Sequences

3) Problems:

1) Let $\{a_n\}_{n\geq 1}$ be an arithmetic sequence and $\{g_n\}_{n\geq 1}$ be a geometric sequence such that the first four terms of $\{a_n + g_n\}$ are $0, 0, 1$, and 0, in that order. What is the 10th term of $\{a_n + g_n\}$.

2) A sequence of numbers $a_1, a_2, a_3, ...$ satisfies $a_1 = \frac{1}{2}$ and $a_1 + a_2 + ... + a_n = n^2 a_n$ for all $n \geq 1$. Determine the value of a_n for $n \geq 1$.

3) Let a_n be a sequence such that $a_0 = 0$, $a_1 = 1$ and for all $n \geq 2$,
$a_n = 2a_{n-1} - a_{n-2} + 2^n$. Find a_{2013}.

3) Harder Problems:

1) A Fibonacci-like sequence of numbers is defined by $a_1 = 1$, $a_2 = 3$ and for$n \geq 3$, $a_n = a_{n-1} + a_{n-2}$. One can compute that $a_{29} = 1149851$ and $a_{30} = 1860498$. What is the value of $\sum_{n=1}^{28} a_n$.

2) A sequence of integers a_i is defined as follows $a_i = i$ for all $1 \leq i \leq 5$, and $a_i = a_1 a_2 ... a_{i-1} - 1$ for all $i > 5$. Evaluate $a_1 a_2 ... a_{2011} - \sum_{i=1}^{2011} a_i^2$

3) The terms of sequence a_i are defined by $a_{n+2} = \frac{a_n + 2009}{1 + a_{n+1}}$ for $n \geq 1$ are positive integers. Find the minimum possible value of $a_1 + a_2$.

4) Let $a_0 = -2$, $b_0 = 1$, and for $n \geq 0$, let

$$a_{n+1} = a_n + b_n + \sqrt{a_n^2 + b_n^2}$$
$$b_{n+1} = a_n + b_n - \sqrt{a_n^2 + b_n^2}$$

Find a_{2012}

3) Harderer Problems:

1) The sequence (a_n) satisfies $a_0 = 0$ and $a_{n+1} = \frac{8}{5} a_n + \frac{6}{5}\sqrt{4^n - a_n^2}$ for $n \geq 0$. Find the greatest integer less than or equal to a_{10}.

2) A sequence is defined over non-negative integral indexes in the following way: $a_0 = a_1 = 3$, $a_{n+1} a_{n-1} = a_n^2 + 2007$. Find the greatest integer that does not exceed $\frac{a_{2006}^2 + a_{2007}^2}{a_{2006} a_{2007}}$

3) Let $a_1 = 3$, and for $n > 1$, let a_n be the largest real number such that

$$4(a_{n-1}^2 + a_n^2) = 10 a_{n-1} a_n - 9$$

What is the largest positive integer less than a_8.