Black-Litterman Model: Portfolio with market views

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Motivation

We have covered different portfolio formulations utilizing returns ($\boldsymbol{\mu}$) as their input, such as the Maximum Sharpe Ratio Portfolio (MSRP) and the Mean-CVaR Portfolio.

- We discussed sample estimator and shrinkage estimator for $\boldsymbol{\mu}$.
- However, both of them do not include any prediction power to future's return.

For active investors, especially fund managers, they would like to incorporate their view on $\boldsymbol{\mu}$ during portfolio optimization.

Black-Litterman Model offers a way by combining

- The market model (starting point).
- Investor's view (extra information).

Model formulation

Goal: find out "actual" μ, by combining market information and our view.

Market model: $\mathbf{m} = \boldsymbol{\mu} + e_m, \quad e_m \sim \mathcal{N}(0,\, \tau\boldsymbol{\Sigma})$

- One example of $\mathbf{m}$ is to directly use the sample estimator $\hat{\mu}$, while factor models (e.g., CAPM) is also appropriate.
- τ corresponds to our uncertainty about the market information. The more uncertain we are, the larger τ we put in the model.

Investor's view: $\mathbf{v} = \mathbf{P}\boldsymbol{\mu} + e_v, \quad e_v \sim \mathcal{N}(0,\, \boldsymbol{\Omega})$

- $\boldsymbol{\Omega}$ corresponds to the uncertainty of our views.

Example

We have stock A and B. If we believe B will outperform A by $x\%$, with variance σ^2, then we have $\mathbf{P} = [\,-1 \quad 1\,], \mathbf{v} = [\,x\%\,], \boldsymbol{\Omega} = [\,\sigma^2\,]$.

Model formulation (con't)

By re-writing the two equations into a block matrix form, we have

$$\begin{bmatrix} \mathbf{m} \\ \mathbf{v} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{P} \end{bmatrix} \boldsymbol{\mu} + e, \quad e \sim \mathcal{N}\left(0, \begin{pmatrix} \tau\boldsymbol{\Sigma} & 0 \\ 0 & \boldsymbol{\Omega} \end{pmatrix}\right)$$

This problem can be viewed as a weighted least square problem, where we underweight the errors caused by observation with high variance.

Alternately, we could utilize the Bayes' Theorem together with the pdf of Normal Distribution to obtain

$$\boldsymbol{\mu}_{\boldsymbol{BL}} = \mathbf{m} + \tau\boldsymbol{\Sigma}\mathbf{P}^T(\tau\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^T + \boldsymbol{\Omega})^{-1}(\mathbf{v} - \mathbf{Pm})$$

$$\boldsymbol{\Sigma}_{\boldsymbol{BL}} = (1+\tau)\boldsymbol{\Sigma} - \tau^2\boldsymbol{\Sigma}\mathbf{P}^T(\tau\mathbf{P}\boldsymbol{\Sigma}\mathbf{P}^T + \boldsymbol{\Omega})^{-1}\mathbf{P}\boldsymbol{\Sigma}$$

Generating investor's view

Obtaining views (which is the main source of alpha) is a very challenging task. Active fund managers usually devote most of the time to this area.

In the literature, there are mainly two classes of methods:

- Time series modelling: predicting returns, volatility, or economic factors (e.g., unemployment rate), based on various time series models (e.g., ARIMA, GARCH), and turn the prediction into views.
- Machine learning techniques: predicting returns based on methods such as ensemble learning (with random forest) and neural network.

I followed the methodology in this paper to generate views and perform an empirical study on the BL model on 8 famous technology stocks - 'AAPL', 'AMZN', 'GOOGL', 'META', 'MSFT', 'NFLX', 'NVDA', 'TSLA', from 2018-01-02 to 2020-01-04. The code can be accessed here.

Empirical study - Methodology

Step 1 - Model the log-return by using a suitable ARMA(p,q) model. Parameters are picked from a validation set by minimizing MSE.

Step 2 - Generate a 21-days ahead forecast for the log-return. Only keep the ticker(s) with non-negative predicted return.

Step 3 - Perform Monte-Carlo simulation based on the assumption of Geometric Brownian Motion. Only keep the simulated paths with positive returns (matching our hypothesis).

Step 4 - Calculate the expected return and variance based on the simulation. Create views correspondingly for each stock.

Step 5: Compute $\boldsymbol{\mu_{BL}}$ and $\boldsymbol{\Sigma_{BL}}$, and fit into MSRP.

Empirical study - Result

The performance of BL is compared to other portfolios we have learnt in class, and the performance is decent for this particular realization:

- Ranked 2^{nd} in terms of Sharpe in the in-sample period
- Ranked 2^{nd} in terms of Sharpe in the out-of-sample period

	Sharpe ratio (in-sample)	Sharpe ratio (out-of-sample)
EWP	0.123	2.21
MSRP	0.918	0.99
GMVP	-0.0296	2.75
mean-CVaR	0.237	2.25
mean-max DD	0.381	2.26
BL	0.641	2.67

Summary

In short, BL model is a great starting point if you want to incorporate future prediction in portfolio construction.

There are some criticisms to the original BL model:

- Naive assumption of normal distribution: Extensions have been developed to handle heavy tailed distribution (e.g., student-t).
- Limited ways of specifying views: More flexible ways to specify views and even views on volatility are suggested by Meucci and others.

Major references:

- Y. Feng and D. P. Palomar, 2016
- A. Meucci, 2010
- B. Kocuk and G. Cornuéjols, 2020