# Cocco, Gomes, and Maenhout (2005) REMARK

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**Abstract**

This paper contains the highlights from the REMARK file in Code>Python folder.

**Keywords** Hic heac hoc

**JEL codes** XXX

`GitHub:` `http://github.com/econ-ark/REMARK/REMARKS/CGMPort`
*(In GitHub repo, see `/Code` for tools for solving and simulating the model)*

CLICK HERE for an interactive Jupyter Notebook that uses the Econ-ARK/HARK toolkit to produce all of the paper's figures (warning: it may take several minutes to launch). Information about citing the toolkit can be found at Acknowleding Econ-ARK.

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All numerical results herein were produced using the Econ-ARK/HARK toolkit; for further reference options see Acknowleding Econ-ARK. Thanks to Chris Carroll and Sylvain Catherine for comments and guidance.

# 1 Introduction

# 2 The Problem

## 2.1 Setup

The consumer solves an optimization problem from period $t$ until the end of life at $T$ defined by the objective

$$\max \ \mathbb{E}_t \left[ \sum_{n=0}^{T-t} \beta^n \mathrm{u}(\mathbf{c}_{t+n}) \right] \tag{1}$$

where $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$ is a constant relative risk aversion utility function with $\rho > 1$.[1,2] The consumer's initial condition is defined by market resources $\mathbf{m}_t$ and permanent noncapital income $\mathbf{p}_t$.

[1] The main results also hold for logarithmic utility which is the limit as $\rho \to 1$ but incorporating the logarithmic special case in the proofs is cumbersome and therefore omitted.

[2] We will define the infinite horizon solution as the limit of the finite horizon problem as the horizon $T-t$ approaches infinity.

# Appendices

## A Equality of Aggregate Consumption Growth and Income Growth with Transitory Shocks

Section ?? asserted that in the absence of permanent shocks it is possible to prove that the growth factor for aggregate consumption approaches that for aggregate permanent income. This section establishes that result.

Suppose the population starts in period $t$ with an arbitrary value for $\text{cov}_t(a_{t+1,i}, \mathbf{p}_{t+1,i})$. Then if $\breve{m}$ is the invariant mean level of $m$ we can define a 'mean MPS away from $\breve{m}$' function

$$\acute{\text{a}}(\Delta) = \Delta^{-1} \int_{\breve{m}}^{\breve{m}+\Delta} \text{a}'(z) dz$$

and since $\psi_{t+1,i} = 1$, $\mathcal{R}_{t+1,i}$ is a constant at $\mathcal{R}$ we can write

$$a_{t+1,i} = \text{a}(\breve{m}) + (m_{t+1,i} - \breve{m})\acute{\text{a}}(\overbrace{\mathcal{R}a_{t,i} + \xi_{t+1,i}}^{m_{t+1,i}} - \breve{m})$$

so

$$\text{cov}_t(a_{t+1,i}, \mathbf{p}_{t+1,i}) = \text{cov}_t \left( \acute{\text{a}}(\mathcal{R}a_{t,i} + \xi_{t+1,i} - \breve{m}), \Gamma \mathbf{p}_{t,i} \right).$$

But since $\mathsf{R}^{-1}(\wp \mathsf{R}\beta)^{1/\rho} < \acute{\text{a}}(m) < \text{\textbf{Þ}}_{\mathsf{R}}$,

$$|\text{cov}_t((\wp\mathsf{R}\beta)^{1/\rho} a_{t+1,i}, \mathbf{p}_{t+1,i})| < |\text{cov}_t(a_{t+1,i}, \mathbf{p}_{t+1,i})| < |\text{cov}_t(\text{\textbf{Þ}} a_{t+1,i}, \mathbf{p}_{t+1,i})|$$

and for the version of the model with no permanent shocks the GIC says that $\text{\textbf{Þ}} < \Gamma$, which implies

$$|\text{cov}_t(a_{t+1,i}, \mathbf{p}_{t+1,i})| < \Gamma |\text{cov}_t(a_{t,i}, \mathbf{p}_{t,i})|.$$

This means that from any arbitrary starting value, the relative size of the covariance term shrinks to zero over time (compared to the $A\Gamma^n$ term which is growing steadily by the factor $\Gamma$). Thus, $\lim_{n\to\infty} \mathbf{A}_{t+n+1}/\mathbf{A}_{t+n} = \Gamma$.

This logic unfortunately does not go through when there are permanent shocks, because the $\mathcal{R}_{t+1,i}$ terms are not independent of the permanent income shocks.

To see the problem clearly, define $\breve{\mathcal{R}} = \mathbb{M}[\mathcal{R}_{t+1,i}]$ and consider a first order Taylor expansion of $\acute{\text{a}}(m_{t+1,i})$ around $\breve{m}_{t+1,i} = \breve{\mathcal{R}} a_{t,i} + 1$,

$$\acute{\text{a}}_{t+1,i} \approx \qquad \acute{\text{a}}(\breve{m}_{t+1,i}) + \acute{\text{a}}'(\breve{m}_{t+1,i}) \left( m_{t+1,i} - \breve{m}_{t+1,i} \right).$$

The problem comes from the $\acute{\text{a}}'$ term. The concavity of the consumption function implies convexity of the a function, so this term is strictly positive but we have no theory to place bounds on its size as we do for its level á. We cannot rule out by theory that a positive shock to permanent income (which has a negative effect on $m_{t+1,i}$) could have an unboundedly positive effect on $\acute{a}'$ (as for instance if it pushes the consumer arbitrarily close to the self-imposed liquidity constraint).

## References