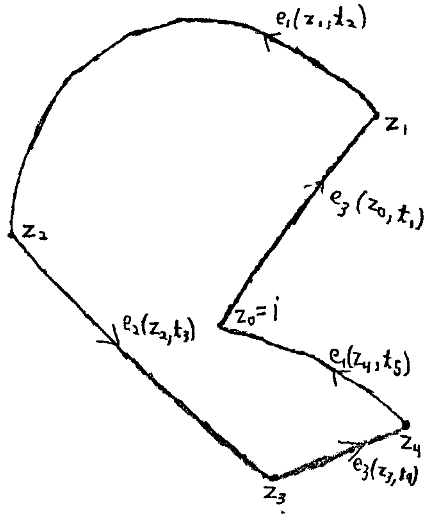
Figure 1 : Generic example of a control trajectory

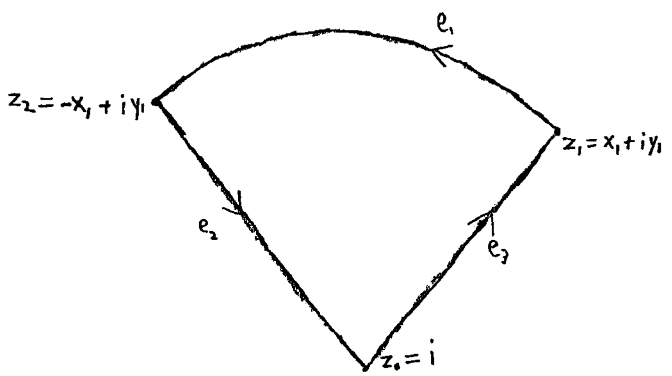


* It takes time t_j to get from z_{j-1} to z_j.

** Switching times are $T_j =: \sum_{n=1}^{j} t_n$

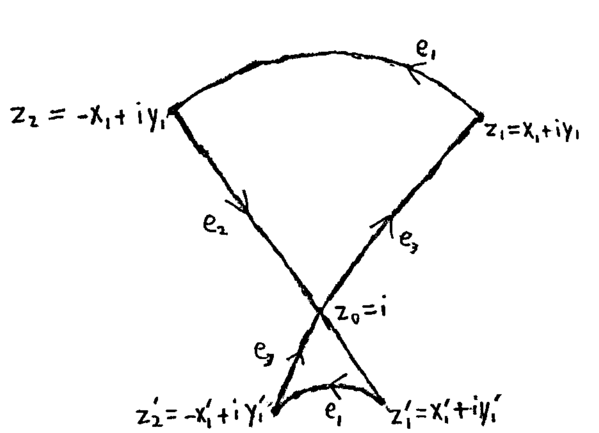
*** $z_j = e_{k_j}(z_{j-1}, t_j)$

Figure 2 : Simple control trajectory



* Requires one parameter, x_1

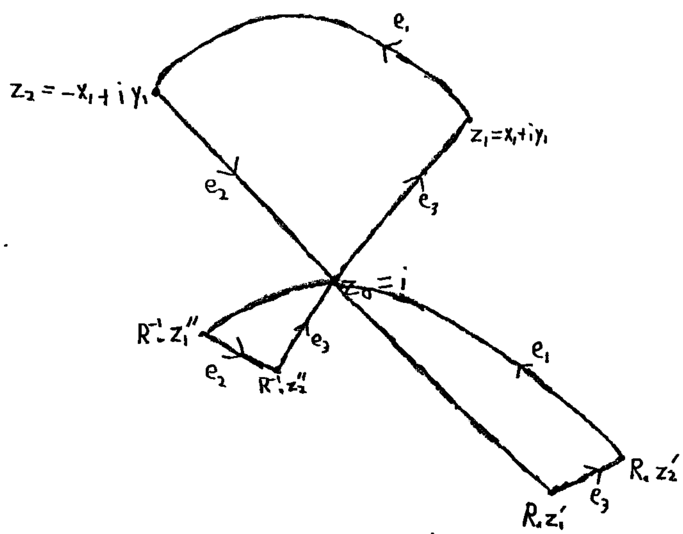
Figure 3 : Two-parameter control trajectory



* Requires two parameters, x_1 and x_1'

** ′ does not indicate a derivative here

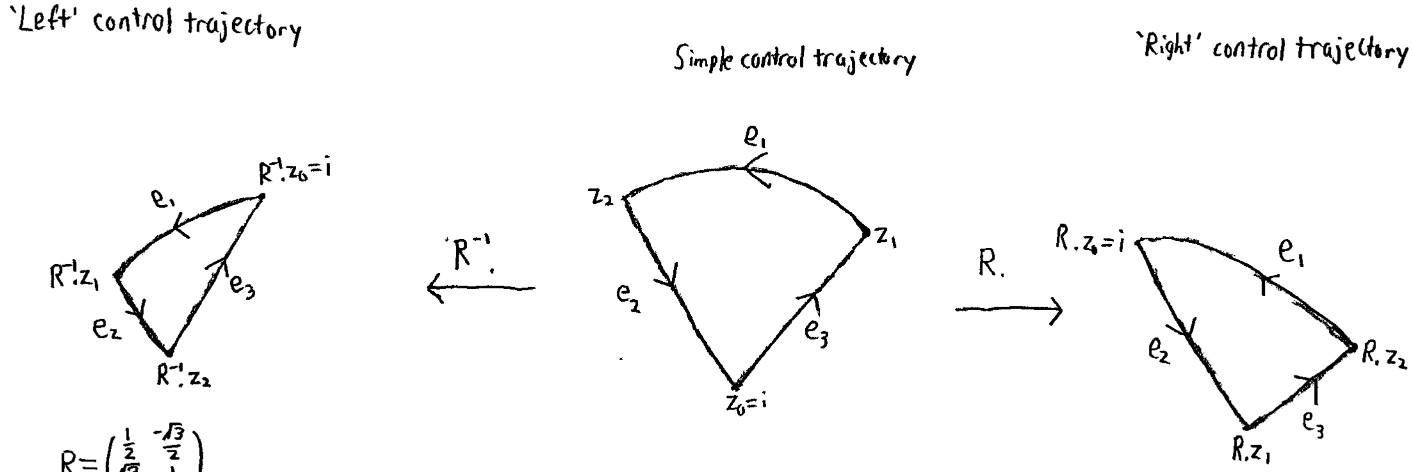
Figure 4 : Three-parameter control trajectory



* Requires three parameters, x_1, x_1', and x_1'' (again, ′ is not a derivative)

** Captures three-piece hyperbolic symmetry

Figure 5: Creating a three-parameter control trajectory

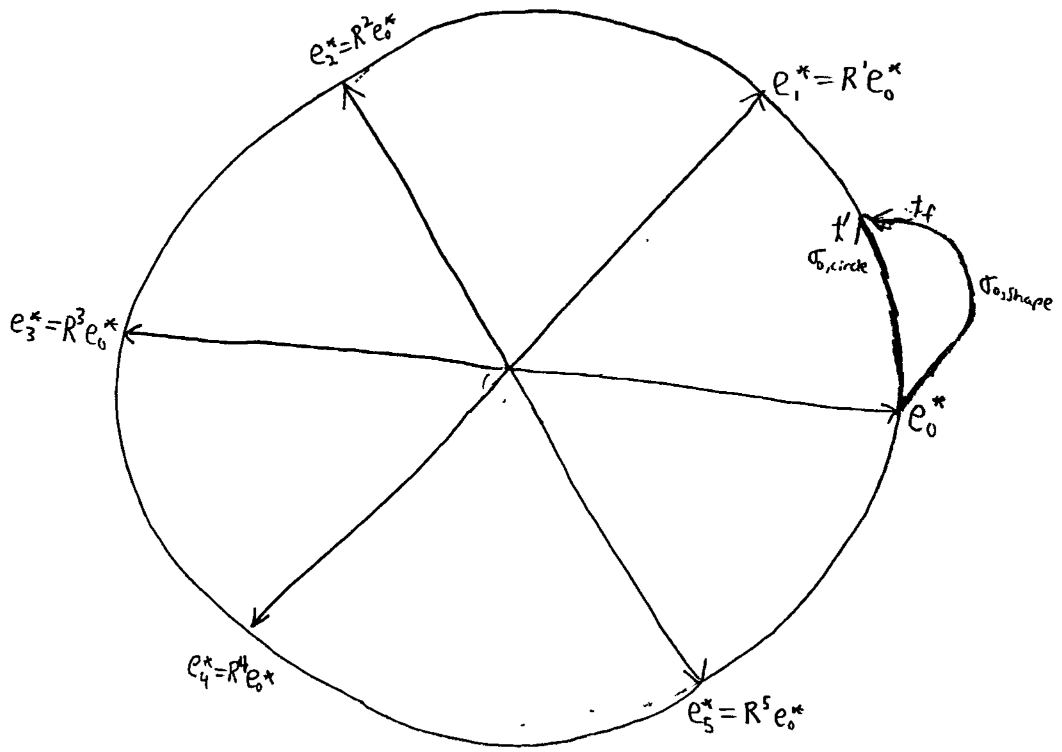


$$R=\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

Note that $R.e_3(z,t)=e_2(z,t)$ and $R^{-1}.e_3(z,t)=e_1(z,t)$

Therefore, the `right' control trajectory is simply the result of acting $R$ on a simple control trajectory, and correspondingly, the `left' control trajectory results from acting R^{-1} on a simple control trajectory. Then to create a three-parameter control trajectory, simply create three independent simple control trajectories from parameters x_1, x_1', and x_1''. Act R on the x_1' simple control trajectory to get the `right' piece, and act $R^{-1}$ on the $x_1''$ simple control trajectory to get the `left' piece. All three pieces share the common point $z_0=i$, since $R.i=i$. Link them together at i to create the three-parameter control trajectory, based on x_1, x_1', and x_1''.

Figure 6: Closure



*Total of 6 hexagonally symmetric deformations corresponding to the single deformation illustrated here

There must be some time t' when traveling along the circle curve, such that $\sigma_{j,circle}(t') = \sigma_{j,shape}(t_f)$ in order for the shape to be closed. From here, simply 'patch' the remainder of the circle curve onto the deformation curve to provide continuity.