final_2_2_2

December 1, 2025

1 Final Project

[11]:
```python
import numpy as np
from matplotlib import pyplot as plt

# function for returning the amplitude in dB
def dB(x):
    return 20 * np.log10(x, dtype=float)
```

1.1 2.2.2

1.1.1 Part A

In the language of dB, a factor of two is "6 dB." In other words, if B_2 is 6 dB bigger than B_1, then it is twice as big (approximately). Explain why this statement is true.

Response: Decibels express ratios using a logarithmic scale: e.g. the definition for aplitude is dB = $20\log_{10}(\frac{B_2}{B_1})$. Therefore, a factor of two would imply that $\frac{B_2}{B_1}$ evaluates to 2.0. So $20\log_{10}(2.0) = 20 \times 0.30103 = 6.021$. Another way to think about this problem is by considering that ratios become differences on dB scale. Thus, $20\log_{10}(\frac{B_2}{B_1}) = 20\log_{10}(B_2) - 20\log_{10}(B_1)$, where $B_2 = 2B_1$. Finally, $20\log_{10}(\frac{B_2}{B_1}) = 20\log_{10}(2B_1) - 20\log_{10}(B_1) = 6.021$ dB $\rightarrow 20\log_{10}(2B_1) = 20\log_{10}(B_1) + 6.021$ dB.

[5]:
```python
# Code that calculates values from above.

print(f"The value 20log_10(2.0) evaluates to: {np.round(20*np.log10(2.0), 3)}␣
↪dB")
```

```
The value 20log_10(2.0) evaluates to: 6.021 dB
```

1.1.2 Part B

The nonzero Fourier coefficients of the triangular wave are $a_k = \frac{-2}{\pi^2 k^2}$. Determine the dB difference between a_1 and a_3. In other words, a_3 is how many dB below a_1. Furthermore, explain why the dB difference depends only on the k indices.

[14]:
```python
# function to evaluate the coefficients of the triangular wave a_k
def get_kth_coef(k):
    a_k = (-2.0 / (np.pi**2 * k**2))
    return a_k
```

```python
a_1 = get_kth_coef(1)
a_3 = get_kth_coef(3)

ratio = a_1 / a_3

print(f"Coefficient k = 1 evaluates to a_1 = {np.round(a_1, 4)}")
print(f"Coefficient k = 3 evaluates to a_3 = {np.round(a_3, 4)}")
print(f"Therefore, the ratio is a_1 / a_3 = {np.round(ratio, 4)}")
print(f"So a_3 is about {np.round(dB(ratio))} dB below a_1.")
```

```
Coefficient k = 1 evaluates to a_1 = -0.2026
Coefficient k = 3 evaluates to a_3 = -0.0225
Therefore, the ratio is a_1 / a_3 = 9.0
So a_3 is about 19.0 dB below a_1.
```

Response: The dB difference depends only on the k indices because the term $\frac{-2}{\pi^2}$ is a constant and does not change across coefficients. As reported above, a_1 is 9.0 times larger than a_3 which translates to a_3 being 19 dB smaller than a_1 as a result of k multiplying $\frac{-2}{\pi^2}$ by the inverse square of itself ($a_k = \frac{-2}{\pi^2}\frac{1}{k^2}$). Thus, the dB difference only depends on k.

1.1.3 Part C

Determine (in dB) how far a_15 is below a_1 for the periodic triangular wave.

[17]:
```python
a_1 = get_kth_coef(1.0)
a_15 = get_kth_coef(15.0)

ratio = a_1 / a_15

print(f"Coefficient k = 1 evaluates to a_1 = {np.round(a_1, 4)}")
print(f"Coefficient k = 15 evaluates to a_15 = {np.round(a_15, 4)}")
print(f"Therefore, the ratio is a_1 / a_15 = {np.round(ratio, 4)}")
print(f"So a_15 is about {np.round(dB(ratio))} dB below a_1.")
```

```
Coefficient k = 1 evaluates to a_1 = -0.2026
Coefficient k = 15 evaluates to a_15 = -0.0009
Therefore, the ratio is a_1 / a_15 = 225.0
So a_15 is about 47.0 dB below a_1.
```