# Part A

## 1)

a) either $x \in \text{dom}\, \rho$ or $x \in \text{dom}\, \xi$

X is defined as either a formal parameter or a global variable

b) $\langle e, \xi, \Phi, \rho \rangle \Downarrow \langle v, \xi, \Phi, \rho \rangle$

expression e evaluates to value v without defining or modifying any variables or parameters

## 2)

a) If global variable x is not defined, it is possible that expression e evaluates

If, for environments $\xi, \Phi, \rho, \xi', \rho'$. $x \notin \text{dom}\, \xi$, $\langle e, \xi, \Phi, \rho \rangle$ could evaluate to $\langle V, \xi', \Phi, \rho' \rangle$

b) If e evaluates successfully, no global variables' value are changed

If $\langle e, \xi, \Phi, \rho \rangle \Downarrow \langle v, \xi', \Phi, \rho' \rangle$, $\xi = \xi'$

c) If e evaluates successfully, no new global variables are defined

If $\langle e, \xi, \Phi, \rho \rangle \Downarrow \langle v, \xi', \Phi, \rho' \rangle$ $\text{dom}\, \xi = \text{dom}\, \xi'$

## 3)

a) These rules are effectively the same, because WhileEnd is saying "if $e_1$ evaluates to $v_1$, and $v_1 = 0$" then the loop terminates, and WhileEnd' states "if $e_1$ evaluates to 0" then the loop terminates. Because of this, the premises and conclusion of both rules effectively are identical, therefore the rules are identical.

b) These rules are not the same, because FormalAssign' is ambiguous as to which environment $\{x \mapsto v\}$ ends up in. In the following code example, two variables, one global and one a formal parameter, both named x both exist. Because FormalAssign' does not specify which environment $x \mapsto 4$ gets allocated to, it would be unable to complete this task.

```
(Val x 6)
(define f (x)
    (set x 5)) ;; FormalAssign results in ρ'{x ↦ 5}
               ;; FormalAssign' does not define which x to update
```

Part B

4)

a)
$$\frac{x \notin \operatorname{dom} \rho \qquad x \notin \operatorname{dom} \xi}{\langle \text{VAR}(x), \xi, \Phi, \rho \rangle \Downarrow \langle 0, \xi'\{x \mapsto 0\}, \Phi, \rho \rangle}$$ GlobalVarUnbound (Added for AwK)

$$\frac{x \notin \operatorname{dom} \rho \qquad \langle e_1, \xi, \Phi, \rho \rangle \Downarrow \langle v_1, \xi', \Phi, \rho' \rangle}{\langle \text{SET}(x, e_1), \xi, \Phi, \rho \rangle \Downarrow \langle v_1, \xi'\{x \mapsto v_1\}, \Phi, \rho' \rangle}$$ GlobalAssign' (Modified for AwK)

b)
$$\frac{x \notin \operatorname{dom} \rho \qquad x \notin \operatorname{dom} \xi}{\langle \text{VAR}(x), \xi, \Phi, \rho \rangle \Downarrow \langle 0, \xi', \Phi, \rho\{x \mapsto 0\} \rangle}$$ LocalVarUnbound (Added for Icon)

$$\frac{x \notin \operatorname{dom} \rho \qquad \langle e_1, \xi, \Phi, \rho \rangle \Downarrow \langle v_1, \xi', \Phi, \rho' \rangle}{\langle \text{SET}(x, e_1), \xi, \Phi, \rho \rangle \Downarrow \langle v_1, \xi', \Phi, \rho'\{x \mapsto v_1\} \rangle}$$ GlobalAssign'' (Modified for Icon)

c) I prefer the global changes, because the scope rules for icon's handling of unbound variables are confusing

d) see code

## Part c)

5)

$$\dfrac{\dfrac{x \in dom\ \rho \quad \dfrac{}{\langle LIT(3), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\rangle}\text{(literal)}}{\langle SET(x, LIT(3)), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\}\rangle}\text{(FormalAssign)} \quad \dfrac{x \in dom\ \rho\{x \mapsto 3\}}{\langle Var(x), \xi, \phi, \rho\{x\mapsto 3\}\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\{x \mapsto 3\}\rangle}\text{(FormalVar)}}{\langle BEGIN(SET\ x\ Lit(3)), VAR(X), \xi, \phi, \rho\rangle \Downarrow \langle 3, \xi, \phi, \rho\{x \mapsto 3\}\rangle}\text{(Begin)}$$

6) We do not know whether x is formal or global, so we show that (if x x 0) evaluates to Var(x) for each of the four possible cases True/x is formal, True/x is Global, false/x is formal, false/x is global.

**Case #1, Formal True**

$$\dfrac{\dfrac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\text{(FormalVar)} \quad Var(x) \neq 0 \quad \dfrac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\text{(FormalVar)}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi', \phi, \rho'\rangle}\text{(ifTrue)}$$

**Case #2 Global True**

$$\dfrac{\dfrac{x \notin dom\ \rho \quad x \in dom\ \xi}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \xi(x), \xi, \phi, \rho\rangle}\text{(GlobalVar)} \quad \dfrac{x \notin dom\ \rho \quad x \in dom\ \xi}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \xi(x), \xi, \phi, \rho\rangle}\text{(GlobalVar)}}{\langle IF(VAR(x), VAR(x), LIT(0)), \xi, \phi, \rho\rangle \Downarrow \langle \xi(x), \xi', \phi, \rho'\rangle}\text{(ifTrue)}$$

In both true cases above, if(x x 0) evaluates to X

**Case #3, Formal False**

$$\dfrac{\dfrac{x \in dom\ \rho}{\langle VAR(x), \xi, \phi, \rho\rangle \Downarrow \langle \rho(x), \xi, \phi, \rho\rangle}\text{(FormalVar)} \quad Var(x) = 0 \quad \dfrac{}{\langle Lit(0), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi, \phi, \rho\rangle}\text{(literal)}}{\langle IF(VAR(x)\ VAR(x), LIT(0)), \xi, \phi, \rho\rangle \Downarrow \langle 0, \xi', \phi, \rho'\rangle}\text{(if false)}$$

$\boxed{\text{Case \#4, Global False}}$

$$\dfrac{\dfrac{x \notin \text{dom } \rho \quad x \in \text{dom } \xi}{\langle \text{VAR}(x), \xi, \Phi, \rho \rangle \Downarrow \langle \xi(x), \xi, \Phi, \rho \rangle}\text{(GlobalVar)} \quad \text{Var}(X) = 0 \quad \dfrac{}{\langle \text{Lit}(0), \xi, \Phi, \rho \rangle \Downarrow \langle 0, \xi, \Phi, \rho \rangle}\text{(literal)}}{\langle \text{IF}(\text{VAR}(x), \text{VAR}(x), \text{LIT}(0)), \xi, \Phi, \rho \rangle \Downarrow \langle 0, \xi', \Phi, \rho' \rangle}\text{(if False)}$$

In both false cases above, if (x x 0) evaluates to 0 which is equivalent to the value of x when (if x $e_1$ $e_2$) returns the false $e_2$ case.

Therefore (if x x 0) always evaluates to VAR(x)