<u>Overview:</u>

- **Chapter 6 - Constraint Satisfaction**
- **Chapter 18 - Learning**
- **Chapter 7 - Logical Agents**

Notes:
??? means do not have notes for this section, do not understand, or return to clarify.
Add comments for possible test question to be included in study guide

<u>Chapter 6 - Constraint Satisfaction</u>

- **Constraint Satisfaction Problems (CSP)**
  - Definition: solutions with caveats
  - Example: find a way to take classes such that I graduate in four years. Constraints are prereq's, course availability, and funding
  - Previously, states were
    - Atomic - don't care about internal rep. except w/ respect to goal/heuristic
    - Mutated by actions to produce new atomic state
  - Now - **Factored representations**
    - States have **internal structure**
    - Structure can be **manipulated**
    - **Constraints** related different parts of the structure to one another and provide legal/illegal configurations
- CSP Definition
  - **Problem = {X, D, C}**
  - **X - set of variables** X = {X1, X2, … Xn}
  - **D - set of domains** D = {D1, D2, … Dn}
    - such that Xi = xi where xi in Di
  - **C - set of constraints** C = {C1, C2, … Cn}
    - such that Ci = < (Ca, Cb), relationship(Ca, Cb)>
- CSP Example: Map Coloring
  - Color territories on a map using 3 colors such that no two colors are adjacent
  - Can be represented as a graph with adjacent area connected by an edge
  - All variables have the same domain (the three colors)
  - Constraint set consists of areas adjacent not being equal Resultant Set
- CSP Example: Scheduling
- **Constraint Types**
  - **Domain values** - time at which task begins {0, 1, 2 …}
  - Precedence constraints - a + b <= c
  - Disjunctive constraints - e.g. can only do one thing at a time => a + 10 <= b or b + 10 <= a
  - Unary - single variable (z <= 10)
  - Binary - between two variables z^2 > y

- ○ **Global** - constraints with 3+ variables can be **reduced to multiple binary/unary constraints**
- Constraint graphs / hyper graphs - see slides for picture
- **Binarization of constraints**
  - ○ Convert n-ary constraints into **unary/binary** ones
  - ○ Any arbitrary n-ary constraint can be converted to equivalent unary constraint
  - ○ <u>Example 1: Individual variables and their domains</u>
    - ■ **Variables: X = {1,2} Y = {3,4} Z = {5,6}**
    - variable & domain X Y Z [illegible]
    - ■ **Encapsulated U = {(1,3,5), (1,3,6), (1,4,5), (1,4,6), (2,3,5), (2,3,6), (2,4,5), (2,4,6)}** all possibilities that are possible
    - ■ Introduce new encapsulated variable that is a Cartesian product of the domains of the individual variables! All possibilities that satisfy throughout All possibilitive
    - ■ Cartesian product U = X x Y x Z
    - ■ This new encapsulated variable U contains all the unique combinations (8) that satisfy constraints
  - ○ <u>Example 2: Original constraint and variables</u>
    - ■ **Constraint: X + Y = Z**
    - ■ **Variables: X = {1,2} Y = {3,4} Z = {5,6}**
    - ■ **Encapsulated U = {(1,3,5), (1,3,6), (1,4,5), (1,4,6), (2,3,5), (2,3,6), (2,4,5), (2,4,6)}** can get rid of some of
    - ■ **Encapsulated U (reduced)= {(1,4,5), (2,3,5), (2,4,6)}**
    - ■ Create the encapsulated variable like before but perform a reduction based on the constraint! It will reduce the domain of U
  - ○ https://ktiml.mff.cuni.cz/~bartak/constraints/binary.html
- CSP Example: House Puzzle
  - ○ Row of houses each one has
    - ■ color
    - ■ Person with nationality
    - ■ favorite candy
    - ■ Favorite drink
    - ■ Pet
  - ○ All attributes distinct
  - ○ Associate variables with a location
    - ■ e.g. milk - 3 for house #3, cat - 4 for house 4
- **Implementing a CSP problem: Representation**
  - ○ **Variables** - simple list
  - ○ **Values** - mapping from variables to value lists e.g. python dictionary
  - ○ **Neighbors** - mapping from variables to list of other variables that participate in constraints
  - ○ **Binary constraints** - explicit value pairs, functions that return a boolean value
- **General Strategies for Solving CSP**
  - ○ **Local consistency:** reduce set of possible values through constraint enforcement and propagation

    - Perform search on **remaining possible states**
- **Node Consistency**
    - A variable is **node-consistent** if all values satisfy all unary constraints
    - Other unary conditions could further restrict the domain
- **Arc Consistency**
    - **Arc-consistent**
        - **Variable** - all binary constraints are satisfied for the variable
        - **Network** - if all variables in CSP are arc-consistent
    - Arc consistency only helps when some combinations of values preclude others
    - **3 Outcomes** (when all arcs arc consistent)
        - One domain is empty => **no solution**
        - Each domain has a single value => **unique solution**
        - Some domains have more than one value => **may or may not be a solution**
            - In this case, arc consistency isn't enough to solve
- **Paths and k-consistency - higher levels of consistency, beyond our scope**
    - **Path consistency**
        - See if pair of variables consistent with 3rd variable, solved similarly to arc consistency
    - **K-consistency**
        - Given k-1 consistent variables, can we make Nth variable consistent (generalization of consistency)
- **Global Constraints**
    - Consider an "all different" constraint
    - Each variable has to have a distinct value
    - Assuming M variables and N distinct values
    - What happens when M > N? Impossible to solve, have to repeat a value
    - Extending this idea: <u>**??? - review this**</u>
        - Find variables constrained to single value
        - Remove these variables and their values from all variables
        - Repeat until no variable is constrained to a single value
        - Constraints cannot be satisfied if
            - Variable remains with empty domain
            - There are more variables than remaining values
- **Resource contains ("at most")**
    - Consistency checks - minimum values of domains satisfy constraint?
    - Domain restriction - are the largest values consistent with the minimum ones?
    - <u>**???**</u>
- **Range Bounds**
    - Impractical to store large integer sets
    - Ranges can be used [min, max] instead
    - Bounds propagation can be used to restrict domains according to constraints
- Once all constraints propagated, search for solution

- **Naive Search**
  - Action picks a variable and a value.
  - N variables, domain size D -> N x D possible search nodes
  - Search on next variables
  - Backtrack when search fails
  - **Problems**
    - N variables with domain of size D
    - NxD choices for first, (N-1)D for second -> N!d^n
    - Only d^n possible assignments
- **Modified Search**
  - CSPs are **commutative** => order of variable selection does not affect correctness
  - Each level of search handles specific variable
  - Levels have d choices, leaving us with d^n leaves
- **Backtracking Search**
- **Select-Unassigned-Variable**
  - Fail-first strategies:
    - Minimum remaining value heuristic:
      - Select the most constrained value; the one with the smallest domain
      - Rationale: probably most likely variable to fail
    - Degree heuristic:
      - Use the variable with the highest number of constraints on other unassigned variables
- **Order-Domain-Values**
  - Order of values within a domain **may or may not make a difference**
  - If goal is to produce all solutions or if there are no solution => order has no consequence
- In other cases, we use a fail-last strategy and pick the value that reduces neighbors' domains as little as possible
- **Why fail-first for variable selection and fail-last for value selection?**
  - For variables, order does not matter. So we want to eliminate as many possible. For values, order can matter so if we eliminate a value, we might hurt ourselves later when we go to check it out.
- **Inference in Search => Forward Checking**
  - Check arc consistency with neighboring variables
  - Not needed if arc-consistency was performed prior to search
- **Maintaining Arc Consistency (MAC)**
  - Algorithm that propagates constraints beyond the node
  - AC3 algorithm with modified initial queue
    - Typical AC3 - all constraints
    - MAC - constraints between selected variable and its neighbors
- **Back-Jumping**

    - Maintain a **conflict set** for each variable X:
        - <u>A set of assignments that restricted values in X's domain</u>
    - When conflict occurs, backtrack to last conflict added
- **Back-Jumping Implementation:**
    - On forward checks of X assigned to x,
        - When X deletes a value form Y's domain, add X=x to Y's conflict set
        - If Y is emptied, add Y's conflict set to X's and backjump
    - Easy to implement, build conflict set during forward check
    - However, what we prune is redundant to what we'd prune from forward checking or MAC searches
- More sophisticated backjumps - add notes here
- Conflict-Directed Back-Jumps - add notes for example?
- **Constraint-Learning and No-Goods**
    - Minimal set of assignments that caused problem - no-goods
    - Avoid running into problem by adding new constraint (or checking no-good cache)
- **Local Search CSPs**
    - Alternative to what we have seen so far
    - Assign everything at once
    - Search changes one variable at a time
        - Which variable? **???**
- **Min-Conflicts Local Search**
    - Pretty effective for many problems, e.g. million queens problem can be solved in about 50 steps
    - Essentially **greedy search**, consequently:
        - **Local extrema**
        - **Can plateau**
        - Techniques discussed for **hill climbing** can be applied (e.g. simulated annealing, plateau search)
- **Structure of CSP Problems**
    - Improve search by **exploiting structure**
    - Independent **subproblem**s - solve separately
    - Tree structure CSP
    - **Not on exam ???**
- **Tree Structure CSP**
    - Basic Ideas
        - Order variables (topological sort) such that constraints form a tree
        - Solve one variable at a time, propagate
    - Not all CSP constraints form trees
    - Transforming graphs with cycles into trees
        - Solve variable that reduces remaining conditions to a tree
        - Select a set of variables, a cutest that reduces problem to a tree after removal and examine with each possible assignment

            - **Not on exam ???**

Ch 18 - Learning

- Agents can learn to improve
    - **Inference from percepts**
    - Information about world evolution as the result of **changing world** or **action**
    - Utility estimators
    - Action choices either update condition-action maps or involve goal modification to maximize utility
- What we want to learn
    - **Mapping function**
        - Inputs are factored representation e.g. vector of values
    - **Outputs are**
        - Discrete (e.g. categorical)
        - Continuous
- **Type of Learning**
    - Inductive - learn map between input / output pairs
    - Deductive - creating rules that are logically entailed
- Learners vary based on feedback
    - **Unsupervised learning**
        - No explicit feedback
        - Goal is to cluster similar things
    - **Reinforcement learning**
        - Learner given rewards/punishments for actions
        - Ex: animal training w rewards
    - **Hybrids** are possible such as semi-supervised learning where a small set of labeled data accompanies large set of unlabeled data
- Caveat about labeled data sets
    - Labels referred to as “ground truth"
    - **Why be cautious with “ground truth”?** Error can be systematic / important?
- Supervised Learning
- How to choose amongst functions?
- Hypothesis spaces
- Supervised Learning
- **Decision Tree Learner**
    - Answers a series of **questions** to arrive at a solution
    - For now, restrict discussion to
        - Questions that have **categorical** (discrete answers)
        - **Binary** classification decisions
    - Decision trees -> good for justifying decision because you can see the decisions (white box)
- Constructing a tree from examples
- Quantity of information

    - Amount of surprise that one sees when observing an event
    - If an event is rare, we can derive a large quantity of information (measured in bits) from it
- Expectation (review)
- Entropy
    - Defined as expected amount of information (average amount of surprise) and is usually denoted by the symbol H
- Example
- Restaurant Example
- Entropy and tree questions
- Tree Qeustions
    - Eq's have binary response
    - Suppose goal: separate mammals from birds
    - Question: does it fly?
- Tree Questions Entropy
    - Entropy is E[I(P(X))] = E[-log2P(X)]
    - For binary categories, define short hand:
        - Q = p / (p + n) => the positive rate
        - 1 - Q = Q / (p + n) => the negative rate
        - B(q) = E[IP(X))] = -qlog2q - (1-q)log2(1-q)
        - Non-binary entropy calc:

          $-\Sigma_{i \in \text{num\_classes}} P(v_i)\log_2(v_i) == \Sigma_{i \in \text{num\_classes}} P(v_i)\log_2(1/P(v_i))$
    - Bird/mammal example
- Entropy and Tree Questions
- **Information Gain**
    - Goal: **reduce** the amount of information needed to represent the problem
    - We can represent the remaining entropy after dividing **data into d groups** with question **A** as follows:
        - **Remainder(A)** = some questions from slide

          $$\text{Remainder}(A) = \sum_{k=1}^{d} \frac{p_k + n_k}{p + n} B\left(\frac{p_k}{p_k + n_k}\right)$$

        - Expected value of entropy for all the categories
    - And information gain as:
        - **Gain(A) = B(p/(p+n)) - Remainder(A)**
    - **B = Binary Entropy**
- Information Gain Examples
- Decision Tree Learner
- Features and Overlearning
    - Useless features are not good for prediction, but a learner may pick up on random patterns in the training data and incorporate into rules

- Generalization and overfitting
    - Learning random patterns that don't affect function is called overfitting
    - For each leaf node, ask ourselves if good information gain
        - If node informative, keep it, otherwise discard
    - How do we know if our decisions were any good?
        - Goal was to **separate into positive and negative classes as well as possible**
        - Can we devise a statistic that lets us know if our observed split is statistically significant from the expected ratio? => Chi squared
- **Chi Squared Test**
    - Supposed decision tree splits a node into V categories
    - If node does not add any new info, we would exotic # of class examples in each split roughly according to same proportion of examples
    - Let's restrict an example to
    - We can look at how much our categories differ from what would be expected if the proportion of categories did not change
    - When value is small, we are close to the original distribution
    - Test stat has distribution related to number of cateogrires - 1 => **degrees of freedom**
    - Cumulative density function (CDF) -> integrate P(x dof) up to delta
    - The chi-squared test is used to determine **whether there is a significant difference** between the **expected frequencies** and the **observed frequencies** in one or more categories.
- **More Thoughts on Decision Trees**
    - Continuous/integer-valued attributes
        - Dont create infinite branches
        - Select split point
            - Sort values
            - Keep running total of number of +/l examples for each point in sorted list and pick separating point that give best separation
- Decision Tree Summary
    - Relatively straightforward learners
    - Recursively portion the feature space into hyperplanes
    - Sensitive to overtraining, have methods to prune
    - Easy for humans to understand
- Do I have a good hypothesis function?
    - Assume that are independent and identically distributed (iid)
        - ???
        - ???
    - We cross-validate the learner on a separate validation set
    - Problem: don't exploit all data
- K-fold cross validation
    - Extreme caseL leave-one-out cross validation (aka jackknife)

- **Model selection**
    - More complex models (e.g. more nodes in a decision tree) learn the training data better, but are they really better?
    - Look at validation error
- **Loss**
    - Loss functions are form o**f utiility function that provide cost for misclassification**
    - Could be good to find whale and noted to misclassify nonwhale
    - Some learners attempt to minimize loss
    - Common loss functions ???

$$L_1(x,y,\hat{y}) = |y - \hat{y}| \qquad \text{absolute loss function}$$

$$L_2(x,y,\hat{y}) = (y - \hat{y})^2 \qquad \text{squared loss function}$$

$$L_{0/1}(x,y,\hat{y}) = \begin{cases} 0 & y = \hat{y} \\ 1 & \text{otherwise} \end{cases} \qquad \text{0/1 loss function}$$

    -
- **Generalization Loss**
    - What is our loss when we use a novel data set e?
    - The expected loss requires the distribution (X,Y) which we probably do not have:

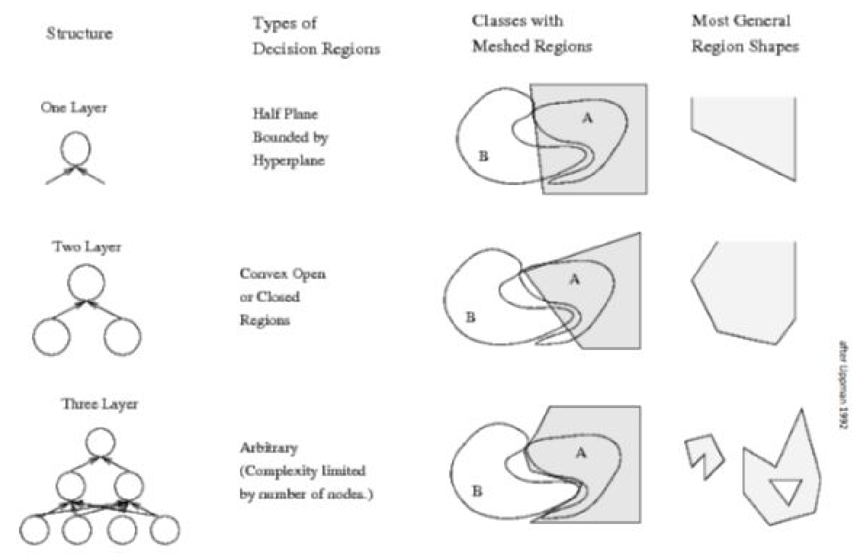
$$GenLoss_L(h) = \sum_{(x,y)\in\epsilon} L(x,y,h(x))P(x,y)$$

    -
    - But we can estimate it empirically on a finite set of examples E of N samples:

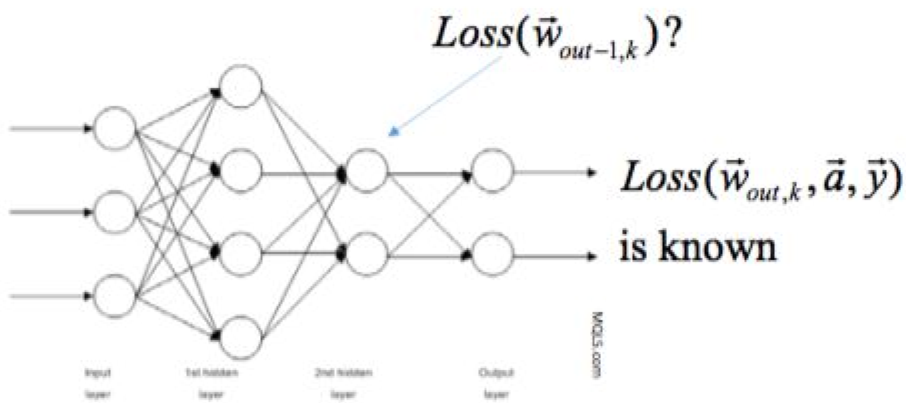
$$EmpLoss_L(h) = \frac{1}{N}\sum_{(x,y)\in E} L(x,y,h(x))$$

    -
    -
    -
        - **Unreliability:** f may not be in H
        - Variance: learners return different f's for different training sets
        - **Noise:**
            - F may be noisy (e.g. stochastic component - different y's for the same x)
            - The training samples may have mis-measured attributes or incorrect labels
            - Might not have measured important attributes
        - **Complexity:** learner may not achieve a global minimum
- Regularization of Regression
    - ???

- **Linear Classification**
  - Regression lines can be used to classify examples
  - We look for a linear separating line (hyperplane when data is in R3 or higher)
  - Suppose we pick a vector w perpendicular to the decision boundary
  - Consider the dot product between w and an arbitrary point
  - Once we have w, we can classify by taking the dot product and looking at the sign
  - How do we choose w?
    - Variant of the gradient descent rule used for regression is applicable
    - Recall regression rule
- **Perceptron Learning Rule**
  - Iterate through data (1 iteration = 1 epoch) and repeat until noe rrors
  - Convergence may be slow, but guaranteed for linearly separable data
  - Most data sets not linear seperable
  - When non linearly separable examples presented in random order, we will converge to a stable classifier if alpha decays linearly with epoch number
- **Learning with Logistic Regression**
  - Remember update rule was:
    - $$w_i = w_i - \alpha \frac{\partial}{\partial w_i} Loss(w)$$
  - When the hard threshold is replaced with logistic regression:
    - Easier if we use labels [0,1] instead of [-1, 1] as this range covered by function
    - Derivative of loss needs to be recomputed
    - $$w_i = w_i - \alpha(y - h_w(x))h_w(x)(1 - h_w(x))x_i$$
    - This results in smoother + more predictable learning curve
- **Connectionist Networks (Artificial Neural Networks)**
  - Activation functions for perceptrons are **nonlinear:**
    - Hard threshold
    - Logistic regression (frequently called sigmoid function)
  - **Linking perceptrons together** provides complex function modeling capability
- **Decision Boundary Capability as a Function of Network Depth**

- 

- An Intuitive View of Neural Nets
  - Suppose we combine two perceptrons whose output functions are reversed
  - This could be used to model a ridge in output space
- Learning in a Neural Network
  - Similar to the regression problem, for output a and desired output y, we can find the loss gradient for each output node
  - $$\frac{\partial}{\partial w} Loss(w) = \frac{\partial}{\partial w} \left| y - h_w(x) \right|^2 = \frac{\partial}{\partial w} \sum_{k=1}^{D} (y_k - a_k)^2 = \sum_{k=1}^{D} \frac{\partial}{\partial w} (y_k - a_k)^2$$
    $$a_k = \frac{1}{1 + e^{-W_{output,k} \cdot input}}$$
  - Use perceptron learning rule for the sum of the gradients at the output layer
- Back-Propagation
  - What should the targets be for the previous input layer?



Multilayer Feed-forward network

- Back-propagating error (overview)
  - Error of the kth output: Errk = yk - ak
  - We can compute gradient for any input node and apply regression rule
  - This gives new set of weights for output node
  - After applying update to the output layer, there still exists loss
  - We assign a portion of the loss to each of the input nodes based on their weight
  - This contribution is computed for each node of current layer
  - Look at sum of losses attributable to each node in previous layer -> sum of these provides us with loss to minimize
  - Repeat recursively
- **… gap in nodes ???**
- **Neural Net Summary**
  - Supervised Learner
    - Training labels either
      - High value for class (n classes -> n output nodes)
      - Encoding of class ifnormation
    - Iterative training typically using a gradient descent algorithm (e.g. back propagation
  - **Classification**
    - Present features to input nodes
    - Interpret output nodes for category
  - Disadvantages
    - Frequently hard to interpret
    - Many parameters require large data sets
    - Bad w imbalanced examples
    - Slow to train
    - Overfits easily regularization important
  - Advantages
    - Flexible, nonlinear learner

        - Deep architectures very powerful
- **Non-Parametric Models**
    - Neural nets and decision trees have models with parameters
    - Decision node parameters: **attribute and cut-point/categories for sub-trees**
    - Neural nets: **weights and connections**
    - **Non-Parametric models:**
        - Cannot be characterized by **bounded set** of parameters
        - Simplest case: look at every example and use it to classify novel examples
        - Called **instance** or **memory-based learning**
- **Nearest Neighbors Models**
    - Use a distance metric to find the k closest neighbors, e.g. for continuous attributes
    - Use the plurality of labels that are the k closest
    - Good
        - Simplicity
        - Effective technique for low-dimensional data
    - Bad - searching is expensive with large training sets, but we can mitigate for this:
        - Trees - similar to decision tree (split on value, may at times need to search both sides)
        - Locally sensitive hash tables
            - Hash functions
                - Set of projections on to lines
                - Line projections discretized into buckets
            - Can be much more effective than tree approach
    - The Ugly
        - N points uniformly distribution in R^D unit hypercube
        - To capture r = 0.01, what edge length would we need in a random sample?
        - Samples are randomly distributed and total volume is 1, so we need a volume of r (0.01)
- **Support Vector Machines**
    - A **margin** is the distance to the closest examples on either side of a hyperplane
    - SVM approaches attempt to **maximize the margin**
    - Can only separate linear problems, but kernel function can project the data into a higher dimensional space where perhaps the data can be better separated
    - Maximal margins computed as functions of training examples
    - Summary
        - **SVM - non parametric technique**
        - In practice, **only small subset of training examples, the support vectors are required**
    - Training algorithm beyond our scope
- **Bias and Variance**

- Error in learning comes from two sources: **bias** and **variance**
- **Bias** - large when learners make consistently incorrect predictions
- **Variance** - large when different training sets result in different predictions

- **Ensemble Learning**
  - **Ensemble learners** are collections of weak learners that are combined to form robust classifier
  - **Weak learner** - simple learning algorithm that is likely to have high bias (e.g. **single node**, or stump, **of decision tree**
  - Ensemble learners typically use collections of weak classifiers to reduce both bias and variance
- **Adaptive Boosting (ADABOOST)**
  - **No notes ???**

<u>**Chapter 7 - Logical Agents**</u>

- **Logical Agents**
  - Two key components:
    - **Representation** of the world
      - **Background information**
      - **Percepts**
    - **Ability to reason** : derive new information based on inference
- **Knowledge Base (KB)**
  - **Sentence** - statement about the agent's world
    - Based on **sensors**
    - Based on **background knowledge** (or possible learning)
- **Satisfaction and Entailment**
  - Suppose a is true in model M, then we state:
    - M satisfies a or equivalent
    - Or: M is a model of a
  - M(a) means all models that satisfy a
  - Reason - entailment
    - $\alpha \models \beta$ if and only if $M(\alpha) \subseteq M(\beta)$
- **Examples of Entailment**
  - House is cornflower blue entails house is a shade of blue
  - X = 0 entails xy = 0
  - Second set must be bigger and contain it (basically subset)
- **Inference Algorithms**
  - Are sound if inference only devices entailed sentences
    - If our algorithm entailed KB |= a2, we might fall into a pit!
  - Are complete if they can service any sentence that is entailed. Becomes an issue when the left-hand side is infinite
  - If KB is well grounded, our entailments should follow in the real world
- **Sentence Construction: Propositional Logic**

- Propositional logic is for the most part the logic you learned to program with
  - Sentence -> AtomicSentence | Complex Sentence
  - AtomicSentence -> true | false | literal
  - ComplexSentence -> (Sentence) | [Sentence]
    - | ~ Sentence => (negation)
    - | Sentence ^ Sentence (conjunction)
    - | Sentence v Sentence (disjunction)
    - | Sentence => Sentence (implication)
    - | Sentence <=> Sentence (biconditional)
  - Operator priority is shown by order
- **Sentence Semantics**
  - Sentences reduced to true or false with respect to specific model
  - In the Wumpus cave
    - we might denote presence or absence of pit by literals indexed by location: Px,y
    - Example: P1,2, P2,2, and P3,1 that have true/false values in any given model
  - Models must specify values for each proposition
  - To resolve sentence: apply logical connectives to truth values
- **Knowledge Base Rules**
  - In the Wumpus save,
    - Denote pit & rumpus presence/absence y $P_{i,j}$ and $W_{i,j}$
  - There is a breeze if and only if there is a neighboring pit:
    - $B_{2,2} \Leftrightarrow P_{2,1} \vee P_{2,3} \vee P_{1,2} \vee P_{3,2}$
    - $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$
  - There is stench if the rumpus is in the next cavern
    - $S_{2,2} \Leftrightarrow W_{2,1} \vee W_{2,3} \vee W_{1,2} \vee W_{3,2}$
    - $S_{1,1} \Leftrightarrow W_{1,2} \vee W_{2,1}$
- **Percepts**
  - There is no pit at 1,1
    - $\sim P_{1,1}$
  - There is no breeze at 1,1
    - $\sim B_{1,1}$
  - There is a breeze at 2,1
    - $B_{2,1}$
- **Simple Entailment Algorithm**

```
def TruthTable-Entails(KB, a):
    symbols = proposition-symbols in KB and a
    return TT-Check-All(KB, a, symbols, {})

def TT-Check-All(KB, a, symbols, model):
    if empty(symbols):
        if pl-true(kb, model): # Does the KB entail the model?
          return pl-true(a, model) # Is  entail the model?
        else return True # return true when KB does not hold

    else: # recursively enumerate the models
     (s, others) = (first(symbols), rest(symbols))
     return TT-Check-ALL(KB, a, rest, model U {s=True}) and
              TT-Check-ALL(KB, a, rest, model U {s=False})
```

- **Summary of Model Checking**
    - Recursively generates all possible combinations of truth values for every symbol
    - Checks if the knowledge base is a subset of a specific symbol value assignment
        - If so, returns if sentence a hold as well
        - Otherwise returns true as we don't care about whether as holds outside the KB (implies)
    - Can we prove things without enumerating everything?
- **Concepts**
    - **Logical equivalence**
    - a = B if they are true in the same set of models
        - Alternative definition:
        - Equivalent if they entail one another
            - a =| B <=> b |= a
    - **Validity**
        - Sentences are valid (called **tautologies**) if they are true in all models
            - e.g. (a^b)v~aV~a
        - **Deduction theorem**
            - For arbitrary sentences a and B, a |= B iff (a => B) is valid
        - So if we can show (a <=> B) is a tautology, we know a |= B
- **Satisfiability**
    - There exists **some model** such that sentence is true
    - Sometimes easier to show something is valid by s**howing its contradiction is not satisfiable**:
        - a is valid iff ~a is not satisfiable
        - if no model satisfies ~a, then a must be true
    - which leads to: a |= B iff (a ^ ~B) is not satisfiable
    - Remember a |= B iff a => === ~a v B
- **Inference Rules**
    - **???**