Summations

1) Evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

2) Evaluate $\sum_{i=1}^{100} \frac{1}{2\lfloor i \rfloor + 1}$

3) Evaluate $\sum_{n=2}^{\infty} \sum_{m=2}^{\infty} \frac{1}{m^n}$

4) Evaluate $\sum_{n=1}^{\infty} \frac{n+1}{n^2(n+2)^2}$

5) Evaluate $\sum_{k=1}^{1000} k(\lceil \log_{\sqrt{2}} k \rceil - \lfloor \log_{\sqrt{2}} k \rfloor)$.

6) Compute $\sum_{n=0}^{\infty} \frac{n}{n^4+n^2+1}$

7) For each positive integer p, let b(p) denote the unique positive integer k such that $|k - \sqrt{p}| < \frac{1}{2}$. For example, $b(6) = 2$ and $b(23) = 5$. If $S = \sum_{p=1}^{2007} b(p)$, find the remainder when S is divided by 1000.

8) Evaluate $\sum_{a=0}^{\infty} \sum_{b=a}^{\infty} \sum_{c=b}^{\infty} \frac{1}{2^{a+b+c}}$.

9) Compute $\sum_{n=1}^{\infty} \sum_{k=1}^{n-1} \frac{k}{2^{n+k}}$

10) Evaluate $\sum_{n=1}^{\infty} \frac{(7n+32) \cdot 3^n}{n(n+2) \cdot 4^n}$

11) We select a real number α uniformly and at random from the interval $(0, 500)$. Define

$$S = \frac{1}{\alpha} \sum_{m=1}^{1000} \sum_{n=m}^{1000} \left\lfloor \frac{m+\alpha}{n} \right\rfloor.$$

Let p denote the probability that $S \geq 1200$. Compute $1000p$.

12) Evaluate $\sum_{\substack{i,j \geq 0 \\ i+j \text{ odd}}} \frac{1}{2^i 3^j} = \frac{m}{n}$.