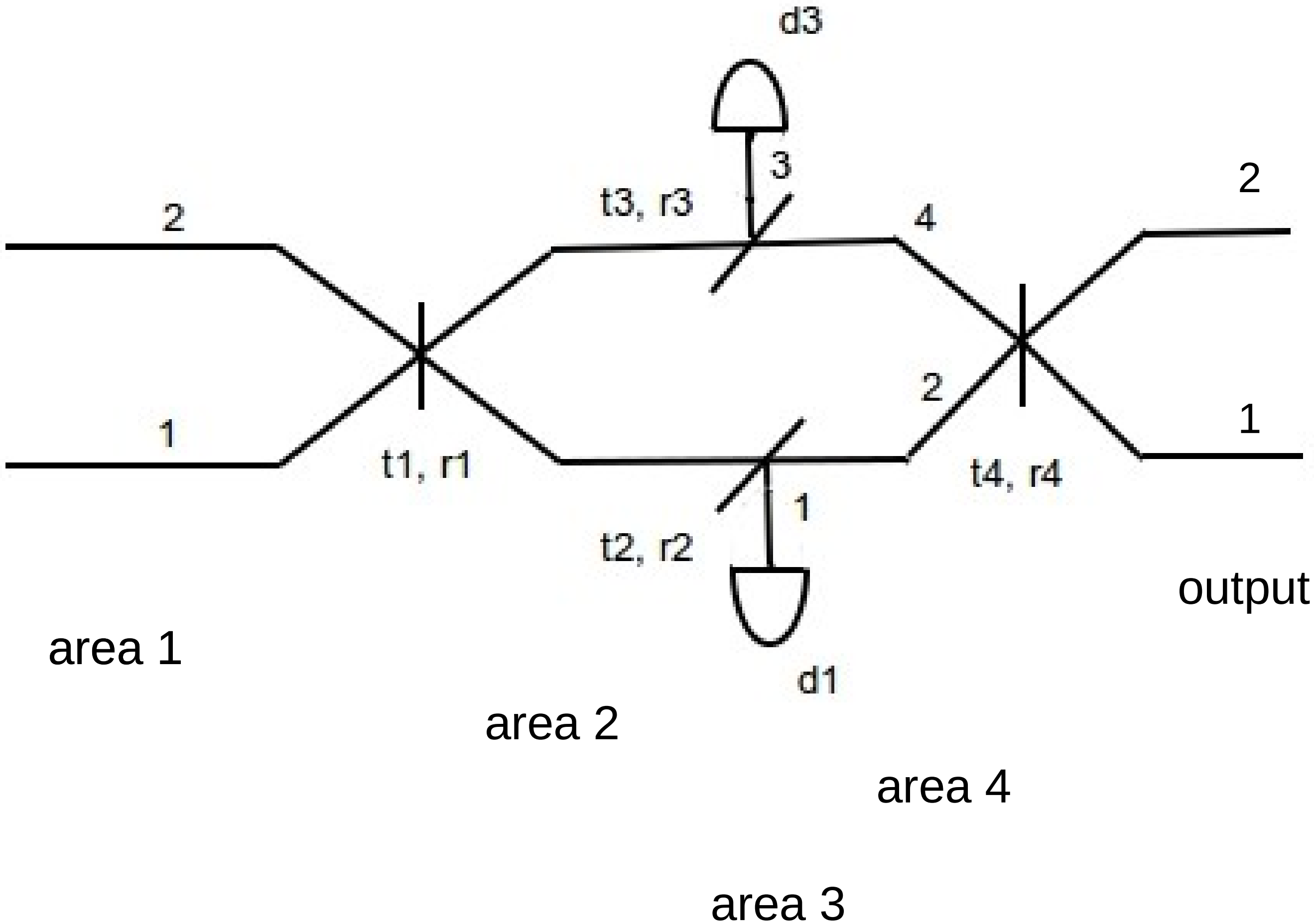
Setup.


d3
3
t3, r3
2
4
2
2
1
t1, r1
1
t4, r4
1
t2, r2
output
area 1
d1
area 2
area 4
area 3

Theory.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle$$

1) State in two channels — area 1:

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

2) Mixed at first BS1:

$$a_1^\dagger \to r_j a_1^\dagger + i t_j a_2^\dagger$$

$$a_2^\dagger \to r_j a_2^\dagger + i t_j a_1^\dagger$$

$$r_j^2 + t_j^2 + a_j^2 = 1$$

3) State in area 2 — after BS1: $|\psi_2\rangle = \sum_{m,n} \alpha_{m,n}^{(2)} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$

$$\sum_{m,n} |\alpha_{m,n}^{(2)}|^2 = 1$$ For ideal BS (a = 0)

4) State in area 3 (4 channels) — after
BS2 and BS3 but before detection:

$$|\psi_3\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (a_1^\dagger)^{p_1} (a_2^\dagger)^{p_2} (a_3^\dagger)^{p_3} (a_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

5) State in area 4 (4 channels) — after detection:

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1-\eta_{SPD})^n |n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1-(1-\eta_{SPD})^n] |n\rangle\langle n|$$

$$|\psi_{out}\rangle = \Pi_{click} |\psi_{in}\rangle$$

5) State in area 4 (4 channels) — after detection:

5.1) Only first detector was clicked:

Detectors are ideal.

$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_4 \\ p_1>0,p_3=0}} \beta_{p_1,p_2,0,p_4}\sqrt{p_1!p_2!p_4!}|p_2,p_4\rangle$$

5.2) Both detectors (1st and 3rd) were clicked:

$$|\psi_4\rangle = \sum_{\substack{p_1,p_2,p_3,p_4 \\ p_1>0,p_3>0}} \beta_{p_1,p_2,p_3,p_4}\sqrt{p_1!p_2!p_3!p_4!}|p_2,p_4\rangle$$

5.3) No detection:

$$|\psi_4\rangle = \sum_{\substack{p_2,p_4 \\ p_1=0,p_3=0}} \beta_{0,p_2,0,p_4}\sqrt{p_2!p_4!}|p_2,p_4\rangle$$

6) State after having been mixed at BS4 (final state):

$$|\psi_{out}\rangle = \sum_{m,n} \alpha^{out}_{m,n}(a_1^\dagger)^m(a_2^\dagger)^n|0\rangle^{\otimes 2}$$

Example with two coherent states:

1) State before BS1:

$$|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle, \quad \alpha = 1$$

2) State after BS1:

$$|\psi_2\rangle \approx e^{-1}(1+\frac{1+i}{\sqrt{2}}a_1^\dagger+\frac{1+i}{\sqrt{2}}a_2^\dagger+ia_1^\dagger a_2^\dagger+\frac{i}{2}(a_1^\dagger)^2+\frac{i}{2}(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}a_1^\dagger(a_2^\dagger)^2+\frac{i-1}{2\sqrt{2}}(a_1^\dagger)^2a_2^\dagger+\ldots)$$