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Computer Models in Room Acoustics: The Ray Tracing Method and the Auralization Algorithms

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Abstract—Computer algorithms are described for constructing virtual acoustic models of various rooms that should satisfy some specific sound quality criteria. The algorithms are based on the ray tracing method, which, in the general case, allows calculation of the amplitude of an acoustic ray that survived multiple reflections from arbitrary curved surfaces. As a result, calculations of room acoustics are reduced to tracing the trajectories of all the acoustic rays in the course of their propagation with multiple reflections from reflecting surfaces to the point of their complete decay. For this approach to be used, the following physical properties of a room should be known: the geometry of the reflecting surfaces, the absorption and diffusion coefficients on each of these surfaces, and the decay law for rays propagating in air. The proposed models allow for the solution of the important problem of architectural acoustics called the auralization problem, i.e., to predict how any given audio segment will sound in any given hall on the basis of computer simulation alone, without any full-scale testing in specific halls.

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1. INTRODUCTION

Calculation of the acoustic properties of rooms is one of the most difficult problems of modern architectural acoustics. The complexity of the problem consists in that the criteria for basic sound quality are rather subjective and cannot be easily interpreted in terms of rigorous quantitative estimates.

If, for simplicity, we consider a room with plane bounding surfaces, its model will have the form of a finite volume bounded by a closed polyhedron. The acoustic properties of such a room are determined by its geometry and by the materials covering the sound-reflecting boundary surfaces.

From the mathematical point of view, the statement of the problem has a classical form. We have a closed polyhedron with a surface S. Inside S, it is necessary to solve the wave equation for acoustic pressure $p(x, y, z, t)$ with initial conditions in time and with boundary conditions on the faces:

$$\Delta p = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}, (x, y, z) \in V; \left.\frac{\partial p}{\partial n}\right|_{S_i} = \gamma_i p, (x, y, z) \in S;$$

$$\left.p\right|_{t=0} = p_0(x, y, z), \left.\frac{\partial p}{\partial t}\right|_{t=0} = v_0,$$

where V is the inner volume of the polyhedron with the surface S, i.e., $S = \partial V$. Here, c is the velocity of sound, $\mathbf{n}$ is the outer normal boundary surface, S_i is an arbitrary ith face of the polyhedron, and the impedance γ_i of the face S_i is governed by the absorbing properties of the coating material on this face.

Within such a classical statement of the problem, is would be natural to apply some direct numerical methods, for example, the finite-element method or the boundary-equation (boundary-element) method, which are often used in solving acoustic boundary-value problems [1–4]. To estimate the efficiency of these methods, we note that, for a correct calculation, it is necessary to consider no less than ten nodes of a grid per wavelength. For example, when the average frequency of sound is $f = 1$ kHz and the velocity of sound is $c = 340$ m/s, the wavelength is $\lambda = c / f = 34$ cm. If we take a hypothetic hall with the dimensions $f = 17 \times 8.5 \times 5.1$ m, ten nodes per wavelength yield in total $M = 500 \times 250 \times 150 \approx 10^7$ nodes in this hall for every instant of time. If we take about 10^3 nodes in time, we arrive at a total number of nodes of about $M = 10^{10}$, which far exceeds the abilities of the most advanced supercomputers. Presumably, this is why the only program based on the finite-element method was rejected at the first stage of the Third Round Robin on Room Acoustical Computer Simulation [5].

Because of the failure of the direct numerical algorithms, engineering methods of calculation have found wide application in room acoustics [6–8]. These methods allow approximate determination of the most important acoustic parameter: the reverberation time (RT). Classical monographs [6–8] describe the Sabine theory, which allows for the determination of this parameter for the case of perfectly diffuse reflecting surfaces. The latter property means that, irrespective of the direction of incidence of an acoustic wave, its reflection from the boundary surface leads to a uniform scattering of the wave energy in all the directions within the corresponding solid angle. The formula based on the statistical pattern of acoustic ray propagation was refined by Eyring and finally was given in the form

$$\mathrm{RT} = \frac{0.161V}{4\mu V + S\ln(1-\alpha^*)}, \quad \alpha^* = \frac{1}{S}\sum_i \alpha_i S_i, \qquad (1)$$

where V and S are the room volume in cubic meters and the boundary surface area in square meters, respectively. Other quantities are as follows: μ is the viscous damping factor in air, α^* is the mean value of the sound absorption coefficient at the reflecting surfaces, and α_i is the sound absorption coefficient at the ith absorbing surface. The latter coefficient determines the share of the incident wave energy that is lost at each single reflection from the given specific surface.

It is well known that the Sabine–Eyring theory underestimates the reverberation time to a considerable extent, because the actual reflection of waves in not perfectly diffuse, but the major part of energy is reflected according to the geometric theory of diffraction, i.e., according to the laws of geometric optics. Maximal deviation from full-scale measurement data occurs in rooms with highly nonuniform absorption. A typical example may be a room with weakly absorbing walls and a strongly absorbing ceiling and floor.

The discrepancy arising in the Sabine theory is mainly due to the deviation of the actual trajectories of the acoustic ray from the perfectly diffuse distribution and, hence, the deviation of the mean distance between two sequential reflections ℓ^* from its true value. For this quantity, Sabine derived the formula

$$\ell^* \approx 4V/S, \qquad (2)$$

which served as the basis for the derivation of Eq. (1). Because of the evident inaccuracy of Eq. (2), attempts were made to determine ℓ^* by direct numerical calculation with the use of the Monte Carlo method [7]. Historically, this was the first attempt of computer simulation, which revealed some statistical regularities in the distribution of the density of propagating acoustic rays.

At the same time, in connection with the flaws in the Sabine theory, other more complicated engineering formulas were proposed. A complete review of these formulas can be found in [9]. Here, we only note the Fitzroy formula, which, in contrast to the Sabine theory, overestimates (as a rule) the value of RT. Thus, in almost all the cases, one can state that the true value of reverberation time lies between those obtained from the Sabine theory and the Fitzroy formula.

The main conclusion that can be derived from the brief review presented above consists in that none of the two opposite approaches to the problem under consideration yields adequate results. Specifically, direct numerical methods could provide accurate results if the calculations based on these methods would be realizable within a reasonable computer time. On the other hand, simple engineering approaches are also of little use because of their low accuracy.

The main purpose of the present study is the development of an alternative approach based on the ray tracing method (the acoustic ray tracing algorithm). This method was described in the literature (see, e.g., [7]) for the case where all the reflecting surfaces are plane. In the present paper, we generalize the approach to the case of curved reflecting surfaces. In Section 2, we introduce the basic result: the formula for an acoustic ray that experiences an arbitrary number of reflections from a set of smooth curved surfaces. We also show that the corresponding formulas for a set of plane reflectors are derived as a particular case from our result. In Section 3, we describe some details of the practical computational algorithm based on the proposed method. In Section 4, we demonstrate the application of our approach to the development of virtual models of sounding in a room. The numerical realization of the proposed models occurs in the real time scale.

2. THE STUDY OF MULTIPLE REFLECTIONS OF AN ACOUSTIC WAVE FROM HARD OBSTACLES IN THE SHORT-WAVE APPROXIMATION

Since neither direct numerical methods nor approximate engineering approaches yield any reliable results, asymptotic methods have become widely used in room acoustics. The most efficient asymptotic method was found to be the ray tracing method (RTM). It is applicable under the condition that the characteristic size of the room L is much greater than the wavelength: $L/\lambda \gg 1$. This condition is satisfied in room acoustics, because the wavelength of sound is on the order of decimeters, while the size of a room is on the order of meters, the frequency bands of the lowest octave being an exception. The theory of the method is based on the analogy between the propagation of beams of sound energy and the propagation of beam of light energy. Classical results lead to the ray approach (not to be confused with the RTM) and are presented in monographs [10–12]. Numerous

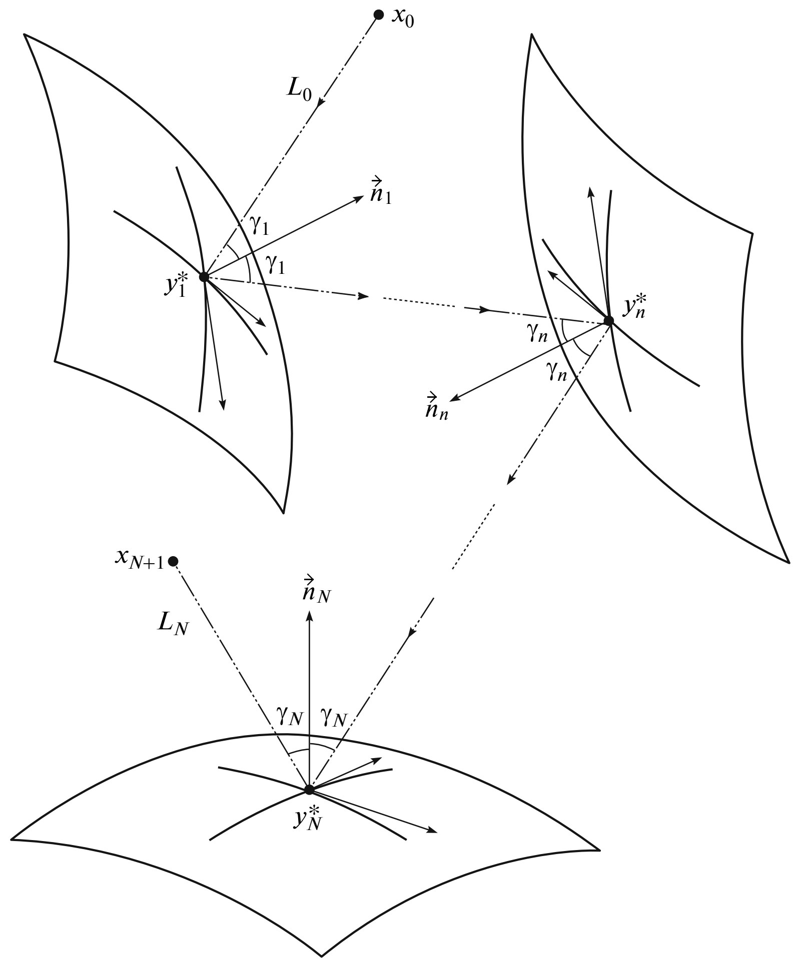

Fig. 1. *N*-fold reflection of a high-frequency acoustic wave propagating along the ray $x_0 - y_1^* - y_2^* - y_3^* - \ldots - y_{N-1}^* - y_N^* - x_{N+1}$ from the boundary surfaces of *N* acoustically hard obstacles placed in an acoustic medium.

attempts to apply the ray approach or the theory of Gaussian beams (related to it) to the problem of multiple reflections of acoustic rays by a set of nonplanar reflectors gave no explicit results. It is only in [12] that a single reflection from an arbitrary smooth surface was described. In view of this situation, in this section of our paper, we apply an alternative approach based on estimating the multidimensional Kirchhoff diffraction integral. The results presented below generalize the results obtained earlier in [13, 14].

Let a high-frequency spherical wave propagate from the point x_0 of an acoustic medium (Fig. 1). We assume that the acoustic wave propagates along the ray $x_0 - y_1^* - y_2^* - y_3^* - \ldots - y_N^* - x_{N+1}$, where the reflection points $y_1^*, y_2^*, y_3^*, \ldots, y_N^*$ belong to boundary surfaces of *N* various reflectors, these surfaces being curved in the general case. The wave is received at the point x_{N+1} of the acoustic medium, in which it is necessary to determine the amplitude of the *N* times reflected wave. This amplitude is governed by wave reflections from small vicinities $S_1^*,\ S_2^*, \ldots,\ S_N^*$ of the reflection points $y_1^*, y_2^*, y_3^*, \ldots, y_N^*$. on the boundary surfaces.

We determine the pressure in the *N* times reflected wave at the point x_{N+1} in the form of the Kirchhoff integral over the vicinity S_N^* of the last reflection point y_N^* of rays obtained after a single reflection from the vicinity S_{N-1}^* of the last-but-one reflection point y_{N-1}^*. Then, the pressure at the point of reception is given by the expression

$$p(x_{N+1}) = \iint\limits_{S_N^*} 2p(y_N)\frac{\partial\Phi(y_N, x_{N+1})}{\partial n_N}\,dS_N,$$

where, according to the theory of short wave diffraction, the total field in the vicinity of the reflection point is identical to twice the incident field, while the function $\Phi(r) = \exp(ikr) / (4\pi r)$ is the Green function for the Helmholtz operator.

In its turn, the pressure $p(y_N)$ is expressed in the form of an integral representation through the wave that is incident on the vicinity S_N^* after being reflected from the vicinity S_{N-1}^*:

$$p(y_N) = \iint\limits_{S_{N-1}^*} 2\,p(y_{N-1}) \frac{\partial \Phi(\,y_{N-1}, y_N\,)}{\partial n_{N-1}}\, dS_{N-1}.$$

Passing sequentially in the opposite direction $x_{N+1} - y_N^* - \ldots - y_2^* - y_1^* - x_0$, we arrive at the final representation for $p(x_{N+1})$ in the form of a $2N$-fold integral:

$$\begin{aligned} p(x_{N+1}) &= 2^N \iint\limits_{S_N^*} \iint\limits_{S_{N-1}^*} \cdots \iint\limits_{S_2^*} \iint\limits_{S_1^*} p^{inc} \frac{\partial \Phi}{\partial n_1} \frac{\partial \Phi}{\partial n_2} \\ &\quad \cdots \frac{\partial \Phi}{\partial n_{N-1}} \frac{\partial \Phi}{\partial n_N}\, dS_1 dS_2 \ldots dS_{N-1} dS_N. \end{aligned} \tag{3}$$

In the case of a wave process at a fixed circular frequency ω with the wave number $k = \omega/c$ in the short-wave region, it is possible to use the asymptotic representation for the Green function at $k \to \infty$:

$$\begin{aligned} \frac{\partial \Phi(\,y_{m-1}, y_m\,)}{\partial n_{m-1}} &= ik \cos\gamma_{m-1} (4\pi)^{-1} |y_{m-1} - y_m|^{-1} \\ &\quad \times e^{ik|y_{m-1} - y_m|} \left[\,1 + \mathrm{O}\left(k^{-1}\right)\right], \\ m &= 1, 2, \ldots, N+1,\ y_0 = x_0,\ y_{N+1} = x_{N+1}, \end{aligned} \tag{4}$$

where γ_{m-1} is the angle between the normal $\mathbf{n}_{m-1}$ and the direction of ray incidence $y_{m-2} - y_{m-1}$, i.e., the angle of incidence at the point y_{m-1}, while $|y_{m-1} - y_m|$ is the distance between the spatial points $y_{m-1} \in S_{m-1}^*$ and $y_m \in S_m^*$. Note that the incident $y_{m-1}^* - y_m^*$ and reflected $y_m^* - y_{m+1}^*$ rays lie in one plane with the normal $\mathbf{n}_m$ at the point y_m^*.

We denote the distances as follows: $\left|x_0 - y_1^*\right| = L_0$, $\left|y_m^* - y_{m+1}^*\right| = L_n$, $\left|y_N^* - x_{N+1}\right| = L_N$, where $m = 1, \ldots, N-1$. Factoring out the slowly varying functions from under the integral in Eq. (3), and taking into account Eq. (4), we obtain the integral representation for the pressure at the reception point:

$$\begin{aligned} p(x_{N+1}) &= \left(\frac{ik}{2\pi}\right)^N L_0^{-1} \prod_{n=1}^{N} L_n^{-1} \cos\gamma_n \\ &\quad \times \iint\limits_{S_N^*} \iint\limits_{S_{N-1}^*} \cdots \iint\limits_{S_1^*} e^{ik\varphi} dS_1 \ldots dS_{N-1} dS_N, \end{aligned} \tag{5}$$

$$\begin{aligned} \varphi &= |x_0 - y_1| + |y_1 - y_2| \\ &\quad + \ldots + |y_{N-1} - y_N| + |y_N - x_{N+1}|. \end{aligned} \tag{6}$$

In the factors before the integral, we take $\cos\gamma_m$ for the incident ray at the reflection point y_m^*.

The asymptotic estimate of integral (5), (6) is obtained by the method of multidimensional stationary phase [15]. In [16] it was shown that the principal term of asymptotics at $k \to \infty$ has the form

$$\begin{aligned} p(x_{N+1}) &= \frac{1}{L_0} \prod_{m=1}^{N} \frac{\cos\gamma_m}{L_m} \\ &\quad \times \frac{\exp\left\{i\left[k \sum\limits_{m=0}^{N} L_m + \frac{\pi}{4}(\delta_{2N} + 2N)\right]\right\}}{\sqrt{\left|\det(D_{2N})\right|}}, \end{aligned} \tag{7}$$

where $D_{2N} = (d_{ij})$, $i, j = 1, 2, \ldots, 2N$, is a $2N \times 2N$ symmetric Hesse matrix of a band structure with a band width of 7. The parameter $\delta_{2N} = \operatorname{sgn} D_{2N}$ is the difference between the number of positive and negative eigenvalues of the matrix D_{2N}. Here, the diagonal elements are

$$\begin{aligned} \begin{Bmatrix} d_{2m-1,\ 2m-1} \\ d_{2m,\ 2m} \end{Bmatrix} &= \left(\,L_{m-1}^{-1} + L_m^{-1}\right) \begin{Bmatrix} \sin^2\alpha_m \\ \sin^2\beta_m \end{Bmatrix} \\ &\quad + 2 \begin{Bmatrix} k_1^{(m)} \\ k_2^{(m)} \end{Bmatrix} \cos\,\gamma_m, \end{aligned} \tag{8}$$

and the off-diagonal elements are

$$\begin{aligned} d_{2m-1,2m} &= -\left(L_{m-1}^{-1} + L_m^{-1}\right)\cos\alpha_m \cos\beta_m, \\ d_{2m-1,2m+1} &= L_m^{-1}\left(\cos\alpha_m \cos\alpha_{m+1} - c_{11}^m\right), \\ d_{2m-1,2m+2} &= L_m^{-1}\left(\cos\alpha_m \cos\beta_{m+1} - c_{21}^m\right), \\ d_{2m,2m+1} &= L_m^{-1}\left(\cos\beta_m \cos\alpha_{m+1} - c_{12}^m\right), \\ d_{2m,2m+2} &= L_m^{-1}\left(\cos\beta_m \cos\beta_{m+1} - c_{22}^m\right). \end{aligned} \tag{9}$$

The angles $\alpha_m, \beta_m, \gamma_m$ determine the directional cosines of the reflected ray at the reflection point y_m^*, and the quantities $k_1^{(m)}, k_2^{(m)}$ are the principal curvatures of the reflecting surface at this reflection point.

The matrix $C^{(m)}$ with the dimensions 2×2 is expressed as a scalar product of the matrices $A^{(m)}$ and $B^{(m)}$:

$$\begin{aligned} c_{11}^m &= a_{11}^m b_{11}^m + a_{21}^m b_{21}^m + a_{31}^m b_{31}^m,\\ c_{12}^m &= a_{12}^m b_{11}^m + a_{22}^m b_{31}^m + a_{32}^m b_{31}^m,\\ c_{21}^m &= a_{11}^m b_{12}^m + a_{21}^m b_{22}^m + a_{31}^m b_{32}^m,\\ c_{22}^m &= a_{12}^m b_{12}^m + a_{22}^m b_{22}^m + a_{32}^m b_{32}^m. \end{aligned} \tag{10}$$

Here, the elements of the matrix $A^{(m)}$ are related to the directional cosines of the reflected ray

$$\begin{aligned} &\begin{Bmatrix} a_{11}^m \\ a_{21}^m \end{Bmatrix} = G_m^{-1}\Bigg[\cos\beta_{m-1}\begin{Bmatrix} \cos\beta_m \\ -\cos\alpha_m \end{Bmatrix} \\ &-\cos\alpha_{m-1}\begin{Bmatrix} \cos\alpha_m \\ \cos\beta_m \end{Bmatrix}\cos(\gamma_{m-1}+\gamma_m)\Bigg],\\ &\begin{Bmatrix} a_{12}^m \\ a_{22}^m \end{Bmatrix} = G_m^{-1}\Bigg[\cos\alpha_{m-1}\begin{Bmatrix} -\cos\beta_m \\ \cos\alpha_m \end{Bmatrix} \\ &-\cos\beta_{m-1}\begin{Bmatrix} \cos\alpha_m \\ \cos\beta_m \end{Bmatrix}\cos(\gamma_{m-1}+\gamma_m)\Bigg],\\ &\begin{Bmatrix} a_{13}^m \\ a_{23}^m \end{Bmatrix} = \begin{Bmatrix} -\cos\alpha_m \\ -\cos\beta_m \end{Bmatrix}\frac{\sin(\gamma_{m-1}+\gamma_m)}{\sin\gamma_m},\\ &\begin{Bmatrix} a_{31}^m \\ a_{32}^m \end{Bmatrix} = \begin{Bmatrix} \cos\alpha_{m-1} \\ \cos\beta_{m-1} \end{Bmatrix}\frac{\sin(\gamma_{m-1}+\gamma_m)}{\sin\gamma_m},\\ &a_{33}^m = -\cos(\gamma_{m-1}+\gamma_m),\quad G_m = \sin\gamma_{m-1}\sin\gamma_m, \end{aligned} \tag{11}$$

and the matrix $B^{(m)}$ determines the rotation of the Cartesian coordinate system related to the principal curvature lines and the normal surface at the point y_m^*, with respect to the corresponding Cartesian system related to the incidence–reflection plane.

In the particular case where all the surfaces are plane, the basic result given by Eq. (7) takes a simpler form, because all the curvatures become zero. For simplicity, we consider the case where the acoustic ray remains in one plane in the course of its reflections. Then, the matrix $B^{(m)}$ becomes an identity matrix and, hence, $C^{(m)} = A^{(m)}$. In this case, the nonzero elements of the symmetric matrix D_{2N} are considerably simplified:

$$d_{2m-1,\,2m-1} = \left(L_{m-1}^{-1} + L_m^{-1}\right)\cos^2\gamma_m,$$

$$d_{2m-1,\,2m+1} = d_{2m+1,\,2m-1} = -L_m^{-1}\cos\gamma_m\cos\gamma_{m+1},$$

$$d_{2m,\,2m} = L_{m-1}^{-1} + L_m^{-1},\ d_{2m,\,2m+2} = d_{2m+2,\,2m} = L_m^{-1},$$

$$m = 1, 2, \ldots, N.$$

Since the two indices of the nonzero elements of this matrix are either even or odd simultaneously, it can be shown that the determinant of the matrix D_{2N} of such a structure is represented in the form of a product of the determinants of two matrices:

$$\det\left(D_{2N}\right) = \det\left(D_N^{(1)}\right)\cdot\det\left(D_N^{(2)}\right), \tag{12}$$

where the matrix $D_N^{(1)}$ with the dimensions $N\times N$ consists of the rows of the matrix D_{2N} with odd numbers, while the matrix $D_N^{(2)}$ with the dimensions $N\times N$ consists of the rows of the matrix D_{2N} with even numbers. In the problem under consideration, after some transformations, we find that the matrices $D_N^{(1)}$ and $D_N^{(2)}$ coincide to within an insignificant factor. As a result, Eq. (12) leads to the following expression for the determinant in Eq. (7):

$$\det D_{2N} = \left(\prod_{m=1}^{N}\cos^2\gamma_m\right)\left(\sum_{m=0}^{N}L_m\right)^2\left(\prod_{m=0}^{N}L_m\right)^{-2}. \tag{13}$$

Finally, the substitution of Eq. (13) into Eq. (7), after simple evaluation of the index δ_{2N}, leads to the following final expression for the pressure at the reception point:

$$p\left(x_{N+1}\right) = \exp\left(ik\sum_{m=0}^{N}L_m\right)\Bigg/\sum_{m=0}^{N}L_m. \tag{14}$$

Thus, when all the reflecting surfaces are plane and perfectly hard and the reflection is perfectly specular, i.e., occurs without energy loss, the propagation of an acoustic ray occurs in the same way as in the case of ray propagation without reflections to a distance identical to the full length of the propagation path. This rule, being natural from the physical point of view, was proven above by rigorous mathematics.

In the general case of curved surfaces, the amplitude of the acoustic pressure at the reception point is calculated in a more complex way, but explicit expressions (7)–(11) allow us to calculate the pressure in this case as well. With the use of the proposed approach, it is also easy to take into account the absorption of sound by the coating of the reflecting surfaces: it is sufficient to introduce the appropriate factors in the above formulas.

We note that, in architectural acoustics, the key point is the practical accuracy of the basic formula (7), which is of an asymptotic nature. Since the estimate of the relative error $O(1/k)$ does not depend on the curvature of the reflecting surfaces while the particular case of plane reflectors (14) was tested in detail by many researchers both theoretically and experimentally, we can expect that, at least in sufficiently regular cases, formula (7) is accurate to within the same order of magnitude.

3. COMPUTER REALIZATION OF THE RAY TRACING METHOD

In a specific computer realization of the model on the basis of the RTM, a great number of acoustic rays is used (in our algorithm, up to 500000 rays), which issue from the source of sound uniformly in all directions when the radiation pattern of the source is spherical. These rays travel over the closed room by losing part of their energy at every reflection according to the absorption coefficient α_m, specified for each of the reflecting planes. In general, these coefficients are different for different reflecting surfaces. We note that this fact alone causes wide deviation of exact calculations from approximate formulas based on the mean absorption coefficient.

If the ray under study experiences the first reflection from the surface S_1 with the absorption coefficient α_1, the second reflection from the surface S_2 with the absorption coefficient α_2, ..., the Nth reflection from the surface S_N with the absorption coefficient α_N, and, after these N reflections, arrives at the receiver, the energy associated with this ray will be

$$E_N = \prod_{m=1}^{N} (1-\alpha_m)$$

under the condition that attenuation due to viscous friction is negligible. The inclusion of viscous friction in the consideration presents no difficulties and depends on how accurate the coefficient of molecular attenuation in air as a function of frequency is.

Since the acoustic ray never falls exactly at the reception point, the usual practice is to combine the contributions of rays that pass through a sphere with a small radius ε and with its center at the point of reception. Then, a simple qualitative analysis of ray paths gives an estimate for the necessary choice of the total number of rays M issued from the source [17, 18]:

$$M \sim 8c^2t^2/\varepsilon^2,$$

where t is the time within which the full set of the contributions of rays arriving at the vicinity of the receiver is calculated (in practice, t is usually taken to be about several seconds).

Summation of the contributions of all the acoustic rays occurs according to the principle of energy summation, because previous practical studies in room acoustics (both experimental and theoretical ones) have shown that the phase information leading to the interference of rays at the receiver produces almost no effect on the total energy level of the received signal.

In the algorithm under consideration, the key role is played by the law of ray reflection. For simplicity, we describe it for the case of a plane reflector. If the reflection is specular, for an incident ray described parametrically in the vector form, we have

$$\mathbf{r} = \mathbf{r}_0 + \tau\mathbf{q},$$

where $\mathbf{r}_0 = (x_0, y_0, z_0)$ is the point of preceding reflection and $\mathbf{q}$ is the unit's directional vector that determines the direction of incidence. First, the point of intersection of this ray with the reflecting plane is determined, namely, the point (x_1, y_1, z_1), and the distance to this plane

$$\tau = \frac{n_x(x_1-x_0)+n_y(y_1-y_0)+n_z(z_1-z_0)}{n_xq_x+n_yq_y+n_zq_z}.$$

Here, $\mathbf{n} = (n_x, n_y, n_z)$ is the unit vector of the inner normal plane under consideration. Then, using the formula

$$\mathbf{q}_1 = \mathbf{q} - 2(\mathbf{n}\cdot\mathbf{q})\mathbf{n}, \tag{15}$$

the new directional vector is determined for the ray reflected at the point (x_1, y_1, z_1).

However, by considering specular reflection alone, we do not take into account the scattering (diffuse) properties of the reflecting surfaces. It should be noted that, while the Sabine–Eyring theory yields underestimated values of the RT because of the hypothesis of a totally diffuse wave field, the RTM used with allowance for specular reflections alone, i.e., without considering the diffuse properties of the reflecting surfaces, considerably overestimates the RT. To overcome this drawback, it is necessary to introduce the diffusion coefficient δ_m, characterizing each of the reflecting surfaces S_m. This coefficient together with the absorption coefficient α_m gives a full characteristic of the physical properties of the mth reflecting surface [7, 8, 18].

Decomposition of the reflected part of sound energy into the specular and diffuse components is determined by the value of the diffusion coefficient δ. The latter shows that the δth fraction of the reflected energy is scattered in a diffuse manner. Hence, the $(1-\delta)$th fraction undergoes specular reflection according to the laws of the geometric (ray) theory. Here, the principal role is played by the choice of the diffusion law, which is ambiguous. After many discussions, the Lambert law borrowed from optics was adopted as the basic law in room acoustics [7]. According to this law, in the case of an arbitrary incidence of an acoustic ray on a plane surface with normal $\mathbf{n}$, the scattering pattern is a function of the polar angle (i.e., the angle measured with respect to the normal) and the azimuth angle (i.e., the angle measured in the tangent plane). It is assumed that the diffuse scattering pattern does not depend on the azimuth angle while the dependence on the polar angle is proportional to its cosine:

$$I(\varphi,\theta) = \frac{\cos\theta}{\pi}, \Rightarrow \int_0^{2\pi}\int_0^{\pi/2} I(\varphi,\theta)d\varphi\sin\theta d\theta d\varphi$$
$$= \frac{1}{\pi}\int_0^{2\pi} d\varphi \int_0^{\pi/2}\cos\theta\sin\theta d\theta = 1.$$

In this case, the distribution of the energy of the incident ray among three components is as follows: the αth fraction of the incident energy is absorbed, the $(1 - \delta)$th fraction is scattered in the diffuse manner according to the Lambert law, and the $(1 - \alpha)(1 - \delta)$th fraction is reflected specularly (the angle of incidence is identical to the angle of reflection). Note that $\alpha + (1-\alpha)\delta + (1-\alpha)(1-\delta) = 1$.

Realization of this method in a computer algorithm is as follows. The algorithm traces each of the rays as they leave the source and wander in the closed room by recording as point signals all the events of the ray passage through a small ε vicinity of the receiver (in practice, in the case of a large room, such as a concert hall, it is possible to take, e.g., ε = 0.1 m). The full path length of each of the rays primarily determines the decrease in its amplitude at reception because of the attenuation in air in the course of its propagation (the attenuation in air μ is assumed to be known for a given temperature, humidity, and frequency). Since at each collision with a reflecting plane, the ray energy also decreases according to the absorption coefficient for the given surface, each current ray is traced until the instant it reaches a certain minimal level (in our algorithm, the minimal level of sound is taken to be −60 dB with respect to the sound level at the source). To determine the direction of propagation of the reflected ray after every collision with a reflecting plane, we generate a random number ζ with a uniform distribution in the interval (0, 1). This number serves to determine whether the ray's reflection is of specular or diffuse character. Specifically, if, for a given reflecting surface, we have $\varsigma \geq \delta$, the ray will be specularly reflected; if we have $\varsigma < \delta$, a diffuse process will take place. In the first case, the direction of the reflected ray is determined by the geometric optics laws, according to Eq. (15). In the second case, the reflection obeys the Lambert law, which is realized in a stochastic manner.

If we deal with a room of a complex shape with reflecting surfaces consisting of different materials, the values of the coefficients α and δ will be different for different reflecting surfaces. The values of the absorption coefficient α for different materials can be found in handbooks [19] and on the Internet in the form of tables. The values of the absorption coefficient strongly depend on the frequency and, in the literature, are usually given within six-octave bands from 125 Hz to 4 kHz. This determines the frequency dependence of the RT at the receiver.

Comparison of calculations by the proposed algorithm with full-scale data at the frequency f = 1 kHz

Parameter	RT	EDT	D50	C80	TS	LF
Results of computations	0.78	0.80	60	4.8	57	25
Measured data	0.82	0.85	61	4.9	56	22

The values of the diffusion coefficient of different materials are not presented in tables and are difficult to find in the literature. Strictly speaking, this coefficient may also strongly depend on frequency. Physically, it is determined by two factors. The first consists in that the reflecting surfaces have a finite size and, therefore, the diffraction by them is accompanied by the edge effect, which leads to reflection that does not obey the laws of the geometric theory. This factor causes a diffusion, which can be integrally estimated through a certain coefficient. However, in engineering acoustics, it is known that one cannot manufacture an actual coating with zero (or close to zero) diffusion because of the effect of roughness. This second factor results in that, even for very long reflectors with very smooth surfaces, the diffusion coefficient should in practice be taken $\delta \approx 0.1$. For a surface covered with many fine reflectors, one can take $\delta \approx 0.8$–0.9. For other intermediate reflecting surfaces, the value of the scattering coefficient lies between these two values.

Figure 2 illustrates the formation of acoustic rays reflected in the specular and diffuse manners, where the full set of these rays forms the true structure of sound reflections, i.e., the impulse response of the room. Note that, from the constructed pulse received by a virtual listener of the acoustic signal, it is easy to determine the basic acoustic parameter, i.e., RT, as well as other acoustic parameters, such as EDT, C80, D50, TS, LF, etc., which are included in the ISO-3382 international standard [20].

These parameters are defined as follows. The EDT parameter differs from RT in that the slope of the curve representing the decay of the impulse response with time is taken within the first 10 dB, rather than within the whole interval of 60 dB. Then, the time within which the new straight line reaches a level of –60 dB with respect to the direct sound level is determined. In some cases, the RT parameter is denoted as $T60$ and the EDT parameter as $T10$. We note that, because of the presence of noise, the measurement accuracy may be insufficient (especially, in the low-frequency region) for correct determination of the instant the level of –60 dB is reached. Therefore, it is common practice to determine the parameter $T30$, which is

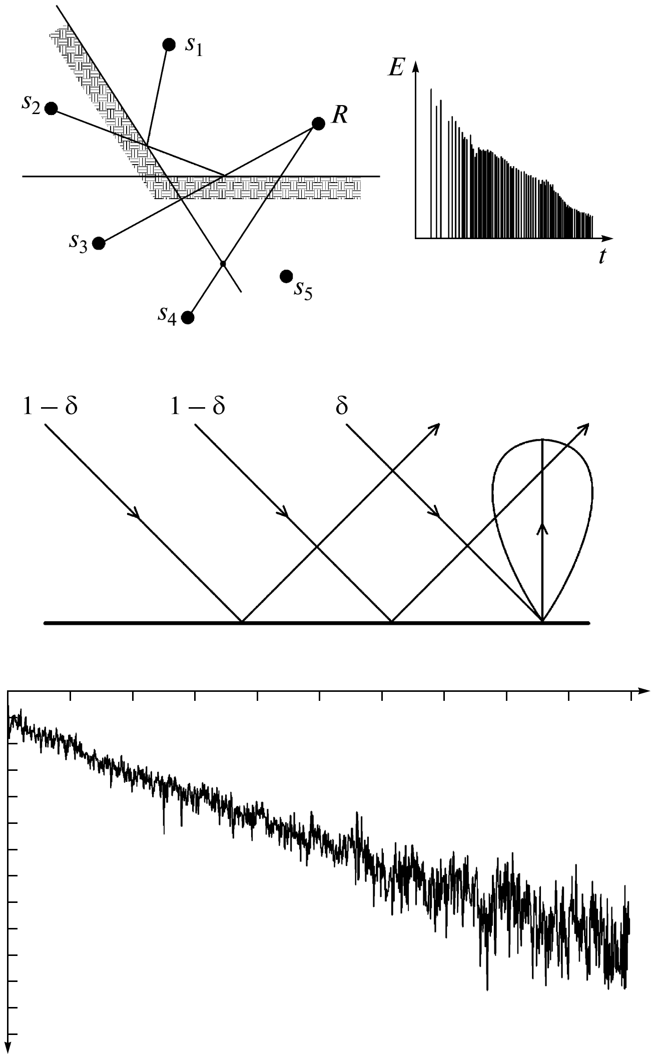

Fig. 2. Specular and diffuse reflections from boundary surfaces, which determine the structure of the received pulse.

obtained by detecting a 30-dB decrease in the impulse response and then multiplying the corresponding time interval by a factor of two. It is commonly believed that RT ≈ T30 with an accuracy sufficient for practical purposes.

The *C*80 parameter characterizes the clarity of sound and is measured in decibels. It is defined as the ratio of the energy of early reflections that occur within the first 80 ms to the energy of late reflections that occur after the first 80 ms:

$$C80 = 10\lg\left[\int_0^{80} p^2(t)dt\right] \Big/ \left[\int_{80}^{\infty} p^2(t)dt\right].$$

The *D*50 parameter characterizes the speech intelligibility and is expressed as a percentage. It is determined as the ratio of the energy of early reflections occurring within the first 50 ms to the total energy of the impulse response signal:

$$D50 = \left[\int_0^{50} p^2(t)dt\right] \Big/ \left[\int_0^{\infty} p^2(t)dt\right].$$

The *TS* parameter characterizes the characteristic duration of sounding and is measured in milliseconds. It is determined as the weighted-mean time from the graphical representation of the impulse:

$$TS = \left[\int_0^{\infty} tp^2(t)dt\right] \Big/ \left[\int_0^{\infty} p^2(t)dt\right].$$

The *LF* parameter refers to the energy of early lateral reflections and is expressed as a percentage:

$$LF = \int_5^{80} p^2(t)\cos^2[\theta(t)]\,dt \Big/ \int_0^{80} p^2(t)\,dt, \qquad (16)$$

where $\theta(t)$ is the angle between the direction of the received acoustic ray and the horizontal line that is perpendicular to the direction toward the source of sound. This parameter characterizes the fraction of energy arriving from lateral directions at the early reflections (within 80 ms), which is important for the subjective perception of sound.

The RTM algorithm was realized by us in the form of the AIST program and was tested in applications in several specific halls. In particular, this program was presented at the Third Round Robin on Room Acoustical Computer Simulation [5], which was held at the Aachen University (Germany) on the basis of the sound-recording studio belonging to this university. In [5], Russia represented by one participant with the AIST program was mentioned among the 15 countries, which were represented by 21 participants of the event. The aforementioned studio is schematically depicted in Fig. 3. The characteristic dimensions of the studio were $9 \times 8 \times 5$ m. The inner volume of the room was 377 m^3, and the total area of reflecting surfaces was 405 m^2. To suppress the standing waves, the rear wall was made with a slope. A diffuser system was fixed on the ceiling. The right-hand wall consisted of small wooden blocks, which simultaneously served as diffusers and absorbers. In the geometry proposed for testing, all the reflecting surfaces were plane. The total number of reflecting surfaces was 260. An example of comparison between our calculations and the data of full-scale measurements for the chosen *S*2–*R*2 source–receiver pair in the case of closed curtains (see [5]) is presented in the table.

Note that the measurement accuracy adopted as acceptable was 5% for the parameters *D*50 and *LF*, 0.05 s for the parameters *RT* and *EDT*, 10 ms for *TS*, and 1 dB for *C*80.

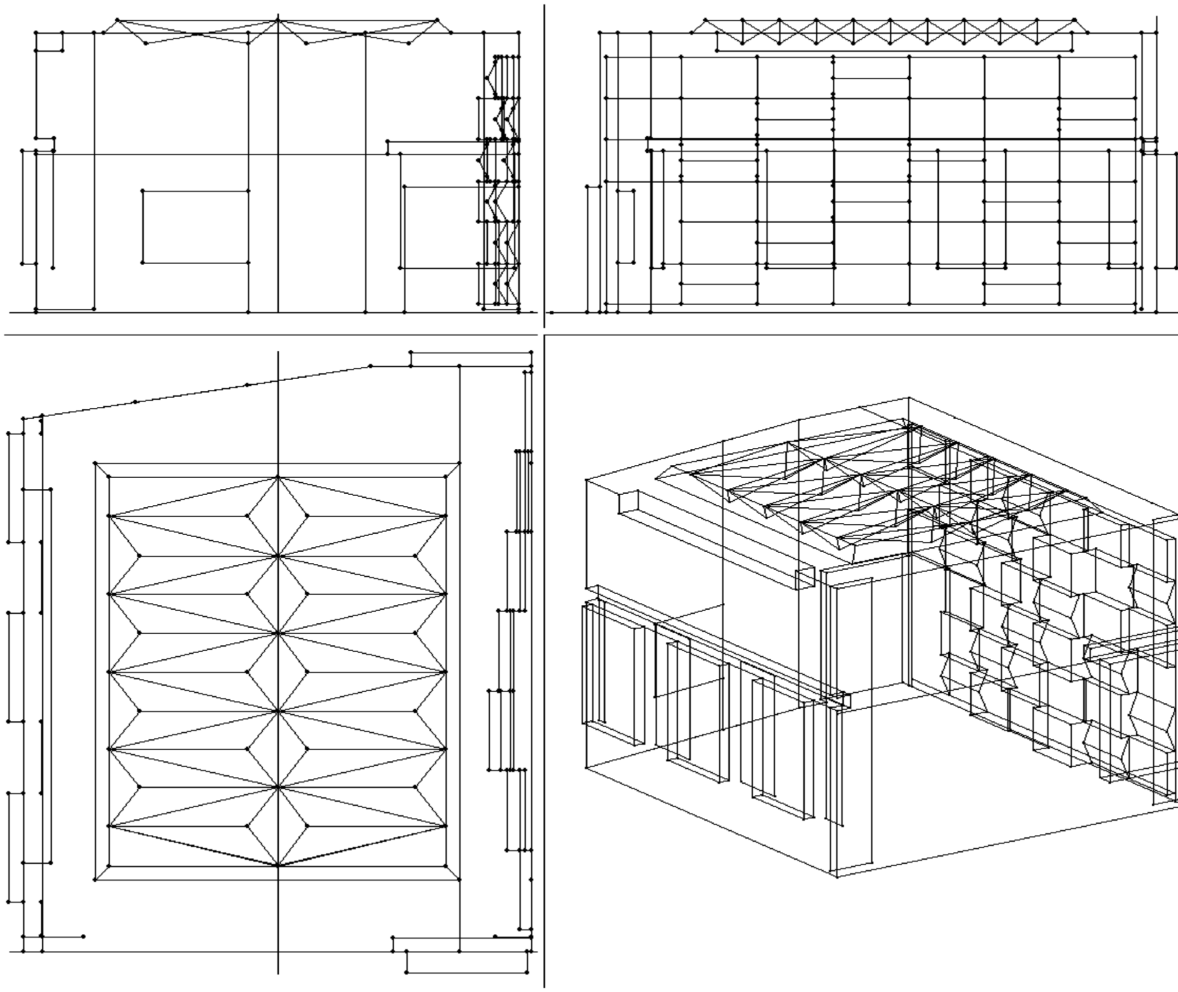

Fig. 3. Sound-recording studio of Aachen University (Germany).

We stress that a number of programs mentioned in [5] do not calculate the whole impulse response curve because of the long computation time required for this purpose. In these programs, a statistical approximation of the impulse response curve by an exponentially decaying tail is used for large time values. An exact calculation is only performed for several first reflections while the remaining part of the curve is approximated by a certain simple function. In contrast to the aforementioned approach, the algorithm developed by us provides the necessary accuracy by calculating the trajectories of the acoustic rays to the point where the energy of each of the rays decreases by 60 dB. In our method, calculation of the trajectories of the acoustic rays in real time is accelerated owing to the use of optimal methods of computational geometry [21].

As for the case of curved reflecting surfaces, the realization of our algorithm basically takes into account the curvature, i.e., the formulas from Section 2 are included in the algorithm. However, the main difficulty of practical application to a specific room's geometry is related to the introduction of the curvature data into the program. The geometry of a surface can seldom be expressed in explicit form; it can be approximated by spline surfaces by using a set of specific points, whose coordinates are not easy to measure. Determination of the principle curvatures at every point of an approximately specified surface is questionable, because it requires calculation of the derivatives of functions given with a certain error, which is an ill-posed problem according to Tikhonov [22]. Testing of the program is in this case possible by approximating the curved surfaces with a set of plane faces (this is the standard approach). However, if the surface curvatures are explicitly involved in the formulas for the amplitude of the reflected wave (Eq. (7)), such an approach is incorrect. Evaluation of the accuracy of this approach in comparison with the method realized on the basis of the formulas given in Section 2 remains open to discussion. It should be noted that, judging from the literature [23–25], this problem is rather intricate. Even in the case of canonic surfaces (a cylinder, a spherical dome, etc.), for which not only an explicit equation of the surface can be written, but also an exact solution to the diffraction problem can be obtained, the available data do not give any unambiguous answer. Determination of the regions of parameter variations within which the approximation by plane faces yields an acceptable accuracy provides qualitative similarity, rather than quantitative closeness [23]. Testing of our algorithm in its exact realization in application to classical structures with curved boundaries, such as, e.g., a dome of a church requires special attention and special investigation, which is planned by us for the near future.

4. AURALIZATION IN THE VIRTUAL COMPUTER MODEL OF ROOM ACOUSTICS

We consider a source of sound that generates an acoustic signal in the form of a sound wave with the time dependence of sound pressure $p_0(t)$. This signal is modified by the room due to sound reflections, attenuation, diffraction, etc., i.e., the processes described above. If we consider the room as a linear filter [26–28] with an amplitude–time energy characteristic $E_r(t) = p_r^2(t)$, the total signal at any spatial point is obtained as a "convolution" of the signal produced by the source and the characteristics of the room:

$$p(t) = \int_{-\infty}^{\infty} p_0(\tau)p_r(t-\tau)d\tau \Rightarrow \tilde{p}(\omega) = \tilde{p}_0(\omega)\tilde{p}_r(\omega), \quad (17)$$

where the tilde sign denotes Fourier transform.

Here, the most difficult problem is the determination of the full spectral characteristic of the room response $\tilde{p}_r(\omega)$. The point is that the practical realization of the RTM for calculating the impulse response (see Fig. 2) depends on the data on the parameters α_m and δ_m for every mth reflecting surface. As a rule, these data are given for six octave carrier frequencies: f = 125, 250, 500, 1000, 2000, and 4000 Hz. Thus, the application of the RTM allows for determination of the impulse characteristic shown in Fig. 2 only for these six frequencies. In discrete form, the impulse response for each of the frequencies f_n is determined by a delta sequence (here, δ is the Dirac delta function):

$$p_r^{(n)}(t) = \sum_{j=1}^{J_n} \sqrt{E_j^{(n)}}\,\delta(t - t_j^{(n)}), \quad (18)$$

where $t_j^{(n)}$ are the instants of arrival of acoustic rays at the receiver and $E_j^{(n)}$ are the corresponding energy values. In the general case of a frequency-dependent room response, function (18) requires an extrapolation from the six octave carrier frequencies to the entire frequency region. However, in the particular case of a room response weakly depending on frequency, the form of the audio signal modified by the room can be represented in explicit form. Indeed, substituting Eq. (18) in Eq. (17) and using the basic property of the delta function, we arrive at the following relation (the frequency dependence is omitted):

$$p(t) = \sum_{j=1}^{J} \sqrt{E_j}\,p_0(t - t_j). \quad (19)$$

Representation (19) has an evident physical meaning. Let the impulse response of the room (Fig. 2) be the same for all the frequencies of sound. Then, the initial audio segment being reproduced in the given hall forms a sum of its initial copies, each of which is shifted by the delay time corresponding to the jth ray arrival and possessing an amplitude that is determined by the energy loss of the jth ray after all of its reflections from the boundary surfaces. We note that such a calculation performed directly in the time domain requires a squared number of arithmetic operations compared to the number of chosen temporal nodes, because, in Eq. (19), for each of the instants of time $t = t_k$, it is necessary to perform summation over j. To reduce the computation time, we change to the spectral domain by applying the convolution theorem to the second relation (17). In this case, the function $\tilde{p}_r(\omega)$ is the Fourier transform of Eq. (18):

$$\tilde{p}_r(\omega) = \sum_{j=1}^{J} \sqrt{E_j}\exp(i\omega t_j). \quad (20)$$

This approach with the use of the fast Fourier transform (FFT) is a linear-logarithmic one, i.e., virtually linear in the number of nodes. Then, the multiplication in the spectral region in Eq. (17) is linear, and the Fourier transform of the function $\tilde{p}(\omega)$ in Eq. (17) is again linear-logarithmic.

In the general case of a frequency-dependent energy response, one may expect that the function $\tilde{p}_r(\omega)$ in Eq. (16) will be a strongly oscillating function of frequency ω. However, relation (20) can in this case be used only for the six octave carrier frequencies. From the physical point of view, the actual frequency dependence should be approximately identical to the frequency dependence of the absorption coefficient; i.e., it should be a rather smooth function. This property was described in detail in the literature (see, e.g., [26]). Hence, we can interpolate the spectral properties to the entire frequency region.

The practical realization of this idea is as follows. If we consider the chosen limited frequency band from $125/2^{1/2} \approx 88$ Hz to $4000 \times 2^{1/2} \approx 5640$ Hz, the initial signal $p_0(t)$ can be represented as a sum of six signals, each of which is filtered within its own octave band:

$$p_0(t) = \sum_{n=1}^{6} p_0^{(n)}(t). \quad (21)$$

We assume that, to a first approximation, the room response is constant within one octave band. Then, with allowance for Eq. (21), convolution (17) can be represented in the form

$$p(t) = \sum_{n=1}^{6} \int_{-\infty}^{\infty} p_0^{(n)}(\tau) p_r^{(n)}(t-\tau) d\tau \Rightarrow \tilde{p}(\omega)$$
$$= \sum_{n=1}^{6} \tilde{p}_0^{(n)}(\omega) \tilde{p}_r^{(n)}(\omega),$$

where the impulse response $p_r^{(n)}(t)$ has a specific form in each of the octave bands. Finally, using Eq. (18), we obtain

$$p(t) = \sum_{n=1}^{6} \sum_{j=1}^{J_n} \sqrt{E_j^{(n)}}\, p_0^{(n)}(t - t_j^{(n)}). \tag{22}$$

This is the basic computational formula. We note that effective computation by Eq. (19) is also based on the application of the FFT with the use of the convolution theorem. In the particular case of a frequency-independent impulse response of the room ($E_j^{(n)} = E_j, t_j^{(n)} = t_j$), in view of relation (21), formula (22) takes the form of Eq. (19).

The proposed algorithm makes it possible to modify any recorded audio segment to the form that it will take when presented in a simulated hall.

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