LaTeX 2_ε Template

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$$u_p^l(r,\phi) \propto r^l L_p^l(2r^2/w^2) e^{-r^2/w^2} e^{-il\phi} \tag{1}$$

1. Low gain.

2. $$H \sim \int d^3\mathbf{r}\chi^{(2)}(\mathbf{r}) E_p^{(+)}(\mathbf{r},t) E_s^{(-)}(\mathbf{r},t) E_i^{(-)}(\mathbf{r},t) + H.c.$$

3. $$H = i\hbar\Gamma \int d\mathbf{q}_s d\mathbf{q}_i F(\mathbf{q}_s,\mathbf{q}_i) a^\dagger_{\mathbf{q}_s} a^\dagger_{\mathbf{q}_i} + H.c.$$

4. $$F(\mathbf{q}_s,\mathbf{q}_i) = C \exp\big\{-\sigma^2 \frac{(\mathbf{q}_s+\mathbf{q}_i)^2}{2}\big\} \mathrm{sinc}\Big(\frac{L(\mathbf{q}_s-\mathbf{q}_i)^2}{4k_p}\Big) \exp\big\{i\frac{L(\mathbf{q}_s-\mathbf{q}_i)^2}{4k_p}\big\}$$

5. $$|\psi\rangle = \exp\big\{-\frac{i}{\hbar}\int_{-\infty}^{\infty} dt H(t)\big\}|0\rangle \approx -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt H|0\rangle$$

6. Shmidt decomposition

7. $$F(q_s,q_i,\phi_s-\phi_i) = \sum_n \chi_n(q_s,q_i) e^{in(\phi_s-\phi_i)}$$

8. $$\chi_n(q_s,q_i) = \sum_m \sqrt{\lambda_{mn}} \frac{u_{mn}(q_s)}{\sqrt{q_s}} \frac{v_{mn}(q_i)}{\sqrt{q_i}}$$

9.
$$F(\mathbf{q}_s, \mathbf{q}_i) = \sum_{m,n} \sqrt{\lambda_{mn}} \frac{u_{mn}(q_s)}{\sqrt{q_s}} \frac{v_{mn}(q_i)}{\sqrt{q_i}} e^{in(\phi_s - \phi_i)}$$

10. broadband modes and hign gain

11.
$$A^\dagger_{mn} = \int d\mathbf{q}_s \frac{u_{mn}(q_s)}{\sqrt{q_s}} e^{in\phi_s} a^\dagger_{\mathbf{q}_s}$$

12.
$$B^\dagger_{mn} = \int d\mathbf{q}_i \frac{v_{mn}(q_i)}{\sqrt{q_i}} e^{-in\phi_i} a^\dagger_{\mathbf{q}_i}$$

13.
$$H = i\hbar\Gamma \sum_{m,n} \sqrt{\lambda_{mn}} (A^\dagger_{mn} B^\dagger_{mn} - A_{mn} B_{mn})$$

14.
$$\frac{dA_{mn}}{dt} = \frac{i}{\hbar}[H, A_{mn}]$$

15.
$$A^{out}_{mn} = A^{in}_{mn} \cosh[G\sqrt{\lambda_{mn}}] + [B^{in}_{mn}]^\dagger \sinh[G\sqrt{\lambda_{mn}}]$$

16.
$$B^{out}_{mn} = B^{in}_{mn} \cosh[G\sqrt{\lambda_{mn}}] + [A^{in}_{mn}]^\dagger \sinh[G\sqrt{\lambda_{mn}}]$$

17.
$$\frac{da_{\mathbf{q}_{s,i}}}{dt} = \Gamma \sum_{m,n} \sqrt{\lambda_{mn}} \frac{u_{mn}(q_s)}{\sqrt{q_s}} [A^\dagger_{mn} e^{-in\phi_{s,i}} + B^\dagger_{mn} e^{in\phi_{s,i}}]$$

18.
$$\langle N_s(\mathbf{q}_s) \rangle = \sum_{m,n} \frac{|u_{mn}(q_s)|^2}{q_s} (\sinh[G\sqrt{\lambda_{mn}}])^2$$

19. three crystals

20.

$$F(\mathbf{q}_s, \mathbf{q}_i) = C \exp\Big\{ -\sigma^2 \frac{(\mathbf{q}_s + \mathbf{q}_i)^2}{2} \Big\} \mathrm{sinc}\Big(\frac{\Delta\widetilde{q}L}{2}\Big)$$
$$\times \Big(\exp\Big\{\frac{i\Delta\widetilde{q}L}{2}\Big\} + \exp\Big\{i(\Delta\widetilde{q}^{air}d_1 + \frac{3}{2}\Delta\widetilde{q}L)\Big\} + \exp\Big\{i(\Delta\widetilde{q}^{air}(d_1+d_2) + \frac{5}{2}\Delta\widetilde{q}L)\Big\}\Big) \tag{2}$$

21.

$$\Delta\widetilde{q} = \frac{(\mathbf{q}_s - \mathbf{q}_i)^2}{2k_p}$$

22.

$$\Delta\widetilde{q}^{air} = \frac{(\mathbf{q}_s - \mathbf{q}_i)^2}{2k_p} n_s + \frac{2\delta n^{air} k_s}{n_s}$$

23. splitters

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle$$
$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle$$
$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$
$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

24. loseless BS

25.

$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

26.

$$\sum_{m,n} |\alpha^{(2)}_{m,n}|^2 = 1$$

$$r_j^2 + t_j^2 + a_j^2 = 1$$
$$a_1^\dagger \to r_j a_1^\dagger + i t_j a_2^\dagger$$
$$a_2^\dagger \to r_j a_2^\dagger + i t_j a_1^\dagger$$

27. state in area 2 after first BS

28.
$$|\psi_2\rangle = \sum_{m,n} \alpha^{(2)}_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

29. 4 channels

30.
$$|\psi_3\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (a_1^\dagger)^{p_1} (a_2^\dagger)^{p_2} (a_3^\dagger)^{p_3} (a_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

31. First and only one ideal detector was clicked

32.
$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_4 \\ p_1>0, p_3=0}} |\beta_{p_1,p_2,0,p_4} \sqrt{p_1! p_2! p_4!}|^2 |p_2, p_4\rangle$$

33. First and third detectors were clicked

34.
$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_3,p_4 \\ p_1>0, p_3>0}} |\beta_{p_1,p_2,p_3,p_4} \sqrt{p_1! p_2! p_3! p_4!}|^2 |p_2, p_4\rangle$$

35. No detection

36.
$$|\psi_4\rangle = \sum_{\substack{p_2,p_4 \\ p_1=0, p_3=0}} |\beta_{0,p_2,0,p_4} \sqrt{p_2! p_4!}|^2 |p_2, p_4\rangle$$

37. final state

38.
$$|\psi_{out}\rangle = \sum_{m,n} \alpha^{out}_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

39. example with two coherent states - alpha=1

40.
$$|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle, \quad \alpha = 1$$

41. two coherent after BS

42.

$$|\psi_2\rangle \approx e^{-1}(1+\frac{1+i}{\sqrt{2}}a_1^\dagger+\frac{1+i}{\sqrt{2}}a_2^\dagger+ia_1^\dagger a_2^\dagger+\frac{i}{2}(a_1^\dagger)^2+\frac{i}{2}(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}a_1^\dagger(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}(a_1^\dagger)^2a_2^\dagger+...)$$

43. auto correlation

$$Autocorr_s(q,q^{'}) = \langle N_s(q)N_s(q^{'})\rangle - \langle N_s(q)\rangle\langle N_s(q^{'})\rangle =$$
$$= \langle a_s^\dagger(q)a_s(q)a_s^\dagger(q^{'})a_s(q^{'})\rangle - \langle a_s^\dagger(q)a_s(q)\rangle\langle a_s^\dagger(q^{'})a_s(q^{'})\rangle$$

44. cross correlation

$$Crosscorr(q,q^{'}) = \langle N_s(q)N_i(q^{'})\rangle - \langle N_s(q)\rangle\langle N_i(q^{'})\rangle =$$
$$= \langle a_s^\dagger(q)a_s(q)a_i^\dagger(q^{'})a_i(q^{'})\rangle - \langle a_s^\dagger(q)a_s(q)\rangle\langle a_i^\dagger(q^{'})a_i(q^{'})\rangle$$

45. POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty}(1-\eta_{SPD})^n|n\rangle\langle n|$$
$$\Pi_{click} = 1-\Pi_{no-click} = \sum_{n=0}^{\infty}[1-(1-\eta_{SPD})^n]|n\rangle\langle n|$$

46. After projection

47. Only first clicked

48.

$$|\psi_{out}\rangle = \Pi_{click}^{(1)}|\psi_{in}\rangle$$

49. Two clicked 1st and 3rd

50.

$$|\psi_{out}\rangle = \Pi_{click}^{(1)}\Pi_{click}^{(3)}|\psi_{in}\rangle$$

51. First and only one ideal detector was clicked

52.

$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_4\\ p_1\neq 0,p_3=0}} \beta_{p_1,p_2,0,p_4}\sqrt{p_1!}(b_2^\dagger)^{p_2}(b_4^\dagger)^{p_4}|0\rangle^{\otimes 2}$$

53. First and third detectors were clicked

54.
$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_3,p_4 \\ p_1\neq 0, p_3\neq 0}} \beta_{p_1,p_2,p_3,p_4} \sqrt{p_1! p_3!} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

55. No detection

56.
$$|\psi_4\rangle = \sum_{\substack{p_2,p_4 \\ p_1=0, p_3=0}} \beta_{0,p_2,0,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

57. Projection operators

58. It starts here

59. States.

$$|\psi_1^{in}\rangle = |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0} \frac{\alpha^n}{n!} (a_1^\dagger)^n |0\rangle$$
$$|\psi_2^{in}\rangle = |1\rangle = a_2^\dagger |0\rangle$$

$$|\psi_{in}\rangle = |\alpha\rangle \otimes |1\rangle$$

60. Beam splitter.

$$T_n + R_n = r_n^2 + t_n^2 = 1$$
$$a_1^\dagger \to r_n a_1^\dagger + i t_n a_2^\dagger$$
$$a_2^\dagger \to r_n a_2^\dagger + i t_n a_1^\dagger$$

61. POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty}(1-\eta_{SPD})^n|n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty}[1-(1-\eta_{SPD})^n]|n\rangle\langle n|$$

η_{SPD} – *single photon detection efficiency.*

$$\hat{\Pi}_{both} = \Pi^{(1)}_{click} \otimes \Pi^{(2)}_{click}$$
$$\hat{\Pi}_{bottom} = \Pi^{(1)}_{click} \otimes \Pi^{(2)}_{no-click}$$
$$\hat{\Pi}_{top} = \Pi^{(1)}_{no-click} \otimes \Pi^{(2)}_{click}$$
$$\hat{\Pi}_{none} = \Pi^{(1)}_{no-click} \otimes \Pi^{(2)}_{no-click}$$

62. State before detection.

63.
$$|\psi_4\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4}(b_1^\dagger)^{p_1}(b_2^\dagger)^{p_2}(b_3^\dagger)^{p_3}(b_4^\dagger)^{p_4}|0\rangle^{\otimes 4}$$

64. Detection

65.
$$\rho_{after\ det.} = Tr_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^\dagger)$$

66. Detection probability.

67.
$$P_{event} = Tr(\rho_{in}\hat{\Pi}^\dagger_{event})$$

68. dens matrix after detection

69.
$$\rho = \sum_{\substack{p_2,p_4\\p_2',p_4'}} \rho_{p_2,p_4,p_2',p_4'}|p_2\rangle\langle p_2'| \otimes |p_4\rangle\langle p_4'|$$

70. Phase modulation:

$$a_2^\dagger \to a_2^\dagger e^{i\phi}$$
$$\rho_{m,n,m',n'} \to \rho_{m,n,m',n'} \exp[i\phi(n-n')]$$

71. Last BS:

72.

$$|p_2, p_4\rangle\langle p_2', p_4'| = \frac{1}{\sqrt{p_2!p_4!p_2'!p_4'!}}(a_2^\dagger)^{p_2}(a_4^\dagger)^{p_4}|0,0\rangle\langle 0,0|(a_2)^{p_2'}(a_4)^{p_4'}$$

73. Density matrix as output.

74.

$$\rho^{out} = \sum_{\substack{p_1,p_2 \\ p_1',p_2'}} \rho^{out}_{p_1,p_2,p_1',p_2'}|p_1\rangle\langle p_1'| \otimes |p_2\rangle\langle p_2'|$$

75. Density matrix of subsystem.

76.

$$\rho_1 = Tr_2(\rho_{12})$$

77. Von Neumann entropy.

$$S_{FN} = -\sum_n \lambda_n \ln(\lambda_n)$$
$$\rho_1 \to \{\lambda_n\},\ eigenvalues$$

78. Linear entropy.

79.

$$S_L = 1 - Tr(\rho_1^2)$$

80. Negativity.

$$N(\rho) = \frac{||\rho^{T_1}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$
$$\rho_{12} \to \{\lambda_n\},\ eigenvalues$$

$$LN(\rho) := \log_2(2N + 1)$$

81. Negativity 2.

$$N(\rho_{AB}) = \frac{||\rho^{T_A}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$
$$\rho^{T_A} \to \{\lambda_n\},\ eigenvalues$$

$$LN(\rho_{AB}) := \log_2(2N + 1)$$

$N = 0 - means\ no\ correlation,$
$subsystems\ are\ \boldsymbol{independent}.$
$N > 0 - subsystems\ are\ \boldsymbol{entangled}.$

82. eigvalues and states.

83.
$$|1\rangle \quad |out\rangle^{\otimes 2}$$

84.
$$|1,1\rangle \quad \frac{1}{\sqrt{2}}(|2,0\rangle + |0,2\rangle)$$

85.
$$\frac{1}{\sqrt{3}}(|2,0\rangle + |0,2\rangle + |1,1\rangle)$$

86. non unit transformations

87.
$$c_0|0\rangle + c_1|1\rangle + c_2|2\rangle \to c_0|0\rangle + c_1|1\rangle - c_2|2\rangle$$

88. eigenvalues

89.
$$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 0$$

90.
$$\lambda_1 = \frac{1}{2} \quad \lambda_2 = 0 \quad \lambda_3 = \frac{1}{2}$$

91.
$$\lambda_1 = \frac{1}{3} \quad \lambda_2 = \frac{1}{3} \quad \lambda_3 = \frac{1}{3}$$

92. beamsplitters

$$T_1 : R_1$$
$$T_2 : R_2$$
$$T_3 : R_3$$
$$T_4 : R_4$$

93. channels

①
②
③
④

94. detectors

D_1

D_2

95. PHASE

$\Delta\phi$

96. states

97.
$$|\alpha = 1\rangle$$

98.
$$|1\rangle$$

99.
$$|\psi_{in}\rangle = |\alpha = 1\rangle \otimes |1\rangle$$

100.
$$|\psi_{in}\rangle = |1\rangle \otimes |1\rangle$$

101.
$$(\Delta X_i)^2 < \frac{1}{4}$$

102.
$$X,\ P$$

103.
$$(X_a - X_b)/\sqrt{2}$$

104.
$$(P_a + P_b)/\sqrt{2}$$

$$(X_a - X_b)/\sqrt{2}$$
$$(P_a + P_b)/\sqrt{2}$$

105. EPR