SUBSTRATE THEORY — SYMBOLIC LOGIC CANONICAL REFERENCE

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TYPES

$$
\begin{aligned}
\text{State} &:= \text{List Bool} \\
\text{Entity} &: \text{opaque type} \\
\Omega &: \text{Entity (axiom)} \\
\text{Substrate} &: \text{Entity (axiom)} \\
\text{Time} &:= \mathbb{R} \\
\text{Phase} &:= \mathbb{R} \\
\text{Nbhd} &:= \text{List State} \\
\text{Precision} &:= \mathbb{N}
\end{aligned}
$$

COMPLEXITY FUNCTIONS

$$\begin{aligned}
K &: \text{Entity} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
K_{\text{sum}} &: \text{List Entity} \to \mathbb{R} \\
K_{\text{sum}}(es) &:= \sum_{e \in es} K(e)
\end{aligned}$$

$$\begin{aligned}
K_{\text{joint}} &: \text{List Entity} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
K_{\text{cond}} &: \text{Entity} \to \text{Entity} \to \mathbb{R} \\
K_{\text{cond}}(e_1, e_2) &:= K_{\text{joint}}([e_1, e_2]) - K(e_1)
\end{aligned}$$

$$\begin{aligned}
C &: \text{Entity} \to \mathbb{N} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
C_{\text{sum}} &: \text{List Entity} \to \mathbb{N} \to \mathbb{R} \\
C_{\text{sum}}(es, p) &:= \sum_{e \in es} C(e, p)
\end{aligned}$$

$$\begin{aligned}
C_{\text{joint}} &: \text{List Entity} \to \mathbb{N} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
C_{\text{cond}} &: \text{Entity} \to \text{Entity} \to \mathbb{N} \to \mathbb{R} \\
C_{\text{cond}}(e_1, e_2, p) &:= C_{\text{joint}}([e_1, e_2], p) - C(e_1, p)
\end{aligned}$$

$$\begin{aligned}
K_{LZ} &: \text{State} \to \mathbb{N} \quad \text{(axiom)} \\
K_{\text{toy}} &: \text{State} \to \mathbb{N} \\
K_{\text{toy}}(s) &:= |\text{dedup}(s)|
\end{aligned}$$

RANK FUNCTIONS

$$\begin{aligned}
\text{rank}_K &: \text{Entity} \to \mathbb{N} \quad \text{(noncomputable axiom)} \\
\text{rank}_C &: \text{Entity} \to \mathbb{N} \to \mathbb{N} \quad \text{(noncomputable axiom)}
\end{aligned}$$

TEMPORAL FUNCTIONS

$$\begin{aligned}
\text{indexed} &: \text{Entity} \to \text{Time} \to \text{Entity} \quad \text{(axiom)} \\
\text{temporal_slice} &: \text{List Entity} \to \text{Time} \to \text{List Entity} \\
\text{slice} &: \text{List (Entity} \times \text{Time)} \to \text{Time} \to \text{List Entity}
\end{aligned}$$

$$\begin{aligned}
\text{join} &: \text{List State} \to \text{State} \quad \text{(axiom)} \\
\text{mode} &: \text{State} \to \text{State} \quad \text{(noncomputable constant)}
\end{aligned}$$

$$\begin{aligned}
\text{traj} &: \text{Entity} \to \text{List (Entity} \times \text{Time)} \\
P_{\text{total}} &: \text{Entity} \to \mathbb{R}
\end{aligned}$$

COHERENCE FUNCTIONS

$$\begin{aligned}
\mathrm{Coh} &: \text{List Entity} \to \text{List Time} \to \mathbb{R} \\
\mathrm{Coh}(es, \mathrm{times}) &:= 1 - K_{\mathrm{joint}}(\mathrm{slice}(es, \mathrm{times}))/K_{\mathrm{sum}}(\mathrm{slice}(es, \mathrm{times}))
\end{aligned}$$

$$\begin{aligned}
\mathrm{Coh}_{\mathrm{op}} &: \text{List Entity} \to \text{List Time} \to \mathbb{N} \to \mathbb{R} \\
\mathrm{Coh}_{\mathrm{op}}(es, \mathrm{times}, p) &:= 1 - C_{\mathrm{joint}}(\mathrm{slice}(es, \mathrm{times}), p)/C_{\mathrm{sum}}(\mathrm{slice}(es, \mathrm{times}), p)
\end{aligned}$$

$$\begin{aligned}
\mathrm{Coh}_{\mathrm{trajectory}} &: \text{Entity} \to \mathbb{R} \\
\frac{d\mathrm{Coh}}{dt} &: \text{Entity} \to \text{Time} \to \mathbb{R}
\end{aligned}$$

$$\text{compression\_ratio} : \text{List Entity} \to \text{List Time} \to \mathbb{R}$$

PARTITION FUNCTIONS

$$\begin{aligned}
Z_{\mathrm{ideal}} &: \text{Finset Entity} \to \mathbb{R} \\
Z_{\mathrm{ideal}}(S) &:= \sum_{e \in S} 2^{-K(e)}
\end{aligned}$$

$$\begin{aligned}
Z_{\mathrm{op}} &: \text{Finset Entity} \to \mathbb{N} \to \mathbb{R} \\
Z_{\mathrm{op}}(S, p) &:= \sum_{e \in S} 2^{-C(e,p)}
\end{aligned}$$

GROUNDING FUNCTIONS

$$\begin{aligned}
\text{grounds_graph} &: \text{Finset Entity} \to \mathbb{N} \to \text{Finset (Entity} \times \text{Entity)} \\
\mathrm{parents}_C &: \text{Entity} \to \text{Finset Entity} \to \mathbb{N} \to \text{Finset Entity} \\
\text{bfs_depth}_C &: \text{Entity} \to \mathbb{N} \to \text{Finset Entity} \to \mathbb{N} \\
\text{bfs_grounding_path} &: \text{Entity} \to \text{Finset Entity} \to \mathbb{N} \to \text{Option (List Entity)}
\end{aligned}$$

CONSTANTS

$$
\begin{aligned}
c_{\text{grounding}} &:= 50 \\
c_{\text{margin}} &:= 5 \\
c_{\text{sub}} &: \mathbb{R} \quad \text{(constant)} \\
c_{\text{single}} &: \mathbb{R} \quad \text{(constant)} \\
C_{\text{mode}} &: \mathbb{R} \quad \text{(constant)}
\end{aligned}
$$

$$
\begin{aligned}
c &\approx 299792458 \text{ m/s} \\
\hbar &: \mathbb{R} \quad (\text{axiom}, \hbar > 0) \\
G &: \mathbb{R} \quad (\text{axiom}, G > 0) \\
k_B &: \mathbb{R} \quad (\text{axiom}, k_B > 0) \\
e &: \mathbb{R} \quad (\text{axiom}, e > 0) \\
\varepsilon_0 &: \mathbb{R} \quad (\text{axiom}, \varepsilon_0 > 0) \\
\alpha &: \mathbb{R} \quad (\text{axiom}, 1/138 < \alpha < 1/137)
\end{aligned}
$$

$$
\begin{aligned}
\ell_{\text{Planck}} &:= \sqrt{\frac{\hbar G}{c^3}} \\
t_{\text{Planck}} &:= \ell_{\text{Planck}}/c \\
M_{\text{Planck}} &:= \sqrt{\frac{\hbar c}{G}} \\
E_{\text{Planck}} &:= M_{\text{Planck}} \cdot c^2 \\
T_{\text{Planck}} &:= E_{\text{Planck}}/k_B
\end{aligned}
$$

$$
\begin{aligned}
\kappa_{\text{energy}} &:= E_{\text{Planck}} \\
\hbar_{\text{eff}} &: \mathbb{R} \quad (\text{axiom}, \hbar_{\text{eff}} > 0) \\
\varepsilon_{\text{geom}} &: \mathbb{R} \quad (\text{axiom}, \varepsilon_{\text{geom}} > 0)
\end{aligned}
$$

$$
\begin{aligned}
H_0 &: \mathbb{R} \quad (67 < H_0 < 74) \\
\Omega_m &: \mathbb{R} \quad (0.3 < \Omega_m < 0.32) \\
\Omega_\Lambda &: \mathbb{R} \quad (0.68 < \Omega_\Lambda < 0.70) \\
\Omega_r &: \mathbb{R} \quad (0 < \Omega_r < 0.0001) \\
\Omega_k &: \mathbb{R} \quad (|\Omega_k| < 0.01) \\
\Omega_{DM} &: \mathbb{R} \quad (0.25 < \Omega_{DM} < 0.27) \\
\Omega_{\text{baryon}} &: \mathbb{R} \quad (0.04 < \Omega_{\text{baryon}} < 0.05) \\
t_{\text{universe}} &: \mathbb{R} \quad (13.7 \times 10^9 < t_{\text{universe}} < 13.9 \times 10^9)
\end{aligned}
$$

CORE AXIOMS

Axiom 1 (K2 (Substrate Minimality)).

$$K(\mathit{Substrate}) = 0$$
$$K(\Omega) = 0$$

Axiom 2 (G1 (Substrate Grounds All)).

$$\forall e.\ \mathit{is\_presentation}(e) \to \mathit{is\_grounded}(e, \mathit{Substrate})$$

where $\mathit{is\_grounded}(e, \mathit{ctx}) := K_{\mathit{cond}}(\mathit{ctx}, e) < K(e) - K(\mathit{ctx}) + c_{\mathit{grounding}}$

Axiom 3 (T7 (Time Arrow)).

$$\forall \mathit{hist},\ \mathit{next}.\ \mathit{hist.length} \geq 2 \to K_{\mathit{joint}}(\mathit{next} :: \mathit{hist}) - K_{\mathit{joint}}(\mathit{hist}) \leq K_{\mathit{joint}}([\mathit{hist.last}, \mathit{hist.init}]) - K(\mathit{hist.init})$$

Axiom 4 (T4 (Emergence/Collapse)).

$$\forall e_{\mathit{classical}}, e_{\mathit{quantum}}.\ \mathit{emergent}(e_{\mathit{classical}}, e_{\mathit{quantum}}) \to \mathit{is\_measurement\_device}(e_{\mathit{classical}}) \vee \mathit{is\_observable}(e_{\mathit{classical}})$$

where $\mathit{emergent}(e_{\mathit{classical}}, e_{\mathit{quantum}}) := K_{\mathit{cond}}(\mathit{Substrate}, e_{\mathit{classical}}) < K(e_{\mathit{quantum}})$

Axiom 5 (C6 (Coherence Preservation)).

$$\forall e.\ \mathit{is\_quantum\_state}(e) \to \mathit{coherent}(e)$$

where $\mathit{coherent}(e) := \forall t_1, t_2.\ t_1 < t_2 \to K_{\mathit{cond}}(\mathit{indexed}(e, t_1), \mathit{indexed}(e, t_2)) = K_{\mathit{cond}}(\mathit{indexed}(e, t_2), \mathit{indexed}(e, t_1))$

AXIOM CONSEQUENCES

Theorem 1 (substrate_ultimate_ground).

$$\forall e.\ \mathit{is\_presentation}(e) \to \exists \mathit{path}.\ \mathit{path.head} = \mathit{Substrate} \wedge \mathit{path.last} = e \wedge \forall i.\ i+1 < \mathit{path.length} \to \mathit{is\_grounded}(\mathit{path}[i+1], \mathit{path}[i])$$

Theorem 2 (decoherence_implies_classical).

$$\forall e.\ \mathit{is\_presentation}(e) \wedge \neg \mathit{coherent}(e) \to \exists t_0.\ \forall t > t_0.\ \neg \mathit{is\_quantum\_state}(\mathit{indexed}(e, t))$$

Theorem 3 (measurement_breaks_coherence).

$$\forall e_q, e_c.\ \mathit{is\_quantum\_state}(e_q) \wedge \mathit{coherent}(e_q) \wedge \mathit{emergent}(e_c, e_q) \to \neg \mathit{coherent}(e_c)$$

BRIDGE AXIOMS

Axiom 6 (BRIDGE1 (Pointwise Convergence)).

$$\forall e, \varepsilon > 0.\, \mathit{is\_presentation}(e) \to \exists p_0.\, \forall p \geq p_0.\, |C(e,p) - K(e)| < \varepsilon$$

Axiom 7 (BRIDGE2 (Uniform Convergence)).

$$\forall S, \varepsilon > 0.\, (\forall e \in S.\, \mathit{is\_presentation}(e)) \to \exists p_0.\, \forall p \geq p_0, e \in S.\, |C(e,p) - K(e)| < \varepsilon$$

Axiom 8 (BRIDGE3 (Probability Convergence)).

$$\forall S, \varepsilon > 0.\, (\forall e \in S.\, \mathit{is\_presentation}(e)) \wedge Z_{\mathit{ideal}}(S) > 0 \to \\ \exists p_0.\, \forall p \geq p_0.\, |Z_{op}(S,p) - Z_{\mathit{ideal}}(S)| / Z_{\mathit{ideal}}(S) < \varepsilon$$

Axiom 9 (BRIDGE4 (Grounding Convergence)).

$$\forall S, \varepsilon > 0, e_1, e_2.\, e_1, e_2 \in S \wedge \mathit{is\_presentation}(e_1) \wedge \mathit{is\_presentation}(e_2) \to \\ \exists p_0.\, \forall p \geq p_0.\, \mathit{grounds}_K(e_1, e_2) \leftrightarrow \mathit{grounds}_C(e_1, e_2, p)$$

where:

$$\mathit{grounds}_K(e_1, e_2) := K_{\mathit{cond}}(e_1, e_2) < K(e_2) - K(e_1) + c_{\mathit{grounding}}$$
$$\mathit{grounds}_C(e_1, e_2, p) := C_{\mathit{cond}}(e_1, e_2, p) < C(e_2, p) - C(e_1, p) + c_{\mathit{grounding}}$$

Axiom 10 (BRIDGE5 (Rank Stability)).

$$\forall S, e.\, e \in S \wedge \mathit{is\_presentation}(e) \to \\ \exists p_0.\, \forall p \geq p_0.\, \mathit{rank}_C(e,p) = \mathit{rank}_K(e)$$

Axiom 11 (BRIDGE6 (Temporal Continuity)).

$$\forall e, \mathit{times}, \varepsilon > 0.\, \mathit{is\_temporal\_presentation}(e) \to \\ \exists p_0.\, \forall p \geq p_0, t \in \mathit{times}.\, |\mathit{Coh}_{op}([e],[t],p) - \mathit{Coh}([e],[t])| < \varepsilon$$

Axiom 12 (BRIDGE7 (Conditional Convergence)).

$$\forall e_1, e_2, \varepsilon > 0.\, \mathit{is\_presentation}(e_1) \wedge \mathit{is\_presentation}(e_2) \to \\ \exists p_0.\, \forall p \geq p_0.\, |C_{\mathit{cond}}(e_1, e_2, p) - K_{\mathit{cond}}(e_1, e_2)| < \varepsilon$$

Axiom 13 (BRIDGE7_joint (Joint Convergence)).

$$\forall es, \varepsilon > 0.\, (\forall e \in es.\, \mathit{is\_presentation}(e)) \to \\ \exists p_0.\, \forall p \geq p_0.\, |C_{\mathit{joint}}(es, p) - K_{\mathit{joint}}(es)| < \varepsilon$$

Axiom 14 (BRIDGE8 (Continuum Limit)).

$$\forall e, \mathit{times}, \varepsilon > 0.\, \mathit{is\_temporal\_presentation}(e) \to \\ \exists p_0, \delta.\, \delta > 0 \wedge \forall p \geq p_0, t \in \mathit{times}. \\ \left| \frac{\mathit{Coh}_{op}([e],[t+\delta],p) - \mathit{Coh}_{op}([e],[t],p)}{\delta} - \frac{d\mathit{Coh}}{dt}(e,t) \right| < \varepsilon$$

CA RULES

$$
\begin{aligned}
F &: \text{State} \to \text{State} \quad \text{(noncomputable)} \\
\text{merge} &: \text{State} \to \text{State} \to \text{State} \quad \text{(noncomputable)}
\end{aligned}
$$

$$
\begin{aligned}
R_{\text{Cohesion}} &: \text{List State} \to \text{State} \to \text{State} \\
R_{\text{Cohesion}}(n, h) &:= \text{merge}(F(\text{join}(n)), h)
\end{aligned}
$$

$$
\begin{aligned}
R_{\text{Reduction}} &: \text{List State} \to \text{State} \\
R_{\text{Reduction}}(n) &:= \text{mode}(\text{join}(n))
\end{aligned}
$$

$$
\begin{aligned}
R_{G1} &: \text{List State} \to \text{State} \to \text{State} \\
R_{G1}(n, h) &:= \begin{cases} R_{\text{Cohesion}}(n, h) & \text{if } K_{LZ}(\text{join}(n)) \leq c_{\text{grounding}} \\ R_{\text{Reduction}}(n) & \text{otherwise} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
\text{coherent\_state} &: \text{State} \to \text{Prop} \\
\text{coherent\_state}(s) &:= K_{LZ}(s) \leq c_{\text{grounding}}
\end{aligned}
$$

CA PRESERVATION THEOREMS

Theorem 4 (P3 (C6 Preservation))**.**

$$\forall n, h.\, \mathit{coherent\_state}(\mathit{join}(n)) \to K_{LZ}(R_{G1}(n, h)) = K_{LZ}(h)$$

Theorem 5 (R_G1_grounding_reduction)**.**

$$\forall n, h.\, K_{LZ}(\mathit{join}(n)) > c_{\mathit{grounding}} \to K_{LZ}(R_{G1}(n, h)) < K_{LZ}(\mathit{join}(n)) + c_{\mathit{grounding}}$$

Theorem 6 (R_G1_preserves_time_arrow)**.**

$$\forall \mathit{hist}, n, h.\, K_{LZ}(\mathit{join}(R_{G1}(n, h) :: \mathit{hist})) \leq K_{LZ}(\mathit{join}(\mathit{hist})) + c_{\mathit{margin}}$$

FUNDAMENTAL THEOREMS

Theorem 7 (E_K (Energy-Complexity Equivalence))**.**

$$
\begin{multline}
\forall e.\, \mathit{is\_presentation}(e) \to \\
(\mathit{has\_mass}(e) \to \exists \Delta > 0.\, K(e) = K(\Omega) + \Delta) \wedge \\
\mathit{energy\_of}(e) = \kappa_{\mathit{energy}} \cdot K(e)
\end{multline}
$$

Theorem 8 (G_Ψ (Grounding Stability))**.**

$$\forall e.\, \mathit{stable}(e) \leftrightarrow K_{\mathit{cond}}(\Omega, e) > c_{\mathit{grounding}}$$

where $\mathit{stable}(e) := \mathit{is\_presentation}(e) \wedge K_{\mathit{cond}}(\Omega, e) > c_{\mathit{grounding}}$

Theorem 9 (B_Ω (Holographic Bound)).

$$\forall region,\ Area.\ is\_presentation(region) \land Area > 0 \rightarrow \\ K(region) \leq \frac{Area}{4\ell_{Planck}^2}$$

Theorem 10 (Ψ_I (Coherence Invariant)).

$$\forall e.\ is\_temporal\_presentation(e) \land coherent(e) \rightarrow \\ Coh_{trajectory}(e) \cdot P_{total}(e) = 1$$

Theorem 11 (U_Ω (Uncertainty Principle)).

$$\forall e, \Delta K, \Delta t.\ is\_temporal\_presentation(e) \land \Delta K > 0 \land \Delta t > 0 \rightarrow \\ \Delta K \cdot \Delta t \geq \hbar_{eff}$$

RANK SYSTEM

$$\begin{aligned} \mathrm{rank}_K(\Omega) &= 0 \\ \mathrm{grounds}(e_1, e_2) &\rightarrow \mathrm{rank}_K(e_2) < \mathrm{rank}_K(e_1) \\ \forall e.\ \exists n.\ \mathrm{rank}_K(e) &= n \end{aligned}$$

$$\mathrm{rank}_C(e, p) = \mathrm{bfs\_depth}_C(e, p, S) \quad \text{for } e \in S$$

UNIVERSAL GROUNDING

Theorem 12.

$$\forall e.\ is\_presentation(e) \rightarrow \\ \exists path.\ path.head = \Omega \land path.last = e \land \\ \forall i.\ i + 1 < path.length \rightarrow grounds(path[i], path[i+1])$$

Theorem 13 (grounding_transitive).

$$\forall e_1, e_2, e_3.\ grounds(e_1, e_2) \land grounds(e_2, e_3) \rightarrow grounds(e_1, e_3)$$

Theorem 14 (grounding_acyclic).

$$\forall e.\ \neg grounds(e, e)$$

COMPLEXITY BOUNDS

$$\begin{gathered} \text{K\_joint\_nonneg} : \forall es.\ 0 \leq K_{\text{joint}}(es) \\ \text{K\_joint\_nil} : K_{\text{joint}}([]) = 0 \\ \text{K\_joint\_singleton} : \forall e.\ K_{\text{joint}}([e]) = K(e) \end{gathered}$$

Theorem 15 (compression_axiom).

$$\forall es.\,(\forall e \in es.\, \mathit{is\_presentation}(e)) \land es.\mathit{length} \geq 2 \rightarrow K_{joint}(es) < K_{sum}(es)$$

Theorem 16 (joint_le_sum).

$$\forall es.\,(\forall e \in es.\, \mathit{is\_presentation}(e)) \rightarrow K_{joint}(es) \leq K_{sum}(es)$$

Theorem 17 (complexity_positive).

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow 0 < K(e)$$

Theorem 18 (substrate_minimal).

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow K(\mathit{Substrate}) \leq K(e)$$

OPERATIONAL BOUNDS

$$\begin{aligned}
\text{C_nonneg} &: \forall e, p.\, 0 \leq C(e, p) \\
\text{C_monotone} &: \forall e, p_1, p_2.\, p_1 \leq p_2 \rightarrow C(e, p_2) \leq C(e, p_1) \\
\text{C_upper_bound} &: \forall e, p.\, \text{is_presentation}(e) \rightarrow K(e) \leq C(e, p)
\end{aligned}$$

$$\text{C\_joint\_nonneg} : \forall es, p.\, 0 \leq C_{\text{joint}}(es, p)$$

$$\begin{aligned}
\text{K_LZ_nonneg} &: \forall s.\, 0 \leq K_{LZ}(s) \\
\text{K_LZ_empty} &: K_{LZ}([]) = 0 \\
\text{K_LZ_monotone} &: \forall s_1, s_2.\, s_1.\text{length} \leq s_2.\text{length} \rightarrow K_{LZ}(s_1) \leq K_{LZ}(s_2)
\end{aligned}$$

TOY COMPRESSOR BOUNDS

$$\begin{aligned}
\text{K_toy_lower_bound} &: \forall s.\, K_{LZ}(s) \leq K_{\text{toy}}(s) \\
\text{K_toy_upper_bound} &: \forall s.\, K_{\text{toy}}(s) \leq K_{LZ}(s) + \log_2(s.\text{length})
\end{aligned}$$

KLZ AXIOMS

Axiom 15 (K_LZ_subadditive_cons).

$$\forall x, xs.\, K_{LZ}(\mathit{join}(x :: xs)) \leq K_{LZ}(\mathit{join}(xs)) + K_{LZ}(x) + c_{sub}$$

Axiom 16 (K_LZ_prefix).

$$\forall b, s.\, K_{LZ}(\mathit{join}([b])) \leq K_{LZ}(\mathit{join}(b :: s))$$

Axiom 17 (K_LZ_singleton_bound).

$$\forall b.\, K_{LZ}(\mathit{join}([b])) \leq c_{single}$$

Axiom 18 (K_LZ_mode_le)**.**

$$\forall s.\, K_{LZ}(mode(s)) \leq K_{LZ}(s) + C_{mode}$$

Axiom 19 (C_mode_lt_c_grounding)**.**

$$C_{mode} < c_{grounding}$$

COHERENCE BOUNDS

Theorem 19 (coherence_bounds)**.**

$$\forall es, times.\, (\forall e \in es.\, is\_presentation(e)) \rightarrow \\ 0 \leq Coh(es, times) \land Coh(es, times) \leq 1$$

Theorem 20 (compression_ratio_ge_one)**.**

$$\forall es, times.\, (\forall e \in es.\, is\_presentation(e)) \rightarrow 1 \leq compression\_ratio(es, times)$$

PHYSICS CORRESPONDENCES

$$\begin{aligned} \text{energy\_of}(e) &= \kappa_{\text{energy}} \cdot K(e) \\ \text{mass}(e) &= \text{energy\_of}(e)/c^2 \\ \text{entropy}(e) &= k_B \cdot \log(2) \cdot K(e) \end{aligned}$$

$$\begin{aligned} \text{is\_quantum}(\text{nbhd}) &:= K_{LZ}(\text{join}(\text{nbhd})) \leq c_{\text{grounding}} \\ \text{is\_classical}(\text{nbhd}) &:= K_{LZ}(\text{join}(\text{nbhd})) > c_{\text{grounding}} \end{aligned}$$

PREDICATES

$$\begin{aligned}
\text{is_substrate} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_emergent} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_temporal_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_static_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_quantum_state} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_measurement_device} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_observable} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)}
\end{aligned}$$

$$\begin{aligned}
\text{phenomenal} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{has_mass} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)}
\end{aligned}$$

$$\begin{aligned}
\text{grounds} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{temporal_grounds} &: \text{Entity} \to \text{Time} \to \text{Entity} \to \text{Time} \to \text{Prop} \quad \text{(axiom)} \\
\text{interacts} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{inseparable} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{emerges_from} &: \text{Entity} \to \text{List Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{phase_coupled} &: \text{Entity} \to \text{Entity} \to \text{Phase} \to \text{Prop} \quad \text{(axiom)}
\end{aligned}$$

$$\begin{aligned}
\text{coherent} &: \text{Entity} \to \text{Prop} \\
\text{decoherent} &: \text{Entity} \to \text{Prop} \\
\text{stable} &: \text{Entity} \to \text{Prop}
\end{aligned}$$

$$\begin{aligned}
\text{is_quantum} &: \text{List State} \to \text{Prop} \\
\text{is_classical} &: \text{List State} \to \text{Prop} \\
\text{coherent_state} &: \text{State} \to \text{Prop}
\end{aligned}$$

ENTITY CLASSIFICATION

Theorem 21 (entity_classification)**.**

$$\forall e.\, (\textit{is\_substrate}(e) \land e = \textit{Substrate}) \lor \textit{is\_presentation}(e) \lor \textit{is\_emergent}(e)$$

$$\begin{aligned}
\text{substrate_not_presentation} &: \forall e.\, \neg(\text{is_substrate}(e) \land \text{is_presentation}(e)) \\
\text{substrate_not_emergent} &: \forall e.\, \neg(\text{is_substrate}(e) \land \text{is_emergent}(e)) \\
\text{presentation_not_emergent} &: \forall e.\, \neg(\text{is_presentation}(e) \land \text{is_emergent}(e))
\end{aligned}$$

Theorem 22 (presentation_temporal_or_static)**.**

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow (\mathit{is\_temporal\_presentation}(e) \lor \mathit{is\_static\_presentation}(e)) \land \neg(\mathit{is\_temporal\_presentation}(e) \land \mathit{is\_static\_presentation}(e))$$

SUBSTRATE PROPERTIES

$$\begin{aligned}
\text{substrate_unique} &: \forall x, y.\, \text{is_substrate}(x) \land \text{is_substrate}(y) \rightarrow x = y \\
\text{substrate_is_Substrate} &: \text{is_substrate}(\text{Substrate}) \\
\text{Omega_is_substrate} &: \text{is_substrate}(\Omega) \\
\text{Omega_eq_Substrate} &: \Omega = \text{Substrate}
\end{aligned}$$

TEMPORAL PRESERVATION

Theorem 23 (indexed_preserves_presentation)**.**

$$\forall e, t.\, \mathit{is\_presentation}(e) \rightarrow \mathit{is\_presentation}(\mathit{indexed}(e, t))$$

Theorem 24 (temporal_slice_preserves_presentation)**.**

$$\forall es, t.\, (\forall e \in es.\, \mathit{is\_presentation}(e)) \rightarrow (\forall e \in \mathit{temporal\_slice}(es, t).\, \mathit{is\_presentation}(e))$$

ASSOCIATIVITY

Theorem 25 (join_associative)**.**

$$\forall s_1, s_2, s_3.\, \mathit{join}([\mathit{join}([s_1, s_2]), s_3]) = \mathit{join}([s_1, \mathit{join}([s_2, s_3])])$$