

Homework #1

$$X_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad X_2 = \begin{bmatrix} 10 \\ 18 \end{bmatrix}$$

1a.) L2 Norm

$$\|X_2 - X_1\|_2 = \sqrt{(10-1)^2 + (18-2)^2} = \sqrt{9^2 + 16^2} = \sqrt{337}$$

1b.) L1 norm

$$\|X_2 - X_1\|_1 = \|10-1\| + \|18-2\| = 9+16=25$$

1c.) L ∞ norm

$$\|X_2 - X_1\|_\infty = \max(\|10-1\|, \|18-2\|) = \max(9, 16) = 16$$

2.) $f(x) = x^T A x$

$$A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

$$x = \begin{bmatrix} a_1 \\ B \end{bmatrix} \quad ** a_1 = \text{greek } \alpha \text{ to help differentiate}$$

show that: $\frac{df}{dx} = 2Ax$

$$f(x) = \begin{matrix} 1 \times 2 & 2 \times 2 & 2 \times 1 \\ [a_1 & B] & \begin{bmatrix} a & c \\ c & b \end{bmatrix} & \begin{bmatrix} a_1 \\ B \end{bmatrix} \end{matrix}$$

$$\frac{df}{dx} = \begin{bmatrix} \frac{df}{da_1} \\ \frac{df}{dB} \end{bmatrix}$$

$$\begin{matrix} 1 \times 2 & 2 \times 1 \\ [a_1 & B] & \begin{bmatrix} a_1 a + c B \\ a_1 c + b B \end{bmatrix} \end{matrix}$$

$$2Ax = 2 \begin{matrix} 2 \times 2 & 2 \times 1 \\ \begin{bmatrix} a & c \\ c & b \end{bmatrix} & \begin{bmatrix} a_1 \\ B \end{bmatrix} \end{matrix} = 2 \begin{bmatrix} a_1 a + c B \\ a_1 c + b B \end{bmatrix}$$

$$\frac{df}{dx} = \begin{bmatrix} \frac{df}{da_1} = 2a_1 a + 2c B \\ \frac{df}{dB} = 2a_1 c + 2b B \end{bmatrix}$$

$$a_1(a_1 a + c B) + B(a_1 c + b B) =$$

$$a_1^2 a + a_1 c B + B a_1 c + B^2 b =$$

$$f(x) = a_1^2 a + 2(a_1 c B) + b B^2$$

- 3.) - Update centers: for each cluster, move the center vector c to the average location of the data points in the cluster
- update labels: for each data point, find the nearest cluster center and then attach a cluster label to the data point

4.) Euclidean distance

5.) There are only a finite # of cluster assignments, so it only needs to pass through each given assignment once. Any extra increases the total loss we'd like the loss curve to be flat.

- 6.) - Clustering result is determined by data distribution and initialization
- initialization is random

7.) - optimal solution has been found for specific problem