

Presented here is a detailed calculation for the intrinsic quality factor for the copper CPW RT set up including calculations for the coupling capacitor and the vacuum voltage and Rabi Frequency.

The capacitance and inductance per unit length of non-superconducting CPW's are functions of geometry alone. There are defined through conformal mapping and take the form

$$C_l = 4\epsilon_0\epsilon_{eff} \frac{K(k)}{K(k')} \quad (1)$$

$$L_l = \frac{\mu_0}{4} \frac{K(k')}{K(k)} \quad (2)$$

$$Z_0 = \sqrt{\frac{L_l}{C_l}} \quad (3)$$

Where:

$$k = \frac{w}{w + 2s} \quad (4)$$

$$k' = \sqrt{1 - k^2} \quad (5)$$

$$K \rightarrow \text{Complete elliptic integrals} \quad (6)$$

For a 20 μm wide center trace, to achieve an impedance of 100 Ω , the ground plane spacing, s , is calculated to be 111 μm yielding

$$C_l = 8.26 \times 10^{-11} \text{F/m} \quad (7)$$

$$L_l = 8.27 \times 10^{-7} \text{H/m} \quad (8)$$

$$(9)$$

Near resonance, a CPW can be approximated by a parallel LCR circuit with characteristic values [Goppl /Gupta]

$$C_{LCR} = \frac{C_l l}{2} \rightarrow 497 \text{fF} \quad (10)$$

$$L_{LCR} = \frac{2L_l l}{n^2 \pi^2} \rightarrow 2.02 \text{nH} \quad (11)$$

$$R = \frac{Z_0}{\alpha_{Loss} l} \quad (12)$$

where:

$$l = \frac{\lambda}{2} = \frac{v_{ph}}{2f} = \frac{c}{\sqrt{\epsilon_{eff}}2f} \quad (13)$$

The loss term in (12) has 3 contributions, namely

$$\alpha_{Loss} = \alpha_{cond} + \alpha_{di} + \alpha_{rad} \quad (14)$$

Of which the conduction and radiation term are negligible (4 orders of magnitude smaller) compared to that of the dielectric term for a copper conductor at room temperature.

Using sapphire as a substrate ($\epsilon_{del} = 9.6$, $\tan(\delta) = 2 \times 10^{-4}$ @ RT), the loss term, as defined by [Gupta] is

$$\alpha_{di} = \frac{2.73\epsilon_{rel}(\epsilon_{eff} - 1)\tan(\delta)}{\sqrt{\epsilon_{rel}}(\epsilon_{rel} - 1)\lambda_0} = 5.25 \times 10^{-2} \quad (15)$$

Which, in turn leads to an LCR resistor values of $R_{LCR} = 158151\Omega$. The internal Q of the resonator can now be calculated using

$$Q_{int} = \omega_n RC = n\omega_0 R_{LCR} C_{LCR} = 2483n \quad (16)$$

The input / output capacitors can be engineered to maintain critical coupling ($Q_{ext} = Q_{int}$) so that the loaded Q, $Q_{load} \approx Q_{int}$. The calculation is omitted here but to achieve critical coupling assuming negligible output capacitance, the input capacitor would have to be 11.2 fF, a number readily achievable with the interdigitated capacitor design.

Moving onto the vacuum voltage, we now set the energy stored in the electric field due to the capacitance of the CPW and set it equal to 1/2 the energy of the vacuum photon state

$$\frac{1}{2}C_{LCR}V_{vac}^2 = \frac{\hbar\omega}{4} \quad (17)$$

$$V_{vac} = \sqrt{\frac{\hbar\omega}{2C_{LCR}}} \quad (18)$$

$$= 1.86\mu V \quad (19)$$

The electric field at the grounded gap capacitor is now

$$E_{vac} = \frac{V_{vac}}{d} \quad (20)$$

where d is the gap spacing. At a spacing of 10 μm the vacuum electric field is .186 V/m which, when combined with a dipole moment of $\sqrt{2/9}8360ea_0$ yields a vacuum Rabi frequency of

$$\frac{\Omega}{2\pi} = \frac{E_{vac}d_{Ryd}}{2\pi\hbar} = 9.22 \text{ MHz}$$

This now scales as $1/d$, the spacing of the tall end gap capacitor, so doubling the gap size to $20 \mu\text{m}$ results in a 4.61 MHz coupling, etc.

The next step is to model how the difference in height between the ground planes and center pin will effect everything detailed here. Just by looking at the relationships, however, I feel that reducing the effective area for the geometric capacitance will by in large increase the impedance of the line which will in turn increase the resistance of the effective LCR circuit. the intrinsic Q scales with this R so we might get a small but noticeable boost in the intrinsic Q. We'll see what HFSS, MEAP, SONNET, etc. deem necessary to tell or not tell us.