When do we use matrix transpose and why?

3 Answers

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Updated Feb 15, 2016

While the answers before me are all technically correct, there isn't much of an answer as to why the idea of matrix transposes exist in the first place, and why people cared enough to invent it.

This is partly because the original question is very close to asking "Why does linear algebra exist as a subject?", which is a little bit abstract. Linear algebra is the study of vectors (a finite sequence of numbers) and *linear functions* that act on those vectors. It's usage can be found in pretty much every endeavor involving mathematics in some way, from computation and statistics to the natural sciences (note that plugging in numbers to equations is **not math**). Why is it so prevalent and what makes it so useful?

Recall that between two vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}^T\mathbf{v}$ is equal to the *dot product* between $\mathbf{u}$ and $\mathbf{v}$.

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \, \|\mathbf{v}\| \cos\theta,$$

where θ is of course the angle between $\mathbf{u}$ and $\mathbf{v}$ and $\|\mathbf{u}\|$ is the length of $\mathbf{u}$, and vice versa. The squared length of a vector (e.g. $\mathbf{u}$) is therefore given by $\mathbf{u} \cdot \mathbf{u}$.

Thus we see immediately that the transposition of vectors are critical for providing the properties of *sizes* and *angles*. In fact, the reason why linear algebra is so useful is that *vectors are the **simplest** mathematical objects for which notions of sizes and angles, and thus **similarity**, can be provided*.

Now, how does this relate to matrices? At this point it is important to consider what a matrix is, for which I present two perspectives:

1. It is a bunch of (column) vectors stacked side by side, OR
2. It represents a *linear* function acting on a vector (e.g. $\mathbf{y} = \mathbf{A}\mathbf{x}$).

If we use the first perspective, suppose that $\mathbf{A} = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_m]$, $\mathbf{B} = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \cdots \quad \mathbf{b}_k]$, and $\mathbf{P} = \mathbf{A}^T\mathbf{B}$. Let the i, j-th entry of $\mathbf{P}$ be given by p_{ij}. Then it shouldn't be too hard to see that

$$p_{ij} = \mathbf{a}_i \cdot \mathbf{b}_j,$$

so the product $\mathbf{A}^T\mathbf{B}$ is nothing more than a table of how the vectors stacked by $\mathbf{A}$ and $\mathbf{B}$ relate to each other through *size and angle*!

But lets try to apply this to the second perspective, by setting $\mathbf{P} = \mathbf{y}$, $\mathbf{M} = \mathbf{A}^T$ and $\mathbf{B} = \mathbf{x}$. Since $\mathbf{P}$ and $\mathbf{B}$ are now just vectors, we can therefore see that each entry of $\mathbf{y} = [y_1 \quad y_2 \quad \cdots \quad y_m]^T$ is just the dot product between the columns of $\mathbf{A}$ and $\mathbf{x}$. This means that **every *linear* function** taking a vector $\mathbf{x}$ as an argument essentially just takes dot products between a collection of

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notions of size and angle, and gave a clue towards why these properties can be so incredibly beneficial.

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Pablo Carneiro Elias, works at Tecgraf/PUC-Rio
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If you have an orthonormal matrix (e.g: a rotation matrix) then you use the transpose to get the inverse transformation (e.g: the inverse rotation). This happens because by switching columns and rows of a orthonormal matrix you are exchanging domain space and image space of the transformation, but since they are both orthonormal there is a simple way of getting between them which is exactly the original matrix and the transpose matrix. So the inverse transform is the transpose in that case and for those cases $RR^T = I$, being R orthonormal. Its computationally cheaper than actually finding the inverse through gaussian reduction or any other method. Thats a very good use of matrix transposition.

Jorge Andres Gonzalez Sierra, studied at National University of Colombia
Answered Mar 3, 2015

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