1 Comparison with CGM

In this section, we compare the policy functions that we obtain using HARK, with those that are obtained using CGM's publicly available `Fortran 90 code`.

We attempted to execute the authors' code as-is, but obtained a "division by zero" error, apparently caused by one of the asset grids starting at 0, allowing the possibility of a 0-level of consumption to be evaluated in the utility function. We fixed the issue by making said grid start at 1. The figures produced in this section use the policy functions outputted by the author's code after this fix.

Figures 1 and 2 display policy functions for consumption and risky portfolio shares obtained with CGM's code and our HARK implementation, and their substantial differences.

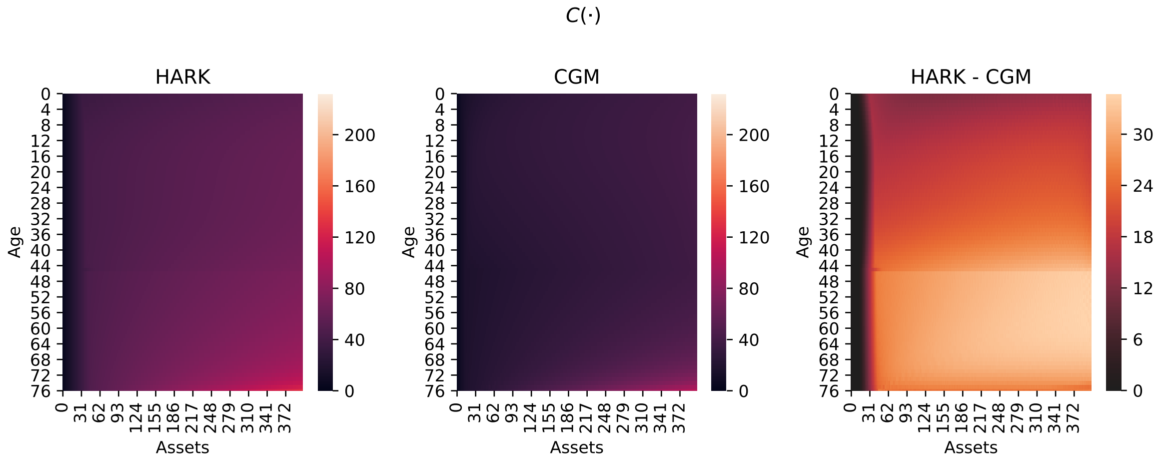

Figure 1 Consumption policy functions in HARK and CGM's Fortran code.

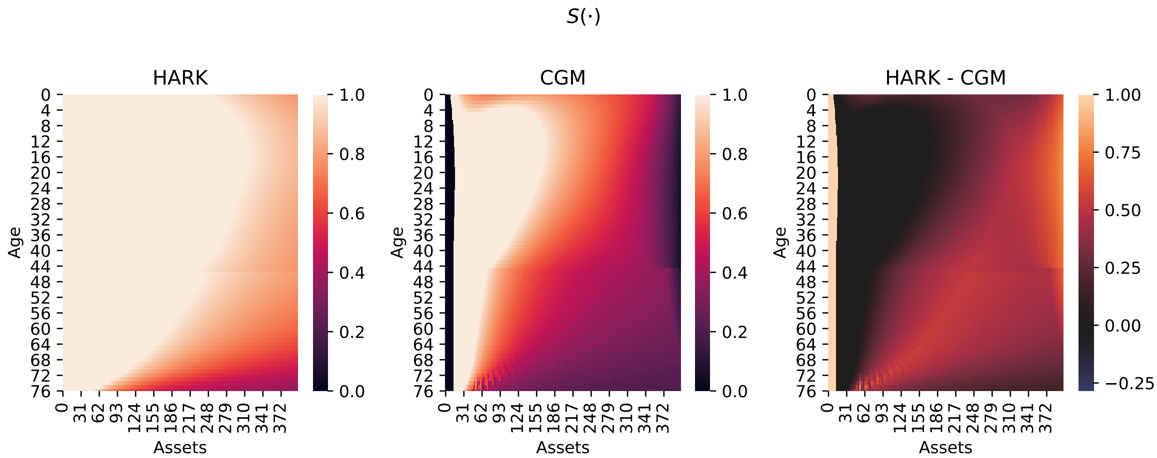

Figure 2 Risky share policy functions in HARK and CGM's Fortran code.

Given the differences, we inspect the second and third-to-last life periods more closely. Figure 3 presents policy functions and their differences for these periods. The discrepancies are evident.

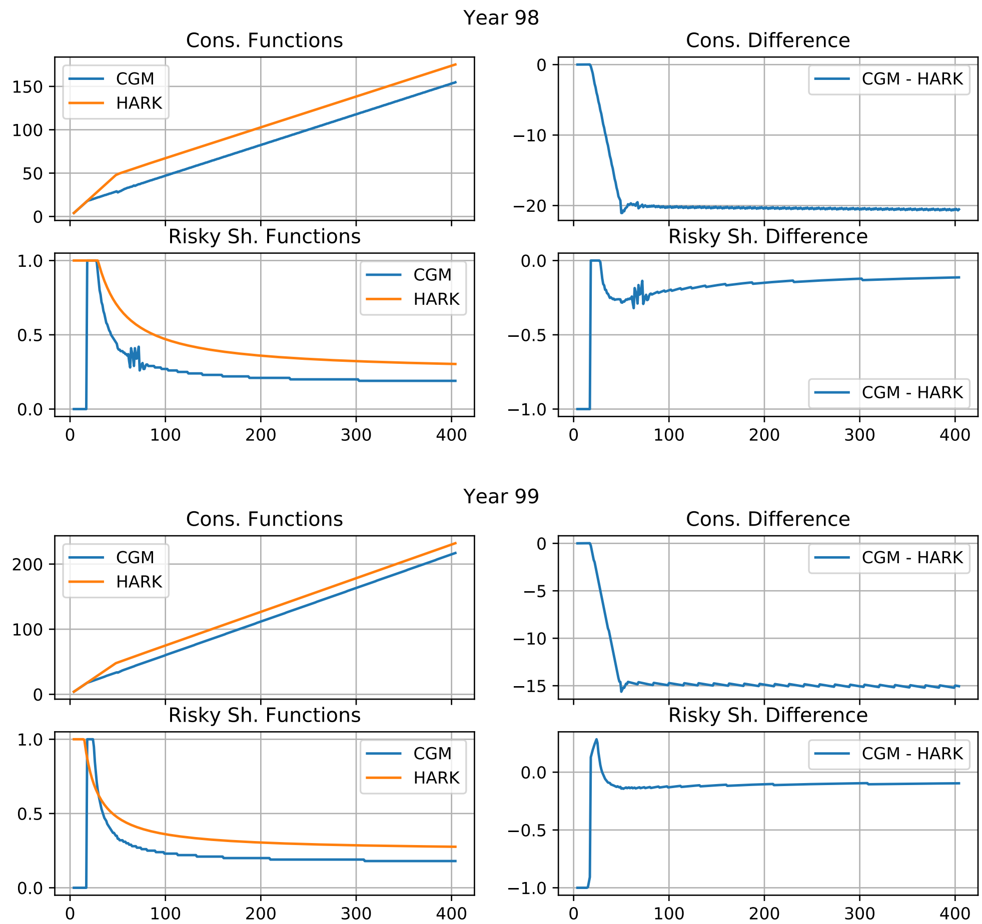

Figure 3 Policy functions in the second and third to last periods of life.

2 Sensitivity analyses

Given that our main set of results do not align with those of CGM, we provide a few tests that compare the behavior of the tools that we are using with well known theoretical results.

2.1 Merton Samuelson's limiting risky share

Merton and Samuelson (TODO: CITE) show that a consumer with constant relative risk aversion ρ, a risky asset with log-normally distributed returns with a risk premium ϕ

and log-standard-deviation σ_r, and no labor income uncertainty will invest the following share of their income:[1]

$$s = \frac{\phi}{\rho\sigma_r^2}. \tag{1}$$

This result holds as long as labor income is an *unimportant* source of wealth for the agent.

To test HARK's implementation, we modify returns to the risky asset to be log-normal and keep the rest of the calibration the same. Since labor income is unimportant for infinitely wealthy agents, our risky portfolio share policy rules must converge to Merton and Samuelson's limit as market resources approach infinity. Figure 4 shows that this is indeed the case.

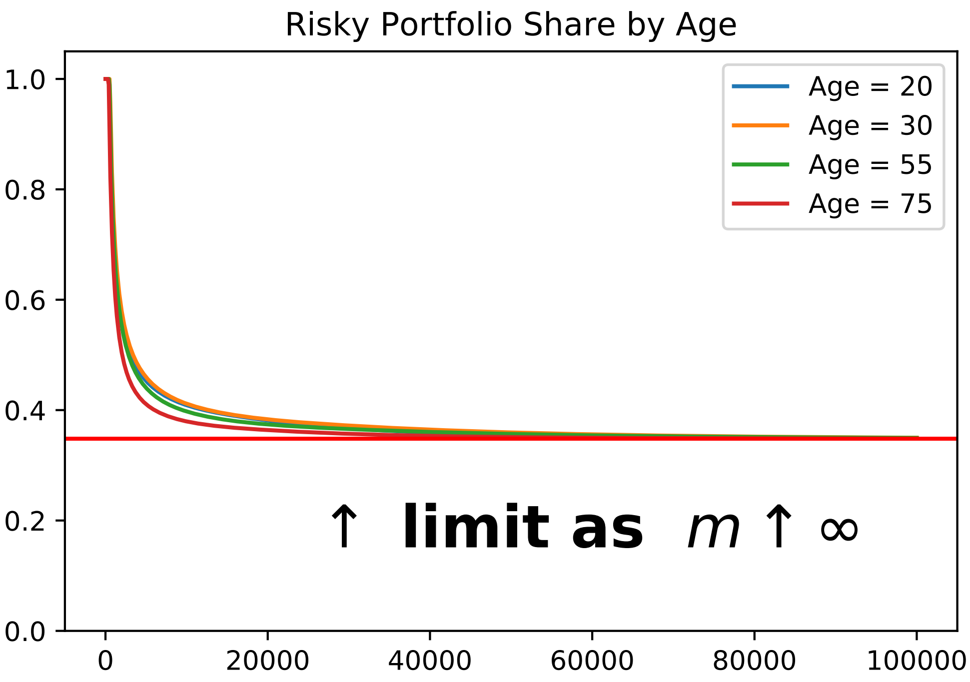

Figure 4 Merton Samuelson as the limit of the risky asset's portfolio share.

2.2 Marginal Propensity to Consume

CRRA-RateRisk shows that an infinitely lived agent with no labor income and who can only store his wealth in a risky asset with log-normally distributed returns must have a

[1]See `Portfolio-CRRA`.

marginal propensity to consume given by:

$$\kappa = 1 - \left(\beta \mathbb{E}_t \left[\mathfrak{R}^{1-\rho}\right]\right)^{1/\rho} \tag{2}$$

where $\mathfrak{R}$ is the risky asset's return factor.

We modify our calibration to have log-normal returns and to ensure that Equation 2 is a positive number. We also set the risk free rate low enough that the agent will allocate all of his wealth to the risky asset. Since the result holds for infinitely lived consumers with no labor income, we should see that our estimated marginal propensity to consume converges to κ for young and wealthy consumers. Figure 5 shows that this is indeed the case.

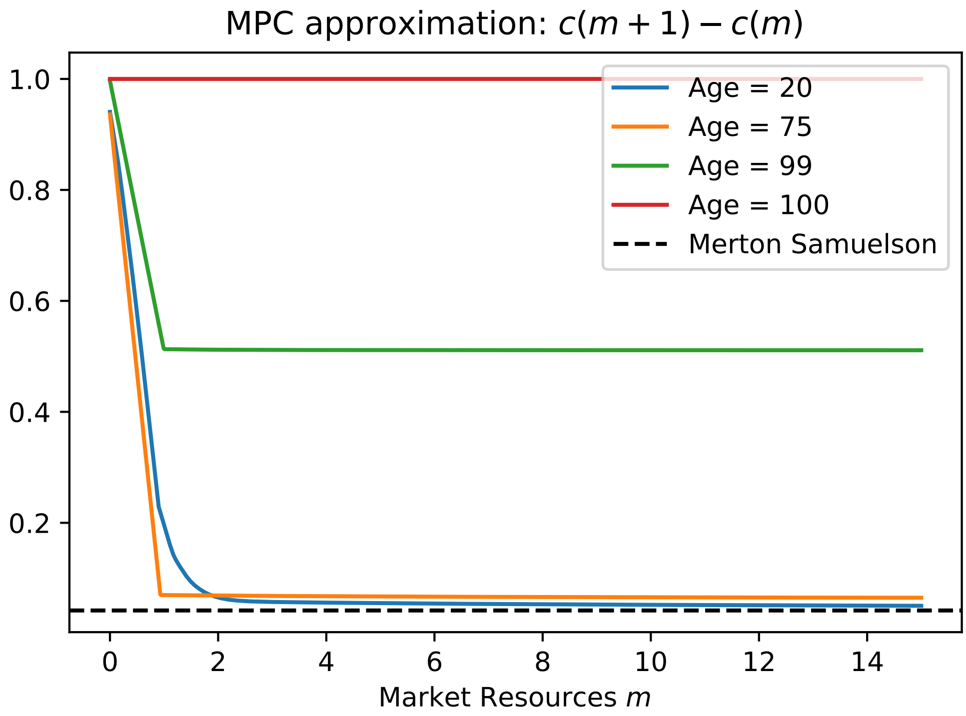

Figure 5 Marginal propensity to consume as $m \to \infty$.

2.3 Perfect Foresight analytical solution

To further verify that HARK's solution algorithm is producing accurate results, we compare the consumption policy function obtained by the `PortfolioConsumerType` class through backward induction with the analytical expression that can be obtained for consumption in a perfect foresight setting.

PrefForesightCRRA shows that a consumer facing no uncertainty will attempt to consume:

$$c_t^* = \frac{1 - \left[R^{-1}\left(R\beta\right)^{1/\rho}\right]}{1 - \left[R^{-1}\left(R\beta\right)^{1/\rho}\right]^{T-t+1}} \times o_t \tag{3}$$

where o_t is his overall (human plus non-human) wealth. In the presence of a liquidity constraint, the agent will then consume:

$$c_t = \min\{m_t, c_t^*\} \tag{4}$$

where m_t are his market resources at time t. We refer to this as the *true* solution.

We shut down all sources of uncertainty in our calibration and, for ease of analytical expressions, assume a constant level of income. We then solve the agent's problem using `PortfolioConsumerType`'s backward induction algorithm and compare its solution both with the true solution and the one obtained by `PerfForesightConsumerType`, another HARK class representing perfect-foresight consumers.

Figure 6 plots consumption functions obtained through the three different methods and three different periods of the agent's life. The `PortfolioConsumerType` policy function aligns with the true solution, but that of `PerfForesightConsumerType` does not.

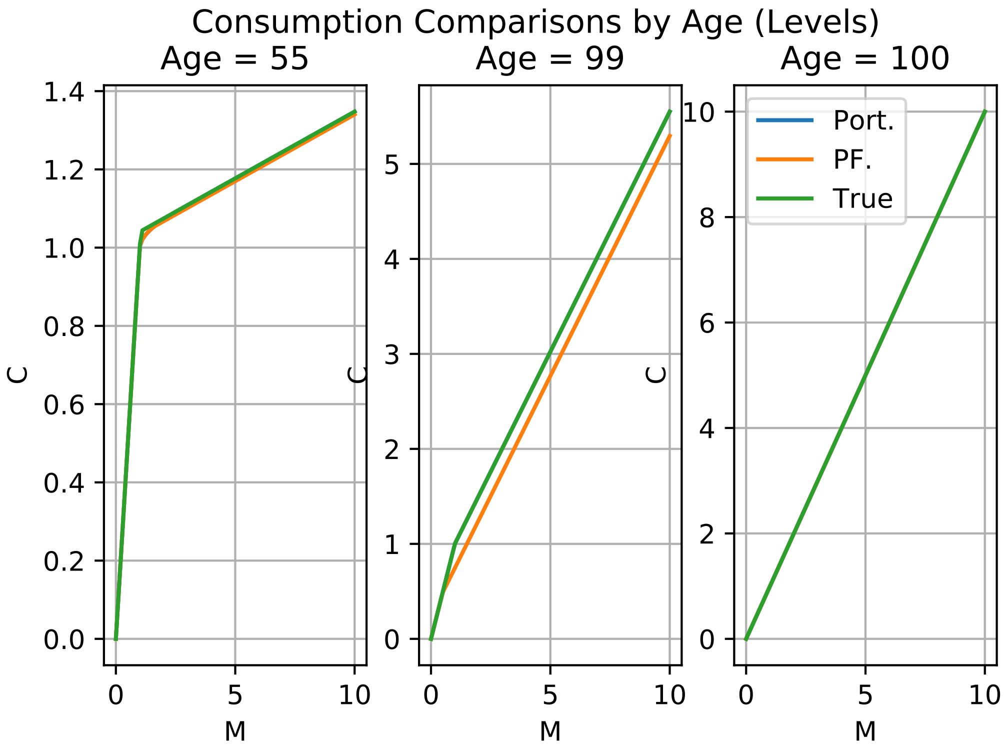

Figure 6 Perfect foresight solutions using different HARK tools.

Figure 7 more closely examines deviations in HARK's solutions from the true solution. `PortfolioConsumerType`'s solution is very close to the true solution at every age and level of wealth with deviations appearing only around the consumption function's kink, which is caused by the liquidity constraint that we impose. On the other hand, the solution provided by `PerfForesightConsumerType` has substantial differences in the level of consumption, which widen with the agent's age.

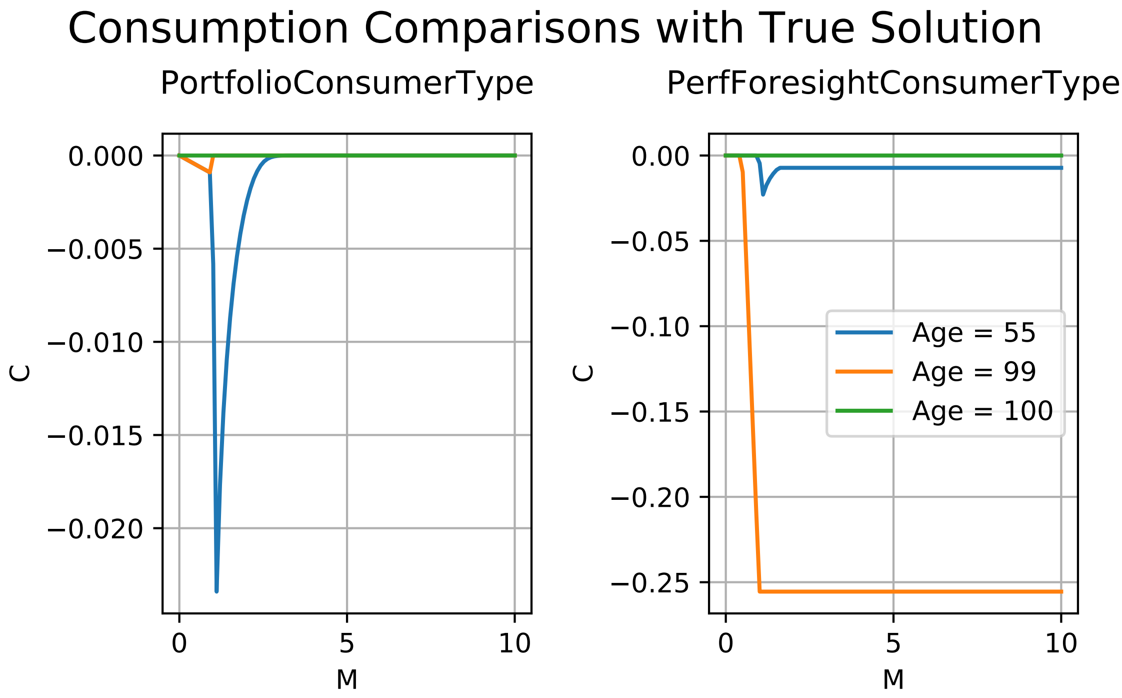

Figure 7 Differences from the true perfect foresight solution.