Search

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Chapter 3, Russell & Norvig

Solving problems through search

- State – atomic representation of world
- Goal formulation
  - What objective(s) are we trying to meet?
  - Can be represented as a set of states that meet objectives: goal states
- Problem formulation
  - Decide actions and states to reach a goal

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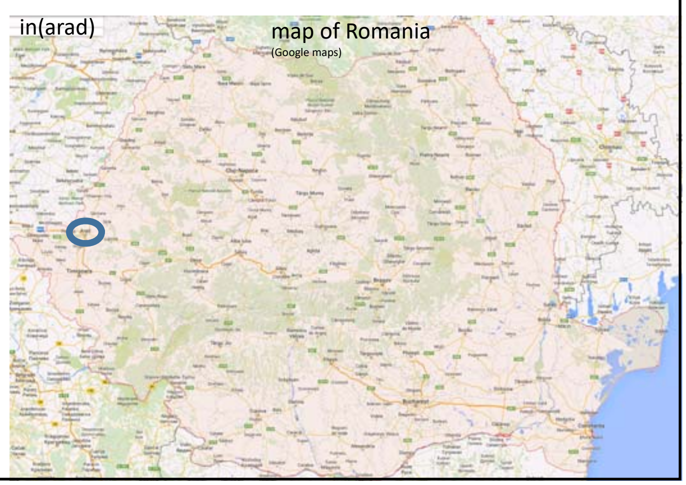
Search

- Assume environment is
  - observable
  - discrete (finite # of actions)
  - deterministic actions
- Search process returns a plan:
  set of states & actions to reach a goal state
- Plan can be executed

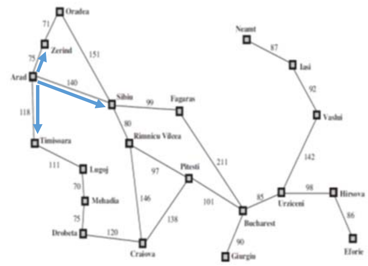
Search problem components

- Initial state



Search problem components

- Initial state
- Actions
  - function that returns set of possible decisions from a given state
  - actions(in(arad)) →{go(sibiu), go(Timisoara), go(zerind)}



Abstract view of Romanian roads
(Russel and Norvig 2010, Fig 3.2)

Note: Abstractions are valid when we can map them onto a more detailed world

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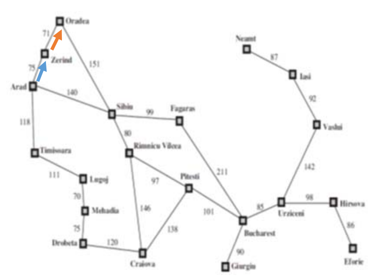
Search problem components

- Initial state
- Cost
  - Each action has a step cost:
    cost(in(arad), go(zerind), in(zerind)) = 75
  - A path has a cost which is the sum of its step costs:

    path: in(arad), in(zerind), in(Oradea)
    has cost: 146 (75+71)



Abstract view of Romanian roads
(Russel and Norvig 2010, Fig 3.2)

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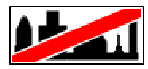
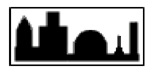
Search problem components

- Initial state
- Actions
- Cost
- Transition model
  Function that reports the result of an action applied to a state:
  result(in(arad),go(zerind) →in(zerind)


Arad


Zerind

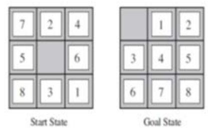
Search problem components

- Initial state
- Actions
- Cost
- Transition model
- Goal predicate
  Is the new state a member of the goal set?
  goal: {in(bucharest)}

Any path that reaches a goal is a *solution*, the lowest cost path is an *optimal solution*.

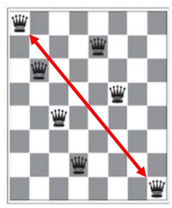
Sample toy problem

- n-puzzle



8-puzzle and one possible goal state
[Figure 3.4 R&N 2010]

- n-queens



8-queens state
[Figure 3.5 R&N 2010]

see text for other examples

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Constructing a problem: n-queens

- States
  1. complete-state:
     - n-queens on board
     - move until no queen can capture another.
  2. Incrementally place queens
     - initial empty board
     - add one queen at a time

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Incremental n-queens

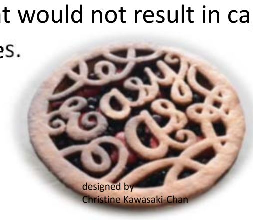
- state: Any arrangement of [0,n] queens
- initial state: empty board
- actions: add queen to empty square
- transition model: new state with additional queen
- goal test: n queens on board, none can attack one another

Incremental n-queens

- A well-designed problem restricts the state space
  - Naïve 8 queens
    1st queen has 64 possibilities
    2nd queen has 63 possibilities…

    $$64 \times 63 \times 62 \ldots \approx 1.8 \times 10^{14}$$

  - Smarter:
    - Actions only returns positions that would not result in capture
    - State space reduced to 2057 states.



Classic real-world problems

- route-finding problem
  - transportation (car, air, train, boat, …)
  - networks
  - operations planning
- touring problem
  Visit a set of states ≥1 time
- traveling salesperson
  Visit a set of state 1 time

- Others: VLSI layout, autonomous vehicle navigation & planning, assembly sequencing, pharmaceutical discovery

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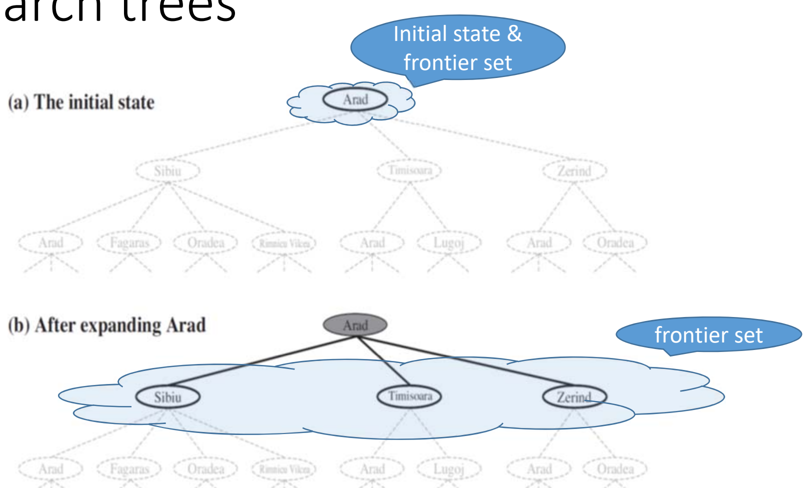
Search trees



[Figure 3.6 R&N 2010]

frontier set also known as an open list

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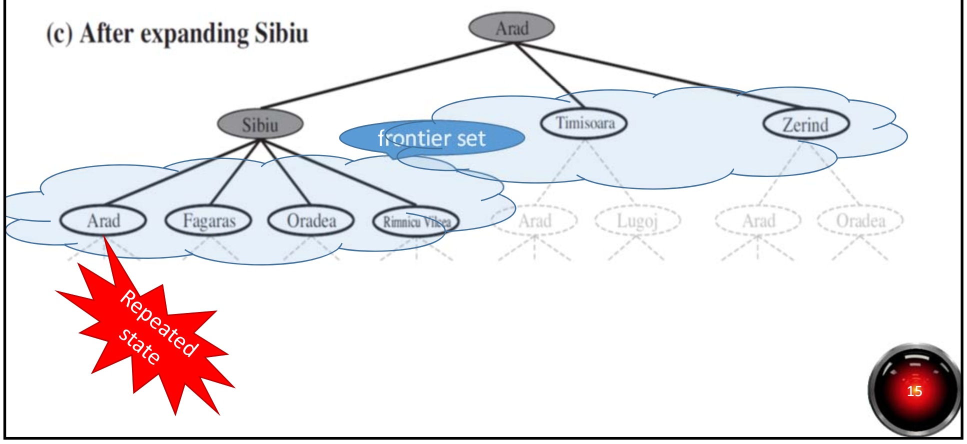
Search trees



Search tree

- Frontier set consists of leaf nodes
- Redundant paths occur when
  - $\exists$ more than 1 path between a pair of states
  - cycles in the search tree (loops) are a special case

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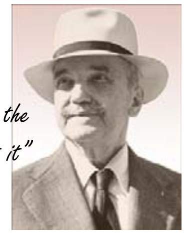
Redundant paths

"Those who cannot remember the past are condemned to repeat it"



George Santayana,
Spanish-American philosopher 1863-1952

- Sometimes, we can define are our problem to avoid cycles
  e.g. n-queens: queen must be placed in the leftmost empty column
- Otherwise: Explored set
  - Track states that have been investigated
  - Don't add any actions that have already occurred

Tree Search

```
function tree-search(problem)
  frontier = problem.initial_state()
  done = found = False
  while not done
    node = frontier.get_node()  # remove state
    if node in problem.goals()
      found = done = True
    else
      frontier.add_nodes(results from actions(node))
      done = frontier.is_empty()
  return solution if found else return failure
```

Graph Search

```
function graph-search(problem)
   frontier = problem.initial_state()
   done = found = False
   explored = {} # keep track of nodes we have checked
   while not done
      node = frontier.get_node()  # remove state
      explored = union(explored, node)
      if node in problem.goals()
         found = done = True
      else
         # only add novel results from the current node
         nodes = setdiff(results from actions(node), union(frontier,explored))
         frontier.add_nodes(nodes)
         done = frontier.is_empty()
   return solution if found else return failure
```

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Search architecture

- Node representation
  - state
  - parent – ancestor in tree
    allows us to find the solution from a goal node by chasing pointers and reversing the path
  - action – Which action was used on parent to generate this node
  - path-cost – What is the cost to reach this node from the tree's root. Usually denoted g(n).

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Search architecture

```
function child-node(problem, node, action)
  child.state = problem.result(node.state, action)
  child.parent = node
  child.path_cost = node.path_cost +
    problem.cost(node.state, action, child.state)
  return child
```

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Search architecture

- frontier set is usually implemented as a queue
  - FIFO – traditional queue
  - LIFO – stack
  - priority

  We will develop a way such that it can always be a priority queue.

- Explored set – Need to make states easily comparable
  - hash the state or
  - store in canonical form (e.g. sort visited cities for traveling salesperson problem)

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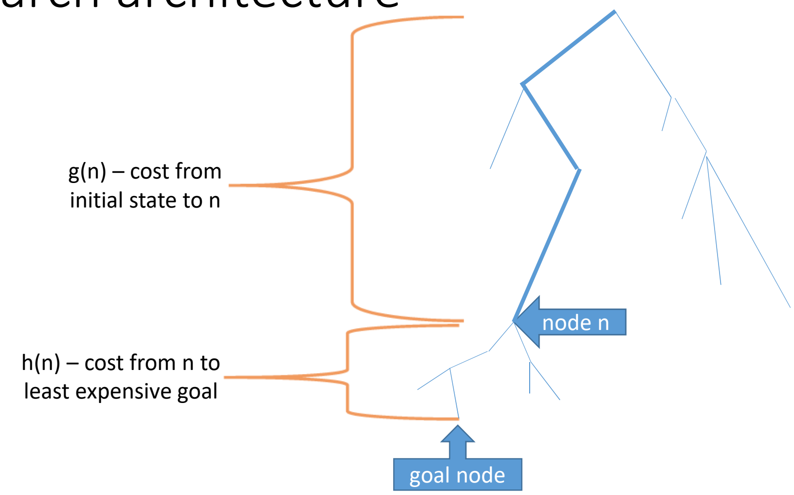
Search architecture



g(n) *and h(n) are frequenty not known precisely.*
Estimates are denoted or $g'(n)$ *&* $h'(n)$ *or* $\hat{g}(n)$ *&* $\hat{h}(n)$

A generic graph search algorithm

```
function tree-search(problem)
    frontier = problem.initial_state()  # priority queue based on lowest cost
    done = found = False
    explored = {} # keep track of nodes we have checked
    while not done
        node = frontier.get_node()  # remove state
        explored = union(explored, node)
        if node in problem.goals()
            found = done = True
        else
            # only add novel results from the current node
            nodes = setdiff(results from actions(node), union(frontier,explored))
            for n in nodes
                estimate a cost g'(n) + h'(n)
            frontier.add_nodes(nodes)  # merge new nodes in by estimated cost
            done = frontier.is_empty()
    return solution if found else return failure
```

Uninformed (blind) search

- No awareness of whether or not a state is promising
- Strategies depend on order of node expansion
  - breadth-first
  - uniform-cost
  - depth-first
  - variants: depth-limited, iterative deepening, bidirectional
- Note: Text uses different queue types for frontier, with generic search algorithm everything is a priority queue, smallest values first.

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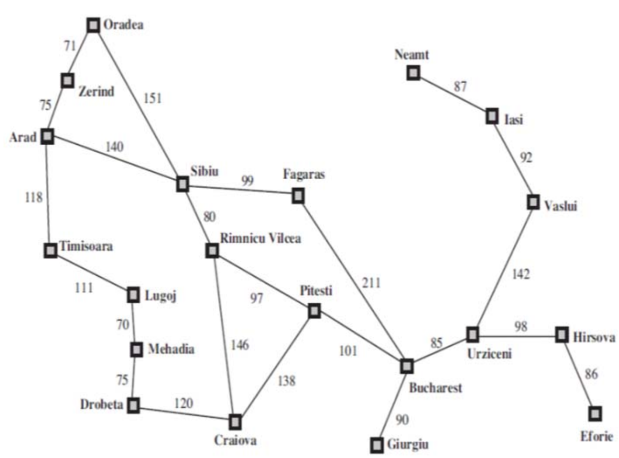
Breadth-first search

- $\forall n\ g'(n) = \text{depth}(n)$ and $h'(n) = k$ (e.g. $k=0$)

Abstract view of Romanian roads

(Russel and Norvig 2010, Fig 3.2)



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Breadth-first search

- Guarantees
  - completeness – will find a solution
  - best path *if cost is a nondecreasing f(depth)*
- How can we measure performance?
  - Time complexity
  - Space complexity

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Complexity (CS 310 material)

- Measure of the number of operations (time) or memory (space) required
- Analysis of performance as the number of items n grows:
  - worst case
  - average case
- Example:

There are $T(n)=4n^2+1$ arithmetic operations

```
def foobar(n):
  x = 0
  for i in xrange(n):
    for j in xrange(n):
      x = x + i*i + j*j
  return x * x
```

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Complexity

- We defined "big oh" of n as follows:

$$T(n) \text{ is } O\big(f(n)\big) \text{ if } T(n) \leq kf(n)$$

$$\text{for some } k \ \& \ \forall n > n_0$$

- Role of k and n_0

  Coefficients of highest order polynomial aren't relevant.

- Implications:
  - T(n) = $4n^2+1$ → $O(n^2)$
  - T'(n) = $500n+8$ → O(n)

  For some small values of n, T(n) is better, but as n increases T(n) will be worse. Using the big-oh notation abstracts this away and we know in general that the second algorithm is better.

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Search complexity

Measured with respect to search tree:

- Complexity is a function of
  - Branch factor – max # of successors
  - Depth of the shallowest goal node
  - Maximum length of a state-space path
- Time measurement: # nodes expanded
- Space measurement: maximum # nodes in memory

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Search complexity

- "Search cost" – time complexity
- "Total cost" – time and space complexity
  Problematic to fuse the metrics…

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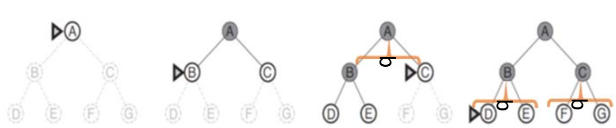
Breadth-first search performance

- Assume branch factor b
- Time complexity:
  $b + b^2 + b^3 + \cdots + b^d = O(b^d)$ *
- Space complexity
  - Every generated node remains in memory, $O(b^{d-1})$ in explored and $O(b^d)$ in frontier.



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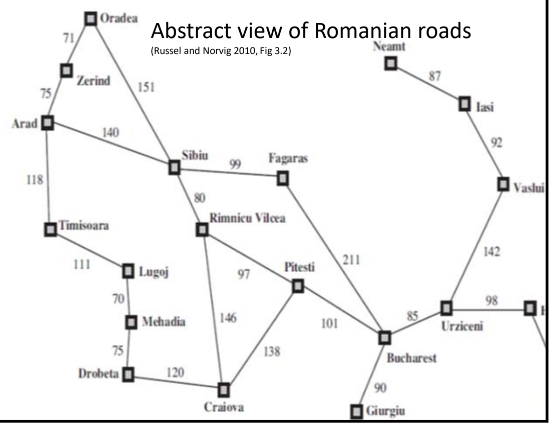
* See text for discussion of $O(b^d)$ vs. $O(b^{d+1})$

Uniform-cost search

- Similar to breadth-first, g’(n) uses edge costs
- $\forall n\ g'(n) = g(n)$ and $h'(n) = k$
- Nodes are expanded in order of optimal cost → optimal solution
- Complexity function of minimum cost for all actions

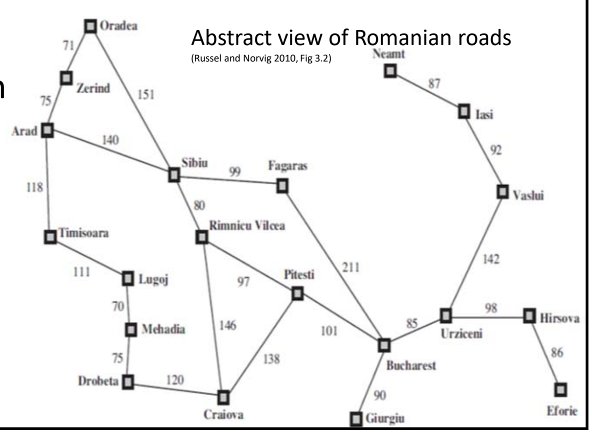
Abstract view of Romanian roads
(Russel and Norvig 2010, Fig 3.2)



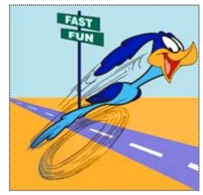
Depth-first search

- Deepest node is expanded first
- $\forall n\ g'(n) = k$ and $h'(n) = -depth(k)$
- Non-optimal
- Incomplete search
- Why bother?

Abstract view of Romanian roads
(Russel and Norvig 2010, Fig 3.2)



Depth-first search (DFS)



- DFS will explore other paths when there are no successors.
- Fast! If you hit the right path… but the average case analysis is $O(b^m)$ where m is maximum depth.
- Space complexity is better: $O(bm)$

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Iterative deepening

- Prevents infinite loops of depth-first search
- Basic idea
  - Depth-first search with a maximum depth
  - If the search fails, repeat with a deeper depth

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Uninformed search

- Other variants exist
- For large search spaces, it is generally a bad idea

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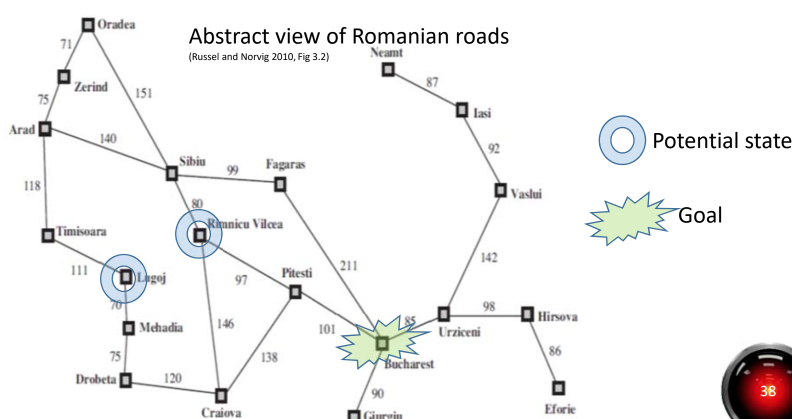
Informed, or heuristic, search

- General idea: Can we guess the cost to a goal based on the current state?



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Heuristic

- h(n) – Actual cost from a search graph node to a goal state *along the cheapest path*.

- h'(n) – An estimate of h(n), known as a heuristic.

Note that your text does not make a notational distinction between the actual cost and the estimated one and always uses h(n), so we will frequently follow suit.

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Heuristic

- h(n) is always ≥ 0
- h(n) is problem specific
- Estimators of h(n) are similar.

- One can think of a heuristic as an educated guess. We will look at how to construct these later…

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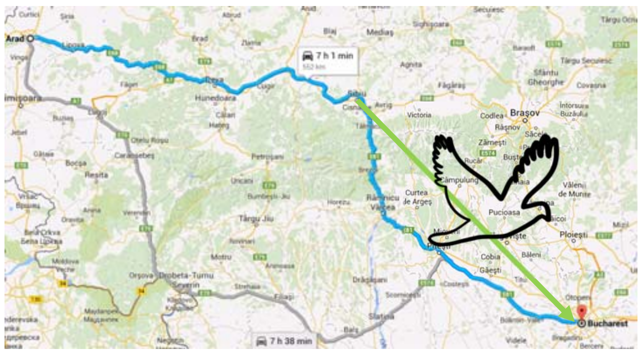
Greedy best-first search

- g(n) = 0, h(n) is heuristic value
- Example h(n) for Romania example:

  as the bird flies distance



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A* Search

- “A-star” search uses:
  - g(n) = cost incurred to n
  - h(n) = estimate to goal

  A* is the estimated cost form start to goal through state n

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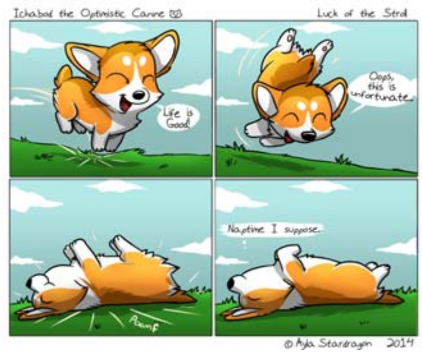
Heuristic properties

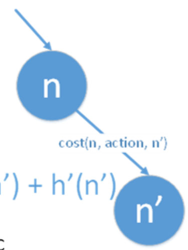


- admissible – h'(n) is admissible if it *never overestimates* the cost to goal

  One can think of it as optimistic: h'(n)

- consistency (aka monotonicity) – h'(n) is consistent if

consistency:
$h'(n) \leq cost(n, action, n') + h'(n')$



Note: We are being careful about distinguishing the heuristic estimator h'(n) from the actual distance h(n)

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Heuristic properties

- Every consistent heuristic is also admissible.
- A* is guaranteed to be:
  - for trees
    A* optimal if h'(n) is admissible
  - for graphs
    A* optimal if h'(n) is consistent

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Understanding A*

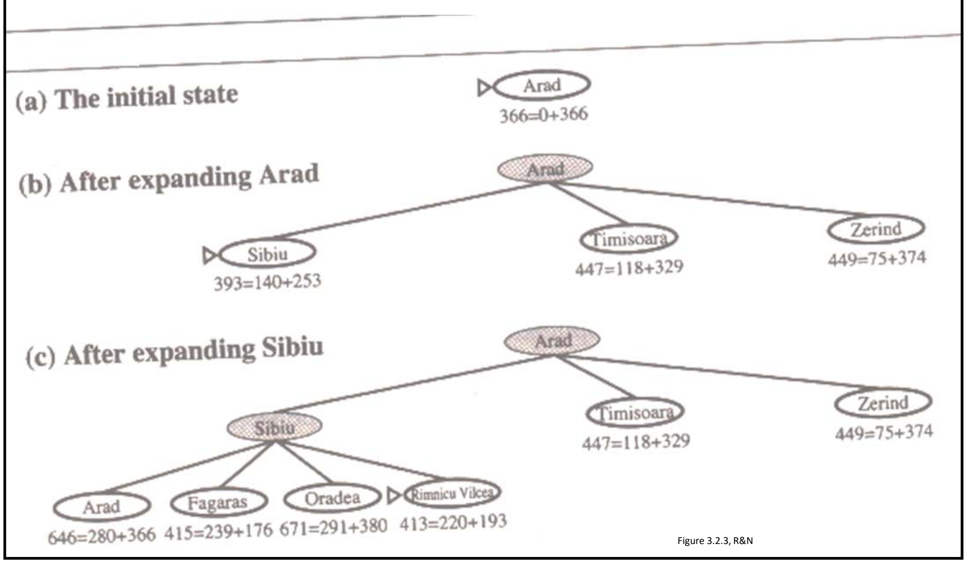
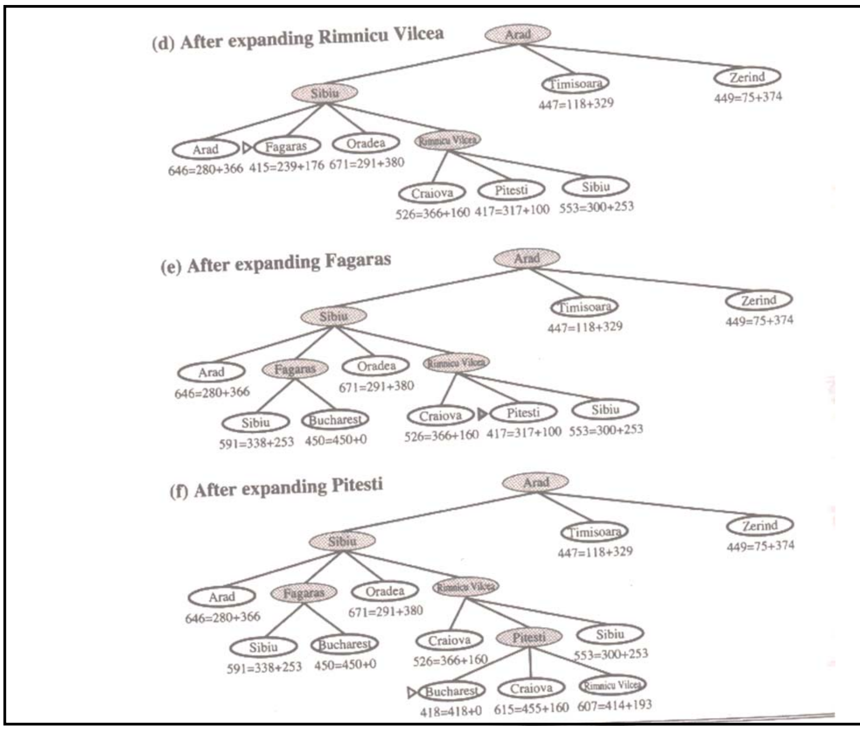


Figure 3.2.3, R&N



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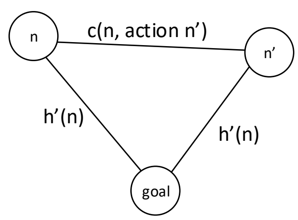
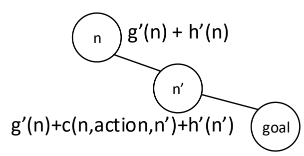
Understanding A* optimality

Consistency revisited: the ▲ inequality – the sum of any two sides ≥ third side



$h'(n) \leq c(n,action,n') + h'(n')$

If h’ consistent and costs are nonnegative, values of f(n) along any path are *nondecreasing*.



Understanding A* optimality

- Suppose we pick node n
- Is the path to node n’s state optimal?

  Proof by contradiction

  Suppose a better path to the same state is in node b.

  As b and n have the same state, so h(n) = h(b).

  Relative position in queue will be driven by g(n) and g(b).

  If b is better, g(b) < g(n) and we would have picked b first

Understanding A* optimality

- When $h(n)$ is consistent, the properties of:
  - nondecreasing values of $f(n)$
  - guarantee that we pick the best path to n

  ensure that the first goal node we find is optimal.

- Completeness holds when there are a finite number of nodes with $f(n) <$ the optimal cost

Limitations of A*

- Need to find a heuristic
- Show it is consistent (for graph search) if optimal goal is required.
- Show the graph is finite for nodes with cost lower than the optimal one if completeness is required

- Note: expanded set requires nodes in memory and is a frequent limitation of A*

A* variants

- iterative deepening A*
  Same idea as iterative depth-first search,
  but we limit on f(n)

- SMA* - simplified memory A*
  - When memory is full
    - drops worst frontier node (highest f(n))
    - stores that value in parent, and will only reconsider branch when everything looks worse than the stored value
  - Details beyond our scope

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Heuristic search summary

- A* can still have problems with space complexity
  - iterative deepening A*
  - other alternatives listed in text

- Complexity of A* search is tricky, but is related to
  - the error in the heuristic, h(n)-h'(n)
  - and solution depth

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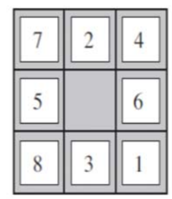
Developing heuristics

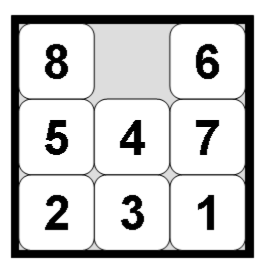
- Requires
  - knowledge of problem domain
  - thinking a bit (usually)
- Effort to show that heuristic is
  - admissible
  - consistent
- What heuristics could we use for the N-puzzle



Start State

N-puzzle heuristics

- Common heuristics
  - $h_1(n)$ – Number of misplaced tiles
  - $h_2(n)$ – Sum of Manhattan[1] distance of tiles to solution
- Are these
  - admissible? (never overestimates)
  - consistent? (non-decreasing path cost)



[1] Also known as city-block distance, the sum of vertical and horizontal displacement.

Heuristics and performance

- Branching factor
  - Measured against a complete tree of solution depth d
  - Suppose A* finds a solution at
    - depth 5
    - 52 nodes expanded (53 with root)
  - A complete tree of depth 5 would have
    $$52 + 1 = b^* + (b^*)^2 + (b^*)^3 + (b^*)^4 + (b^*)^5$$
    where b* is the branch factor
  - Using a root finder for $1(b^*)^5 + 1(b^*)^4 + 1(b^*)^3 + 1(b^*)^2 + 1(b^*)^1 - 53(b^*)^0 = 0$ we see b*≈1.92

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Heuristics and performance

- 8-puzzle example averaged over 100 instances

	Search Cost (nodes generated)			Effective Branching Factor		
d	IDS	$A^*(h_1)$	$A^*(h_2)$	IDS	$A^*(h_1)$	$A^*(h_2)$
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	3644035	227	73	2.78	1.42	1.24
14	–	539	113	–	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

- branch factors closer to one are better

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Finding heuristics

- Okay, developing a heuristic is hard
- Can we make it easier?

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Relaxed problem heuristics

- Let's return to the N-puzzle
- Suppose we allowed
  - A tile to move onto the next square regardless of whether or not it was empty.
  - A tile to move anywhere.
- These are relaxations of the rules

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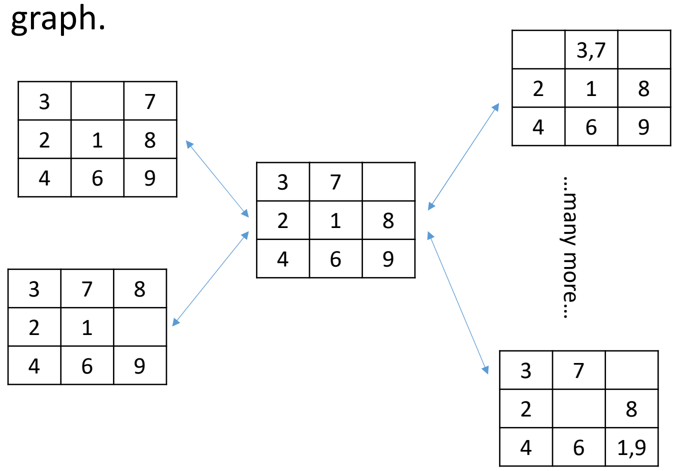
Relaxed problems

We can think of these as expanding the state space graph.



Relaxed problem heuristics

- The original state space is a subgraph of the new one.
- Heuristics on relaxed state space
  - Frequently easier to develop
  - If admissible/consistent properties hold in relaxed space, they also hold in the problem state space.

Relaxation

- Can specify problem in a formal language, e.g.
  - move(A,B) if
    - verticalAdjacent(A,B) and
    - horizontalAdjacent(A,B) and
    - isempty(B)
- Possible relaxations
  - move(A,B) if adjacent(A,B)
  - move(A,B) if isempty(B)
  - move(A,B)

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Automatically generated heuristics

With a formal specification of the problem there exist algorithms to find heuristics (beyond our scope, e.g. ABSOLVER)

Machine Learning, 12, 117–141 (1993)

Machine Discovery of Effective Admissible Heuristics

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Multiple heuristics

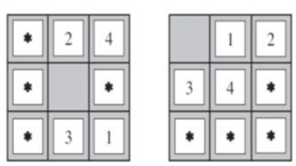
- Regardless of how generated, one may develop multiple heuristics for a problem
- We can merge them

$$h'(n) = \max\left(h'_1(n), h'_2(n), \ldots, h'_i(n)\right)$$

why maximum?

Pattern database heuristics

- Can we solve a subproblem?



Start State　　Goal State

- If we can, we can store its h(n)

Pattern database heuristics

- Cost usually found by searching back from goal nodes.
- Worth it if the search will be executed many times.
- Sometimes patterns are disjoint. If so, the heuristic costs may be added (doesn't work for 8 puzzle)

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Learning heuristics

- Use experience to learn heuristics
- Beyond our reach for now… (machine learning)

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Heuristic summary

(rough outline, no substitute for a little thought)