Substrate Theory: Formal Specification (Lean 4 Verified)

Matthew Scherf

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1 Types

$$
\begin{aligned}
\text{State} &:= \text{List Bool} \\
\text{Entity} &: \text{opaque type} \\
\Omega &: \text{Entity (axiom)} \\
\text{Substrate} &: \text{Entity (axiom)} \\
\text{Time} &:= \mathbb{R} \\
\text{Phase} &:= \mathbb{R} \\
\text{Nbhd} &:= \text{List State} \\
\text{Precision} &:= \mathbb{N} \\
\text{KLZ.State} &: \text{Type (axiom)}
\end{aligned}
$$

2 Complexity Functions

$$
\begin{aligned}
K &: \text{Entity} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
K_{\text{sum}} &: \text{List Entity} \to \mathbb{R} \\
K_{\text{sum}}(es) &:= \sum_{e \in es} K(e) \\
K_{\text{joint}} &: \text{List Entity} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
K_{\text{cond}} &: \text{Entity} \to \text{Entity} \to \mathbb{R} \\
K_{\text{cond}}(e_1, e_2) &:= K_{\text{joint}}([e_1, e_2]) - K(e_1) \\
C &: \text{Entity} \to \mathbb{N} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
C_{\text{sum}} &: \text{List Entity} \to \mathbb{N} \to \mathbb{R} \\
C_{\text{sum}}(es, p) &:= \sum_{e \in es} C(e, p) \\
C_{\text{joint}} &: \text{List Entity} \to \mathbb{N} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
C_{\text{cond}} &: \text{Entity} \to \text{Entity} \to \mathbb{N} \to \mathbb{R} \\
C_{\text{cond}}(e_1, e_2, p) &:= C_{\text{joint}}([e_1, e_2], p) - C(e_1, p) \\
K_{LZ} &: \text{State} \to \mathbb{N} \quad \text{(noncomputable axiom, operational)} \\
K_{LZ} &: \text{KLZ.State} \to \mathbb{N} \quad \text{(noncomputable axiom, KLZ module)} \\
K_{\text{toy}} &: \text{State} \to \mathbb{N} \\
K_{\text{toy}}(s) &:= |\,\text{dedup}(s)|
\end{aligned}
$$

3 Rank Functions

$$
\begin{aligned}
\text{rank}_K &: \text{Entity} \to \mathbb{N} \quad \text{(noncomputable axiom)} \\
\text{rank}_C &: \text{Entity} \to \mathbb{N} \to \mathbb{N} \quad \text{(noncomputable axiom)}
\end{aligned}
$$

4 Temporal Functions

$$
\begin{aligned}
\text{indexed} &: \text{Entity} \to \text{Time} \to \text{Entity} \quad \text{(axiom)} \\
\text{temporal\_slice} &: \text{List Entity} \to \text{Time} \to \text{List Entity} \\
\text{slice} &: \text{List(Entity} \times \text{Time)} \to \text{Time} \to \text{List Entity} \\
\text{join} &: \text{List State} \to \text{State} \quad \text{(axiom)} \\
\text{join} &: \text{List KLZ.State} \to \text{KLZ.State} \quad \text{(axiom, KLZ module)} \\
\text{mode} &: \text{KLZ.State} \to \text{KLZ.State} \quad \text{(noncomputable axiom)} \\
\text{traj} &: \text{Entity} \to \text{List(Entity} \times \text{Time)} \\
\text{traj}(e) &:= (\text{List.range}\,1000).\text{map}\,(\lambda n.\,(\text{indexed}\ e\,n, n)) \\
P_{\text{total}} &: \text{Entity} \to \mathbb{R} \\
P_{\text{total}}(e) &:= 1 \quad \text{(sum of uniform weights over trajectory)}
\end{aligned}
$$

5 Coherence Functions

$$\begin{aligned}
\mathrm{Coh} &: \text{List Entity} \to \text{List Time} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
\mathrm{Coh}_{\mathrm{op}} &: \text{List Entity} \to \text{List Time} \to \mathbb{N} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
\mathrm{Coh}_{\mathrm{trajectory}} &: \text{Entity} \to \mathbb{R} \\
\frac{d\,\mathrm{Coh}}{dt} &: \text{Entity} \to \text{Time} \to \mathbb{R} \quad \text{(noncomputable axiom)} \\
\text{compression_ratio} &: \text{List Entity} \to \text{List Time} \to \mathbb{R}
\end{aligned}$$

6 Partition Functions

$$\begin{aligned}
Z_{\mathrm{ideal}} &: \text{Finset Entity} \to \mathbb{R} \\
Z_{\mathrm{ideal}}(S) &:= \sum_{e \in S} 2^{-K(e)} \\
Z_{\mathrm{op}} &: \text{Finset Entity} \to \mathbb{N} \to \mathbb{R} \\
Z_{\mathrm{op}}(S, p) &:= \sum_{e \in S} 2^{-C(e,p)}
\end{aligned}$$

7 Grounding Functions

$$\begin{aligned}
\text{grounds} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{temporal_grounds} &: \text{Entity} \to \text{Time} \to \text{Entity} \to \text{Time} \to \text{Prop} \quad \text{(axiom)} \\
\text{bfs_depth_C} &: \text{Entity} \to \mathbb{N} \to \text{Finset Entity} \to \mathbb{N} \quad \text{(noncomputable axiom)} \\
\text{bfs_grounding_path} &: \text{Entity} \to \text{Finset Entity} \to \mathbb{N} \to \text{Option(List Entity)}
\end{aligned}$$

8 Physical Constants

$$
\begin{aligned}
c &:= 299792458\,\mathrm{m/s} \\
\hbar &: \mathbb{R} \quad (\text{axiom, } \hbar > 0) \\
G &: \mathbb{R} \quad (\text{axiom, } G > 0) \\
k_B &: \mathbb{R} \quad (\text{axiom, } k_B > 0) \\
e &: \mathbb{R} \quad (\text{axiom, } e > 0) \\
\varepsilon_0 &: \mathbb{R} \quad (\text{axiom, } \varepsilon_0 > 0) \\
\alpha &: \mathbb{R} \quad (\text{axiom, } 1/138 < \alpha < 1/137) \\
\ell_{\text{Planck}} &:= \sqrt{\frac{\hbar G}{c^3}} \\
t_{\text{Planck}} &:= \frac{\ell_{\text{Planck}}}{c} \\
M_{\text{Planck}} &:= \sqrt{\frac{\hbar c}{G}} \\
E_{\text{Planck}} &:= M_{\text{Planck}} \cdot c^2 \\
T_{\text{Planck}} &:= \frac{E_{\text{Planck}}}{k_B} \\
\kappa_{\text{energy}} &:= E_{\text{Planck}} \\
\hbar_{\text{eff}} &: \mathbb{R} \quad (\text{axiom, } \hbar_{\text{eff}} > 0) \\
\varepsilon_{\text{geom}} &: \mathbb{R} \quad (\text{axiom, } \varepsilon_{\text{geom}} > 0)
\end{aligned}
$$

9 Cosmological Parameters

$$
\begin{aligned}
H_0 &: \mathbb{R} \quad (67 < H_0 < 74) \\
\Omega_m &: \mathbb{R} \quad (0.3 < \Omega_m < 0.32) \\
\Omega_\Lambda &: \mathbb{R} \quad (0.68 < \Omega_\Lambda < 0.70) \\
\Omega_r &: \mathbb{R} \quad (0 < \Omega_r < 0.0001) \\
\Omega_k &: \mathbb{R} \quad (|\Omega_k| < 0.01) \\
\Omega_{DM} &: \mathbb{R} \quad (0.25 < \Omega_{DM} < 0.27) \\
\Omega_{\text{baryon}} &: \mathbb{R} \quad (0.04 < \Omega_{\text{baryon}} < 0.05) \\
t_{\text{universe}} &: \mathbb{R} \quad (13.7 \times 10^9 < t_{\text{universe}} < 13.9 \times 10^9 \text{ years}) \\
N_e &: \mathbb{R} \quad (50 < N_e < 70, \text{ e-folds of inflation}) \\
n_s &: \mathbb{R} \quad (0.96 < n_s < 0.97, \text{ scalar spectral index}) \\
r_s &: \mathbb{R} \quad (0 \leq r_s < 0.036, \text{ tensor-to-scalar ratio}) \\
z_{\text{transition}} &: \mathbb{R} \quad (0.6 < z_{\text{transition}} < 0.8, \text{ matter-}\Lambda \text{ transition})
\end{aligned}
$$

10 Grounding Constants

$$
\begin{aligned}
c_{\text{grounding}} &:= 50 \quad \text{(bits)} \\
c_{\text{margin}} &:= 5 \quad \text{(bits)}
\end{aligned}
$$

11 KLZ Constants

$$
\begin{aligned}
c_{\text{sub}} &: \mathbb{R} \quad \text{(constant)} \\
c_{\text{single}} &: \mathbb{R} \quad \text{(constant)} \\
C_{\text{mode}} &: \mathbb{R} \quad \text{(constant)} \\
C_{\text{coh}} &: \mathbb{R} \quad \text{(constant)} \\
c_{\text{time\_reduction}} &:= c_{\text{sub}} + C_{\text{mode}} \\
c_{\text{time\_cohesion}} &:= c_{\text{sub}} + C_{\text{coh}}
\end{aligned}
$$

12 Threshold Parameters

$$
\begin{aligned}
\text{measured\_inseparability\_threshold} &: \mathbb{R} \quad (0 < \cdot < 1) \\
\text{strong\_compression\_threshold} &: \mathbb{R} \quad (0 < \cdot < \text{measured\_inseparability\_threshold}) \\
\text{temporal\_coherence\_threshold} &: \mathbb{R} \quad (0 < \cdot < 1) \\
\text{phase\_coupling\_threshold} &: \mathbb{R} \quad (0 < \cdot \leq 1) \\
\text{baseline\_maximal\_compression} &: \mathbb{R} \quad (1 < \cdot) \\
\text{weak\_coupling\_threshold} &: \mathbb{R} \\
\text{moderate\_coupling\_threshold} &: \mathbb{R} \\
\text{strong\_coupling\_threshold} &: \mathbb{R}
\end{aligned}
$$

Hierarchy:

$$1 < \text{weak\_coupling\_threshold} < \text{moderate\_coupling\_threshold} < \text{strong\_coupling\_threshold} < \text{baseline\_maximal\_con}$$

13 Core Axioms

Axiom 1 (K2: Substrate Minimality)**.**

$$
\begin{aligned}
K(\mathit{Substrate}) &= 0 \\
K(\Omega) &= 0
\end{aligned}
$$

Axiom 2 (G1: Substrate Grounds All)**.**

$$\forall e.\ \mathit{is\_presentation}(e) \rightarrow \mathit{is\_grounded}(e, \mathit{Substrate})$$

where

$$\mathit{is\_grounded}(e, \mathit{ctx}) := K_{cond}(\mathit{ctx}, e) < K(e) - K(\mathit{ctx}) + c_{grounding}$$

Axiom 3 (T7: Time Arrow).

$$\forall hist,\ next.\ hist.length \geq 2 \to \\ (\forall e \in hist.\ is\_temporal\_presentation(e)) \to \\ is\_temporal\_presentation(next) \to \\ K_{joint}(next :: hist) - K_{joint}(hist) \leq K_{joint}([hist.last, hist.init]) - K(hist.init)$$

Axiom 4 (T4: Emergence/Collapse).

$$\forall e_{classical},\ e_{quantum}.\ is\_presentation(e_{classical}) \to is\_quantum\_state(e_{quantum}) \to \\ emergent(e_{classical}, e_{quantum}) \to is\_measurement\_device(e_{classical}) \lor is\_observable(e_{classical})$$

where

$$emergent(e_{classical}, e_{quantum}) := K_{cond}(Substrate, e_{classical}) < K(e_{quantum})$$

Axiom 5 (C6: Coherence Preservation).

$$\forall e.\ is\_quantum\_state(e) \to coherent(e)$$

where

$$coherent(e) := \forall t_1,\ t_2.\ t_1 < t_2 \to \\ K_{cond}(\text{indexed}(e, t_1), \text{indexed}(e, t_2)) = K_{cond}(\text{indexed}(e, t_2), \text{indexed}(e, t_1))$$

14 Axiom Consequences

Theorem 1 (substrate_ultimate_ground).

$$\forall e.\ is\_presentation(e) \to \exists path.\ path.head = Substrate \land path.last = e \land \\ \forall i.\ i + 1 < path.length \to is\_grounded(path[i + 1], path[i])$$

Theorem 2 (decoherence_implies_classical).

$$\forall e.\ is\_presentation(e) \land \neg coherent(e) \to \exists t_0.\ \forall t > t_0.\ \neg is\_quantum\_state(\text{indexed}(e, t))$$

Theorem 3 (measurement_breaks_coherence).

$$\forall e_q,\ e_c.\ is\_quantum\_state(e_q) \quad \land \quad coherent(e_q) \quad \land \quad emergent(e_c, e_q) \qquad \to \qquad \neg coherent(e_c)$$

Theorem 4 (indexed_preserves_presentation).

$$\forall e,\ t.\ is\_presentation(e) \to is\_presentation(\text{indexed}(e, t))$$

15 Bridge Axioms

Axiom 6 (BRIDGE1: Pointwise Convergence).

$$\forall e,\ \varepsilon > 0.\ is\_presentation(e) \to \exists p_0.\ \forall p \geq p_0.\ |C(e, p) - K(e)| < \varepsilon$$

Axiom 7 (BRIDGE2: Uniform Convergence)**.**

$$\forall S,\, \varepsilon > 0.\, (\forall e \in S.\, \mathit{is\_presentation}(e)) \rightarrow \\ \exists p_0.\, \forall p \geq p_0,\, e \in S.\, |C(e,p) - K(e)| < \varepsilon$$

Axiom 8 (BRIDGE3: Probability Convergence)**.**

$$\forall S,\, \varepsilon > 0.\, (\forall e \in S.\, \mathit{is\_presentation}(e)) \wedge Z_{ideal}(S) > 0 \rightarrow \\ \exists p_0.\, \forall p \geq p_0.\, \frac{|Z_{op}(S,p) - Z_{ideal}(S)|}{Z_{ideal}(S)} < \varepsilon$$

Axiom 9 (BRIDGE4: Grounding Convergence)**.**

$$\forall S,\, \varepsilon > 0,\, e_1, e_2.\, e_1, e_2 \in S \wedge \mathit{is\_presentation}(e_1) \wedge \mathit{is\_presentation}(e_2) \rightarrow \\ \exists p_0.\, \forall p \geq p_0.\, \mathrm{grounds}_K(e_1,e_2) \leftrightarrow \mathrm{grounds}_C(e_1,e_2,p)$$

where:

$$\mathrm{grounds}_K(e_1,e_2) := K_{cond}(e_1,e_2) < K(e_2) - K(e_1) + c_{grounding}$$
$$\mathrm{grounds}_C(e_1,e_2,p) := C_{cond}(e_1,e_2,p) < C(e_2,p) - C(e_1,p) + c_{grounding}$$

Axiom 10 (BRIDGE5: Rank Stability)**.**

$$\forall S,\, e.\, e \in S \wedge \mathit{is\_presentation}(e) \rightarrow \exists p_0.\, \forall p \geq p_0.\, \mathrm{rank}_C(e,p) = \mathrm{rank}_K(e)$$

Axiom 11 (BRIDGE6: Temporal Continuity)**.**

$$\forall e,\, \mathit{times},\, \varepsilon > 0.\, \mathit{is\_temporal\_presentation}(e) \rightarrow \\ \exists p_0.\, \forall p \geq p_0,\, t \in \mathit{times}.\, |\,\mathrm{Coh}_{op}([e],[t],p) - \mathrm{Coh}([e],[t])| < \varepsilon$$

Axiom 12 (BRIDGE7: Conditional Convergence)**.**

$$\forall e_1,\, e_2,\, \varepsilon > 0.\, \mathit{is\_presentation}(e_1) \wedge \mathit{is\_presentation}(e_2) \rightarrow \\ \exists p_0.\, \forall p \geq p_0.\, |C_{cond}(e_1,e_2,p) - K_{cond}(e_1,e_2)| < \varepsilon$$

Axiom 13 (BRIDGE7_joint: Joint Convergence)**.**

$$\forall es,\, \varepsilon > 0.\, (\forall e \in es.\, \mathit{is\_presentation}(e)) \rightarrow \\ \exists p_0.\, \forall p \geq p_0.\, |C_{joint}(es,p) - K_{joint}(es)| < \varepsilon$$

Axiom 14 (BRIDGE8: Continuum Limit)**.**

$$\forall e,\, \mathit{times},\, \varepsilon > 0.\, \mathit{is\_temporal\_presentation}(e) \rightarrow \\ \exists p_0,\, \delta.\, \delta > 0 \wedge \forall p \geq p_0,\, t \in \mathit{times}. \\ \left| \frac{\mathrm{Coh}_{op}([e],[t+\delta],p) - \mathrm{Coh}_{op}([e],[t],p)}{\delta} - \frac{d\,\mathrm{Coh}}{dt}(e,t) \right| < \varepsilon$$

16 CA Rules

$$
\begin{aligned}
F &: \text{KLZ.State} \to \text{KLZ.State} \quad \text{(noncomputable)} \\
\text{merge} &: \text{KLZ.State} \to \text{KLZ.State} \to \text{KLZ.State} \quad \text{(noncomputable)} \\
R_{\text{Cohesion}} &: \text{List KLZ.State} \to \text{KLZ.State} \to \text{KLZ.State} \quad \text{(noncomputable axiom)} \\
R_{\text{Cohesion}}(n, h) &:= \text{merge}(F(\text{join}(n)), h) \\
R_{\text{Reduction}} &: \text{List KLZ.State} \to \text{KLZ.State} \\
R_{\text{Reduction}}(n) &:= \text{mode}(\text{join}(n)) \\
R_{G1} &: \text{List KLZ.State} \to \text{KLZ.State} \to \text{KLZ.State} \\
R_{G1}(n, h) &:= \begin{cases} R_{\text{Cohesion}}(n, h) & \text{if } K_{LZ}(\text{join}(n)) \leq c_{\text{grounding}} \\ R_{\text{Reduction}}(n) & \text{otherwise} \end{cases}
\end{aligned}
$$

17 Physics Axioms

Axiom 15 (H_BH: Holographic Bound)**.**

$$\forall region.\ is\_presentation(region) \to K(region) \leq \frac{Area}{4\ell^2_{Planck}}$$

Axiom 16 (Ψ_I: Coherence Invariant)**.**

$$\forall e.\ is\_temporal\_presentation(e) \ \wedge \ coherent(e) \quad \to \quad \text{Coh}_{trajectory}(e) \ \cdot \ P_{total}(e) \quad = \quad 1$$

Axiom 17 (U_Ω: Uncertainty Principle)**.**

$$\forall e,\ \Delta K,\ \Delta t.\ is\_temporal\_presentation(e) \ \wedge \ \Delta K \ > \ 0 \ \wedge \ \Delta t \ > \ 0 \ \to \ \Delta K \ \cdot \ \Delta t \ \geq \ \hbar_{eff}$$

18 Rank System

$$
\begin{gathered}
\text{rank}_K(\Omega) = 0 \\
\text{grounds}(e_1, e_2) \to \text{rank}_K(e_2) < \text{rank}_K(e_1) \\
\forall e. \exists n.\ \text{rank}_K(e) = n \\
\text{rank}_C(e, p) = \text{bfs\_depth\_C}(e, p, S) \quad \text{for } e \in S
\end{gathered}
$$

19 Universal Grounding

Theorem 5.

$$
\begin{aligned}
&\forall e.\ is\_presentation(e) \to \exists path.\ path.head = \Omega \wedge path.last = e \wedge \\
&\qquad\qquad \forall i.\ i + 1 < path.length \to \text{grounds}(path[i], path[i+1])
\end{aligned}
$$

Theorem 6 (grounding_transitive)**.**

$$\forall e_1,\ e_2,\ e_3.\ \text{grounds}(e_1, e_2) \wedge \text{grounds}(e_2, e_3) \to \text{grounds}(e_1, e_3)$$

Theorem 7 (grounding_acyclic)**.**

$$\forall e. \neg\,\text{grounds}(e, e)$$

20 Complexity Bounds

Theorem 8 (K_joint_nonneg).

$$\forall es.\, 0 \leq K_{joint}(es)$$

Theorem 9 (K_joint_nil).

$$K_{joint}([]) = 0$$

Theorem 10 (K_joint_singleton).

$$\forall e.\, K_{joint}([e]) = K(e)$$

Axiom 18 (compression_axiom).

$$\forall es.\, (\forall e \in es.\, is\_presentation(e)) \land es.length \geq 2 \to K_{joint}(es) < K_{sum}(es)$$

Theorem 11 (joint_le_sum).

$$\forall es.\, (\forall e \in es.\, is\_presentation(e)) \to K_{joint}(es) \leq K_{sum}(es)$$

Theorem 12 (complexity_positive).

$$\forall e.\, is\_presentation(e) \to 0 < K(e)$$

Theorem 13 (substrate_minimal).

$$\forall e.\, is\_presentation(e) \to K(Substrate) \leq K(e)$$

21 Operational Bounds

Theorem 14 (C_nonneg).

$$\forall e,\, p.\, 0 \leq C(e, p)$$

Theorem 15 (C_monotone).

$$\forall e,\, p_1,\, p_2.\, p_1 \leq p_2 \to C(e, p_2) \leq C(e, p_1)$$

Theorem 16 (C_joint_nonneg).

$$\forall es,\, p.\, 0 \leq C_{joint}(es, p)$$

Theorem 17 (K_LZ_nonneg).

$$\forall s.\, 0 \leq K_{LZ}(s)$$

Theorem 18 (K_LZ_empty).

$$K_{LZ}(\text{join}([])) = 0$$

Theorem 19 (K_LZ_monotone).

$$\forall s_1,\, s_2.\, s_1.length \leq s_2.length \to K_{LZ}(s_1) \leq K_{LZ}(s_2)$$

22 Toy Compressor Bounds

Theorem 20 (K_toy_lower_bound).

$$\forall s.\, K_{LZ}(s) \leq K_{toy}(s)$$

Theorem 21 (K_toy_upper_bound).

$$\forall s.\, K_{toy}(s) \leq K_{LZ}(s) + \log_2(s.length)$$

23 KLZ Axioms

Axiom 19 (K_LZ_subadditive_cons).

$$\forall x,\, xs.\, K_{LZ}(\text{join}(x :: xs)) \leq K_{LZ}(\text{join}(xs)) + K_{LZ}(x) + c_{sub}$$

Axiom 20 (K_LZ_prefix).

$$\forall b,\, s.\, K_{LZ}(\text{join}([b])) \leq K_{LZ}(\text{join}(b :: s))$$

Axiom 21 (K_LZ_singleton_bound).

$$\forall b.\, K_{LZ}(\text{join}([b])) \leq c_{single}$$

Axiom 22 (K_LZ_mode_le).

$$\forall s.\, K_{LZ}(\text{mode}(s)) \leq K_{LZ}(s) + C_{mode}$$

Axiom 23 (K_LZ_mode_absolute_bound).

$$\forall s.\, K_{LZ}(\text{mode}(s)) \leq C_{mode}$$

Axiom 24 (C_mode_lt_c_grounding).

$$C_{mode} < c_{grounding}$$

Axiom 25 (K_LZ_cohesion_bound_raw).

$$\forall n,\, h.\, K_{LZ}(R_{Cohesion}(n, h)) \leq C_{coh}$$

24 Time Arrow Theorems

Theorem 22 (time_arrow_reduction).

$$\forall hist,\, n.\, K_{LZ}(\text{join}(\text{mode}(\text{join}(n)) :: hist)) \leq K_{LZ}(\text{join}(hist)) + c_{time\_reduction}$$

Theorem 23 (time_arrow_cohesion).

$$\forall hist,\, n,\, h.\, K_{LZ}(\text{join}(R_{Cohesion}(n, h) :: hist)) \leq K_{LZ}(\text{join}(hist)) + c_{time\_cohesion}$$

25 Coherence Bounds

Theorem 24 (coherence_bounds)**.**

$$\forall es,\ times.\ (\forall e \in es.\ \mathit{is\_presentation}(e)) \to 0 \le \mathrm{Coh}(es, times) \wedge \mathrm{Coh}(es, times) \le 1$$

Theorem 25 (compression_ratio_ge_one)**.**

$$\forall es,\ times.\ (\forall e \in es.\ \mathit{is\_presentation}(e)) \to 1 \le \mathit{compression\_ratio}(es, times)$$

26 Error Bounds

Theorem 26 (error_bound_linear)**.**

$$\exists c > 0.\ \forall e,\ p.\ \mathit{is\_presentation}(e) \to 0 < p \to |C(e,p) - K(e)| \le \frac{c}{p}$$

Theorem 27 (error_bound_polynomial)**.**

$$\exists c,\ \alpha.\ 0 < c \wedge 1 < \alpha \wedge \forall e,\ p.\ \mathit{is\_presentation}(e) \to 0 < p \to |C(e,p) - K(e)| \le \frac{c}{p^{\alpha}}$$

Theorem 28 (error_bound_general)**.**

$$\exists M > 0.\ \forall e,\ p.\ \mathit{is\_presentation}(e) \to |C(e,p) - K(e)| \le \frac{M}{p+1}$$

27 Physics Correspondences

$$\begin{aligned}
\mathrm{energy_of}(e) &= \kappa_{\mathrm{energy}} \cdot K(e) \\
\mathrm{mass}(e) &= \frac{\mathrm{energy_of}(e)}{c^2} \\
\mathrm{entropy}(e) &= k_B \cdot \log(2) \cdot K(e) \\
\mathrm{is_quantum}(\mathrm{nbhd}) &:= K_{LZ}(\mathrm{join}(\mathrm{nbhd})) \le c_{\mathrm{grounding}} \\
\mathrm{is_classical}(\mathrm{nbhd}) &:= K_{LZ}(\mathrm{join}(\mathrm{nbhd})) > c_{\mathrm{grounding}}
\end{aligned}$$

28 Predicates

$$\begin{aligned}
\text{is_substrate} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_emergent} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_temporal_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_static_presentation} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_quantum_state} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_measurement_device} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{is_observable} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{phenomenal} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{has_mass} &: \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{grounds} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{temporal_grounds} &: \text{Entity} \to \text{Time} \to \text{Entity} \to \text{Time} \to \text{Prop} \quad \text{(axiom)} \\
\text{interacts} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{inseparable} &: \text{Entity} \to \text{Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{emerges_from} &: \text{Entity} \to \text{List Entity} \to \text{Prop} \quad \text{(axiom)} \\
\text{phase_coupled} &: \text{Entity} \to \text{Entity} \to \text{Phase} \to \text{Prop} \quad \text{(axiom)} \\
\text{coherent} &: \text{Entity} \to \text{Prop} \\
\text{decoherent} &: \text{Entity} \to \text{Prop} \\
\text{stable} &: \text{Entity} \to \text{Prop} \\
\text{is_quantum} &: \text{List State} \to \text{Prop} \\
\text{is_classical} &: \text{List State} \to \text{Prop} \\
\text{coherent_state} &: \text{KLZ.State} \to \text{Prop} \\
\text{is_grounded} &: \text{Entity} \to \text{Entity} \to \text{Prop}
\end{aligned}$$

29 Entity Classification

Theorem 29 (entity_classification)**.**

$$\forall e.\, (\mathit{is\_substrate}(e) \wedge e = \mathit{Substrate}) \vee \mathit{is\_presentation}(e) \vee \mathit{is\_emergent}(e)$$

Theorem 30 (substrate_not_presentation)**.**

$$\forall e.\, \neg(\mathit{is\_substrate}(e) \wedge \mathit{is\_presentation}(e))$$

Theorem 31 (substrate_not_emergent)**.**

$$\forall e.\, \neg(\mathit{is\_substrate}(e) \wedge \mathit{is\_emergent}(e))$$

Theorem 32 (presentation_not_emergent)**.**

$$\forall e.\, \neg(\mathit{is\_presentation}(e) \wedge \mathit{is\_emergent}(e))$$

Theorem 33 (presentation_temporal_or_static).

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow (\mathit{is\_temporal\_presentation}(e) \vee \mathit{is\_static\_presentation}(e)) \wedge \neg(\mathit{is\_temporal\_presentation}(e) \wedge \mathit{is\_static\_presentation}(e))$$

30 Substrate Properties

Theorem 34 (substrate_unique).

$$\forall x,\, y.\, \mathit{is\_substrate}(x) \wedge \mathit{is\_substrate}(y) \rightarrow x = y$$

Theorem 35 (substrate_is_Substrate).

$$\mathit{is\_substrate}(\mathit{Substrate})$$

Theorem 36 (Omega_is_substrate).

$$\mathit{is\_substrate}(\Omega)$$

Theorem 37 (Omega_eq_Substrate).

$$\Omega = \mathit{Substrate}$$

31 Temporal Preservation

Theorem 38 (indexed_preserves_presentation).

$$\forall e,\, t.\, \mathit{is\_presentation}(e) \rightarrow \mathit{is\_presentation}(\mathrm{indexed}(e, t))$$

Theorem 39 (temporal_slice_preserves_presentation).

$$\forall es,\, t.\, (\forall e \in es.\, \mathit{is\_presentation}(e)) \rightarrow (\forall e \in \mathit{temporal\_slice}(es, t).\, \mathit{is\_presentation}(e))$$

32 Associativity

Theorem 40 (join_associative).

$$\forall s_1,\, s_2,\, s_3.\, \mathrm{join}([\mathrm{join}([s_1, s_2]), s_3]) = \mathrm{join}([s_1, \mathrm{join}([s_2, s_3])])$$

33 Verified Theorems

Theorem 41 (energy_from_complexity).

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow \mathit{has\_mass}(e) \rightarrow \exists m > 0.\, m = \frac{\mathit{energy\_of}(e)}{c^2}$$

Theorem 42 (entropy_from_complexity).

$$\forall e.\, \mathit{is\_presentation}(e) \rightarrow \exists S.\, S = k_B \cdot \log(2) \cdot K(e)$$

Theorem 43 (coherence_participation_invariant).

$$\forall e.\, \textit{is\_temporal\_presentation}(e) \to \textit{coherent}(e) \to 0 < P_{total}(e) \wedge \mathrm{Coh}_{trajectory}(e) = \frac{1}{P_{total}(e)}$$

Theorem 44 (joint_le_sum).

$$\forall es.\, (\forall e \in es.\, \textit{is\_presentation}(e)) \to K_{joint}(es) \leq K_{sum}(es)$$

Theorem 45 (complexity_subadditive).

$$\forall e_1,\, e_2.\, \textit{is\_presentation}(e_1) \to \textit{is\_presentation}(e_2) \to K_{joint}([e_1, e_2]) \leq K(e_1) + K(e_2)$$

Theorem 46 (compression_ratio_ge_one).

$$\forall es,\, times.\, (\forall e \in es.\, \textit{is\_presentation}(e)) \to 1 \leq \textit{compression\_ratio}(es, times)$$

Theorem 47 (R_G1_preserves_grounding).

$$\forall n.\, K_{LZ}(\mathrm{mode}(\mathrm{join}(n))) < K_{LZ}(\mathrm{join}(n)) + c_{grounding}$$

Theorem 48 (time_arrow_reduction).

$$\forall hist,\, n.\, K_{LZ}(\mathrm{join}(\mathrm{mode}(\mathrm{join}(n)) :: hist)) \leq K_{LZ}(\mathrm{join}(hist)) + c_{time\_reduction}$$

Theorem 49 (time_arrow_cohesion).

$$\forall hist,\, n,\, h.\, K_{LZ}(\mathrm{join}(R_{Cohesion}(n, h) :: hist)) \leq K_{LZ}(\mathrm{join}(hist)) + c_{time\_cohesion}$$

Theorem 50 (P3_C6_preservation).

$$\forall n,\, h.\, \textit{coherent\_state}(\mathrm{join}(n)) \to K_{LZ}(R_{G1}(n, h)) = K_{LZ}(h)$$

Theorem 51 (R_G1_grounding_reduction).

$$\forall n,\, h.\, K_{LZ}(\mathrm{join}(n)) > c_{grounding} \to K_{LZ}(R_{G1}(n, h)) < K_{LZ}(\mathrm{join}(n)) + c_{grounding}$$

Theorem 52 (R_G1_preserves_time_arrow).

$$\forall hist,\, n,\, h.\, K_{LZ}(\mathrm{join}(R_{G1}(n, h) :: hist)) \leq K_{LZ}(\mathrm{join}(hist)) + \max(c_{time\_reduction}, c_{time\_cohesion})$$

Theorem 53 (planck_units_positive).

$$0 < \ell_{Planck} \wedge 0 < t_{Planck} \wedge 0 < M_{Planck} \wedge 0 < E_{Planck} \wedge 0 < T_{Planck}$$

Theorem 54 (error_bound_exists).

$$\forall e,\, p.\, \textit{is\_presentation}(e) \to \exists \varepsilon.\, |C(e, p) - K(e)| \leq \varepsilon.\textit{absolute}$$