Non-Duality: A Minimal Isabelle/HOL Formalization

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Abstract

This entry presents a concise formalization of an empirically oriented non-dual ontology in Isabelle/HOL. The development isolates a minimal core comprising a unique ground (denoted Ω), phenomena, an inseparability relation, and a symmetry action on appearances. Within Higher-Order Logic, we state conservative axioms and derive representative consequences such as the dependence of phenomena on the ground, the non-duplication of ground, and the stability of inseparability under symmetry. The theory is designed to be small, transparent, and reusable, providing a neutral spine that can be compared with neighboring non-dual formalisations (e.g. Daoism, Dzogchen) in the same technical framework.

Contents

1 Introduction

Non-dual traditions often assert that appearances are not ultimately separate from their ground, and that the ground is unique and inexpressible by the categories that classify appearances. This formalization extracts a neutral, logically explicit fragment of such views. The aim is expository and pragmatic: to state a modest family of axioms in HOL, study their immediate consequences, and make the assumptions and inferential pathways mechanically auditable.

2 Entry Overview

The session consists of a single theory `NonDuality` importing `Main`. At a high level it introduces:

- a background type of entities;
- a distinguished constant Ω intended as the unique ground;

- predicates capturing appearances (`Phenomenon`) and inseparability (`Inseparable`);
- a (group-like) action `act` for symmetry on appearances;
- axioms expressing uniqueness of ground, dependence of phenomena on the ground, and stability properties such as closure and preservation under action.

The development proves elementary consequences and keeps proofs short and readable. It avoids interactive tool invocations in batch builds.

3 Logical Design

3.1 Core Symbols (Sketch)

We work in classical HOL with a small signature. A typical abridged header:

```
typedecl entity

consts
  Omega        :: entity
  Phenomenon   :: "entity  bool"
  Inseparable  :: "entity  entity  bool"
  act          :: "('g)  entity  entity"   (* symmetry action on
    appearances *)
```

Nothing in the entry depends on a particular representation of the carrier for symmetries; the action is treated abstractly.

3.2 Axiom Sketch

The axioms are minimal and phenomenology-friendly. Representative principles:

- *Unique ground:* Ω is the ground and there is no other distinct ground entity.
- *Ground–appearance relation:* every phenomenon is inseparable from Ω.
- *Closure and preservation:* the symmetry action maps phenomena to phenomena and preserves inseparability with Ω.

The theory presents these as plain HOL formulas without requiring non-classical logic or additional meta-theory.

4 Illustrative Results

Dependence of Phenomena on the Ground. Every appearance is inseparable from Ω:

```
lemma phenomenon_inseparable_from_Omega:
  assumes "Phenomenon x"
  shows   "Inseparable x "
```

Symmetry Preserves Non-Two. Inseparability is stable under the action:

```
lemma symmetry_preserves_inseparability:
  assumes "Phenomenon x"
  shows   "Inseparable (act g x) "
```

No Duplication of Ground. There is no second ground distinct from Ω:

```
lemma ground_is_unique:
  assumes "Inseparable  y" "Inseparable y "
  shows   "y = "
```

These statements serve as precise surrogates for familiar non-dual claims. They are proven under the minimal axioms and can be strengthened or varied by altering the axiomatic core.

5 Methods and Sanity Checks

Proofs prefer `auto`, `blast`, and `metis` and avoid non-deterministic or long-running tools in batch mode. During development, finite-scope model exploration (via Nitpick) and proof search (via Sledgehammer) were used interactively to guide axiom tuning and lemma selection; the final submitted theory contains only stable proofs.[1]

6 Relation to Prior Work

This entry complements other small, reusable non-dual formalisations (e.g. Daoism, Dzogchen) by offering a secular, ontology-first presentation focused on a unique ground Ω and an inseparability relation. Placing these within the same HOL substrate supports side-by-side comparison and future unification work.

[1] Typical interactive settings: small cardinalities and short timeouts for model exploration; proofs subsequently recorded with `metis` or `blast`.

7 Limitations and Future Work

The axioms intentionally abstract away from interpretive nuance. Natural extensions include:

- enriching `Inseparable` with modal or temporal structure;
- constraining `act` via group laws and equivariance properties;
- relating the core to epistemic surrogates of direct knowing;
- comparing conservativity across neighboring entries within a shared interface theory.

8 How to Build

From the session directory:

```
isabelle build -c -o document=pdf NonDuality
```

To emit the PDF alongside sources:

```
isabelle build -c -D . -o document=pdf -o document_output=public
    NonDuality
```

```
theory NonDuality
  imports Main
begin
```

9 Core Ontology

```
typedecl E

consts
  Phenomenon :: "E ⇒ bool"
  Substrate  :: "E ⇒ bool"
  Presents   :: "E ⇒ E ⇒ bool"
  Inseparable :: "E ⇒ E ⇒ bool"

axiomatization where
  A1_existence:     "∃s. Substrate s" and
  A2_uniqueness:    "∀a b. Substrate a ⟶ Substrate b ⟶ a = b" and
  A3_exhaustivity:  "∀x. Phenomenon x ∨ Substrate x" and
  A4_presentation:  "∀p s. Phenomenon p ∧ Substrate s ⟶ Presents p s"
and
   A5_insep_def:       "∀x y. Inseparable x y ⟷ (∃s. Substrate s ∧
Presents x s ∧ y = s)"
```

```
lemma unique_substrate: "∃!s. Substrate s"
  using A1_existence A2_uniqueness by (metis)

definition TheSubstrate :: "E"  ("Ω")
  where "Ω = (SOME s. Substrate s)"

lemma substrate_Omega: "Substrate Ω"
  unfolding TheSubstrate_def using A1_existence someI_ex by metis

lemma only_substrate_is_Omega: "Substrate s ⟹ s = Ω"
  using substrate_Omega A2_uniqueness by blast

lemma consistency_witness: True by simp
```

10 Non-Duality

```
theorem Nonduality:
  "∀p. Phenomenon p ⟶ Inseparable p Ω"
proof (intro allI impI)
  fix p assume P: "Phenomenon p"
  from P substrate_Omega A4_presentation have "Presents p Ω" by blast
  hence "∃s. Substrate s ∧ Presents p s ∧ Ω = s"
    using substrate_Omega by blast
  thus "Inseparable p Ω"
    using A5_insep_def by blast
qed
```

11 Causality (Phenomenon-Level)

```
consts CausallyPrecedes :: "E ⇒ E ⇒ bool"

axiomatization where
   C1_only_phenomena: "∀x y. CausallyPrecedes x y ⟶ Phenomenon x ∧
Phenomenon y" and
  C2_irreflexive:    "∀x. Phenomenon x ⟶ ¬ CausallyPrecedes x x" and
  C3_transitive:     "∀x y z. CausallyPrecedes x y ∧ CausallyPrecedes y
z ⟶ CausallyPrecedes x z"

lemma Causal_left_NotTwo:
  assumes "CausallyPrecedes x y" shows "Inseparable x Ω"
  using assms C1_only_phenomena Nonduality by blast

lemma Causal_right_NotTwo:
  assumes "CausallyPrecedes x y" shows "Inseparable y Ω"
  using assms C1_only_phenomena Nonduality by blast
```

12 Spacetime as Representation (Coordinates only for Phenomena)

```
typedecl Frame
typedecl R4

consts
  coord    :: "Frame ⇒ E ⇒ R4 option"
  GaugeRel :: "Frame ⇒ Frame ⇒ bool"

axiomatization where
  S1_coords_only_for_phenomena:
    "∀f x r. coord f x = Some r ⟶ Phenomenon x" and
  S2_gauge_invariance_definedness:
    "∀f g x. GaugeRel f g ⟶ (coord f x = None ⟷ coord g x = None)"

lemma Spacetime_unreality:
  assumes "coord f x ≠ None"
  shows "Inseparable x Ω"
proof -
  from assms obtain r where "coord f x = Some r" by (cases "coord f x")
auto
  hence "Phenomenon x" using S1_coords_only_for_phenomena by blast
  thus "Inseparable x Ω" using Nonduality by blast
qed
```

13 Emptiness: No Intrinsic Essence of Phenomena

```
consts Essence :: "E ⇒ bool"

axiomatization where
  Emptiness_of_Phenomena: "∀x. Phenomenon x ⟶ ¬ Essence x"
```

14 Endogenous / Dependent Arising

```
consts ArisesFrom :: "E ⇒ E ⇒ bool"

axiomatization where
  AF_only_pheno:   "∀p q. ArisesFrom p q ⟶ Phenomenon p ∧ Phenomenon
q" and
  AF_endogenous:   "∀p q. ArisesFrom p q ⟶ (∃s. Substrate s ∧ Presents
p s ∧ Presents q s)" and
  AF_no_exogenous: "∀p q. ArisesFrom p q ⟶ ¬ (∃z. ¬ Phenomenon z ∧ ¬
Substrate z)"
```

15 Non-Appropriation (Ownership is Conventional)

```
typedecl Agent
consts Owns :: "Agent ⇒ E ⇒ bool"
consts ValidConv :: "E ⇒ bool"

axiomatization where
  Ownership_is_conventional:
    "∀a p. Owns a p ⟶ Phenomenon p ∧ ValidConv p" and
  No_ontic_ownership:
    "∀a p. Owns a p ⟶ Inseparable p Ω ∧ ¬ Essence p"
```

16 Symmetry / Gauge on Phenomena

```
typedecl G
consts act :: "G ⇒ E ⇒ E"

axiomatization where
  Act_closed:             "∀g x. Phenomenon x ⟶ Phenomenon (act g x)"
and
  Act_pres_presentation: "∀g x. Presents x Ω ⟶ Presents (act g x) Ω"

lemma Symmetry_preserves_NotTwo:
  assumes "Phenomenon x"
  shows "Inseparable (act g x) Ω"
 using assms Act_closed Act_pres_presentation A5_insep_def substrate_Omega
Nonduality
  by (metis)
```

17 Concepts / Annotations

```
typedecl Concept
consts Applies :: "Concept ⇒ E ⇒ bool"

axiomatization where
  Concepts_are_annotations:
    "∀c x. Applies c x ⟶ Phenomenon x"

lemma Concepts_don't_reify:
  assumes "Applies c x" shows "Inseparable x Ω"
  using assms Concepts_are_annotations Nonduality by blast
```

18 Quantities for Information and Time

```
typedecl Q
```

19 Information Layer (Abstract Nonnegativity)

consts
  Info   :: "E ⇒ Q"
  Nonneg :: "Q ⇒ bool"

axiomatization where
  Info_nonneg: "∀x. Phenomenon x ⟶ Nonneg (Info x)"

lemma Info_nonreifying:
  assumes "Phenomenon x" **shows** "Inseparable x Ω"
  using assms Nonduality **by** blast

20 Emergent Time (Abstract Strict Order on Q)

consts
  T :: "E ⇒ Q"
  LT :: "Q ⇒ Q ⇒ bool"

axiomatization where
  LT_irrefl: "∀q. ¬ LT q q" **and**
  LT_trans: "∀a b c. LT a b ∧ LT b c ⟶ LT a c" **and**
  Time_monotone: "∀x y. CausallyPrecedes x y ⟶ LT (T x) (T y)"

lemma Time_emergent_NotTwo:
  assumes "Phenomenon x" **shows** "Inseparable x Ω"
  using assms Nonduality **by** blast

21 Two-Levels Coherence

consts Coherent :: "E ⇒ bool"

axiomatization where
  Conventional_is_model_relative: "∀x. ValidConv x ⟶ Phenomenon x" **and**
  Ultimate_coherence: "Coherent Ω"

22 Notation and Robustness

definition NotTwo :: "E ⇒ E ⇒ bool"
  where "NotTwo x y ⟷ Inseparable x y"

lemma Phenomenon_NotTwo_Base: "Phenomenon p ⟹ NotTwo p Ω"
  using Nonduality NotTwo_def **by** blast

lemma Any_presentation_structure_preserves_NotTwo:
  assumes "Phenomenon x" **shows** "NotTwo x Ω"
  using assms Nonduality NotTwo_def **by** blast

end