Sequences

1 Lecture

1. Let a_n be a sequence such that $a_0 = 0, a_1 = 1$ and for all $n \geq 2$, $a_n = 2a_{n-1} - a_{n-2} + 3^n$. Find a_{2013}.

2. A sequence of numbers $a_1, a_2, a_3, ...$ satisfies $a_1 = \frac{1}{2}$ and $a_1 + a_2 + ... + a_n = n^2 a_n$ for all $n \geq 1$. Determine the value of a_n for $n \geq 1$.

2 Problems

1. Let $a_0, a_1, a_2, \ldots$ denote the sequence of real numbers such that $a_0 = 2$ and $a_{n+1} = \frac{a_n}{1+a_n}$ for $n \geq 0$. Find a_{2012}.

2. Given that
$$\begin{aligned} x_1 &= 211, \\ x_2 &= 375, \\ x_3 &= 420, \\ x_4 &= 523, \text{ and} \\ x_n &= x_{n-1} - x_{n-2} + x_{n-3} - x_{n-4} \text{ when } n \geq 5, \end{aligned}$$
find the value of $x_{531} + x_{753} + x_{975}$.

3. Let m be a positive integer, and let $a_0, a_1, \ldots, a_m$ be a sequence of reals such that $a_0 = 37$, $a_1 = 72$, $a_m = 0$, and $a_{k+1} = a_{k-1} - \frac{3}{a_k}$ for $k = 1, 2, \ldots, m-1$. Find m.

4. The sequence (a_n) satisfies $a_1 = 1$ and $5^{(a_{n+1} - a_n)} - 1 = \dfrac{1}{n + \frac{2}{3}}$ for $n \geq 1$. Let k be the least integer greater than 1 for which a_k is an integer. Find k.

5. Define the sequence $\{x_i\}_{i \geq 0}$ by $x_0 = 2009$ and $x_n = -\frac{2009}{n} \sum_{k=0}^{n-1} x_k$ for all $n \geq 0$. Find a closed form for x_n.

6. A sequence of real numbers $\{a_n\}_{n=1}^{\infty}$ $(n = 1, 2, \ldots)$ has the following property for $n \geq 3$:
$$6a_n + 5a_{n-2} = 20 + 11a_{n-1}$$
The first two elements are $a_1 = 0, a_2 = 1$. Find the integer closest to a_{2011}.

7. Define two sequences of rational numbers as follows. Let $a_0 = 2$ and $b_0 = 3$, then recursively define
$$a_{n+1} = \frac{a_n^2}{b_n} \quad \text{and} \quad b_{n+1} = \frac{b_n^2}{a_n}$$
for $n \geq 0$. Determine b_8.

8. Define the sequence a_i as follows: $a_1 = 1, a_2 = 2015$, and $a_n = \frac{na_{n-1}^2}{a_{n-1}+na_{n-2}}$

9. Let $a_0 = -2$, $b_0 = 1$, and for $n \geq 0$, let

$$a_{n+1} = a_n + b_n + \sqrt{a_n^2 + b_n^2}$$
$$b_{n+1} = a_n + b_n - \sqrt{a_n^2 + b_n^2}$$

   Find a_{2012}

10. Let a_n be a sequence such that $a_0 = 0$ and:

$$a_{3n+1} = a_{3n} + 1 = a_n + 1$$
$$a_{3n+2} = a_{3n} + 2 = a_n + 2$$

   for all natural numbers n. How many n less than 2012 have the property that $a_n = 7$.

11. Let a sequence be defined as follows: $a_1 = 3$, $a_2 = 3$, and for $n \geq 2$, $a_{n+1}a_{n-1} = a_n^2 + 2007$. Find the largest integer less than or equal to $\frac{a_{2007}^2+a_{2006}^2}{a_{2007}a_{2006}}$.

12. The sequence (a_n) satisfies $a_0 = 0$ and $a_{n+1} = \frac{8}{5}a_n + \frac{6}{5}\sqrt{4^n - a_n^2}$ for $n \geq 0$. Find the greatest integer less than or equal to a_{10}.

13. Let $x_1 = y_1 = x_2 = y_2 = 1$, then for $n \geq 3$ let $x_n = x_{n-1}y_{n-2} + x_{n-2}y_{n-1}$ and $y_n = y_{n-1}y_{n-2} - x_{n-1}x_{n-2}$. What are the last two digits of $|x_{2012}|$?