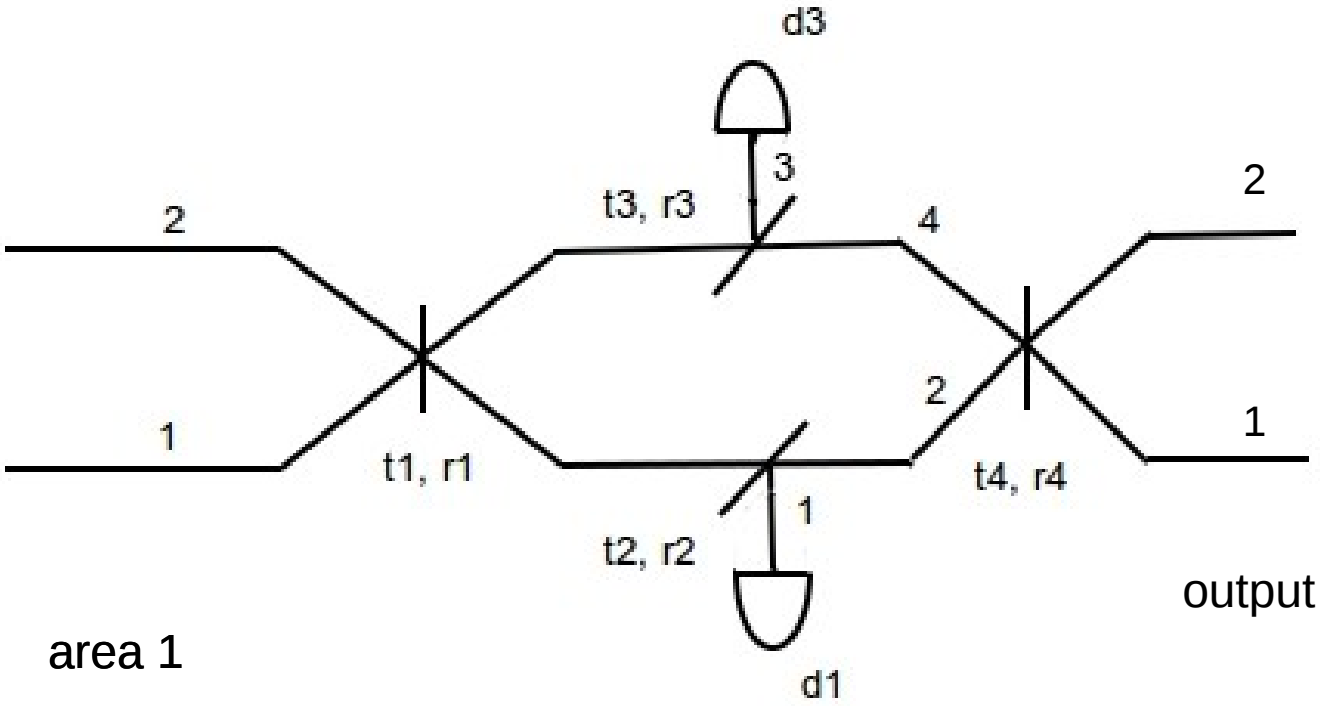
Setup.


d3
3
t3, r3
2
4
2
2
1
1
t1, r1
t4, r4
1
t2, r2
output
d1
area 1
area 2
area 4
area 3

Theory.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle$$

1) State in two channels — area 1:

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

2) Mixed at first BS1: $a_1^\dagger \to r_j a_1^\dagger + i t_j a_2^\dagger$

$a_2^\dagger \to r_j a_2^\dagger + i t_j a_1^\dagger$

$$r_j^2 + t_j^2 + a_j^2 = 1$$

3) State in area 2 — after BS1: $|\psi_2\rangle = \sum_{m,n} \alpha^{(2)}_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$

$\sum_{m,n} |\alpha^{(2)}_{m,n}|^2 = 1$ - For ideal BS (a = 0)

4) State in area 3 (4 channels) — after BS2 and BS3 but before detection:

$$|\psi_3\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (a_1^\dagger)^{p_1} (a_2^\dagger)^{p_2} (a_3^\dagger)^{p_3} (a_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

5) State in area 4 (4 channels) — after detection:

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1 - \eta_{SPD})^n |n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1 - (1 - \eta_{SPD})^n] |n\rangle\langle n|$$

$|\psi_{out}\rangle = \Pi_{click}^{(1)} |\psi_{in}\rangle$ - only first was clicked.

$|\psi_{out}\rangle = \Pi_{click}^{(1)} \Pi_{click}^{(3)} |\psi_{in}\rangle$ - first and third were clicked.

5) State in area 4 (4 channels) — after detection:

Detectors are ideal!

5.1) Only first detector was clicked:

$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_4 \\ p_1\neq 0, p_3=0}} \beta_{p_1,p_2,0,p_4} \sqrt{p_1!} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

5.2) Both detectors (1st and 3rd) were clicked:

$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_3,p_4 \\ p_1\neq 0, p_3\neq 0}} \beta_{p_1,p_2,p_3,p_4} \sqrt{p_1!p_3!} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

5.3) No detection:

$$|\psi_4\rangle = \sum_{\substack{p_2,p_4 \\ p_1=0, p_3=0}} \beta_{0,p_2,0,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

6) State after having been mixed at BS4 (final state):

$$|\psi_{out}\rangle = \sum_{m,n} \alpha_{m,n}^{out} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

Example with two coherent states:

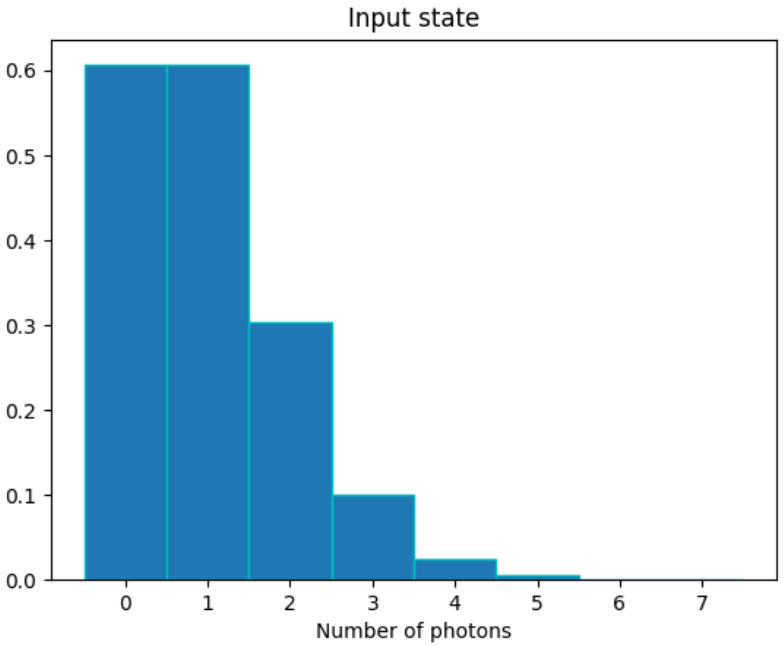
1) State before BS1:

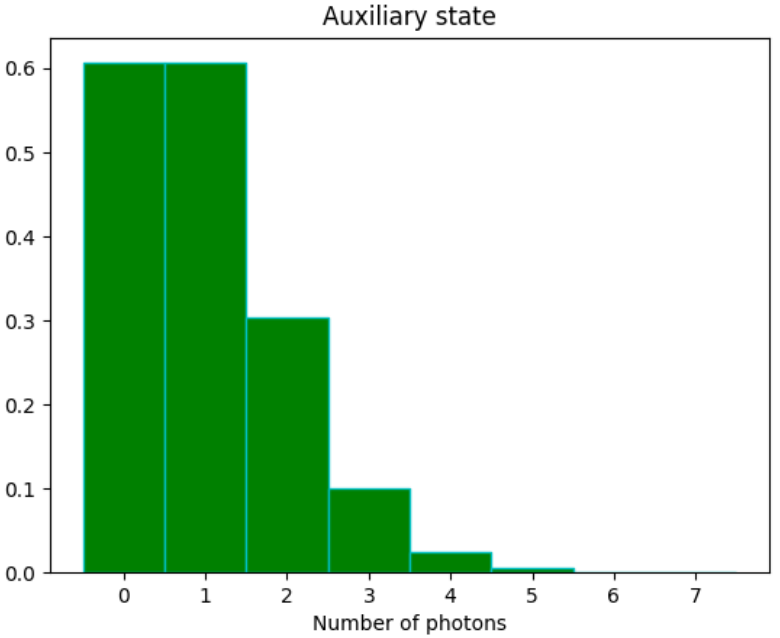
$$|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle, \quad \alpha = 1$$

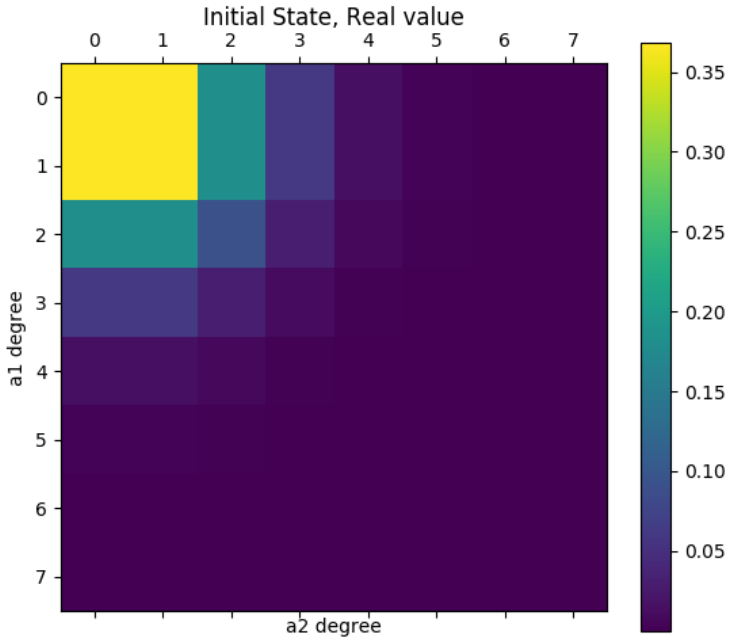
2) State after BS1:

$$|\psi_2\rangle \approx e^{-1}(1+\frac{1+i}{\sqrt{2}}a_1^\dagger+\frac{1+i}{\sqrt{2}}a_2^\dagger+ia_1^\dagger a_2^\dagger+\frac{i}{2}(a_1^\dagger)^2+\frac{i}{2}(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}a_1^\dagger(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}(a_1^\dagger)^2a_2^\dagger+...)$$

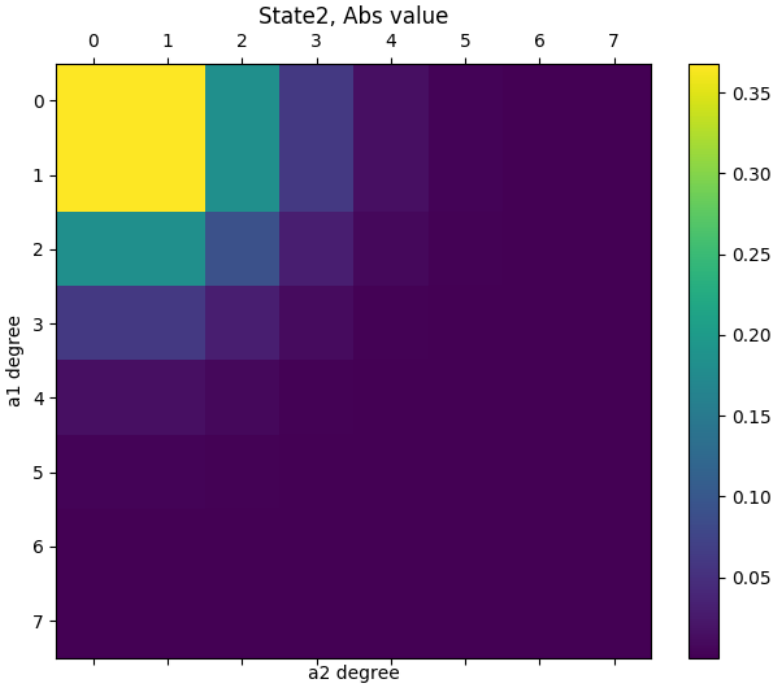
1) Input — coherent state. Aux — coherent state.

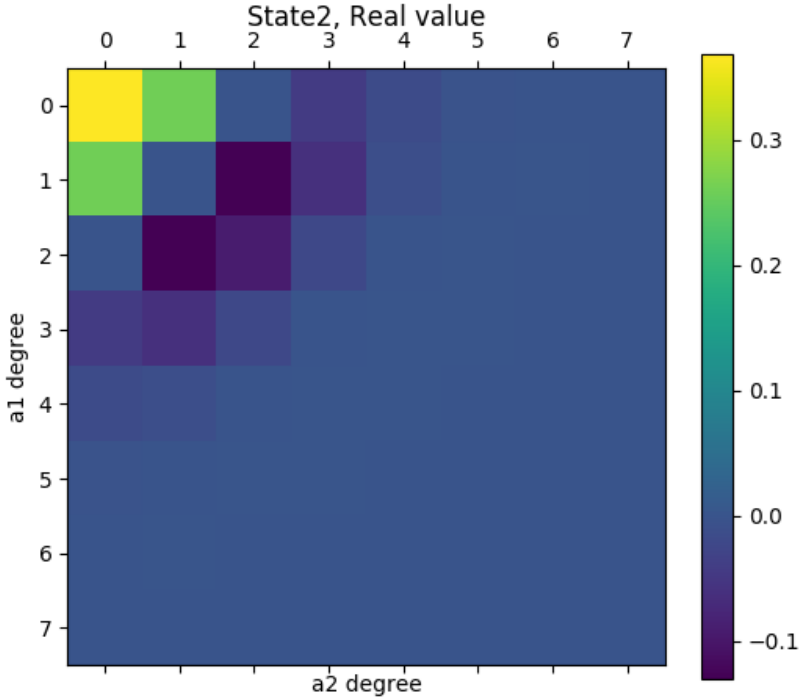


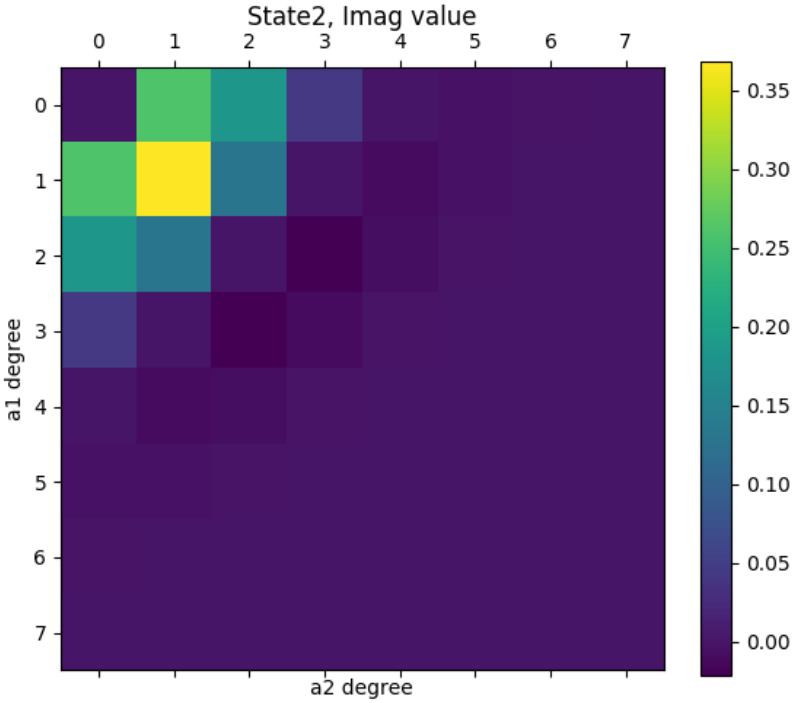




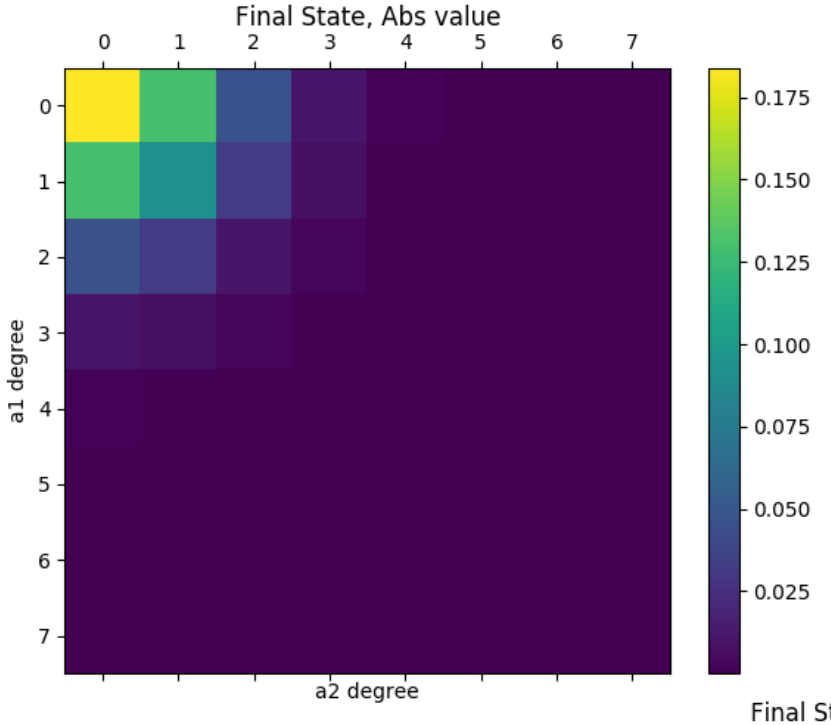
State in area2.

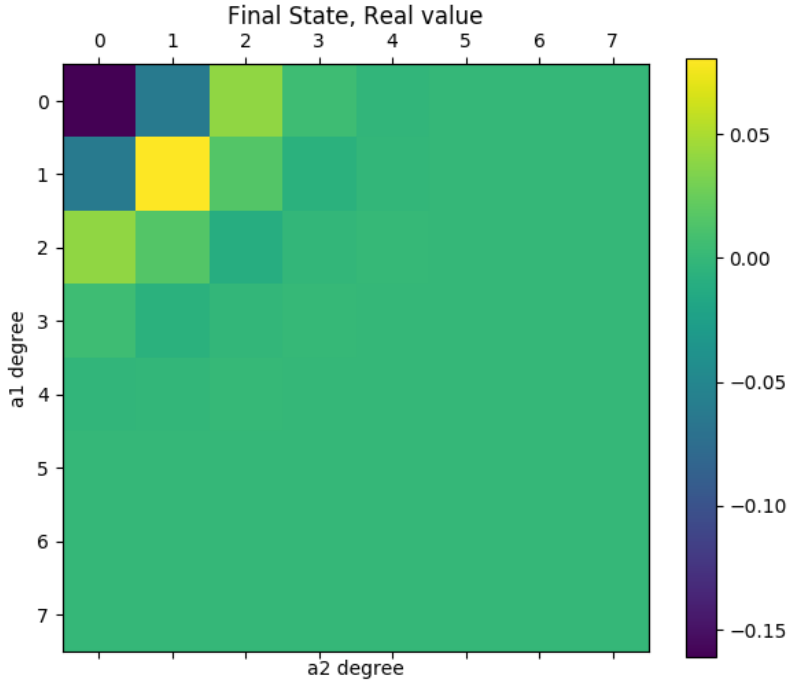


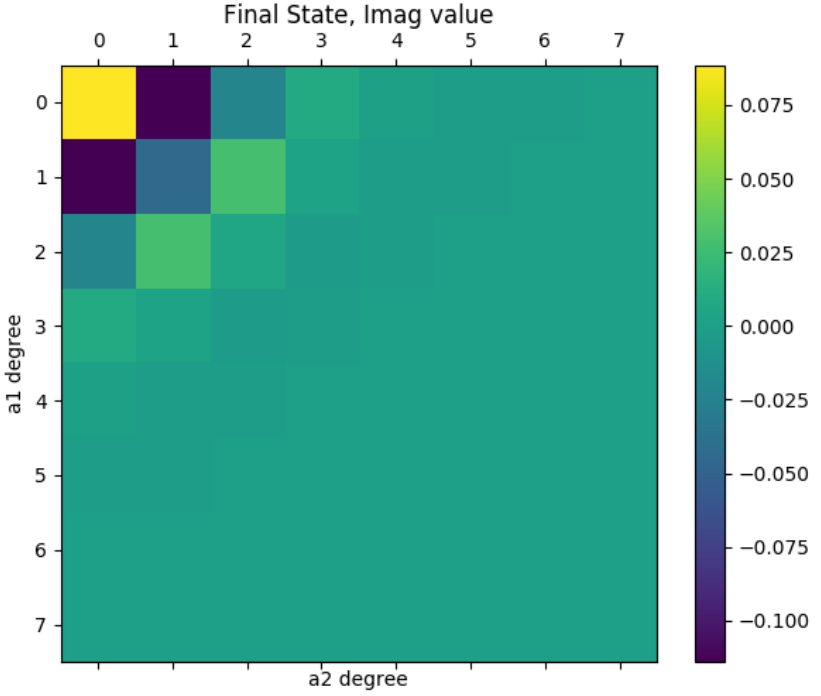




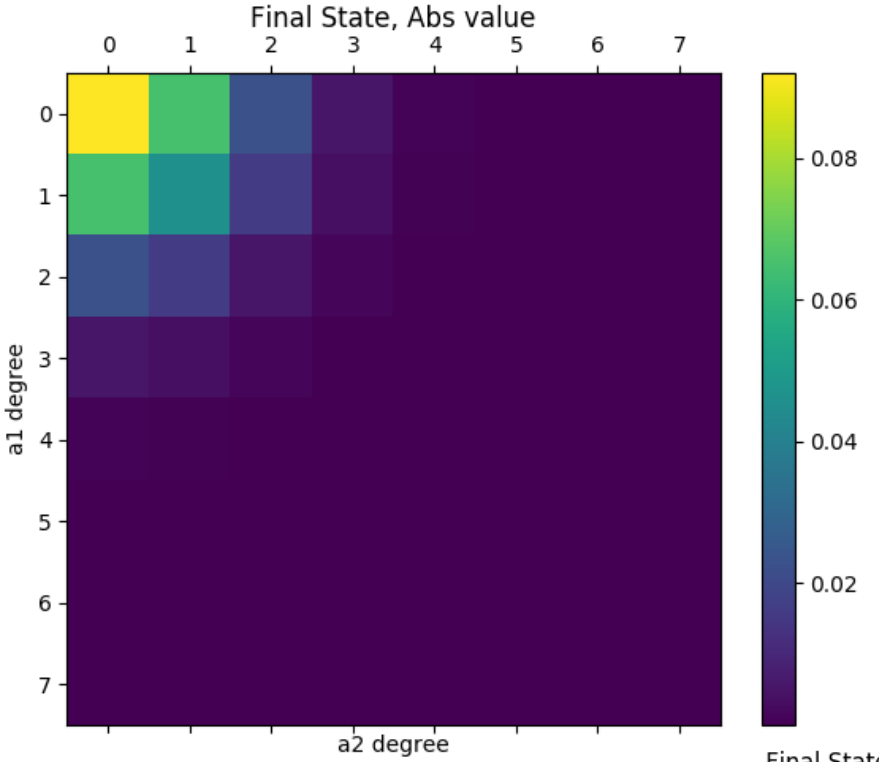
Output(final) state. First detector was clicked.

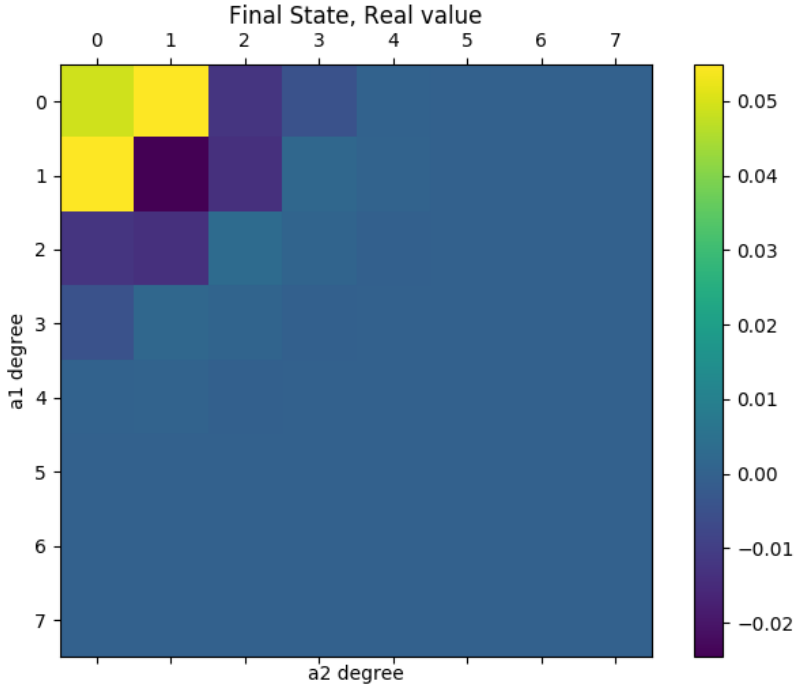


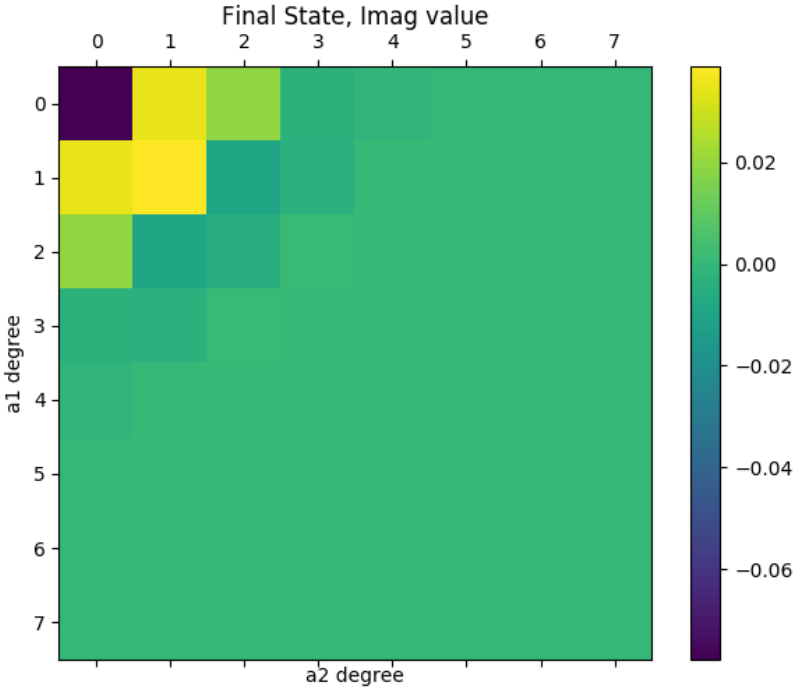




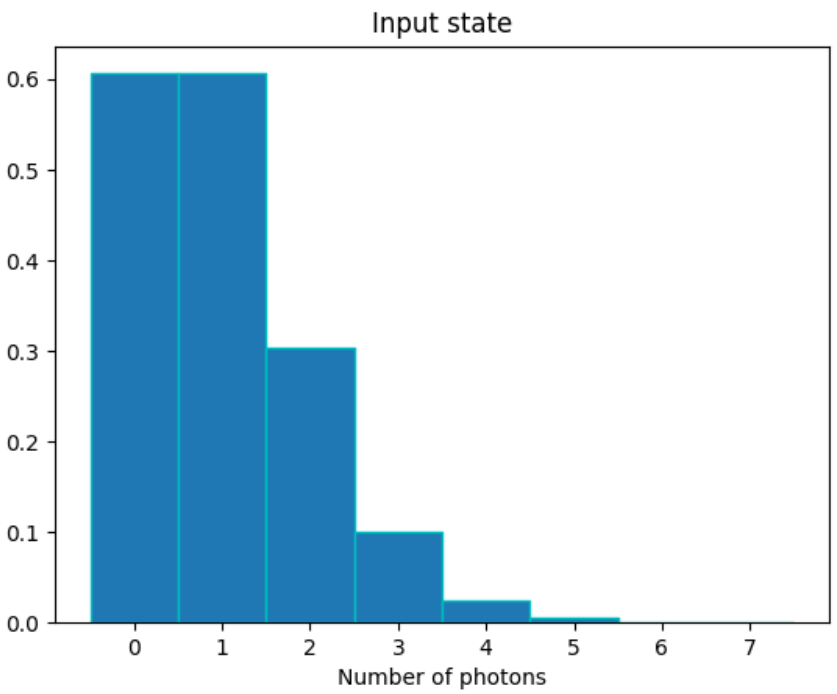
Output(final) state. Both (1st and 3rd) detectors were clicked.

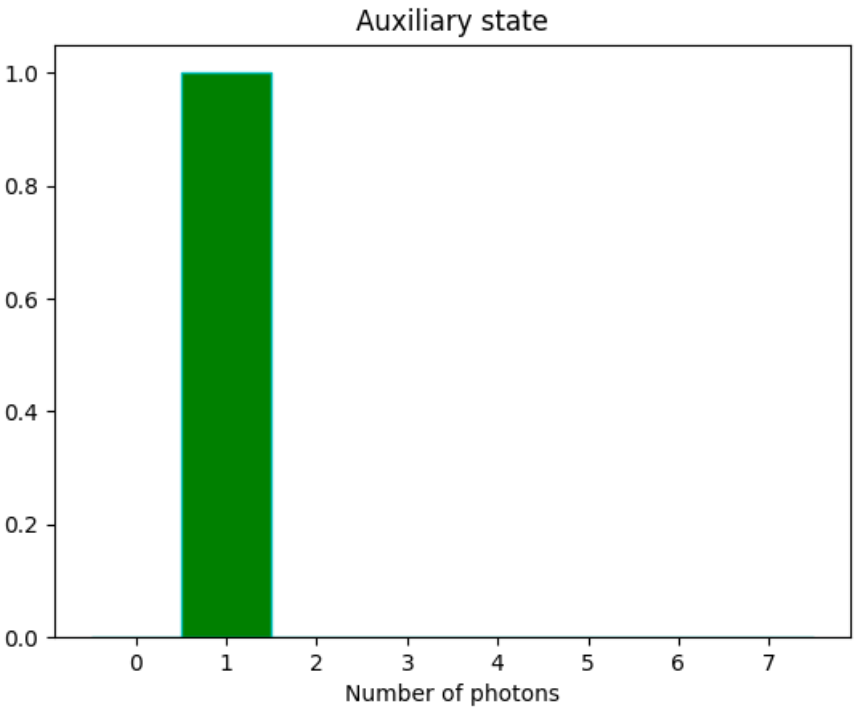


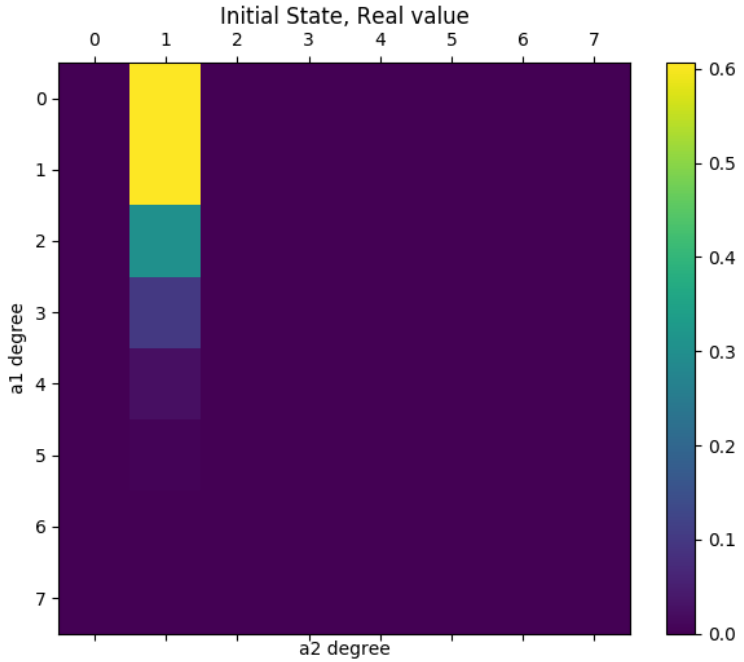




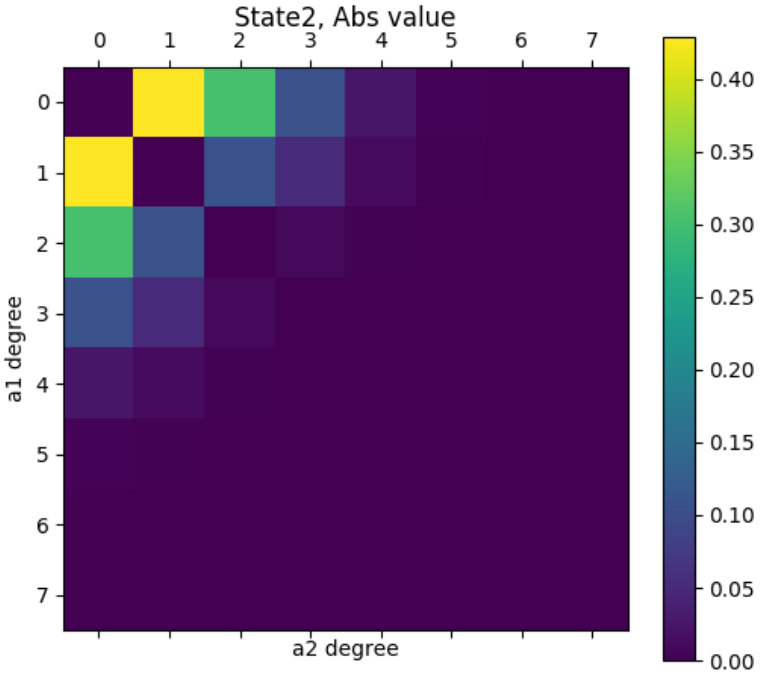
2) Input — coherent state. Aux — single photon.

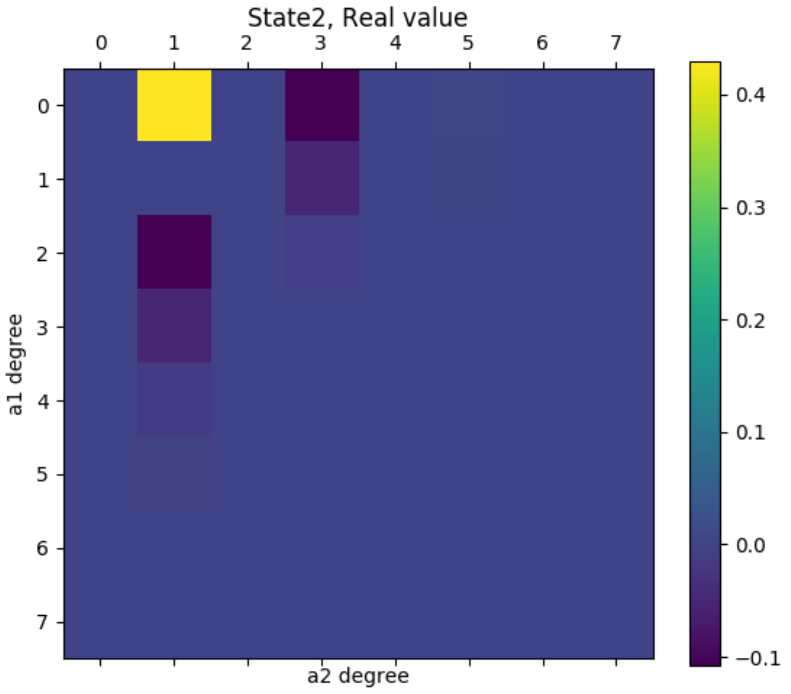


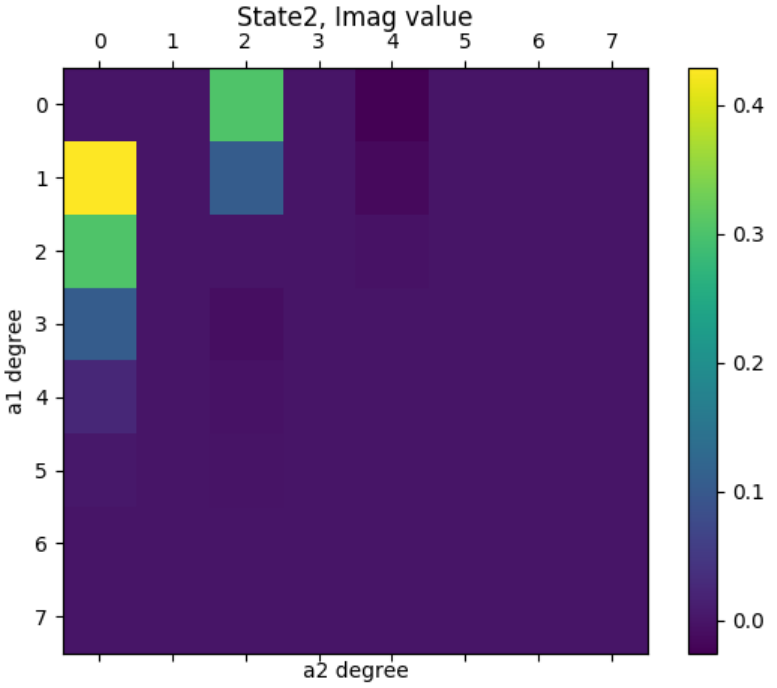




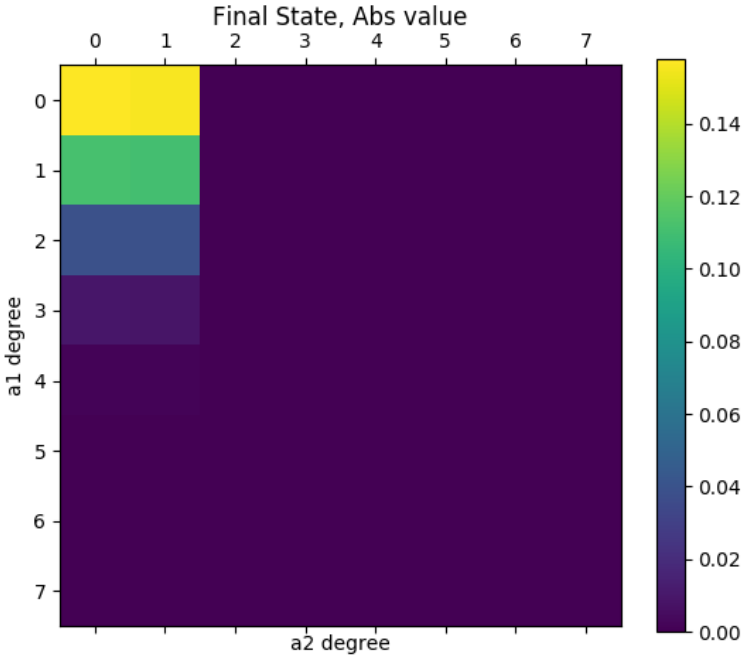
State in area2.

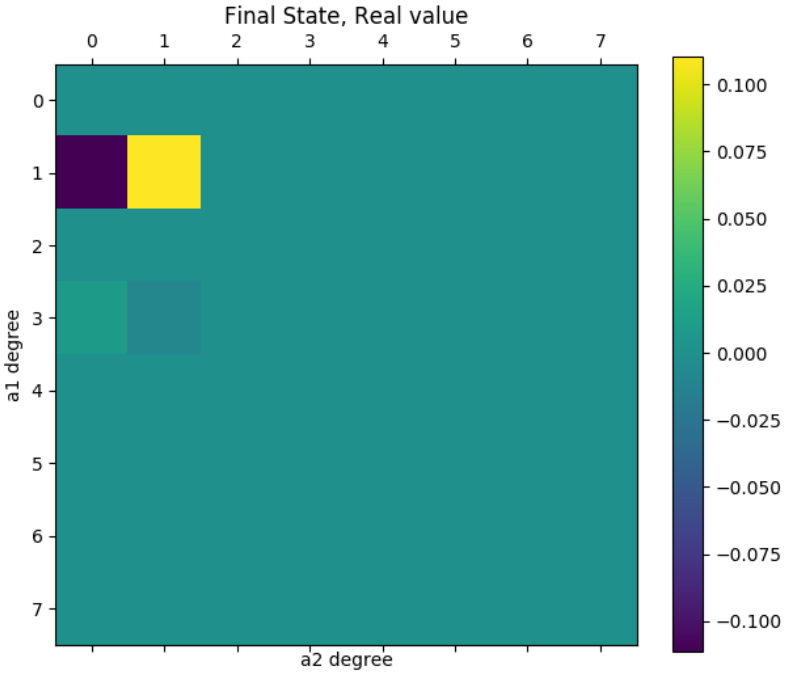


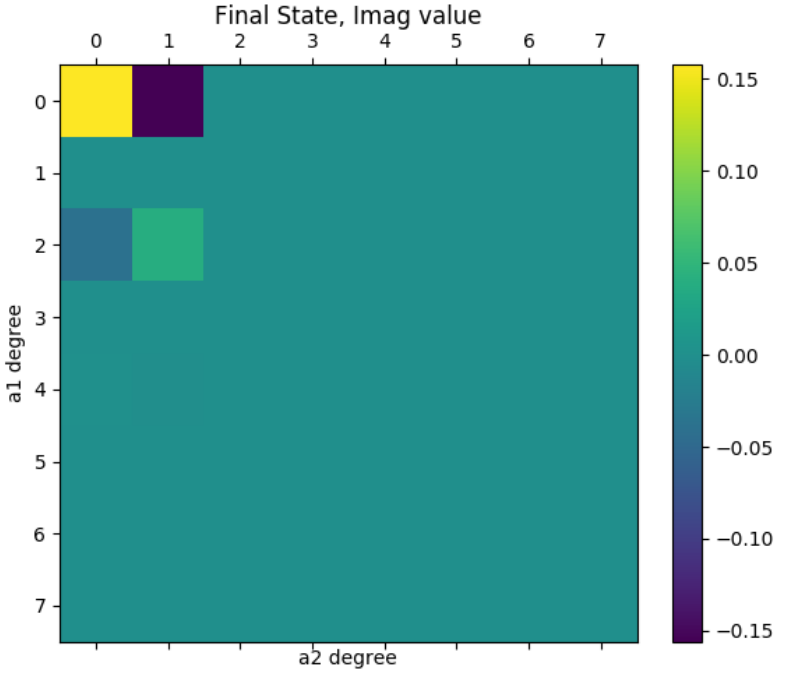




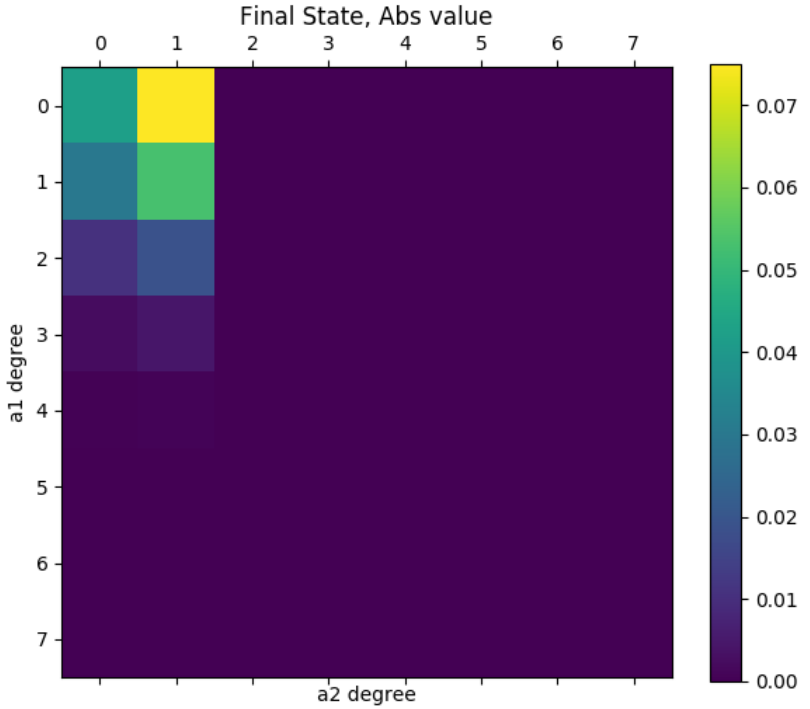
Output(final) state. First detector was clicked.

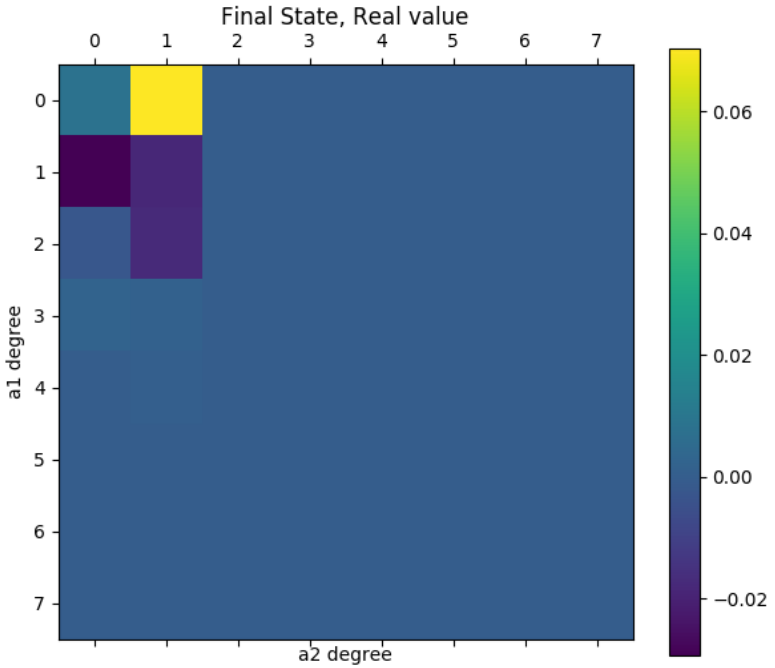


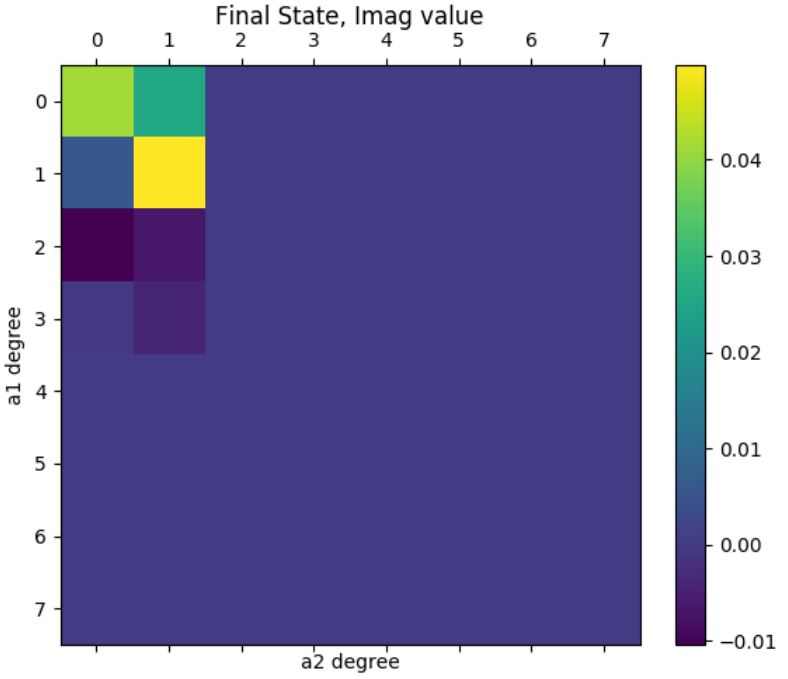




Output(final) state. Both (1st and 3rd) detectors were clicked.







State after detection but before final BS. First detector was clicked.