

1 Uncertainty Principle

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

consequence of wave like nature of particle from de Broglie hypothesis

Non relativistic de Broglie relation

$$\hbar k = mv$$

$$\hbar \frac{2\pi}{\lambda} = mv$$

$$\lambda = \frac{h}{mv}$$

Momentum and position are conjugate variables of one another. Wave function in either space is related to the other via a Fourier Transform.

$$\psi(x) = \int_{-\infty}^{\infty} \phi(p) e^{ipx} dp$$

Variance in a variable

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi(x)|^2 dx$$

$$f = x \cdot \psi(x)$$

$$g = p \cdot \phi(p)$$

Cauchy-Schwartz Inequality

$$\sigma_x^2 \sigma_p^2 \geq |\langle f|g \rangle|^2$$

$$z = \langle f|g \rangle$$

$$|\langle f|g \rangle|^2 = |z|^2 = \Re[z]^2 + \Im[z]^2 \geq \Im[z]^2 = \left(\frac{z - z^*}{2i} \right)^2$$