INDUCTIVE AND CAPACITIVE COUPLING

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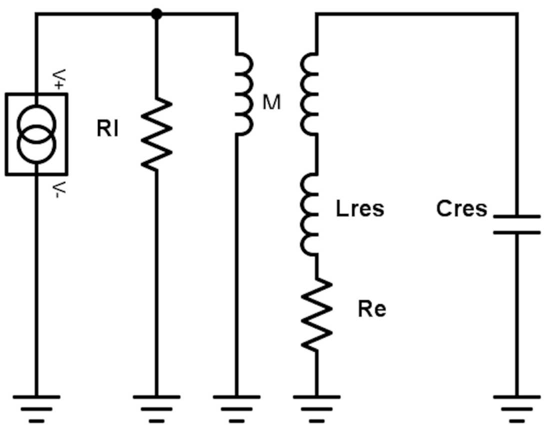
Nomenclature

ω_r	Resonant cavity frequency
C_{Res}	Resonator Capacitance
C_c	Coupling Capacitance
L_{Res}	Resonator Inductance
M	Coupling Inductance
Q_c^C	Capacitive coupling quality factor
Q_c^L	Inductive Coupling quality factor

Inductive Coupling



We begin by defining the noise current power spectral density for the input side of the circuit.

$$S_I = \frac{4k_bT}{R_l} \tag{1}$$

The induced noise voltage on the resonator is then

$$S_V = 4k_bTR_e \tag{2}$$

We can relate (1) and (2) via the mutual inductance, M and the frequency.

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$$\frac{4k_bT}{R_l}(\omega M)^2 = 4k_bTR_e \tag{3}$$

Solving for R_e yields

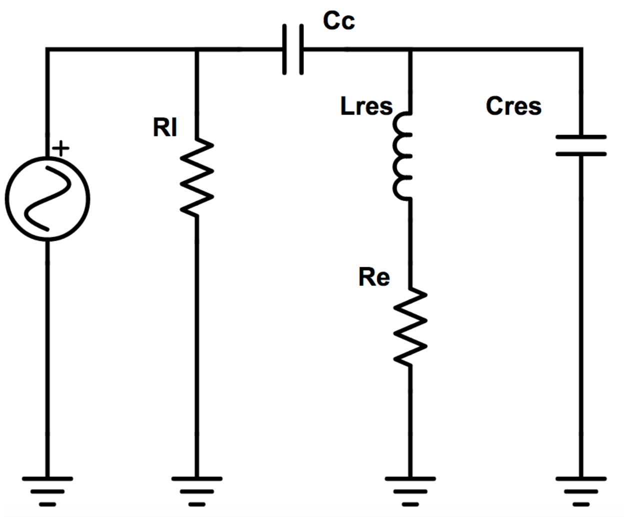
$$Re = \frac{(\omega M)^2}{R_l} \tag{4}$$

The inductive coupling quality factor is then defined as

$$Q_c^L = \frac{\omega L_{\text{res}}}{R_e} = \frac{L_{\text{res}}R_l}{\omega M^2} = \frac{2Z_0L_{\text{res}}}{\omega M^2} \tag{5}$$

Where the substitution $R_l = 2Z_0$ has been made.

Capacitive Coupling



We begin by defining the noise voltage power spectral density for the input side of the circuit

$$S_V = 4k_bTR_l \tag{6}$$

This induces a noise current in the resonator of the form

$$S_I = \frac{4k_bT}{R_e} \tag{7}$$

This can be equated via the coupling impedance

$$\frac{S_I}{(\omega C_c)^2} = S_V \tag{8}$$
$$\frac{4k_bT}{R_e(\omega C_c)^2} = 4k_bTR_l \tag{9}$$
$$R_e = \frac{1}{(\omega C_c)^2 R_l} \tag{10}$$

We can then define the coupling quality factor as

$$Q_c^C = \omega R_e C \tag{11}$$
$$= \frac{\omega C}{(\omega C_c)^2 R_l} \tag{12}$$
$$= \frac{C}{2Z_0\omega C_c^2} \tag{13}$$

Where we have again made the substitution $R_l = 2Z_0$. One can see that (13) matches with the result obtained in Goppl, et al. in the limit that $(\omega C_c R_l)^2 \ll 1$