Stochastic Processes Assignment 4

Matthew Yu Student ID: 0553501

2017/10/16

1. Problem 3.49

A and B play a series of games with A winning each game with probability p. The overall winner is the first player to have won two more games than the other.

(a) Find the probability that A is the overall winner.

(b) Find the expected number of games played.

(a)

Let A denote the event A wins afterall.
Let T denote the number of games A wins within the first 2 games.
Then, conditioning A on T we get

$$\begin{aligned} P(A) &= P(A|T=0)P(T=0) + P(A|T=1)P(T=1) + P(A|T=2)P(T=2) \\ &= 0 + P(A) \times 2p(1-p) + p^2 \end{aligned}$$

After arranging,$1 - 2p(1-p) \times P(A) = p^2$

$$P(A) = \frac{p^2}{1 - 2p(1-p)}$$

(b)

Let X denote total games played.
Applying notions from **(a)**,

$$\begin{aligned} E(X) &= E(X|T=0)P(T=0) + E(X|T=1)P(T=1) + E(X|T=2)P(T=2) \\ &= 2(1-p)^2 + [2 + E(X)] \times 2p(1-p) + 2p^2 \\ &= 2 + E(X) \times 2p(1-p) \end{aligned}$$

Then,

$$E(X) = \frac{2}{1 - 2p(1-p)}$$

2. Problem 3.54

A coin is randomly selected from a group of ten coins, that nth coin having a probability of $n/10$ of coming up heads. The coin is then repeatedly flipped until a head appears. Let N denote the number of flip necessary. What is the probability distribution of N? Is N a geometric random variable? When would N be a geometric random variable; that is, what would have to be done differently?

Since $P(N=j|n=x) = \frac{x}{10} \times (1-\frac{x}{10})^{j-1}$

$$P(N=j) = \frac{1}{10} \times \frac{n}{10} \times \sum_{n=1}^{10}(1-\frac{n}{10})^{j-1}$$

Therefore N is not a geometric variable for the pmf doesn't necessarily match. However if the coin is sampled with replacement after each flip, N will follow geometric distribution.

3. Problem 3.62

A, B and C are evenly matched tennis players. Initially A and B play a set, and the winner then plays C. This continues, with the winner always playing the waiting player, until one of the players has won two sets in a row. That player is the declared the overall winner. Find the probability that A is the overall winner.

W_k: The event of player k winning a game.
L_k: The event of player k losing a game.
$P(k)$: Player k wins overall.
Then, for player A

$$\begin{aligned} P(A) &= \frac{1}{2}P(A|W_A) + \frac{1}{2}P(A|L_A) \\ &= \frac{1}{2} \times (\frac{1}{2} + \frac{1}{2}P(A|W_A L_A)) + \frac{1}{2} \times \frac{1}{2}P(C) \\ &= \frac{1}{4} + \frac{1}{4} \times \frac{1}{2}P(C) + \frac{1}{4}P(C) \\ &= \frac{1}{4} + \frac{3}{8} \times P(C) \end{aligned}$$

Since the case of B is similar to A, $P(B) = \frac{1}{4} + \frac{3}{8} \times P(C)$
While $P(A) + P(B) + P(C) = 1$,

$$\frac{1}{4} + \frac{3}{8} \times P(C) + \frac{1}{4} + \frac{3}{8} \times P(C) + P(C) = 1$$

$P(C) = \frac{2}{7}$, $P(A) = P(B) = \frac{5}{14}$