Parallel LCR Resonator S11 rework

$$Z_{r} = \frac{R_{r}}{1 + 2iQ_{r}x}$$

$$Z_{r} = \frac{R_{r}}{1 + 4Q_{r}^{2}x^{2}} - \frac{2iR_{r}Q_{r}x}{1 + 4Q_{r}^{2}x^{2}}$$

$$R_{t} = \frac{1 + \omega^{2}C_{c}^{2}R_{l}^{2}}{\omega^{2}C_{c}^{2}R_{l}}$$

$$Q_{c} = \omega R_{t}C$$

$$R_{t} = \frac{Q_{c}}{\omega C} = \frac{Q_{c}R_{r}}{Q_{r}}$$

$$Z_{0} \equiv R_{l} \rightarrow R_{t}$$

$$S_{11} = \frac{Z_{r} - R_{t}}{Z_{r} + R_{t}}$$

$$S_{11} = \frac{\frac{R_{r}}{1 + 4Q_{r}^{2}x^{2}} - \frac{Q_{c}R_{r}}{Q_{r}} - \frac{2iR_{r}Q_{r}x}{1 + 4Q_{r}^{2}x^{2}}}{\frac{R_{r}}{1 + 4Q_{r}^{2}x^{2}} + \frac{Q_{c}R_{r}}{Q_{r}} - \frac{2iR_{r}Q_{r}x}{1 + 4Q_{r}^{2}x^{2}}}$$

Factor out all R_r 's

$$S_{11} = \frac{\frac{1}{1+4Q_r^2x^2} - \frac{Q_c}{Q_r} - \frac{2iQ_rx}{1+4Q_r^2x^2}}{\frac{1}{1+4Q_r^2x^2} + \frac{Q_c}{Q_r} - \frac{2iR_rx}{1+4Q_r^2x^2}}$$

factor out $\frac{1}{1+4Q_r^2x^2}$

$$S_{11} = \frac{1 - \frac{Q_c}{Q_r}(1 + 4Q_r^2x^2) - 2iQ_rx}{1 + \frac{Q_c}{Q_r}(1 + 4Q_r^2x^2) - 2iQ_rx}$$
$$S_{11} = \frac{1 - (\frac{Q_c}{Q_r} + 4Q_cQ_rx^2) - 2iQ_rx}{1 + (\frac{Q_c}{Q_r} + 4Q_cQ_rx^2) - 2iQ_rx}$$