. Implied volatility is obtained by inverting the Black-Scholes formula numerically (since no closed-form exists for σ).

- **Pricing Model:** *Black-Scholes (1973) formula* – Uses the Black-Scholes-Merton differential equation to price European calls and puts on a non-dividend-paying underlying [1]. This model assumes no dividends during the option's life [2] and that exercise can only occur at expiration (European-style) [3]. The model requires five inputs: underlying price, strike, time to expiration, risk-free rate, and volatility [1]. The call price formula is:

$$C = S\,N(d_1) - Ke^{-rT}N(d_2),$$

with $d_{1,2} = \frac{\ln(S/K)+(r\pm\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, and $N(\cdot)$ the standard normal CDF. A put price can be obtained via put-call parity or the analogous formula for puts.
Function Signature (Pricing): `bs_price(S, K, T, r, sigma, option_type)` → float
(returns the theoretical option price for a call or put).
- **Implied Volatility:** *Black-Scholes implied volatility* – Solve for σ by numerically inverting the Black-Scholes pricing function. Given a market option price, one can use a root-finding method (e.g. Newton-Raphson) to find the σ that reproduces this price in the Black-Scholes formula. There is no closed-form solution for implied vol in the Black-Scholes model, so iterative methods are used.
Function Signature (Implied Vol): `bs_implied_vol(price, S, K, T, r, option_type)` → float
(returns the implied volatility that yields the given option price).
- **Formula Inputs:** Spot price S ; Strike K ; Time to maturity T (in years); Risk-free interest rate r (continuous compounding); Volatility σ (annualized); Option type (call or put). These five inputs are the standard parameters for Black-Scholes [1].
- **Option Type:** European Call **or** Put (no early exercise). The Black-Scholes formula prices calls and puts on non-dividend stocks; it is only valid for European-style options (it does *not* account for early exercise) [3]. Both call and put prices can be obtained (either directly or via put-call parity).

2. European Options on Dividend-Paying Stocks

For European options on stocks that pay dividends, the pricing is adjusted to account for continuous dividend yield (or expected discrete dividends). The standard approach is the Black-Scholes-Merton model, which extends Black-Scholes by including the dividend yield in the formulation [4]. Implied volatility is computed by inverting this dividend-adjusted pricing model.

- **Pricing Model:** *Black-Scholes-Merton model (1973, Merton's extension)* – This is the Black-Scholes formula modified to include continuous dividend yield q [4]. The effect of dividends is to reduce the effective underlying price or forward price. In practice, one can incorporate a continuous yield by replacing S with Se^{-qT} in the formula (intuitively, the stock's present value is reduced by the expected dividends) [5]. The call price under continuous dividend yield is:

$$C = Se^{-qT}N(d_1) - Ke^{-rT}N(d_2),$$

  where $d_{1,2} = \frac{\ln(S/K)+(r-q\pm\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$. This formula (sometimes called the Merton model) assumes a continuous dividend yield; for discrete dividends, one often approximates by adjusting S by the present value of expected dividends [6].
  Function Signature (Pricing): `bsm_div_price(S, K, T, r, q, sigma, option_type)` → float.
  (Prices a European call or put given continuous dividend yield `q`.)
- **Implied Volatility:** *Dividend-adjusted implied volatility* – Compute σ by numerically inverting the dividend-augmented pricing function. Given an option's market price, one finds the σ such that the Black-Scholes-Merton formula (with the known dividend yield) matches the price. This typically requires iterative solvers (no closed-form).
  Function Signature (Implied Vol): `bsm_div_implied_vol(price, S, K, T, r, q, option_type)` → float.
  (Finds the implied vol for a European option with continuous yield `q`.)
- **Formula Inputs:** Spot price S ; Strike K ; Time to maturity T ; Risk-free rate r ; Continuous dividend yield q (annualized); Volatility σ ; Option type (call/put). The dividend yield enters the pricing as described above, effectively reducing the carry of the underlying [5].
- **Option Type:** European Call **or** Put. (Early exercise is not considered — the formula is for European-style options.) Both calls and puts are handled. The model is specifically designed for dividend-paying underlyings; if $q = 0$ it reduces to the standard Black-Scholes (non-dividend) case [7].

3. European Options on Commodities (Futures/Forwards)

European options on commodities are often priced using **Black-76 (Black's 1976) model**, especially when the underlying is a futures or forward contract [8]. Black-76 is essentially the Black-Scholes formula applied to forward prices, treating the forward/futures price as the underlying and discounting the payoff at risk-free rate. Implied volatility in this context (sometimes called futures implied vol) is computed by inverting the Black-76 pricing formula.

- **Pricing Model:** *Black-76 model (F. Black, 1976)* – A lognormal model for options on futures or forwards [8]. Black's formula is very similar to Black-Scholes, except the underlying spot price is replaced by the futures price F (and the growth rate of the underlying is accounted for by the forward price itself) [9]. The call option on a futures (maturing at or after the option) is:

$$C = e^{-rT}\big[FN(d_1) - KN(d_2)\big],$$

$$d_{1,2} = \frac{\ln(F/K) + (\pm\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}},$$

where F is the current forward/futures price for maturity $T' \geq T$ and r is the risk-free rate (for discounting) [10] [11]. Put options have a similar formula: $P = e^{-rT}[KN(-d_2) - FN(-d_1)]$. Black-76 is widely used for commodity options (as well as interest rate caps/floors and futures options) [8].
Function Signature (Pricing): `black76_price(F, K, T, r, sigma, option_type)` → float. (Prices a European call/put on a futures/forward with price `F`.)

- **Implied Volatility:** *Black-76 implied volatility* – Solve for σ given an option price by inverting Black's formula. Market quotes for commodity options on futures are often given in terms of Black implied vol. The implied volatility is obtained numerically, similar to Black-Scholes inversion.
Function Signature (Implied Vol): `black76_implied_vol(price, F, K, T, r, option_type)` → float.
(Finds the implied σ that reproduces the observed price using Black-76.)
- **Formula Inputs:** Futures/Forward price F (for the underlying commodity contract, for delivery at or just after option expiration); Strike K; Time to option expiration T; Risk-free rate r (to discount the payoff); Volatility σ (of the underlying futures price); Option type (call or put). Note: If working with a commodity spot that has a **continuous net cost-of-carry** (storage cost minus convenience yield), that can be treated analogously to a dividend yield in determining the forward price. In practice, one often inputs the current futures price directly, as it already reflects cost-of-carry and convenience yield.
- **Option Type:** European Call **or** Put. (No early exercise feature.) The Black-76 formula prices European-style options on commodity futures or forwards [8]. Both calls and puts are supported via the above formulas.

4. European Options on Foreign Exchange (FX)

Foreign exchange options (currency options) use an extension of Black-Scholes known as the **Garman-Kohlhagen model**. This model (Garman & Kohlhagen, 1983) adapts Black-Scholes to two interest rates: one for domestic currency and one for foreign currency, effectively treating the foreign interest rate as a continuous dividend yield on the underlying foreign currency [12]. Implied volatility is typically quoted by inverting the Garman-Kohlhagen formula, similar to equity options.

- **Pricing Model:** *Garman–Kohlhagen model (1983)* – This is the closed-form solution for European currency options, analogous to Black-Scholes but accounting for continuous yields in two currencies [12]. The formula for a call option to buy foreign currency (domestic currency strike) is:

$$C = Se^{-r_f T}N(d_1) - Ke^{-r_d T}N(d_2),$$

where S is the current spot FX rate (domestic per unit foreign), r_d is the domestic risk-free rate, r_f is the foreign risk-free rate, and

$$d_{1,2} = \frac{\ln(S/K) + (r_d - r_f \pm \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}.$$

This formula is essentially the Black-Scholes-Merton formula with r_d in place of r and r_f in place of a dividend yield [12]. Intuitively, paying foreign interest is like receiving a continuous dividend yield on the foreign currency. The put option formula is similarly: $P = Ke^{-r_dT}N(-d_2) - Se^{-r_fT}N(-d_1)$.
Function Signature (Pricing): `garman_kohlhagen_price(S, K, T, r_dom, r_for, sigma, option_type)` → float.
(Prices a European call/put on FX, given spot `S`, domestic rate `r_dom`, foreign rate `r_for`.)

- **Implied Volatility:** *FX implied volatility (GK model)* – Determined by inverting the Garman-Kohlhagen pricing equation. In FX markets, implied vol is standard and is solved such that plugging it into the GK model yields the market option price [13]. Numerical methods (e.g. bisection/Newton) are used to find σ.
Function Signature (Implied Vol):
`garman_kohlhagen_implied_vol(price, S, K, T, r_dom, r_for, option_type)` → float.
- **Formula Inputs:** Spot FX rate S (units of domestic per one unit of foreign currency, e.g. USD per EUR); Strike K (in same units as S); Time to maturity T; Domestic interest rate r_d; Foreign interest rate r_f; Volatility σ (vol of the FX rate); Option type (call/put, where call = right to buy foreign currency). Note: r_f acts like a continuous yield on the underlying asset (the foreign currency) [14].
- **Option Type:** European Call **or** Put on the FX rate. (No early exercise.) The Garman-Kohlhagen formula handles both calls and puts. In practice, one must be careful about which currency is underlying vs. numeraire; the above formulation assumes the option payoff is in domestic currency.

5. Asian Options on Commodities (Average Price Options)

Asian options are options where the payoff depends on the average price of the underlying over a period (rather than the price at expiration). Here we consider **arithmetic Asian options** (which are common in commodity markets, e.g. average price options on oil or gas). There is no exact closed-form for arithmetic Asian options, so one uses analytical approximations or numerical methods. A well-known approximation for European Asian options is the **Turnbull-Wakeman (1991) model**. We assume the underlying is a commodity or commodity future and the option payoff is based on the average price over the option's life. Implied volatility for Asian options is typically obtained by numerical inversion of the pricing model (since no closed-form IV formula exists).

- **Pricing Model:** *Turnbull–Wakeman approximation (1991)* – An analytical approximation for the price of a **European arithmetic-average** option. This model provides a closed-form estimate for options on an underlying with continuous averaging (or discretely with many points) by adjusting the Black-Scholes/Black-76 framework to account for averaging. In essence, it computes an effective forward price and an effective volatility for the average, then uses a Black–Scholes-type formula [15]. The Turnbull-Wakeman formula is often applied to commodity Asian options (and can be specifically adjusted for options on futures) [15]. (For example, in the case of an average price option on a futures contract, the formula modifies Black-76 by using the average's variance formula.) The result is an approximate call price C_{Asian}; a put price can be obtained similarly or via parity. *(Kemna & Vorst's geometric Asian formula (1990) provides an exact solution for geometric-average options, but for*

arithmetic average, Turnbull-Wakeman is a standard approximation.)
Function Signature (Pricing): `asian_price_TW(S_or_F, K, T, sigma, r, [q or yield], option_type)` → float.
(Prices a European Asian call/put. If underlying is a futures, use forward price F and r; if spot with yield, include yield like dividend.) For example, `asian_price_TW(F, K, T, sigma, r, option_type)` for an option on a futures.

- **Implied Volatility:** *Asian option implied volatility* – There is no unique closed-form implied vol formula. Instead, implied vol is found by numerical methods: one seeks the volatility such that the Turnbull-Wakeman pricing equals the market price. Often practitioners will quote an “implied vol” for an Asian option by equating the option’s price to that given by a Black-76 formula with some effective volatility. In essence, one can solve for σ such that:

$$\text{TW\_Price}(\sigma) = \text{Market Price},$$

  using an iterative solver. This yields the implied volatility consistent with the TW model (valid for that option’s specific averaging features). **Function Signature (Implied Vol):** `asian_implied_vol(price, S_or_F, K, T, r, [q], option_type)` → float (uses numerical root-finding with the TW pricing function).
- **Formula Inputs:** Underlying price or forward S or F (if underlying is a spot commodity, one may include a convenience yield or use the forward price for the averaging period; if underlying is a futures contract, use current futures price); Strike K ; Time to maturity T ; Volatility σ (volatility of the underlying’s price); Risk-free rate r ; and if applicable, underlying yield (e.g. convenience yield or dividend analog) which affects the forward price. Additionally, details of the averaging (start and end of averaging period, discrete vs continuous average) are inputs to the Turnbull-Wakeman formula, but for design purposes these can be parameters inside the pricing function.
- **Option Type:** European Call **or** Put (average price style). We assume the payoff is based on the arithmetic average price over a defined period (often the length of the option). The pricing model here is for European exercise (the option pays at maturity the difference between the average and strike, if positive, for a call). Only plain calls and puts on the average are considered (no exotic path-dependent triggers beyond the averaging).

6. Spread Options (Commodity Spread Options)

A spread option is an option on the **spread between two prices** – for example, the difference between two commodity prices (like the spread between two different crude oil benchmarks, or between a commodity and its refined product). The payoff might be $(S_1 - S_2 - K)^+$ for a call on the spread (or the reverse for a put). There is no closed-form solution in general for spread options (since it’s a multi-asset option), but a commonly used approximation for European spread options is **Kirk’s approximation (1995)** [16] . Implied volatility for spread options is not as straightforward as single-asset options; typically, one may solve for an implied **combined volatility** (or an implied correlation or individual vol) that, when plugged into the pricing model, yields the market price [17] .

- **Pricing Model:** *Kirk’s approximation (1995) for European spreads* – Kirk’s formula provides an approximate analytic price for a European call or put on the spread between two futures (or forwards) [16] . It assumes lognormal dynamics for each underlying and uses an approximate adjustment to reduce the two-factor problem to a one-factor Black-Scholes-type formula. Specifically, Kirk's approximation prices a call on $F_1 - F_2$ as:

$$C_{\text{spread}} \approx e^{-rT}\left[F_1 N(d_1) - (F_2+K)N(d_2)\right],$$

where $d_{1,2} = \frac{\ln(\frac{F_1}{F_2+K}) + (\pm\frac{1}{2}\sigma_{\text{comb}}^2)T}{\sigma_{\text{comb}}\sqrt{T}}$. Here σ_{comb} is an *effective combined volatility* of the spread, calculated from σ_1, σ_2, and the correlation ρ (for example, one common form is $\sigma_{\text{comb}}^2 = \sigma_1^2 + \frac{\sigma_2^2 F_2^2}{(F_2+K)^2} - 2\rho\sigma_1\sigma_2\frac{F_2}{F_2+K}$; this arises from a first-order expansion) [18] . The call and put formulas under Kirk's method resemble a Black-Scholes formula on a synthetic underlying [19] . This approximation is most accurate when the strike K is not too large relative to F_2 (e.g., for near-zero or small strikes) [20] .
Function Signature (Pricing): `spread_price_kirk(F1, F2, K, T, vol1, vol2, rho, r, option_type)` → float.
(Returns the approximate price of a European call/put on $F_1 - F_2$ using Kirk's model. Here `vol1`, `vol2` are σ_1, σ_2 and `rho` is the correlation between dln(F1) and dln(F2).)

- **Implied Volatility:** *Implied parameters for spread options* – Since a spread option depends on two volatilities and a correlation, the notion of a single implied volatility is not as direct. However, traders sometimes speak of an implied vol for the *spread* under a one-factor model. In practice, one can solve for the implied **combined volatility** (σ_comb) that equates Kirk's formula to the market price [17] . This is analogous to solving for implied vol in Black-Scholes, except using Kirk's pricing function. If instead the two individual vols (σ_1, σ_2) are known, one might solve for an implied correlation ρ that matches the spread option's price [21] . These implied values are found via numerical root-finding on the pricing equation (no closed-form).
Function Signature (Implied Vol):
`spread_implied_comb_vol(price, F1, F2, K, T, vol1, vol2, r, option_type)` → float (solves for σ_comb assuming ρ known), or one could have `spread_implied_correlation` solving for ρ given vols. The implementation will depend on which parameter is treated as unknown to be implied. In all cases, the solver iteratively adjusts the parameter until the Kirk pricing equals the observed price.
- **Formula Inputs:** Futures prices F_1, F_2 for the two related commodities (or underlying assets) defining the spread; Strike K for the spread payoff $F_1 - F_2$ (for a call, payoff = max(F1 - F2 - K, 0)); Time to maturity T ; Volatilities σ_1, σ_2 for each commodity's price (under lognormal assumption); Correlation ρ between the two underlying price log-returns; Risk-free rate r for discounting. These inputs feed into Kirk's formula for the spread price [22] . (If the option is on spot commodities with storage costs/convenience yields, those would be reflected in forward prices F1, F2 given to the model.)
- **Option Type:** European Call **or** Put on the spread. Kirk's approximation gives both call and put prices (it satisfies a form of put-call parity for spread options). A "call on the spread" is an option that pays $(S_1 - S_2 - K)^+$; a "put on the spread" pays $(K - (S_1 - S_2))^+ = (S_2 + K - S_1)^+$. The model can handle both (the function would switch signs accordingly or use parity). Only European exercise is considered (American spread options require numerical methods like Monte Carlo or multi-dimensional trees, beyond this scope).

7. American Options on Dividend-Paying Stocks

American options (which can be exercised before maturity) on dividend-paying stocks require models that account for early exercise optimality, especially around dividend ex-dates. There is **no closed-form exact solution** for American puts and calls with dividends, but there are well-known approximations. A commonly used one is the **Bjerksund-Stensland 2002 model**, which provides an analytic approximation for American options with continuous dividend yield [23]. Alternatively, numerical lattice methods (binomial trees) can be used. Implied volatility for American options must be found by iterative methods using the chosen pricing model (since even the pricing is an approximation or numerical).

- **Pricing Model:** *Bjerksund-Stensland (2002) approximation* – This model is an extension of the Black-Scholes-Merton framework designed to handle American-style exercise and dividends [24] [23]. It breaks the option's life into two phases and derives an approximate critical stock price (early exercise boundary) at which it becomes optimal to exercise early. Bjerksund-Stensland is particularly used for American calls/puts on stocks with continuous dividend yield (it can also accommodate discrete dividends by piecewise constant yields or adjustment) [25]. The model yields a closed-form formula involving the cumulative normal distribution and the dividend yield, plus terms to account for early exercise premium. It is more accurate than older approximations like Barone-Adesi–Whaley (1987) in many cases. *(Other approaches: Binomial trees (e.g. Cox-Ross-Rubinstein) can price American options to arbitrary accuracy given enough steps; these are often used in practice as well.)*
  Function Signature (Pricing): `american_price_bjs(S, K, T, r, q, sigma, option_type)` → float.
  (Prices an American call or put given continuous dividend yield `q`. If discrete dividends need to be considered, one might adjust `q` or the underlying price accordingly or use a piecewise approach.)
- **Implied Volatility:** *American option implied vol (Bjerksund-Stensland or lattice)* – Implied volatility is determined by finding the σ such that the American option pricing model's output matches the market price. With Bjerksund-Stensland, one can use a numerical solver to invert the closed-form approximation (MathWorks, for example, provides an `impvbybjs` function to do this [26]). If using a binomial model, one similarly plugs in vol guesses and converges on the vol that reproduces the observed price. There is no analytical inversion, so this is done via iteration (e.g., using Newton's method while re-pricing the option at each step).
  Function Signature (Implied Vol): `american_implied_vol(price, S, K, T, r, q, option_type)` → float.
  (Returns the implied σ for an American option, using either the Bjerksund-Stensland pricing or a binomial model in each iteration.)
- **Formula Inputs:** Spot price S ; Strike K ; Time to maturity T ; Risk-free rate r ; Dividend yield q (or dividend schedule, which can be approximated by an equivalent continuous yield for the model); Volatility σ ; Option type (call or put). These inputs feed the American option pricing routine. If using Bjerksund-Stensland, q is treated as continuous yield [23]. If using a binomial tree, one would input the dividend schedule or yield into the tree (e.g., as continuous yield or by adjusting underlying at ex-dividend points).
- **Option Type:** American Call **or** Put (on dividend-paying stock). Both are supported by the approximation model. Early exercise is possible: for example, it may be optimal to exercise an American call just before a large dividend. The model accounts for this by computing an optimal exercise boundary [27]. For puts, early exercise is more likely even without dividends (due to cost of carry), and the model captures that as well. *(Note: For American calls without dividends, it's never*

optimal to exercise early, and indeed the model will converge to the European price in that case. But with dividends, early exercise can be rational, which this model handles.)

8. American Options on Commodities

American options on commodities add another wrinkle: the underlying may have storage costs and a **convenience yield** (benefit of holding the physical commodity), or the option may be on a futures contract. Both scenarios can be handled by extending the approaches for American options on stocks and futures. If the option is on a commodity **spot** with a continuous convenience yield (analogous to a dividend yield), one can use an American option model with that yield parameter (much like the dividend case) [28]. If the option is on a **futures contract**, one can use a lattice model with cost-of-carry = 0 (since the futures' forward price grows at the risk-free rate minus cost-of-carry, which for a futures is zero net cost in risk-neutral terms) [29]. There isn't a unique closed-form solution, so numerical methods or approximations are employed. Implied volatility is again solved via numerical inversion.

- **Pricing Model:** *American option model with commodity carry* – Two common approaches: (a) **Binomial Tree (CRR)** calibrated for commodities, or (b) **Bjerksund-Stensland/Barone-Adesi-Whaley** with a yield. For a commodity with a known continuous **convenience yield** `y` (and possibly storage cost), we can treat `y` analogous to a dividend yield in an American option model [28]. In practice, one might plug `y` into the Bjerksund-Stensland formula (or similar) in place of q. This yields an approximate price for an American call/put on the commodity spot. If the American option is on a **futures** (e.g., an American option on a futures contract, which is common in commodity markets), one can use a binomial lattice with **b = 0** as the cost-of-carry (since the forward price of a futures is martingale under risk-neutral measure) [29]. The Cox-Ross-Rubinstein model with $b = 0$ effectively assumes the underlying price grows at 0 (which is appropriate for futures) and can price American options on futures consistently. *(American calls on futures are sometimes exercised early if deep ITM and rates are high, because early exercise locks in the futures P/L immediately; the binomial model will capture this.)*
Function Signature (Pricing): `american_commodity_price(S_or_F, K, T, r, y, sigma, option_type)` → float.
- If `S_or_F` is a spot price S and `y` is the convenience yield (continuous), the function prices an American option on the spot (using an approximation or binomial with $b = r - y$ as cost of carry [29]).
- If `S_or_F` is a futures price F (and thus effectively $y = 0$ in terms of cost-of-carry since $b = 0$ for futures), the function prices an American option on the futures (e.g., via a binomial tree or by treating it as a no-dividend case in an American model).
- **Implied Volatility:** *American commodity option implied vol* – Determined by numerical inversion of the chosen pricing approach. For instance, if using a binomial model for an American option on a futures, one must trial-and-error different σ in the lattice until the model price matches the market price. Similarly, if using an analytic approximation (like Bjerksund-Stensland with yield), one uses a root-finding algorithm to find the σ that makes the formula equal the market price. The process is analogous to the equity American case.
Function Signature (Implied Vol):
`american_commodity_implied_vol(price, S_or_F, K, T, r, y, option_type)` → float.
(Solves for σ given the pricing model. If `y` (convenience yield) is known, it remains fixed during the iteration.)

- **Formula Inputs:** If underlying is a **spot commodity**: Spot price S ; Convenience yield y (continuous annual rate) – this plays the role of a negative cost-of-carry (or a dividend-like yield) [28] ; Storage cost (if modeled, could be netted against y); Risk-free rate r ; Strike K ; Time T ; Volatility σ . If underlying is a **futures**: Current futures price F (for expiration $\geq$T); Risk-free rate r ; Strike K ; Time T ; Volatility σ . In a binomial pricing context, one sets the cost-of-carry $b = r - y$ for a commodity spot, or $b = 0$ for a futures [29] , to construct the lattice. These inputs feed into the pricing algorithm (approximation formula or tree).
- **Option Type:** American Call **or** Put. Both are considered. For example, an American call on a commodity might be exercised early to take possession if the carry costs are low and convenience yield is high (analogous to exercising before a dividend). An American put on a commodity might be exercised early if carrying the commodity is costly relative to interest. The models (binomial or Bjerksund-style) will decide early exercise by comparing continuation value vs. immediate exercise at each step [27] . Both calls and puts are handled by the general approach (the function would incorporate early-exercise logic for both).

Sources: The above mappings are based on well-known theoretical models: Black-Scholes and Merton's dividend extension [1] [4] , Black (1976) for futures [8] [9] , Garman-Kohlhagen for FX [30] [12] , Turnbull-Wakeman for Asian options [15] , Kirk's formula for spreads [16] , and Bjerksund-Stensland and related methods for American options [23] [29] . Each pricing model's function expects specific inputs (spots, forwards, rates, yields, etc.) as noted, and each implied volatility function must invert the respective pricing equation, typically via numerical methods. This guide can serve as a design skeleton for implementing the models and their implied volatility solvers in Python, ensuring the correct model is used for each option type and underlying scenario.

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