Proof: (append (append xs ys) zs) == (append xs (append ys zs)

Case #1: null case

Given xs = '() , ys= '()

(append (append xs ys) zs)

= {By assumption xs= '()}

(append (append '() ys) zs)

= {append-nil law}

(append ys zs)

= {append-nil law}

(append '() (append ys zs))

= {substitution, xs = '()

(append xs (append ys zs)

Case #2: Inductive case for non-empty lists

Inductive hypothesis: Assume (append (append xs ys) zs)

== (append xs (append ys zs) holds true for all xs, ys, zs.

(append (append xs ys) zs))

= {by assumption, xs = (cons a as)}

(append (append (cons a as) ys) zs)

= {append cons rule}

z zs ys

(append (cons a (append as ys) zs))

= {append cons rule}

((cons a (append (append as ys) zs))

= {inductive hypothesis}

zs ys

(cons a (append as (append ys zs)))

= {append-cons rule}

(append (cons a as) (append ys zs)))

$= \{$by assumption, cons a as = xs
(append xs (append ys zs))

Therefore, by induction, (append (append xs ys) zs) == (append xs (append ys zs)) for all xs, ys, zs.