5-1

```python
# 計算解析解
y_exact = exact_solution(t_euler)

# 比較結果
print("t\tEuler\tTaylor\tExact\tEuler Error\tTaylor Error")
for i in range(len(t_euler)):
    euler_error = abs(y_euler[i] - y_exact[i])
    taylor_error = abs(y_taylor[i] - y_exact[i])
    print(f"{t_euler[i]:.1f}\t{y_euler[i]:.6f}\t{y_taylor[i]:.6f}\t{y_exact[i]:.6f}\t{euler_error:.6f}\t{taylor_error:.6f}")

# 繪圖
plt.plot(t_euler, y_euler, 'o-', label='Euler')
plt.plot(t_taylor, y_taylor, 's-', label='Taylor Order 2')
plt.plot(t_euler, y_exact, '-', label='Exact')
plt.xlabel('t')
plt.ylabel('y')
plt.legend()
plt.grid(True)
plt.show()
```

input

```
t       Euler   Taylor  Exact   Euler Error     Taylor Error
1.0     0.000000        0.000000        0.000000        0.000000        0.000000
1.1     0.100000        0.105000        0.105160        0.005160        0.000160
1.2     0.209917        0.220919        0.221243        0.011325        0.000324
1.3     0.330471        0.348612        0.349121        0.018651        0.000509
1.4     0.462354        0.488954        0.489682        0.027328        0.000728
1.5     0.606285        0.642883        0.643875        0.037590        0.000993
1.6     0.763041        0.811438        0.812753        0.049711        0.001315
1.7     0.933475        0.995787        0.997494        0.064019        0.001707
1.8     1.118537        1.197252        1.199439        0.080902        0.002187
1.9     1.319293        1.417344        1.420116        0.100823        0.002772
2.0     1.536943        1.657795        1.661282        0.124338        0.003487
/home/main.py:63: UserWarning: Matplotlib is currently using agg, which is a non-GUI backend, so cannot show the figure.
  plt.show()
```

5-2

```python
plt.show()
```

input

```
h = 0.1
t       u1_RK4  u2_RK4  u1_Exact        u2_Exact        u1_Error        u2_Error
0.0     1.333333        0.666667        1.333333        0.666667        0.000000        0.000000
0.1     -3.052437       8.989305        1.788581        -1.023039       4.841018        10.012345
0.2     -23.847795      51.192704       1.423701        -0.874278       25.271496       52.066982
0.3     -130.165202     269.269193      1.131570        -0.724985       131.296771      269.994178
0.4     -680.231485     1399.368584     0.909408        -0.608214       681.140893      1399.976797
0.5     -3531.299585    7258.241839     0.738788        -0.515658       3532.038373     7258.757497
0.6     -18312.795052   37634.955483    0.605710        -0.440411       18313.400762    37635.395894
0.7     -94951.331907   195131.871735   0.499860        -0.377404       94951.831768    195132.249139
0.8     -492306.465639  1011721.872078  0.413671        -0.322954       492306.879311   1011722.195031
0.9     -2552513.623867 5245578.826590  0.341614        -0.274409       2552513.965482  5245579.100999
1.0     -13234278.789168        27197287.206587 0.279675        -0.229888       13234279.068843 27197287.436475

h = 0.05
t       u1_RK4  u2_RK4  u1_Exact        u2_Exact        u1_Error        u2_Error
0.00    1.333333        0.666667        1.333333        0.666667        0.000000        0.000000
0.05    1.721880        -0.499599       1.897096        -0.879150       0.175215        0.379551
0.10    1.726915        -0.832598       1.788581        -1.023039       0.061666        0.190442
0.15    1.617161        -0.890373       1.600959        -0.959444       0.016201        0.069071
0.20    1.481687        -0.861042       1.423701        -0.874278       0.057986        0.013236
0.25    1.348945        -0.807505       1.267608        -0.795145       0.081337        0.012359
0.30    1.227063        -0.750341       1.131570        -0.724985       0.095494        0.025356
0.35    1.117478        -0.695886       1.012997        -0.663057       0.104481        0.032829
0.40    1.019525        -0.645732       0.909408        -0.608214       0.110117        0.037518
0.45    0.931977        -0.599934       0.818629        -0.559389       0.113347        0.040545
0.50    0.853541        -0.558092       0.738788        -0.515658       0.114753        0.042435
0.55    0.783017        -0.519706       0.668275        -0.476225       0.114743        0.043482
0.60    0.719337        -0.484290       0.605710        -0.440411       0.113627        0.043880
0.65    0.661560        -0.451407       0.549909        -0.407635       0.111651        0.043772
0.70    0.608868        -0.420673       0.499860        -0.377404       0.109007        0.043269
0.75    0.560547        -0.391754       0.454695        -0.349296       0.105852        0.042459
0.80    0.515980        -0.364365       0.413671        -0.322954       0.102309        0.041411
0.85    0.474633        -0.338259       0.376158        -0.298076       0.098475        0.040183
0.90    0.436043        -0.313226       0.341614        -0.274409       0.094428        0.038817
0.95    0.399812        -0.289089       0.309583        -0.251739       0.090229        0.037351
1.00    0.365600        -0.265698       0.279675        -0.229888       0.085925        0.035810
/home/main.py:83: UserWarning: Matplotlib is currently using agg, which is a non-GUI backend, so cannot show the figure.
  plt.show()
```