SFQ - Qubit Parameters

July 22, 2016

Nomenclature

ω_0	Angular frequency
C_c^{Driver}	Coupling capacitance between CPW resonator and SFQ driver
L_c	Coupling inductance between resonator and feedline
Q_c^{ind}	Inductive coupling quality factor
S	CPW Gap width
W	CPW Center trace width
Z_{CPW}	CPW Impedance

For our CPW resonators, W and S are fixed at 10 and 6 μm, respectively. On silicon ($\epsilon_{\text{Rel}} = 11.7$), this gives an impedance of

$$W = 10\,\mu\text{m} \tag{1}$$
$$S = 6\,\mu\text{m} \tag{2}$$
$$Z_{\text{CPW}} \approx 51\,\Omega \tag{3}$$

as calculated with QUCS transmission line software.
We want to aim for frequencies of 6.55 and 6.45 GHz. The correct lengths of CPW lines for these bare frequencies for a relative dielectric constant $\epsilon_{\text{rel}} = 11.7$ are

Freq (GHz)	**l (mm)**	**angle (deg)**
6.65	4.50	89.94
6.55	4.57	90.14

These frequencies will be pulled down by both the capacitive coupling to the sfq driver and inductive coupling to the feedline. The equivalent capacitance and inductance for a quarter-wavelength CPW line as given by Pozar is

$$C_{\text{Equiv}} = \frac{\pi}{4\omega_0 Z_{\text{CPW}}} = 3.85 \times 10^{-13}\,\text{F} \tag{4}$$
$$L_{\text{Equiv}} = \frac{1}{\omega_0^2 C_{\text{Equiv}}} = \frac{4Z_{\text{CPW}}}{\omega_0 \pi} = 1.56 \times 10^{-9}\,\text{H} \tag{5}$$

Shaojiang asked to have $C_c^{\text{Driver}} = 6$, 12 fF coupling to the SFQ driver. These are $C_c^{\text{Driver}}/C_{\text{Equiv}} \times 100 = 1.56, 3.12\%$ corrections to the total capacitance. Additionally, a desired coupling Q of 1×10^5 yields for its coupling inductance

$$Q_c^{\text{ind}} = \frac{2Z_{\text{CPW}} L_{\text{Equiv}}}{\omega_0 L_c^2} \rightarrow L_c = \sqrt{\frac{2Z_{\text{CPW}} L_{\text{Equiv}}}{\omega_0 Q_c^{\text{ind}}}} = 6.18\,\text{pH} \tag{6}$$

This inductance is a $L_c/L_{\text{Equiv}} \times 100 = 0.4\%$ correction to the total inductance. We can sub the added reactive elements and calculate a new resonator frequency.

$$
\begin{aligned}
\omega^2 &= \frac{1}{L_T C_T} = \frac{1}{(L_{\text{Equiv}} + L_c)(C_{\text{Equiv}} + C_c^{\text{Driver}})} && (7)\\
\omega^2 &= \frac{1}{(L_{\text{Equiv}} + L_{\text{Equiv}}/250)(C_{\text{Equiv}} + C_{\text{Equiv}}/40)} && (8)\\
\omega^2 &= \frac{1}{(251/250)L_{\text{Equiv}}(41/40)C_{\text{Equiv}}} && (9)\\
\omega^2 &= \left(\frac{100}{103}\right)\frac{1}{L_{\text{Equiv}}C_{\text{Equiv}}} && (10)\\
\omega^2 &= \left(\frac{100}{103}\right)\omega_0^2 && (11)\\
\frac{\omega}{\omega_0} &= 0.985 && (12)
\end{aligned}
$$

We see now that there is a 1.5% correction to the resonant frequency of the circuit due the the capacitive and inductive loading. Since the total geometic inductance and capacitance scales with length, we see that resonant frequency must also scale with length thus the 1.5% correction can be applied directly to the length of the resonator yielding

Freq (GHz)	$\mathbf{l_{\text{Corrected}}}$ **(mm)**
6.55	4.925
6.45	5.004