Proof: (o ((curry map) f) ((curry map) g)) = ((curry map) (o f g))

Case #1: null case

given x = '(),

((o ((curry map) f) ((curry map) g)) x)

= {by substitution}

((o ((curry map) f) ((curry map) g)) '())

= {apply-compose}

(((curry map) f) (((curry map) g) '()))

= {apply-curry}

(((curry map) f) (map g '()))

= {apply-curry}

(map f (map g '()))

= {map-nil}

'()

= {map-nil}

(map (o f g) '())

= {substitution

(map (o f g) x)

= {apply-curry}

(((curry map) (o f g)) x)

Case #2: Inductive case

Inductive hypothesis:

((o ((curry map) f) ((curry map) g)) x) = (((curry map) (o f g)) x)

holds true for all x of the form (cons a as)

Therefore, by substitution:

((o ((curry map) f) ((curry map) g)) (cons a as))

= (((curry map) (o f g)) (cons a as)) holds true for all a, as.

((o ((curry map) f) ((curry map) g)) x)

= {By assumption}

((o ((curry map) f) ((curry map) g)) (cons a as))

= {apply-compose}

(((curry map) f) (((curry map) g) (cons a as)))

= {apply-curry}

(((curry map) f) (map g (cons a as)))

= {apply-curry}

(map f (map g (cons a as)))

= {map-cons}

(map f (cons (g a) (map g as)))

= {map-cons}

(cons (f (g a)) (map f (map g as)))

= {apply-compose}

(cons ((o f g) a) (map f (map g as)))

= {apply-curry}

(cons ((o f g) a) (map f (((curry map) g) as)))

= {apply-curry}

((cons ((o f g) a) (((curry map) f) (((curry map) g) as))

= {inductive hypothesis, because as is of the form (cons b bs)}

((cons ((o f g) a) (((curry map) (o f g)) as)

= {apply-curry}

((cons ((o f g) a) (map (o f g) as)

= {map-cons}

(map (o f g) (cons a as))

= {apply-curry}

(((curry map) (o f g)) (cons a as))

Therefore, our inductive hypothesis holds for all x.