November 30, 2018

1. Input states.

$$|\psi_1^{in}\rangle = |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0} \frac{\alpha^n}{n!} (a_1^\dagger)^n |0\rangle$$
$$|\psi_2^{in}\rangle = |1\rangle = a_2^\dagger |0\rangle$$

$$|\psi_{in}\rangle = |\alpha\rangle \otimes |1\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$
$$\alpha_{m,n} - \textit{defines the mutual state and will be transformed.}$$

2. 1st beam splitter transformation (very fast).

$$T_n + R_n = r_n^2 + t_n^2 = 1$$

$$a_1^\dagger \to t_1 a_1^\dagger + i r_1 a_2^\dagger$$

$$a_2^\dagger \to t_1 a_2^\dagger + i r_1 a_1^\dagger$$

$$\alpha_{m,n}^{in} \to \alpha_{m,n}^{out}$$

$$|\psi_1\rangle = \sum_{m,n} \alpha_{m,n}^{out} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

3. 2nd and 3rd beam splitters transformations (very fast).

$$a_1^\dagger \to t_2 a_2^\dagger + i r_2 a_1^\dagger$$

$$a_2^\dagger \to t_3 a_4^\dagger + i r_3 a_3^\dagger$$

$$\alpha_{m,n}^{(1)} \to \beta_{p_1,p_2,p_3,p_4}$$

$$|\psi_2\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

4. POVM acts on the state.

$$|\psi_{in}\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

$$\hat{\Pi}_{both}|\psi_{in}\rangle = \sum_{\substack{p_1,p_2,p_3,p_4 \\ p_1 \neq 0, p_3 \neq 0}} \beta_{p_1,p_2,p_3,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

$$\hat{\Pi}_{none}|\psi_{in}\rangle = \sum_{\substack{p_2,p_4 \\ p_1 = 0, p_3 = 0}} \beta_{0,p_2,0,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

$$\hat{\Pi}_{top}|\psi_{in}\rangle = \sum_{\substack{p_1,p_2,p_4 \\ p_3 \neq 0, p_1 = 0}} \beta_{0,p_2,p_3,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4}$$

$$\hat{\Pi}_{bottom}|\psi_{in}\rangle = \sum_{\substack{p_1,p_2,p_4 \\ p_1 \neq 0, p_3 = 0}} \beta_{p_1,p_2,0,p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

5. Detection (computationally heavy).

$$\rho_{after\ det.} = Tr_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^\dagger)$$

$Tr_{1,3}()$ *– a partial trace over 1st and 3rd channels.*

$$Tr_{1,3}(\rho) = \sum_{p_1 = p_1', p_3 = p_3'} \sum_{\substack{p_2,p_4 \\ p_2',p_4'}} \rho_{p_1,p_2,p_3,p_4}^{p_1',p_2',p_3',p_4'} |p_2\rangle\langle p_2'| \otimes |p_4\rangle\langle p_4'|$$

6. Detection probability.

$$P_{event} = Tr(\rho_{in}\hat{\Pi}^{\dagger}_{event})$$

7. dens matrix after detection.

$$\rho = \sum_{\substack{p_2,p_4 \\ p'_2,p'_4}} \rho_{p_2,p_4,p'_2,p'_4} |p_2\rangle\langle p'_2| \otimes |p_4\rangle\langle p'_4|$$

8. Phase modulation (very fast):

$$a_2^{\dagger} \rightarrow a_2^{\dagger} e^{i\phi}$$
$$\rho_{m,n,m',n'} \rightarrow \rho_{m,n,m',n'} \exp[i\phi(n - n')]$$

9. Last, 4th BS (computationally heavy).

$$\rho = \sum_{\substack{p_2,p_4 \\ p_2',p_4'}} \rho_{p_2,p_4,p_2',p_4'} |p_2, p_4\rangle\langle p_2', p_4'|$$

Each density matrix element is tansformed:

$$|p_2, p_4\rangle\langle p_2', p_4'| = \frac{1}{\sqrt{p_2!p_4!p_2'!p_4'!}} (a_2^\dagger)^{p_2} (a_4^\dagger)^{p_4} |0,0\rangle\langle 0,0| (a_2)^{p_2'} (a_4)^{p_4'}$$

$$a_2^\dagger \to t_4 b_4^\dagger + i r_4 b_2^\dagger$$

$$a_4^\dagger \to t_4 b_2^\dagger + i r_4 b_4^\dagger$$

Pseudo code:

$$\begin{aligned}
&Input : [\rho_{in}, t, r] \\
&for\ p_1 \in [0, N] : \\
&for\ p_2 \in [0, N] : \\
&for\ p_1' \in [0, N] : \\
&for\ p_2' \in [0, N] : \\
&for\ n \in [0, p_1] : \\
&for\ k \in [0, p_2] : \\
&for\ n' \in [0, p_1'] : \\
&for\ k' \in [0, p_2'] : \\
&\quad d_1 = p_1 - n + k \\
&\quad d_2 = n + p_2 - k \\
&\quad d_1' = p_1' - n' + k' \\
&\quad d_2' = n' + p_2' - k' \\
&\quad c_1 = t^{p1-n+p2-k} (ir)^{n+k} \sqrt{d_1!d_2!p_1!p_2!} / (n!(p1-n)!k!(p2-k)!) \\
&\quad c_2 = t^{p_1'-n'+p_2'-k'} (-ir)^{n'+k'} \sqrt{d_1'!d_2'!p_1'!p_2'!} / (n'!(p_1'-n')!k'!(p_2'-k')!) \\
&\quad \rho_{out}[d_1, d_2, d_1', d_2'] = \rho_{out}[d_1, d_2, d_1', d_2'] + \rho_{in}[p_1, p_2, p_1', p_2'] c_1 c_2 \\
&return\ \rho_{out}
\end{aligned}$$

10. Density matrix as output.

$$\rho^{out} = \sum_{\substack{p_1,p_2 \\ p'_1,p'_2}} \rho^{out}_{p_1,p_2,p'_1,p'_2} |p_1\rangle\langle p'_1| \otimes |p_2\rangle\langle p'_2|$$

11. Calculating negativity (very fast).

$$\boldsymbol{N}(\rho_{AB}) = \frac{||\rho^{T_A}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$
$$\rho^{T_A} \to \{\lambda_n\},\ eigenvalues$$

$$\boldsymbol{LN}(\rho_{AB}) := \log_2(2N + 1)$$

12. Calculating EPR variance (very fast).

$$epr = EPR(\rho_{out})$$

13. The result is one function F that takes input state and all parameters and returns final density matrix form which we can directly calculate one of variables that we need to minimize, for example EPR, see description below.

$$F(T_1, R_1, T_2, R_2, T_3, R_3, T_4, R_4, \phi, detection) \to \rho_{out}$$
$$epr = EPR(\rho_{out})$$

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In the code, function F() is called:
process_all(input_state, bs_params, phase_diff, phase_mod_channel, det_event).
EPR() fuction is called: erp_squeezing_correlations(final_dens_matrix)
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