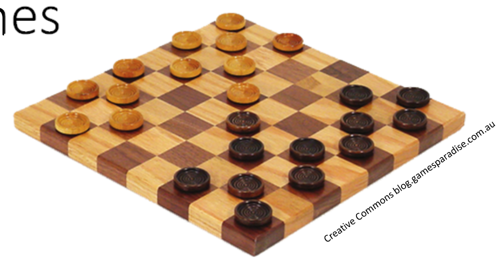
Adversarial Search

Professor Marie Roch

Chapter 5, Russell & Norvig

Two player games



Primary focuses

- Zero-sum games: What's good for you is bad for me
- turn-based (alternating)
- perfect information
- pruning: Removing portions of search tree

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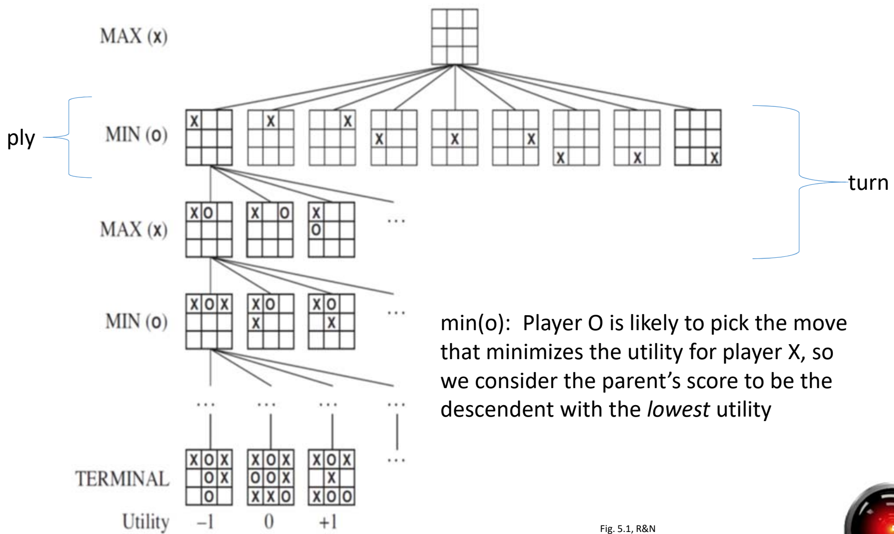
Game search problem

- initial state
- player – turn indicator
- actions – Set of legal actions
- result(state, action) – Result of player taking action
- terminal-test(state) – game over predicate
- utility(state, player) – utility of state for specified player

Game tree

- Game tree – exploration of the game search space
- Can be large
  - Chess – average branch factor 35
    (about 10^{40} distinct nodes, and 10^{154} states in a 50 move game)
  - Checkers, about 10^{40}
- Turn consists of two plys
- Frequently use the same utility function, maximizing for one player an minimizing for the other

Tic-tac-toe



min(o): Player O is likely to pick the move that minimizes the utility for player X, so we consider the parent's score to be the descendent with the *lowest* utility

Fig. 5.1, R&N

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Minimax algorithm

```
minimax(state)
   return arg(max_{a∈actions}(state) minval(result(state,a)))

maxval(state)
   if terminal(state), v = utility(state)
   else
      v = -∞
      for a in actions(state)
         v = max(v, minval(result(state, a)))
   return v

minval(state)
   if terminal(state), v = utility(state)
   else
      v = ∞
      for a in actions(state)
         v = min(v, maxval(result(state, a)))
   return v
```

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Trivial game tree

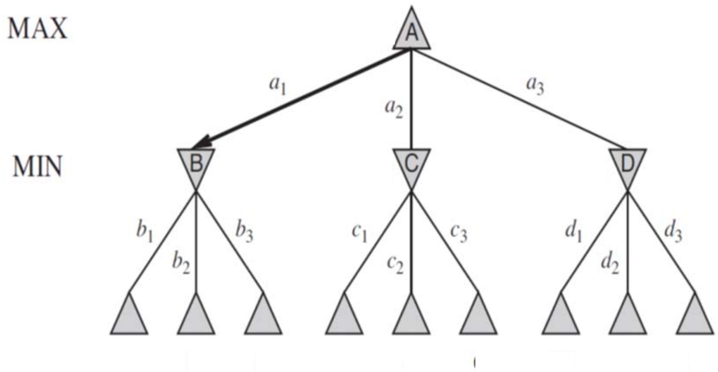

Fig. 5.2, R&N

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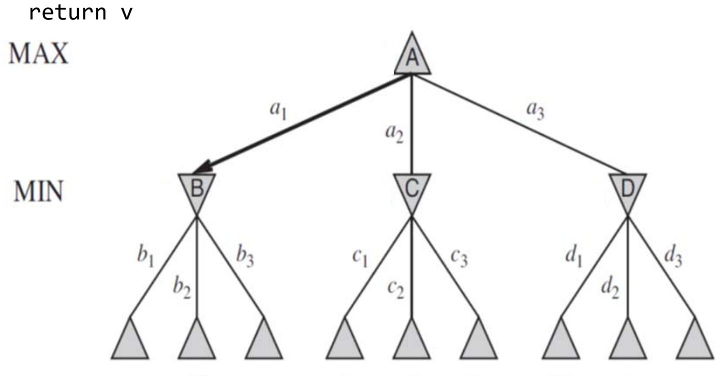
Trivial game tree

```
minimax(A)
   return arg(max_{a∈actions={a1,a2,a3}}(A) minval(result(A,a)))

minval(B)
   if terminal(B), v = utility(B)
   else
      v = ∞
      for a in {b_1,b_2,b_3}
         v = min(v, maxval(result(B, a)))
   return v
```



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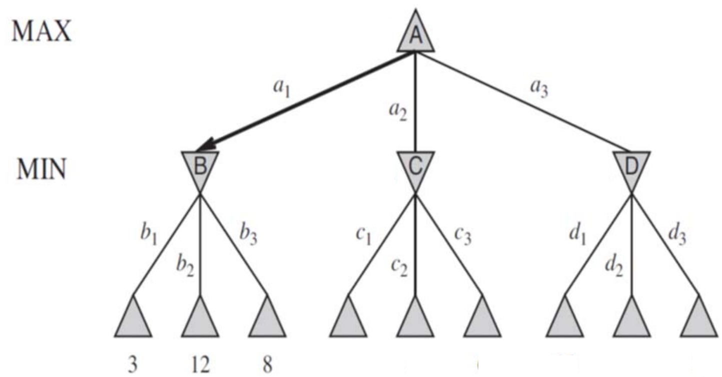
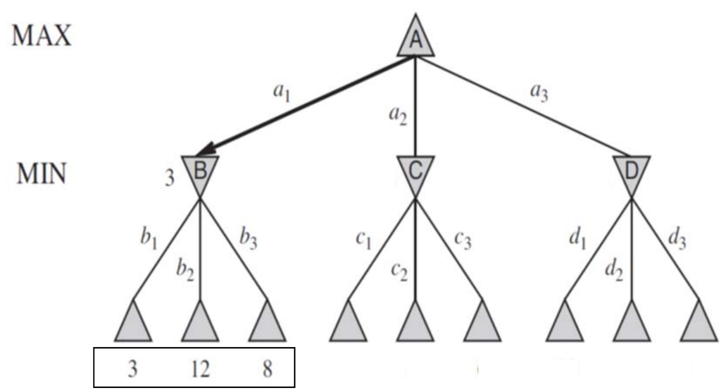
Trivial game tree

```
maxval(state)
   if terminal(state), v = utility(state)
   else
      v = -∞
      for a in actions(state)
         v = max(v, minval(result(state, a)))
   return v
```



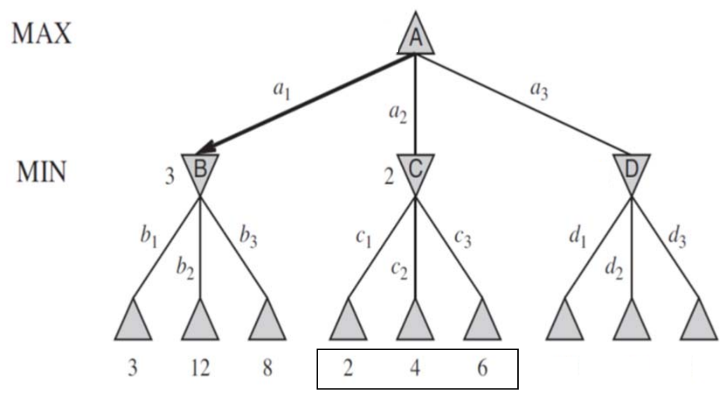
Trivial game tree

```
minval(B)
   if terminal(B), v = utility(B)
   else
      v = ∞
      for a in {b1,b2,b3}
         v = min(v, maxval(result(B, a)))
   return v
```



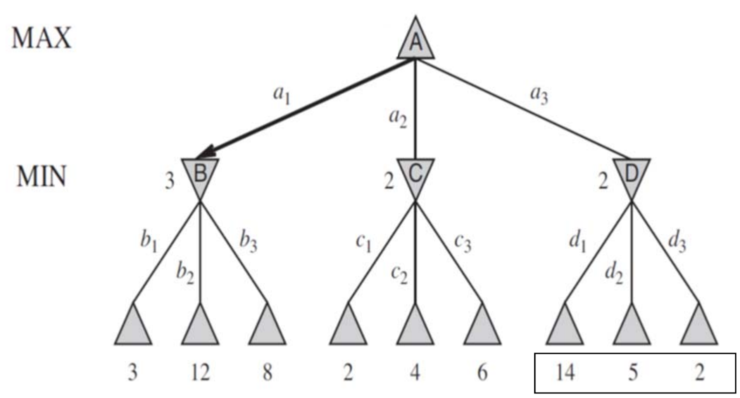
Trivial game tree

```
minval(C)
   if terminal(C), v = utility(C)
   else
      v = ∞
      for a in {c1,c2,c3}
         v = min(v, maxval(result(C, a)))
   return v
```



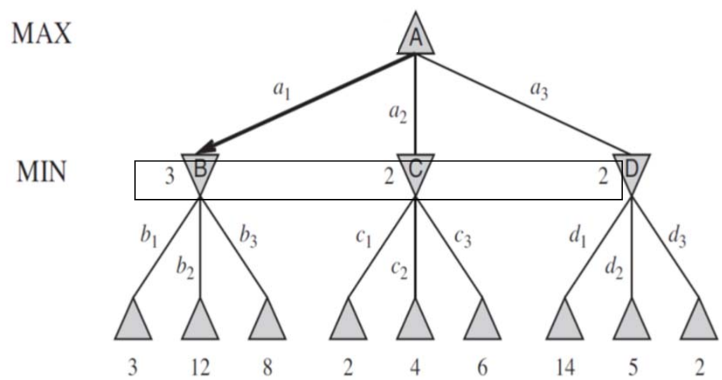
Trivial game tree

```
minval(C)
   if terminal(C), v = utility(C)
   else
      v = ∞
      for a in {c1,c2,c3}
         v = min(v, maxval(result(C, a)))
   return v
```



```
minimax(A)
   return arg(max_{a∈{a1,a2,a3}}(A) minval(result(A,a)))

best move a_1!
```



Trivial game tree

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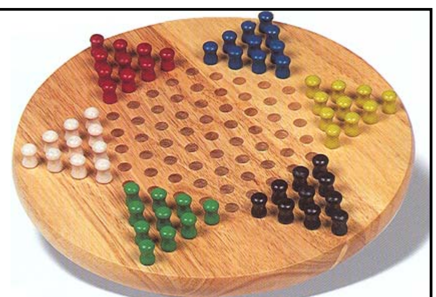
Multiplayer games



- Replace utility value with utility vector: $[u_1, u_2, \ldots, u_N]$
- When evaluating nodes, utility is interpreted as a function of the utility vector and the agent that produced the node.
  - Every player for themselves: $u(i) = u_i$
  - Alliances $u(i) = f([u_1, u_2, \ldots, u_N])$

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Multiplayer games (u_A, u_B, u_C)

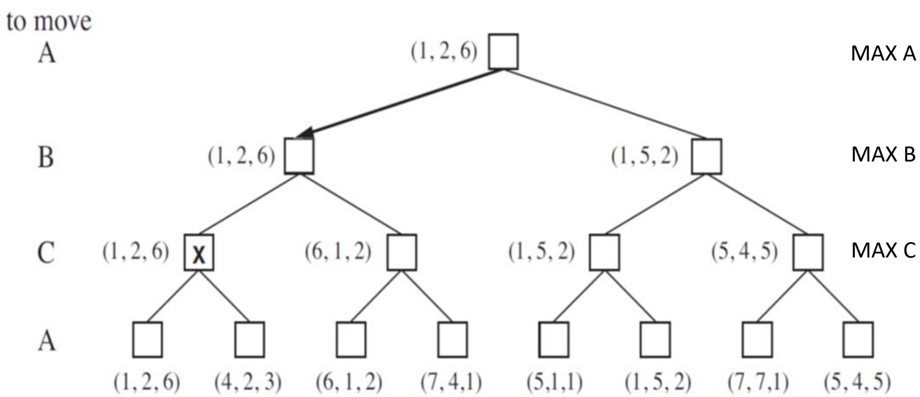

Fig. 5.3, R&N

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Game tree size

- Non-trivial trees are large
- Strategies to reduce search size:
  - alpha-beta pruning – Remove subtrees that can't influence the decision
  - move-ordering –order affects pruning performance
  - cutoff – Don't evaluate all the way to terminal nodes

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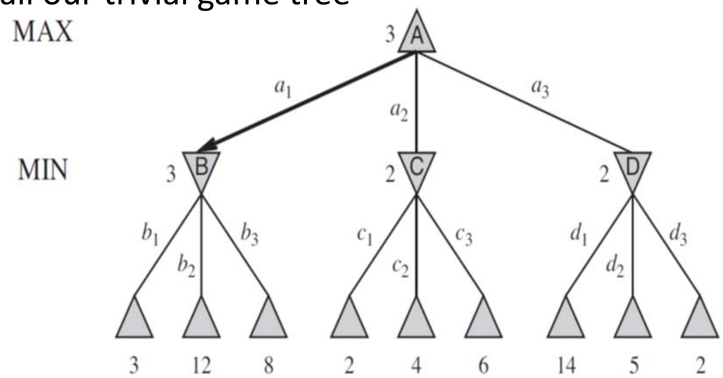
alpha-beta pruning

Recall our trivial game tree



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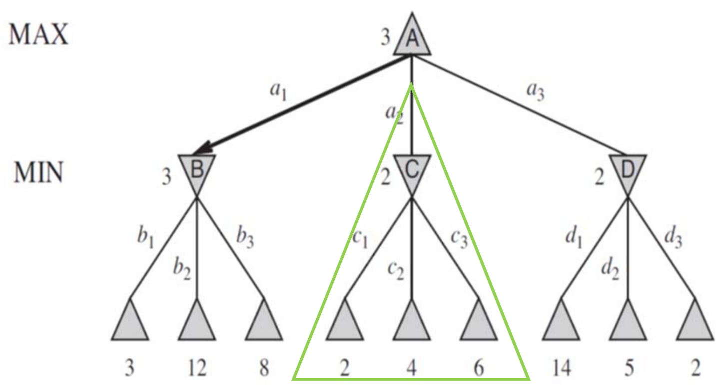
alpha-beta pruning



depth 1
max(3, 2, 2)
max(3, min(2,4,6), 2)

once we know 2, we don't need to look at siblings…

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alpha-beta pruning

- Prune partial computations
  - max(a_{min}, min(… <a_{min} …),)
    example from earlier slide
    max(min(3,12,8), min(2,4,6), min(14,5,2))

  - min(a_{max}, max(… >a_{max} …))
- Can be applied at any depth in tree
- Does not change minimax decision

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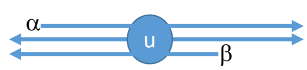
Bounding a node

- Possible values for a node
  - α - lower bound
  - β - upper bound



- α-β pruning looks for situations where $\alpha > \beta$

as these are not viable solutions

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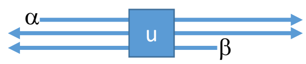
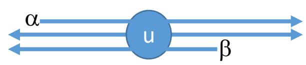
Bounding a node

- Try to increase lower bound of max nodes

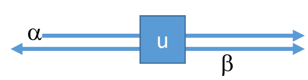


- Try to decrease upper bound of min nodes



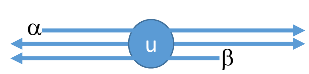
Bounding a node

- Max nodes – loop children



  If min node child value ≥ β
    return child value
    else see if we can increase lower bound α

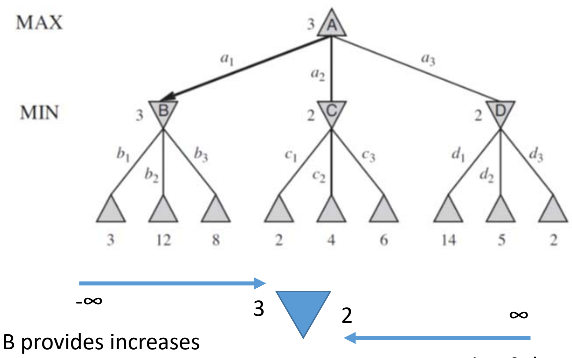
- Min nodes – loop children



  if max node child value ≤ α
    return child value
    else see if we can decrease upper bound β

Formal algorithm later, this is just to build your intuition

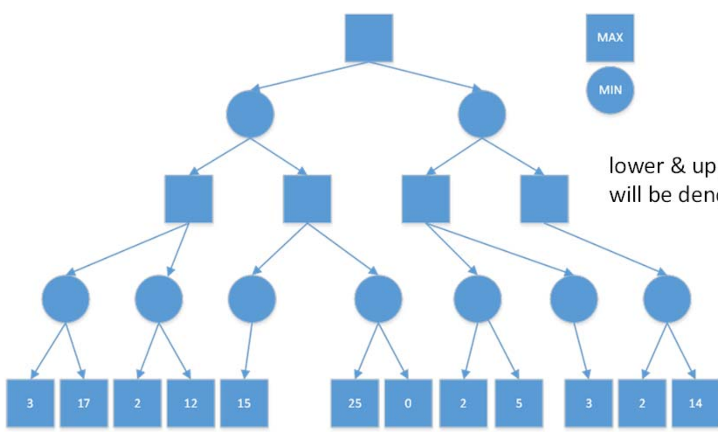
# Bounding example:
max(min(3,12,8), min(2,4,6), min(14,5,2))



Processing B provides increases the lower bound α to 3 for C

Processing C decreases the upper bound β to 2

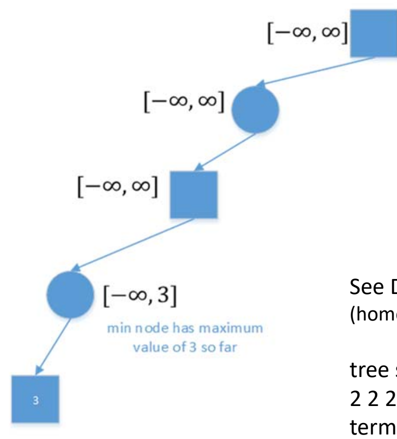
game tree



lower & upper bounds will be denoted [α,β]

Game tree credit: Bruce Rosen, UCLA

bounding a node



See Dr. José Manuel Torres's [alpha-beta animation](homepage.ufp.pt/jtorres/ensino/ia/alfabeta.html)
(homepage.ufp.pt/jtorres/ensino/ia/alfabeta.html)

tree structure:
2 2 2 2 2 2 1 2 2 1 2 2 1 2
terminal values:
3 17 2 12 15 25 0 2 5 3 2 14

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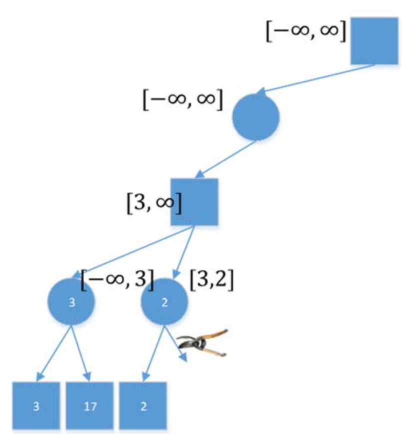
bounding a node

each node has $[\alpha,\beta]$
α - lower bound
β - upper bound



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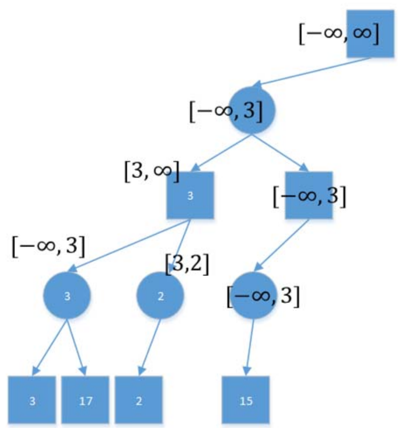
bounding a node

each node has [α,β]
α - lower bound
β - upper bound



This case is tricky, let's look at the algorithm in detail before we proceed.

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alpha-beta algorithm

```
alpha-beta search(state)
   v = maxvalue(state, α=-∞, β=∞)
   return action in actions(state) with value v
```

```
maxvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = -∞
      for a ∈ actions(state)
         v = max(v, minvalue(result(state, a), α, β)
         if v ≥ β then break else α = max(α, v)
   return v
```

```
minvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = ∞
      for a ∈ actions(state)
         v = min(v, maxvalue(result(state, a), α, β)
         if v ≤ α then break else β = min(β, v)
   return v
```

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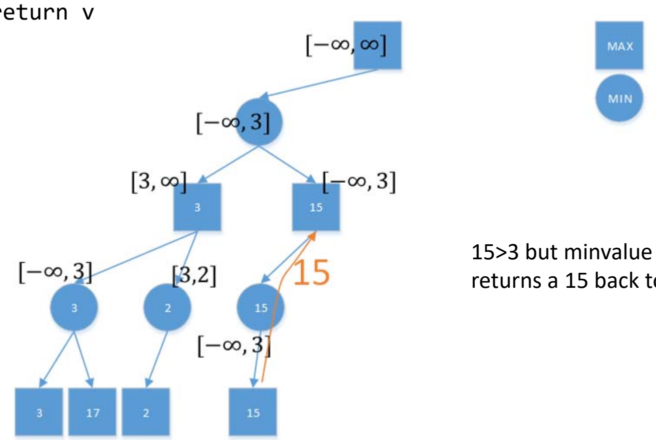
```
minvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = ∞
      for a ∈ actions(state)
         v = min(v, maxvalue(result(state, a), α, β)
         if v ≤ α then break else β = min(β, v)
   return v
```



15>3 but minvalue only checks α
returns a 15 back to parent

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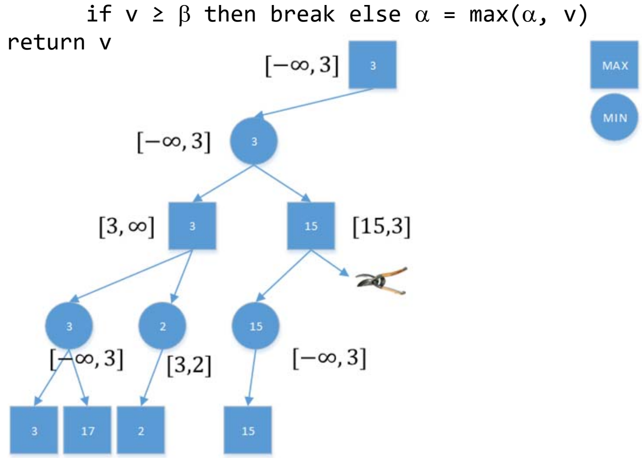
```
maxvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = -∞
      for a ∈ actions(state)
         v = max(v, minvalue(result(state, a), α, β)
         if v ≥ β then break else α = max(α, v)
   return v
```



proceed along right subtree at home

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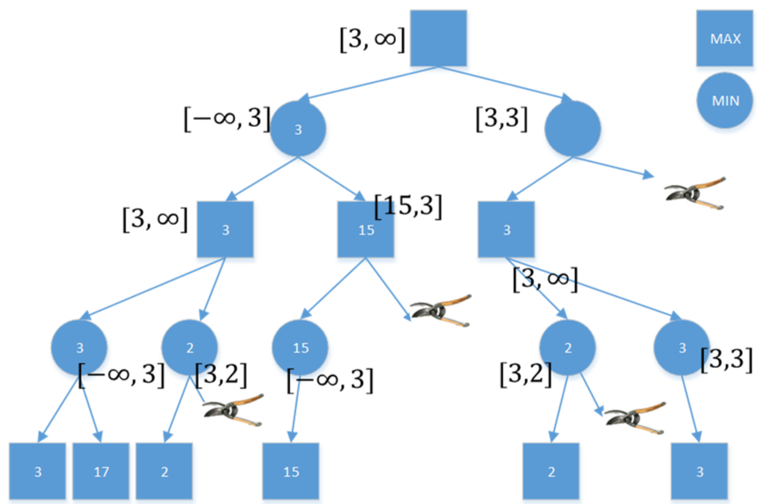
final tree



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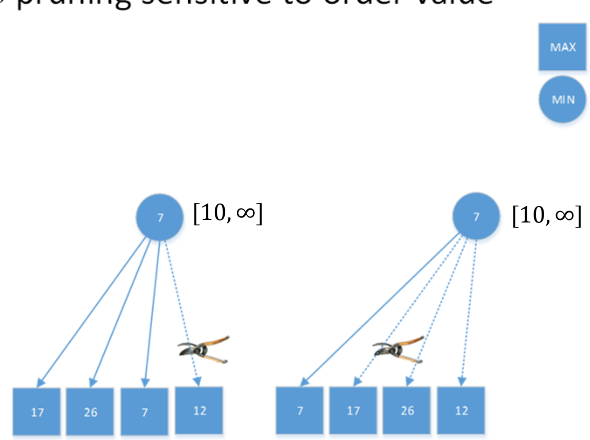
Move ordering

- α-β *pruning sensitive to order value*



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Move ordering

- killer move heuristic
  - iterative deepening search to ply above
  - use heuristic value to order nodes
- games frequently have repeated states
  - *transposition table* stores heuristic of visited states
  - only store good states (heuristic required)
  - favor states in transposition table

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Okay, so you're not Brad Pitt

Imperfect real-time decisions

- Replace utility function with an evaluation function
  - examines non-terminals
  - heuristic giving strength of current state
- How deep should we search?
  - cutoff test provides decision of whether to explore or apply evaluation function

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Evaluation function

- Estimate of the expected utility
- Performance strongly linked to evaluation function choice.
- Guidelines
  - Order terminal states in the same order as the utility function, e.g. $u(a) \leq u(b) \leq u(c) \rightarrow e(a) \leq e(b) \leq e(c)$
  - Estimator should be
    - correlated with odds of winning
    - fast

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Features…

help us identify something based on state or percept.

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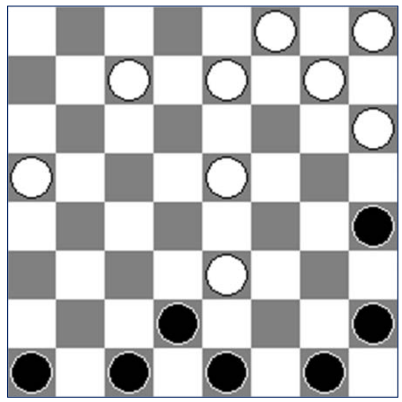
Who has the better position?



Possible features for checkers?

Most features are relative to the opponent.
Count player & opponent and subtract one from other

- Number of pieces
- Number of kings
- Offense
  - Advance of pawns (# moves away from being kinged… sum, mean, max)
  - Number of possible: moves, captures (by pawn/king?)

Possible features for checkers?

- Defense, are pieces
  - On edges of board? (better defended)
  - protected by neighbors of the same color (good) or menaced by neighbors of the opposite color (bad)?
- These features need to be weighted and combined, typically as a wieighted linear combination

$$f_{eval}(board) = \sum_{i=1}^{\#\text{features}} w_i f_i(board)$$

- Determine weights?
  - genetic algorithms?
  - other learning techniques? (beyond our reach for now)

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Pruning search

- Game-tree search → $$$
- Replace terminal-test with

```
if cutoff-test(state, depth), return f_eval(board)
```

- How do we define this?

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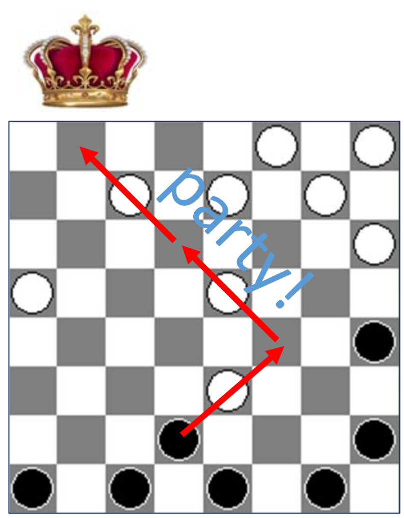
cutoff-test

- Fixed depth – simple
- Timed iterative deepening
  - Run consecutively deeper searches
  - When time runs out, use deepest completed search

cutoff-test – Prefer quiescent states

- Sometimes, a state can change value radically in a few moves.
- Quiescence search
  - Search a few plys from a candidate cutoff node.
  - Look for relative stability of evaluation function → quiet state



Opening moves


Ruy Lopez – 1400s


Sicilian defense 1600s


Queens gambit ca. 1490


Modern defense


Dutch defense


Stonewall attack

excerpted from chess.com blog post by MonsterKing 2011/7/26

Lookup

- Reasonable to search billion game-tree nodes to move a pawn at beginning of game?
- Common to rely on table lookup for openings. Rely on human experience

Lookup and retrograde search

- Endgames
  - Have a reduced search space
  - *retrograde search* - *B*ackwards minimax search backwards from terminal states

Stochastic games

- Consist of
  - probability distributions
  - strategy contingent on probabilities
- Game trees need to somehow incorporate chance
- We will review a few concepts about probability before diving in

Probability Distributions

- Show the probability of an event happening.
  - Distributions are nonnegative
  - Must sum to 1
- Example with a die
  - P(X=3) denotes the probability of rolling a 3
  - Fair die → P(X=3) = 1/6
- Distributions of continuous variables are sometimes described by parameterized formulae

Probability distributions

- Some games depend on independent events
- Example – Roll two dice
- Probabilities that are independent can be multiplied.

$$P(R=1)P(G=6)=\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$$

- Many times, we only care about the probability of 1 and 6, not which die produced it. How do we do this?

Expectation operator

$$E[u(X)] = \sum_{x} u(X = x)\,\mathrm{P}(X = x)$$

When u(X) = X, we call this the mean, or average value:

$$\mu = E[X]$$

Notes

- Distinguish between a random variable X and an instance of a random variable x.
- The X=x notation is frequently dropped and just x is used.
- P(x) is frequently denoted as f(x)

Backgammon

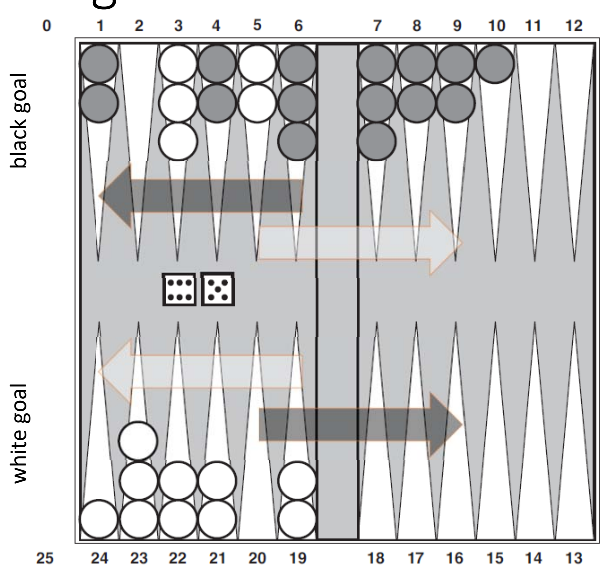
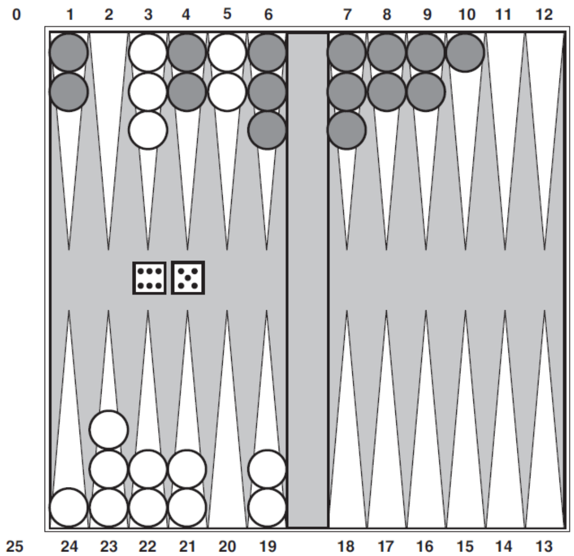


Fig. 5.10, R&N

Backgammon



Rules in a nutshell
(not everything)

- Move into goal by exact count
- Can capture opponent and send to bar by landing on a *single* opponent.
- Roll can be divided amongst pieces or combined

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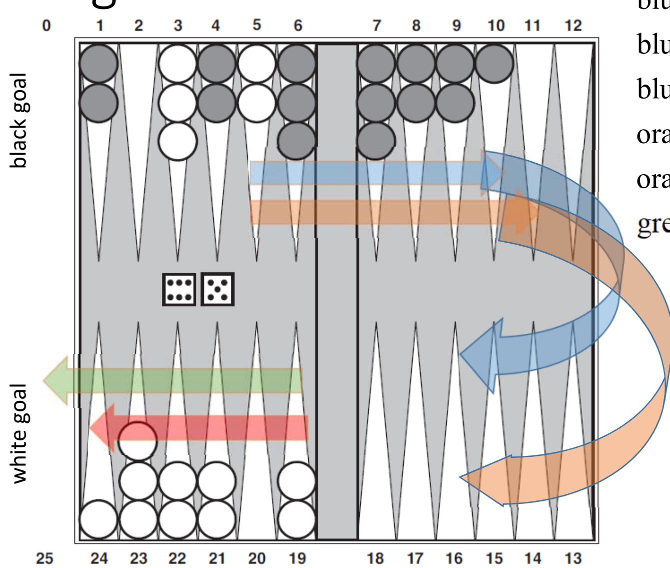
Backgammon



moves
blue + orange
blue + blue
blue + green
orange + orange
orange + red
green + red

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Stochastic game search

- Incorporate chance nodes into min/max search tree
- Each edge denotes an outcome with a probability

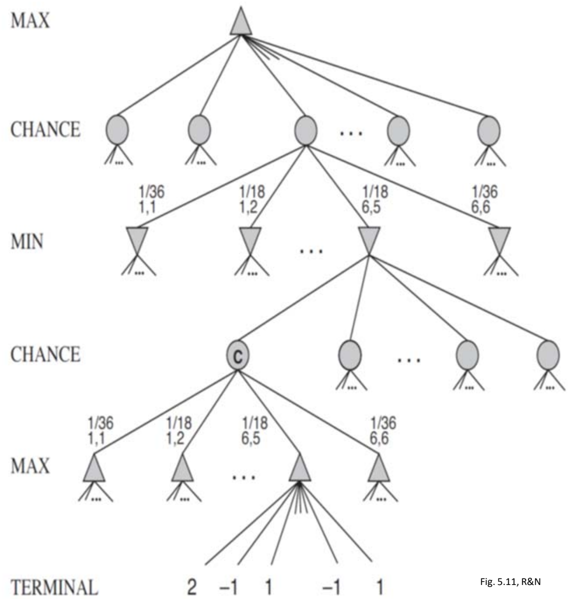

Fig. 5.11, R&N

Stochastic game search

- Minimax search is still the goal
  but how do we pick extrema in the face of uncertainty?
- For each chance node, compute the expected outcome

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Stochastic minimax

```
Eminimax(searchnode):
   switch type of searchnode:
      case terminal:
         return utility(searchnode.state)
      case maxnode:
         return arg max_{a∈actions}(Eminimax(result(searchnode, a)))
      case minnode:
         return arg min_{a∈actions}(Eminimax(result(searchnode, a)))
      case chance:
         Eminimax = Σ_{r∈roll} P(r) Eminimax(result(searchnode, r)))
```

note: multiple actions may be associated with a roll, so last case is a little more complicated.

Stochastic games and evaluation

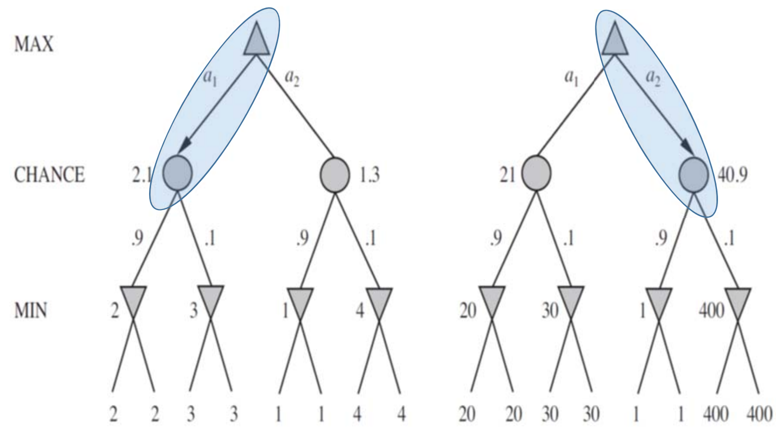
- Not as clear as with deterministic games
- Consider possible values for three game states

State	1	2	3	4
eval fn A	1	2	3	4
eval fn B	10	20	30	400

In a standard minimax routine, we would come up with the same solution, it is the relative ordering that is important.

What about here?

Stochastic games and evaluation



note similarity to and-or trees

State	1	2	3	4
eval fn A	1	2	3	4
eval fn B	10	20	30	400

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Stochastic minimax search

- Chance dramatically increases the branching factor
- Common not to search for more than a few plys
- It is possible to modify alpha-beta pruning to work on bounds of expected values [beyond our scope]

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Stochastic search

Monte-Carlo simulations are an alternate strategy

- Play millions of games using some search algorithm
- Assign state values based on results of wins/losses of the millions of simulated games

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Partial information

you will not be held responsible for this material

- Not all games fully observable
- Basic ideas
  - maintain a belief state
  - use and-or trees to represent possible states
    - problematic: lots of states

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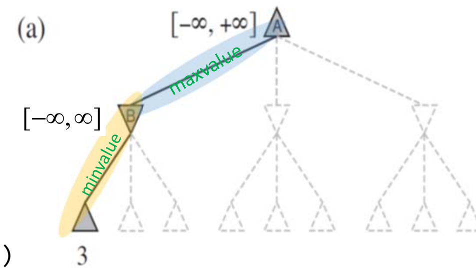
additional example
alpha-beta pruning

```
maxvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = -∞
      for a ∈ actions(state)
         v = max(v, minvalue(result(state, a), α, β)
         if v ≥ β then break else α = max(α, v)
   return v
```




```
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         v = min(v, maxvalue(result(state, a), α, β)
         if v ≤ α then break else β = min(β, v)
   return v
```
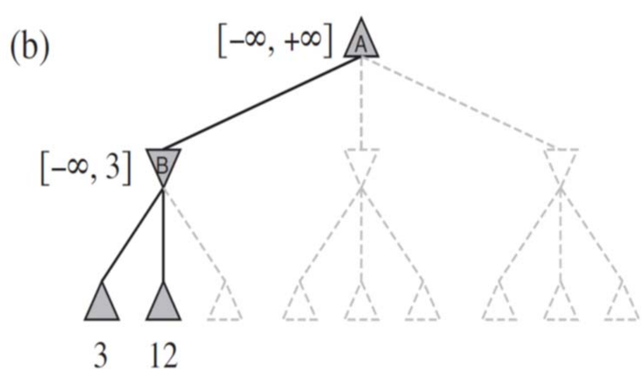

63

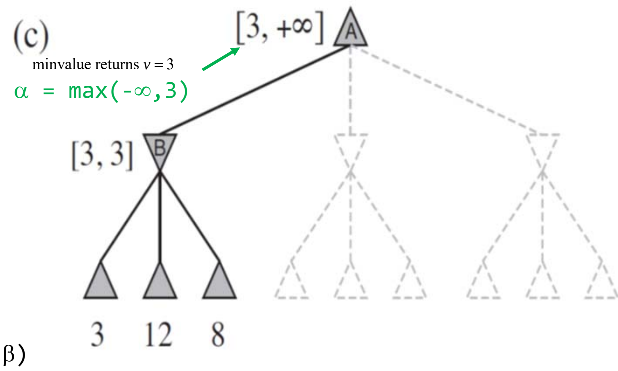
alpha-beta pruning example



Note: When minvalue/maxvalue break their loop, they do not actually set α and β but for emphasis, we will show the values at nodes as if they did.

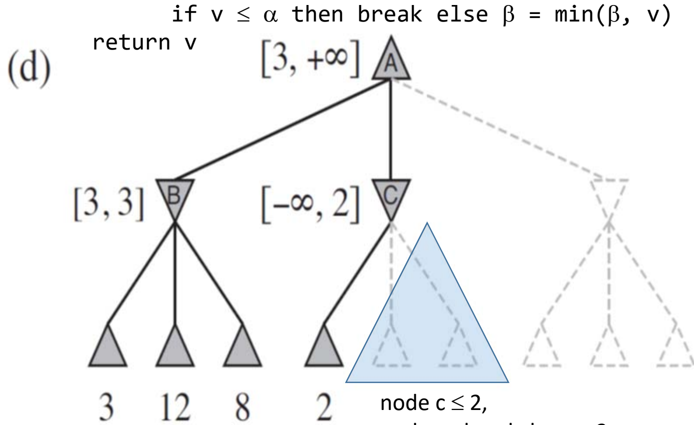
64

alpha-beta pruning example



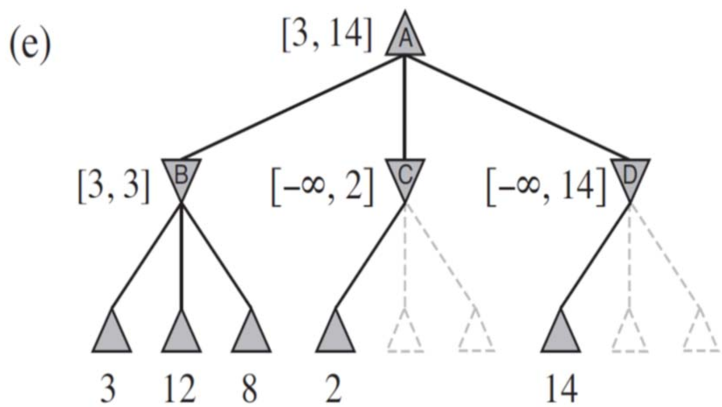

```
maxvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = -∞
      for a ∈ actions(state)
         v = max(v, minvalue(result(state, a), α, β)
         if v ≥ β then break else α = max(α, v)
   return v
```

```
minvalue(state, α, β)
   if terminal(state) then v = utility(state)
   else
      v = ∞
      for a ∈ actions(state)
         v = min(v, maxvalue(result(state, a), α, β)
         if v ≤ α then break else β = min(β, v)
   return v
```

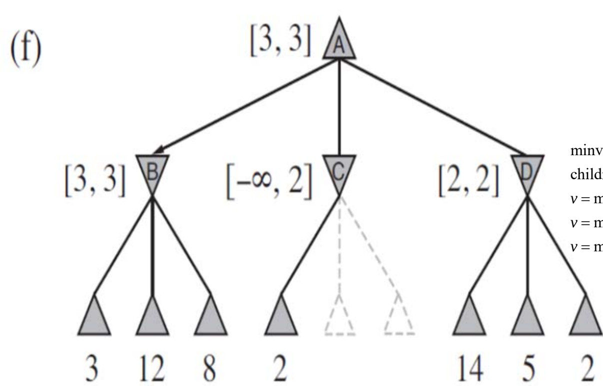


node c ≤ 2,
node a already has α=3
no need to look further…

alpha-beta pruning example



alpha-beta pruning example



minvalue(result(A,actionAtoD),3,
children of D have utilities [14, 5, 2]

$v = \min(\infty, 14) = 14$

$v = \min(14, 5) = 5$

$v = \min(5, 2) = 2$