CQED PROBLEMS

MATTHEW BECK

1. Prove $e^{i\delta}\left|n\right\rangle = \left|n+1\right\rangle$

Beginning with the commutation relation $[\hat{\delta}, \hat{n}] = i$ and $\hat{n} = \frac{1}{i}\frac{\partial}{\partial\delta}$

$$[e^{i\hat{\delta}}, \hat{n}]\left|\psi\right\rangle = (e^{i\hat{\delta}}\hat{n} - \hat{n}e^{i\hat{\delta}})\left|\psi\right\rangle \tag{1}$$

$$= e^{i\hat{\delta}}\frac{1}{i}\frac{\partial\left|\psi\right\rangle}{\partial\delta} - e^{i\hat{\delta}}\frac{1}{i}\frac{\partial\left|\psi\right\rangle}{\partial\delta} - e^{i\hat{\delta}}\left|\psi\right\rangle \tag{2}$$

$$= -e^{i\hat{\delta}}\left|\psi\right\rangle \tag{3}$$

We can now use the results of (3) we can find $\hat{n}e^{i\hat{\delta}}\left|n\right\rangle$

$$\hat{n}e^{i\hat{\delta}}\left|n\right\rangle = \hat{n}e^{i\hat{\delta}}\left|n\right\rangle - e^{i\hat{\delta}}\hat{n}\left|n\right\rangle + e^{i\hat{\delta}}\hat{n}\left|n\right\rangle \tag{4}$$

$$= [\hat{n}, e^{i\hat{\delta}}] + e^{i\hat{\delta}}\hat{n}\left|n\right\rangle \tag{5}$$

$$= e^{i\hat{\delta}}\left|\psi\right\rangle + e^{i\hat{\delta}}\hat{n}\left|n\right\rangle \tag{6}$$

$$= e^{i\hat{\delta}}\left|\psi\right\rangle + e^{i\hat{\delta}}n\left|n\right\rangle \tag{7}$$

$$= (n+1)e^{i\hat{\delta}}\left|n\right\rangle \tag{8}$$

We can equate (8) as

$$(n+1)e^{i\hat{\delta}}\left|n\right\rangle \equiv \hat{n}\left|n+1\right\rangle \tag{9}$$

which inherently defines

$$e^{i\hat{\delta}}\left|n\right\rangle = \left|n+1\right\rangle \tag{10}$$

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