Nonlinear transformations of quantum light.

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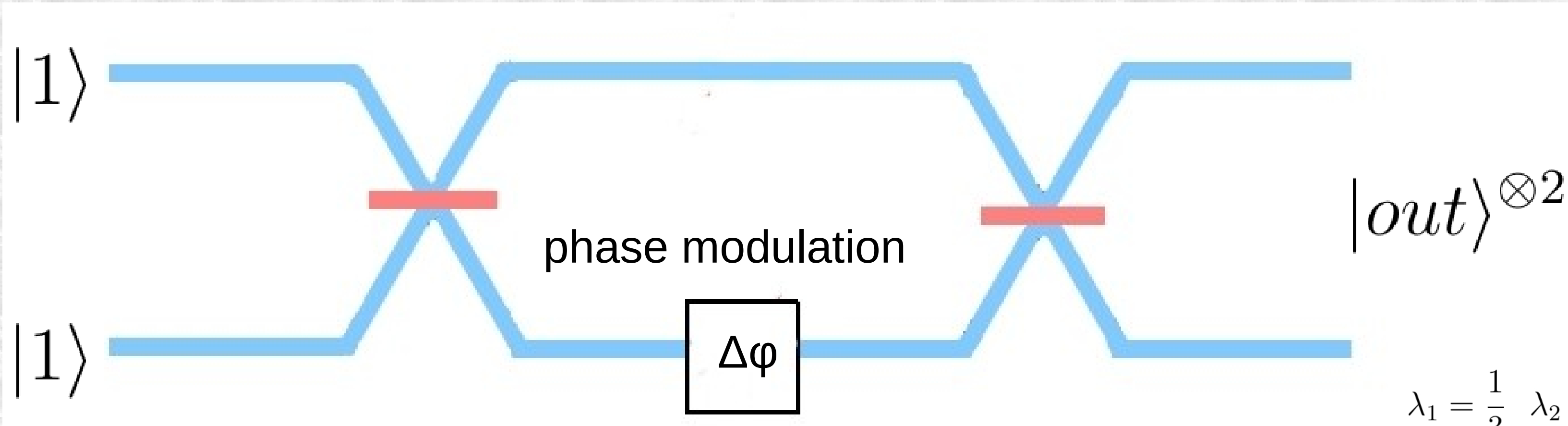
Motivation.

Stydying non linear transformations of quantum light, applying **non-unitary** operators.

$$c_0|0\rangle + c_1|1\rangle + c_2|2\rangle \rightarrow c_0|0\rangle + c_1|1\rangle - c_2|2\rangle$$

Such transformations could be achived by applying detection.

Simple setup.



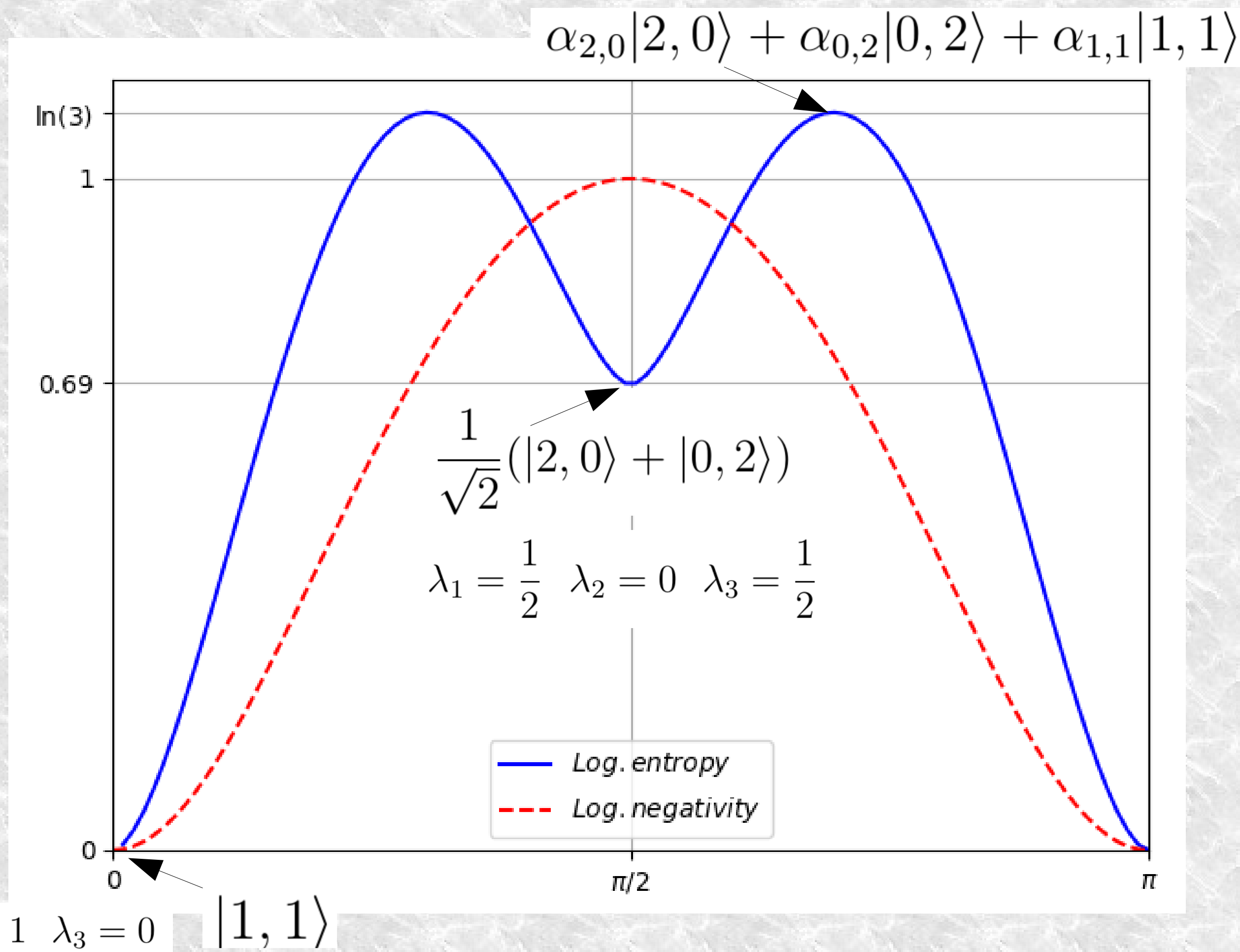
$$S_{FN} = -\sum_n \lambda_n \ln(\lambda_n)$$

$$\rho_1 \rightarrow \{\lambda_n\},\ eigenvalues$$

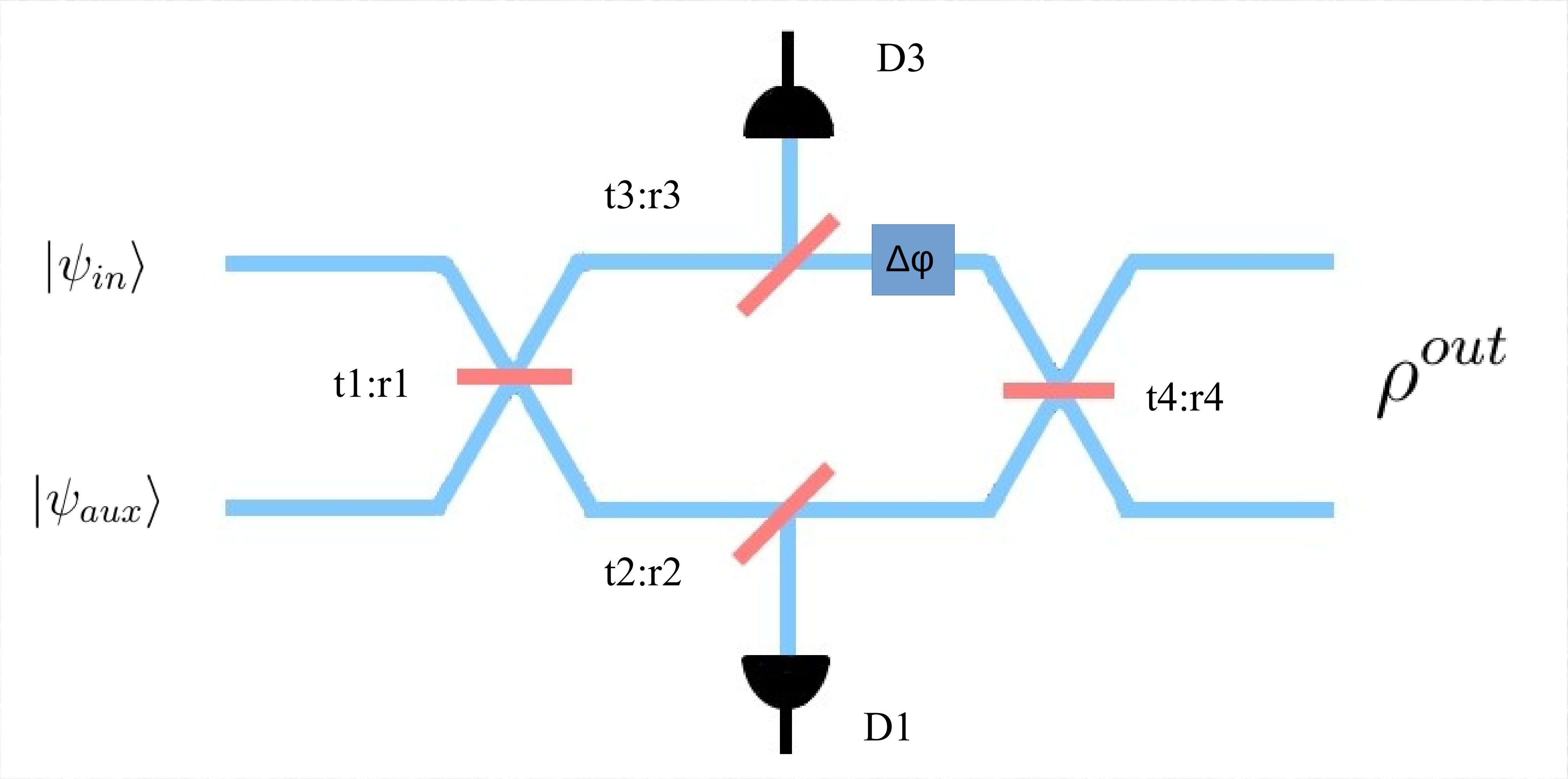
$$N(\rho) = \frac{||\rho^{T_1}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

$$\rho_{12} \rightarrow \{\lambda_n\},\ eigenvalues$$

$$LN(\rho) := \log_2(2N+1)$$



Scheme with detection.



Theoretical approach.

- As an input for system there could be sent different quantum light states in two channels.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle$$

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

- Mutual state is transformed at beam splitter:

$$r_j^2 + t_j^2 + a_j^2 = 1$$
$$a_1^\dagger \to r_j a_1^\dagger + i t_j a_2^\dagger$$
$$a_2^\dagger \to r_j a_2^\dagger + i t_j a_1^\dagger$$

- After transformation at first, second and third BS, the state in 4 channels will look like this:

$$|\psi_4\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

Detection.

- Detection is described in terms of POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1-\eta_{SPD})^n |n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1-(1-\eta_{SPD})^n] |n\rangle\langle n|$$

- Four possibilities of detectors behaviour in total.

$\hat{\Pi} = \Pi^{(1)}_{click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{click} \otimes \mathrm{I}^{(4)}$ - both are clicked.

$\hat{\Pi} = \Pi^{(1)}_{click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{no-click} \otimes \mathrm{I}^{(4)}$ - first is clicked, third is silent.

$\hat{\Pi} = \Pi^{(1)}_{no-click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{click} \otimes \mathrm{I}^{(4)}$ - first is silent, third is clicked.

$\hat{\Pi} = \Pi^{(1)}_{no-click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{no-click} \otimes \mathrm{I}^{(4)}$ - both are silent.

- After detection state is collapsed into density matrix:

$$\rho = Tr_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^\dagger)$$

$$\rho = \sum_{\substack{p_2,p_4 \\ p'_2,p'_4}} \rho_{p_2,p_4,p'_2,p'_4} |p_2\rangle\langle p'_2| \otimes |p_4\rangle\langle p'_4|$$

- Density matrix is transformed at last beam splitter:

$$|p_2, p_4\rangle\langle p'_2, p'_4| = \frac{1}{\sqrt{p_2!p_4!p'_2!p'_4!}}(a_2^\dagger)^{p_2}(a_4^\dagger)^{p_4}|0,0\rangle\langle 0,0|(a_2)^{p'_2}(a_4)^{p'_4}$$

- And transform operators:

$$a_1^\dagger \rightarrow r_j a_1^\dagger + it_j a_2^\dagger$$
$$a_2^\dagger \rightarrow r_j a_2^\dagger + it_j a_1^\dagger$$

Entanglement.

- Output is found in terms of density matrix for two channels:

$$\rho^{out} = \sum_{\substack{p_1,p_2 \\ p_1',p_2'}} \rho^{out}_{p_1,p_2,p_1',p_2'} |p_1\rangle\langle p_1'| \otimes |p_2\rangle\langle p_2'|$$

- Information about subsystem represented by reduced density matrix:

$$\rho_1 = Tr_2(\rho_{12})$$

Von Neumann entropy.

$$S_{FN} = -\sum_n \lambda_n \ln(\lambda_n)$$

$$\rho_1 \to \{\lambda_n\},\ eigenvalues$$

Linear entropy.

$$S_L = 1 - Tr(\rho_1^2)$$

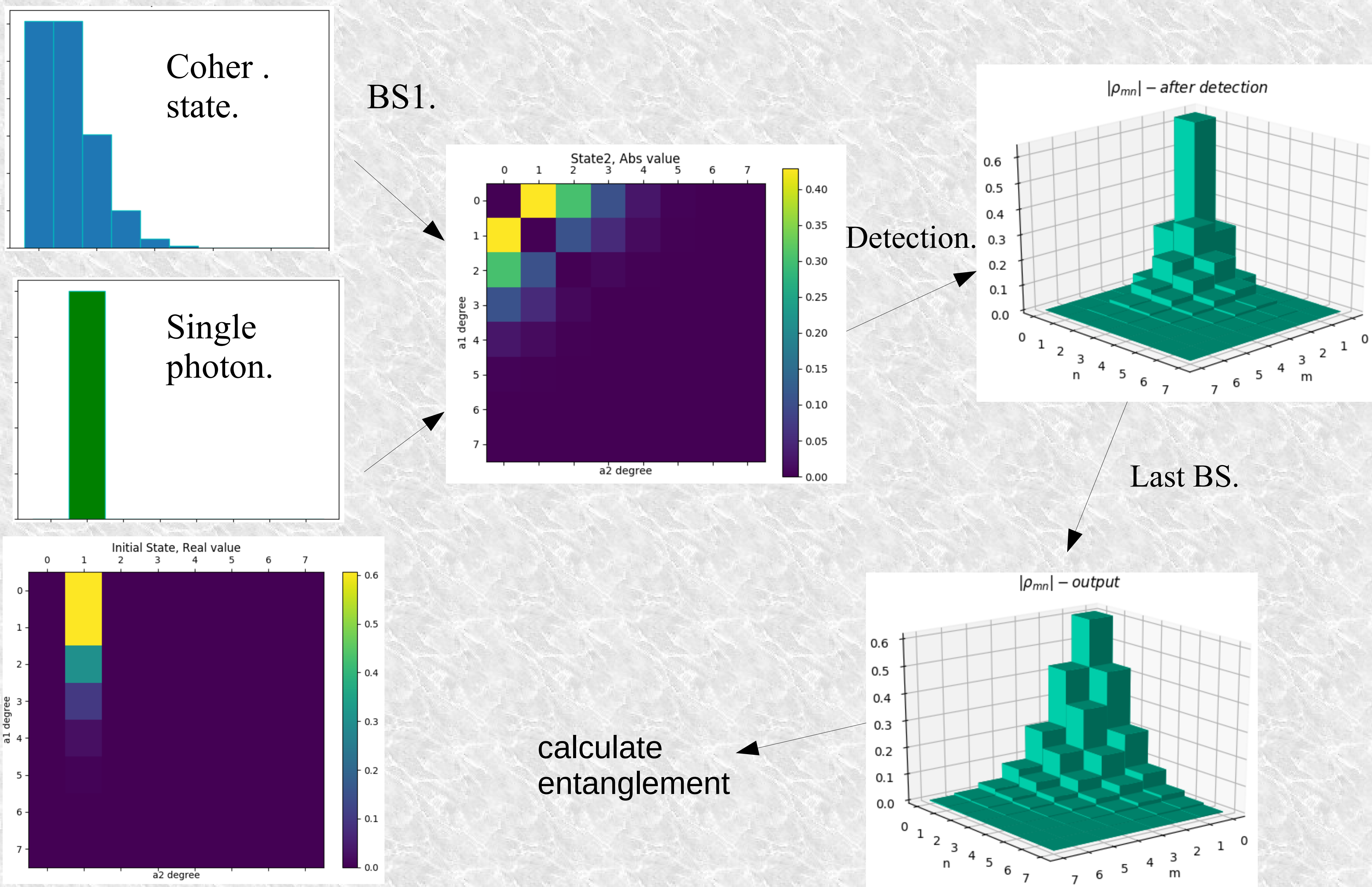
Negativity.

$$N(\rho) = \frac{||\rho^{T_1}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

$$\rho_{12} \to \{\lambda_n\},\ eigenvalues$$

$$LN(\rho) := \log_2(2N+1)$$

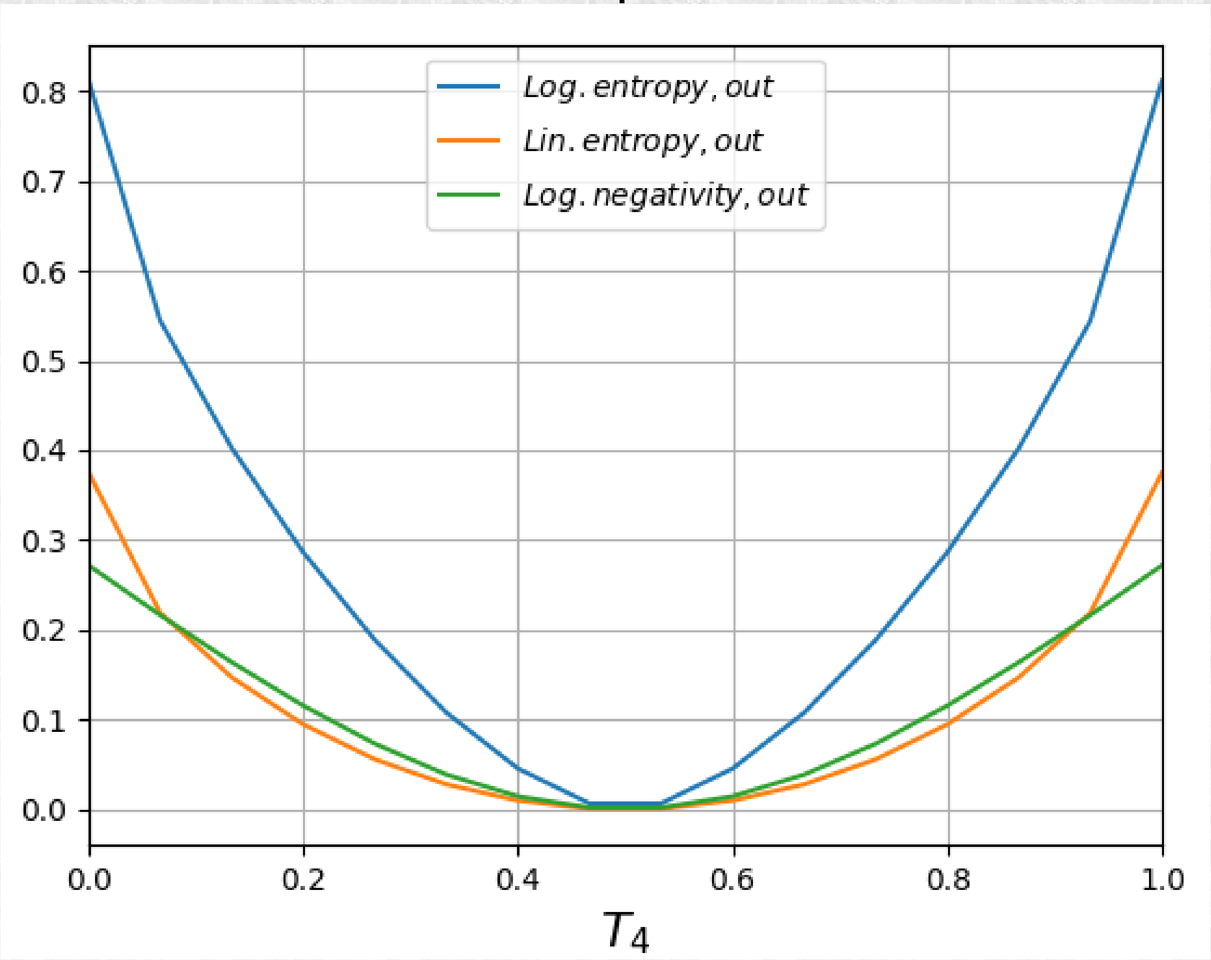
Scheme of calculation.



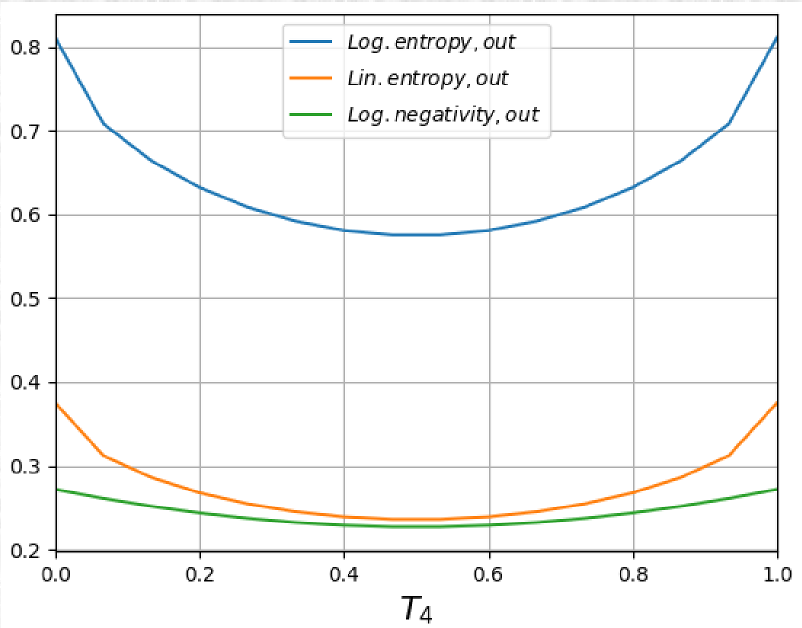
Entanglement.

As input there are coherent state with phot. number equals 1 and single photon. Considered that **both** detectors is clicked.

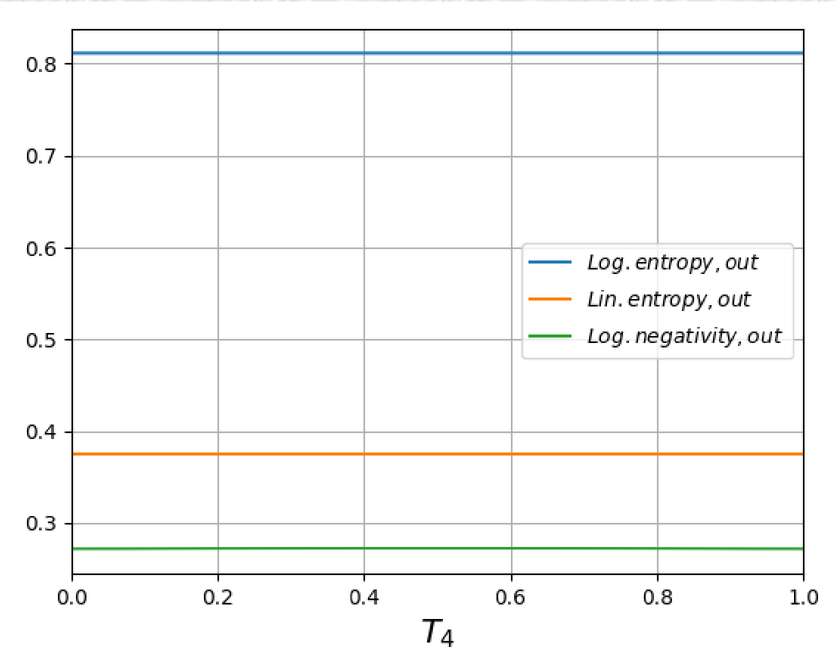
$\Delta\varphi = 0$



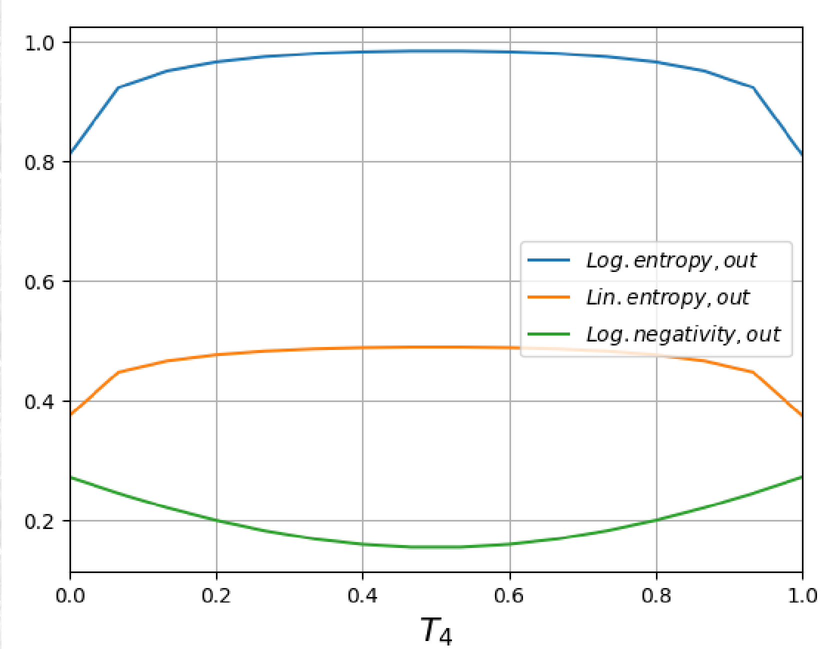
$\Delta\varphi = 0.35\pi$



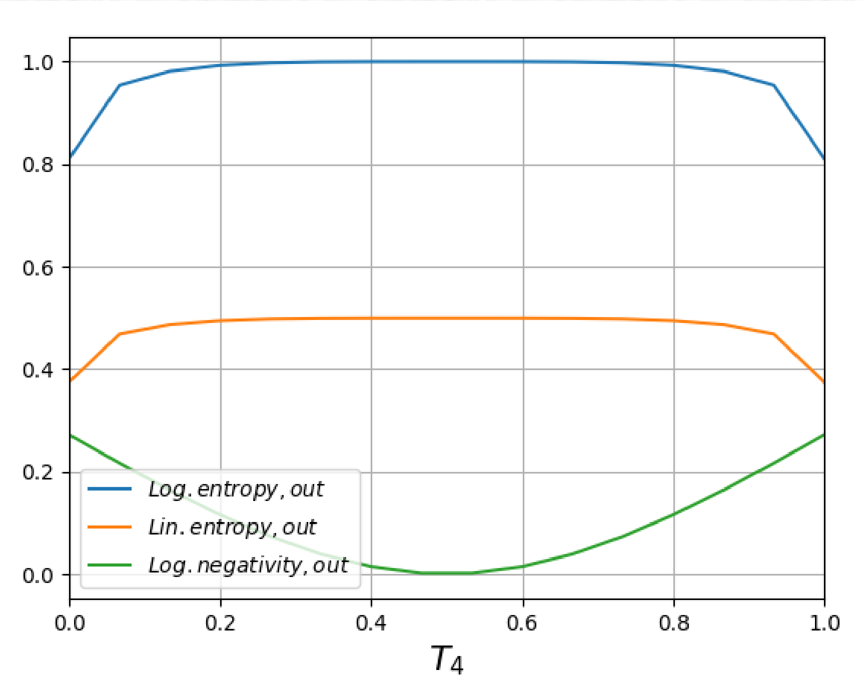
$\Delta\varphi = 0.5\pi$



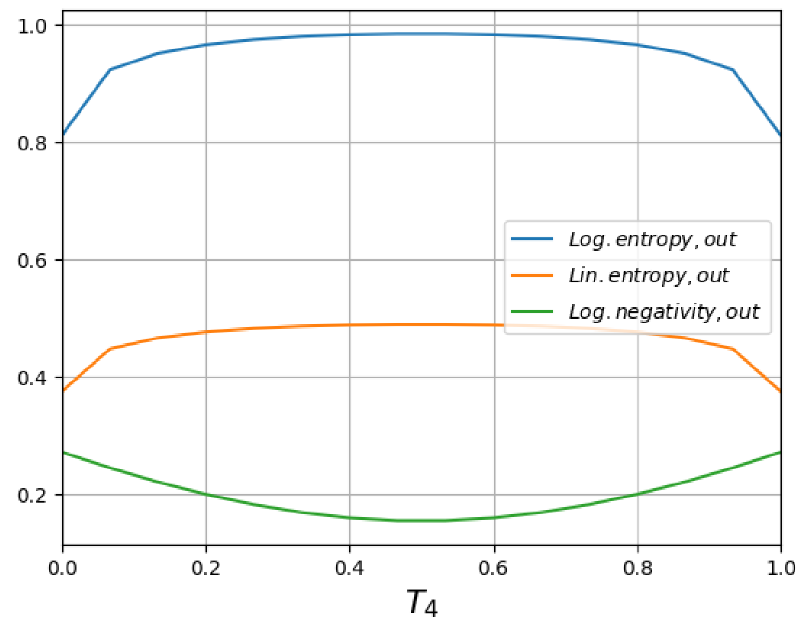
$\Delta\varphi = 0.75\pi$



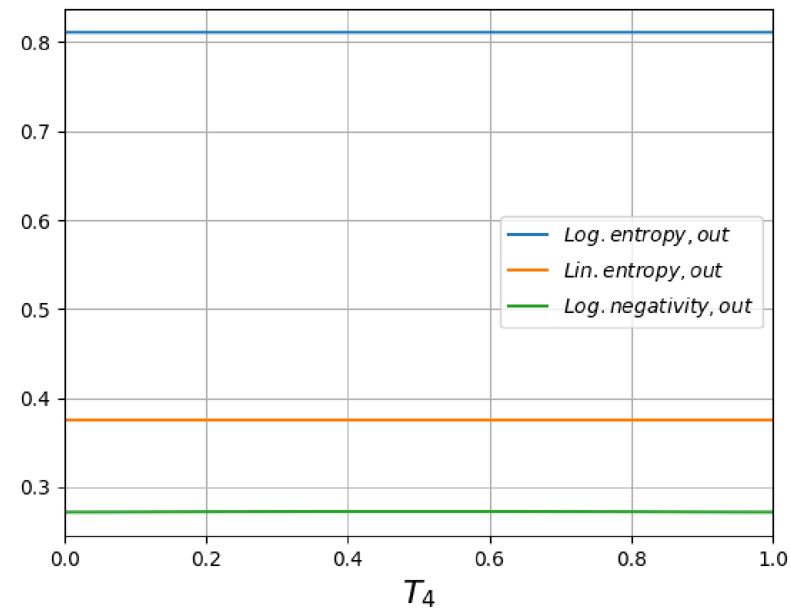
$\Delta\varphi = \pi$



Δφ = 1.25π



Δφ = 1.5π



Δφ = 1.75π