

# Nonlinear transformations of quantum light.

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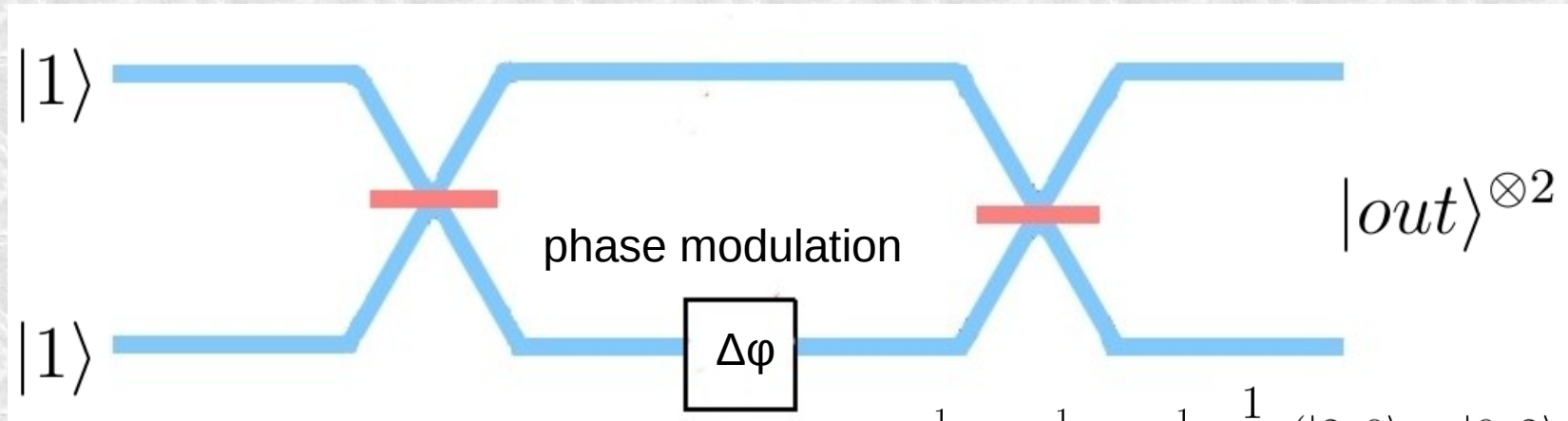
## Motivation.

Studying non linear transformations of quantum light, applying **non-unitary** operators.

$$c_0|0\rangle + c_1|1\rangle + c_2|2\rangle \rightarrow c_0|0\rangle + c_1|1\rangle - c_2|2\rangle$$

Such transformations could be achieved by applying detection.

# Simple setup.



$$\lambda_1 = \frac{1}{3} \quad \lambda_2 = \frac{1}{3} \quad \lambda_3 = \frac{1}{3} \quad \frac{1}{\sqrt{3}} (|2, 0\rangle + |0, 2\rangle + |1, 1\rangle)$$

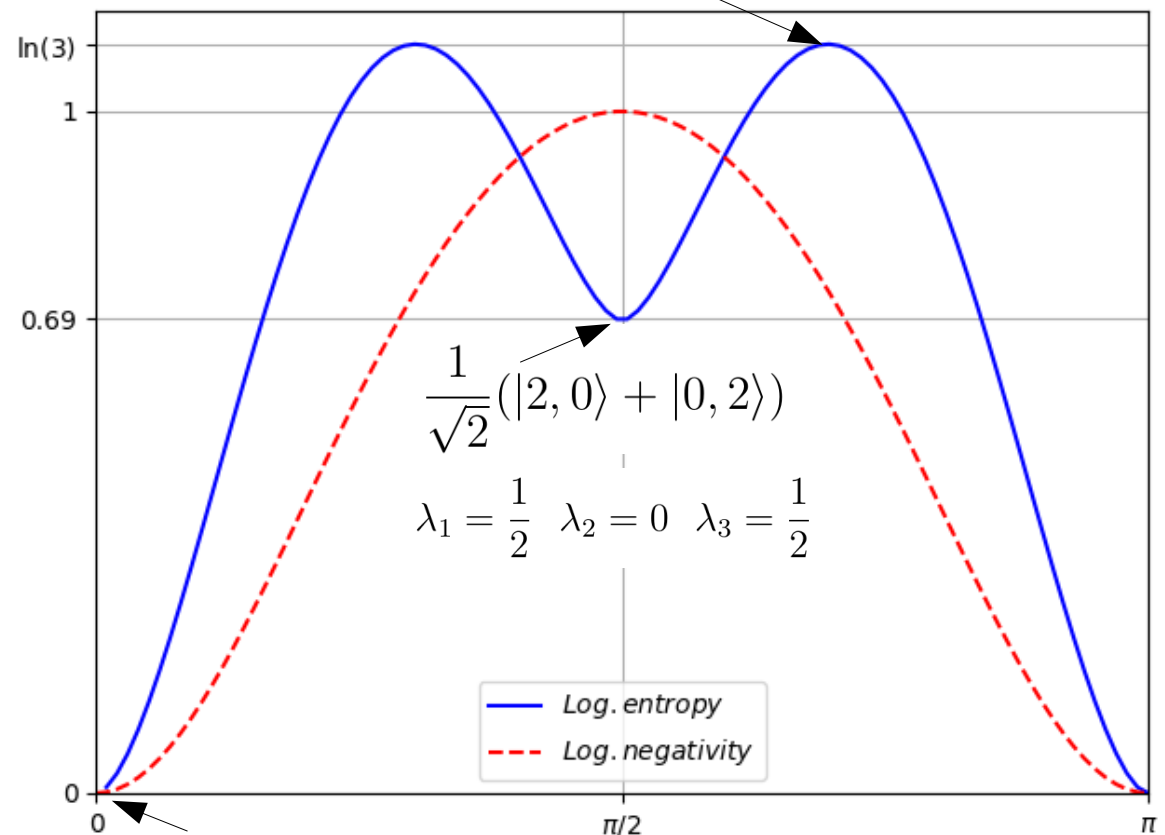
$$S_{FN} = - \sum_n \lambda_n \ln(\lambda_n)$$

$\rho_1 \rightarrow \{\lambda_n\}$ , eigenvalues

$$N(\rho) = \frac{\|\rho^{T_1}\|_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

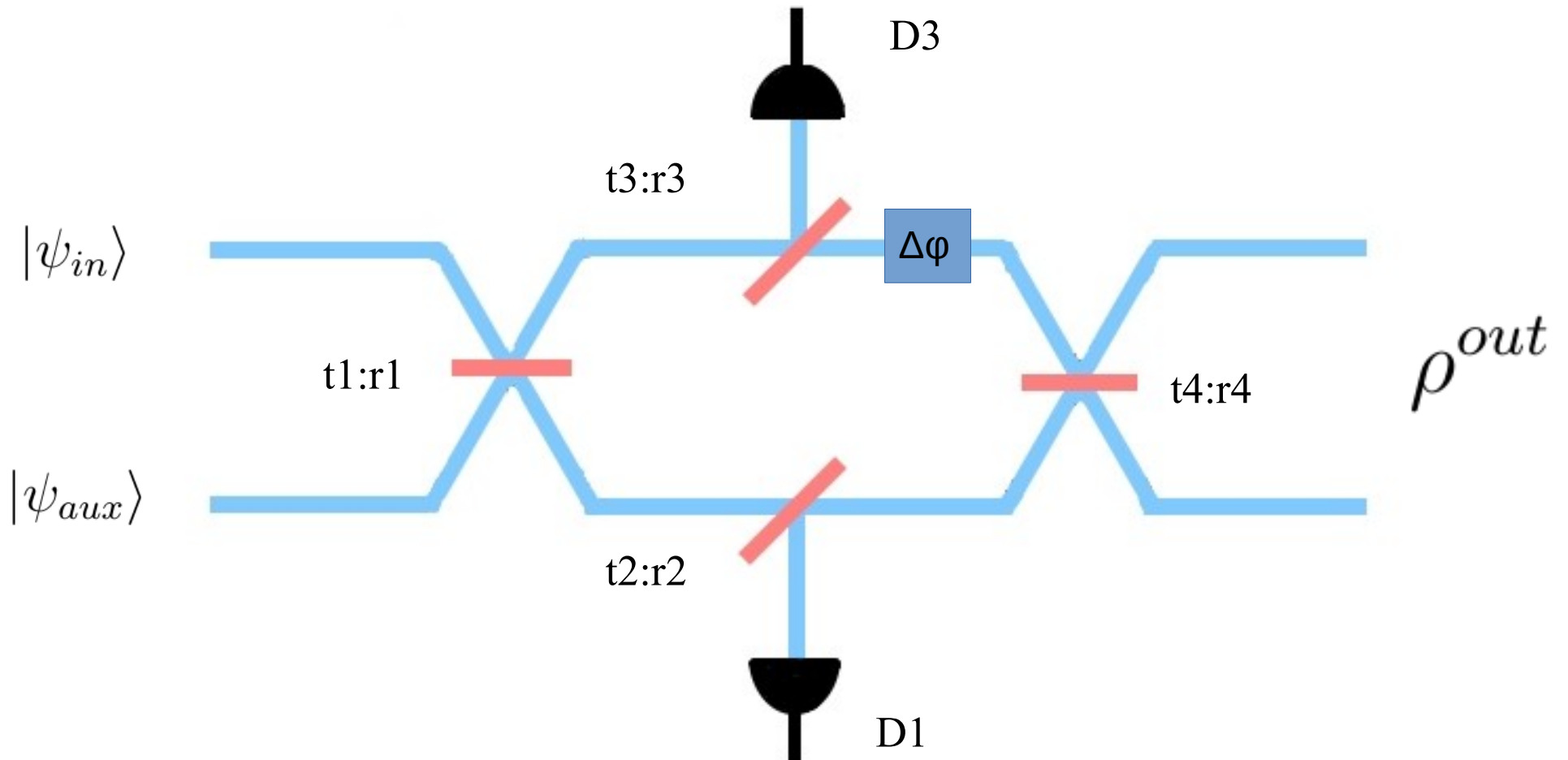
$\rho_{12} \rightarrow \{\lambda_n\}$ , eigenvalues

$$LN(\rho) := \log_2(2N + 1)$$



$$\lambda_1 = 0 \quad \lambda_2 = 1 \quad \lambda_3 = 0 \quad |1, 1\rangle$$

## Scheme with detection.



# Theoretical approach.

- As an input for system there could be sent different quantum light states in two channels.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n(a_1^\dagger)^n|0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n(a_2^\dagger)^n|0\rangle$$

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

- Mutual state is transformed at beam splitter:

$$\begin{aligned}r_j^2 + t_j^2 + a_j^2 &= 1 \\a_1^\dagger &\rightarrow r_j a_1^\dagger + i t_j a_2^\dagger \\a_2^\dagger &\rightarrow r_j a_2^\dagger + i t_j a_1^\dagger\end{aligned}$$

- After transformation at first, second and third BS, the state in 4 channels will look like this:

$$|\psi_4\rangle = \sum_{p_1, p_2, p_3, p_4} \beta_{p_1, p_2, p_3, p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

# Detection.

- Detection is described in terms of POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1 - \eta_{SPD})^n |n\rangle \langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1 - (1 - \eta_{SPD})^n] |n\rangle \langle n|$$

- Four possibilities of detectors behaviour in total.

$$\hat{\Pi} = \Pi_{click}^{(1)} \otimes I^{(2)} \otimes \Pi_{click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{click}^{(1)} \otimes I^{(2)} \otimes \Pi_{no-click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{no-click}^{(1)} \otimes I^{(2)} \otimes \Pi_{click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{no-click}^{(1)} \otimes I^{(2)} \otimes \Pi_{no-click}^{(3)} \otimes I^{(4)}$$

- both are clicked.
- first is clicked, third is silent.
- first is silent, third is clicked.
- both are silent.

- After detection state is collapsed into density matrix:

$$\rho = \text{Tr}_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^\dagger)$$

$$\rho = \sum_{\substack{p_2, p_4 \\ p'_2, p'_4}} \rho_{p_2, p_4, p'_2, p'_4} |p_2\rangle\langle p'_2| \otimes |p_4\rangle\langle p'_4|$$

- Density matrix is transformed at last beam splitter:

$$|p_2, p_4\rangle\langle p'_2, p'_4| = \frac{1}{\sqrt{p_2!p_4!p'_2!p'_4!}} (a_2^\dagger)^{p_2} (a_4^\dagger)^{p_4} |0, 0\rangle\langle 0, 0| (a_2)^{p'_2} (a_4)^{p'_4}$$

- And transform operators:

$$\begin{aligned} a_1^\dagger &\rightarrow r_j a_1^\dagger + it_j a_2^\dagger \\ a_2^\dagger &\rightarrow r_j a_2^\dagger + it_j a_1^\dagger \end{aligned}$$



# Entanglement.

- Output is found in terms of density matrix for two channels:

$$\rho^{out} = \sum_{\substack{p_1, p_2 \\ p'_1, p'_2}} \rho_{p_1, p_2, p'_1, p'_2}^{out} |p_1\rangle \langle p'_1| \otimes |p_2\rangle \langle p'_2|$$

- Information about subsystem represented by reduced density matrix:

$$\rho_1 = Tr_2(\rho_{12})$$

Von Neumann entropy.

$$S_{FN} = - \sum_n \lambda_n \ln(\lambda_n)$$

$\rho_1 \rightarrow \{\lambda_n\}$ , *eigenvalues*

Linear entropy.

$$S_L = 1 - \text{Tr}(\rho_1^2)$$

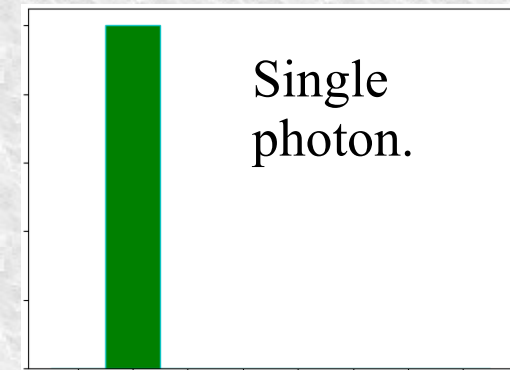
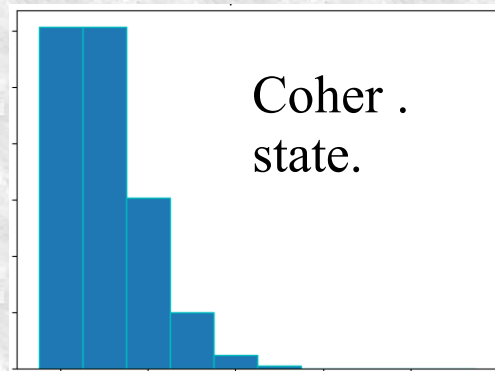
# Negativity.

$$N(\rho) = \frac{||\rho^{T_1}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

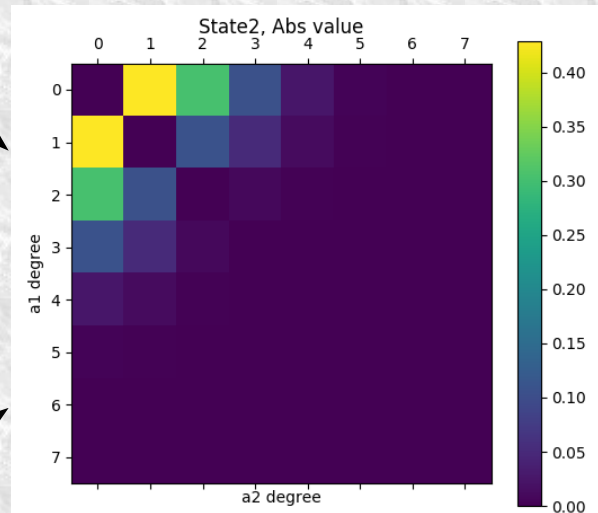
$\rho_{12} \rightarrow \{\lambda_n\}$ , *eigenvalues*

$$LN(\rho) := \log_2(2N + 1)$$

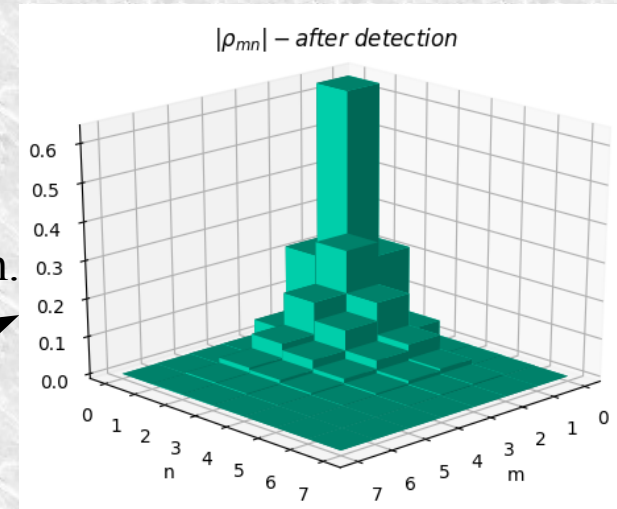
# Scheme of calculation.



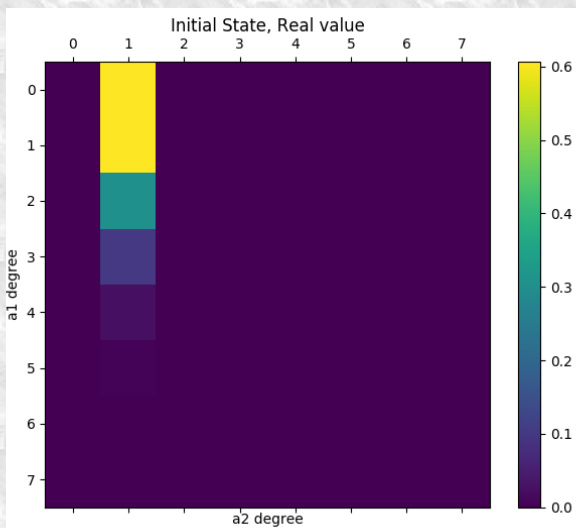
BS1.



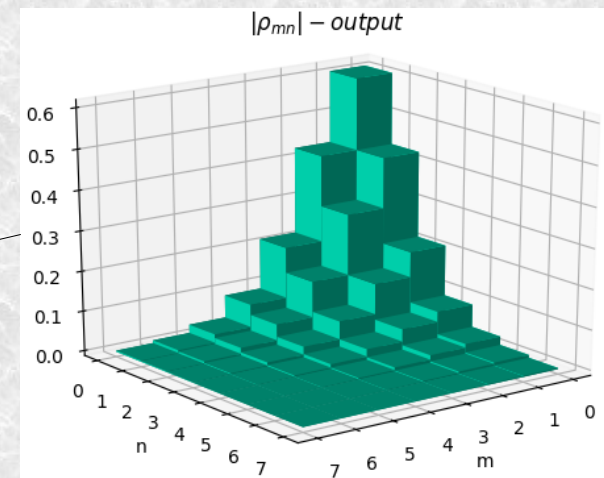
Detection.



Last BS.



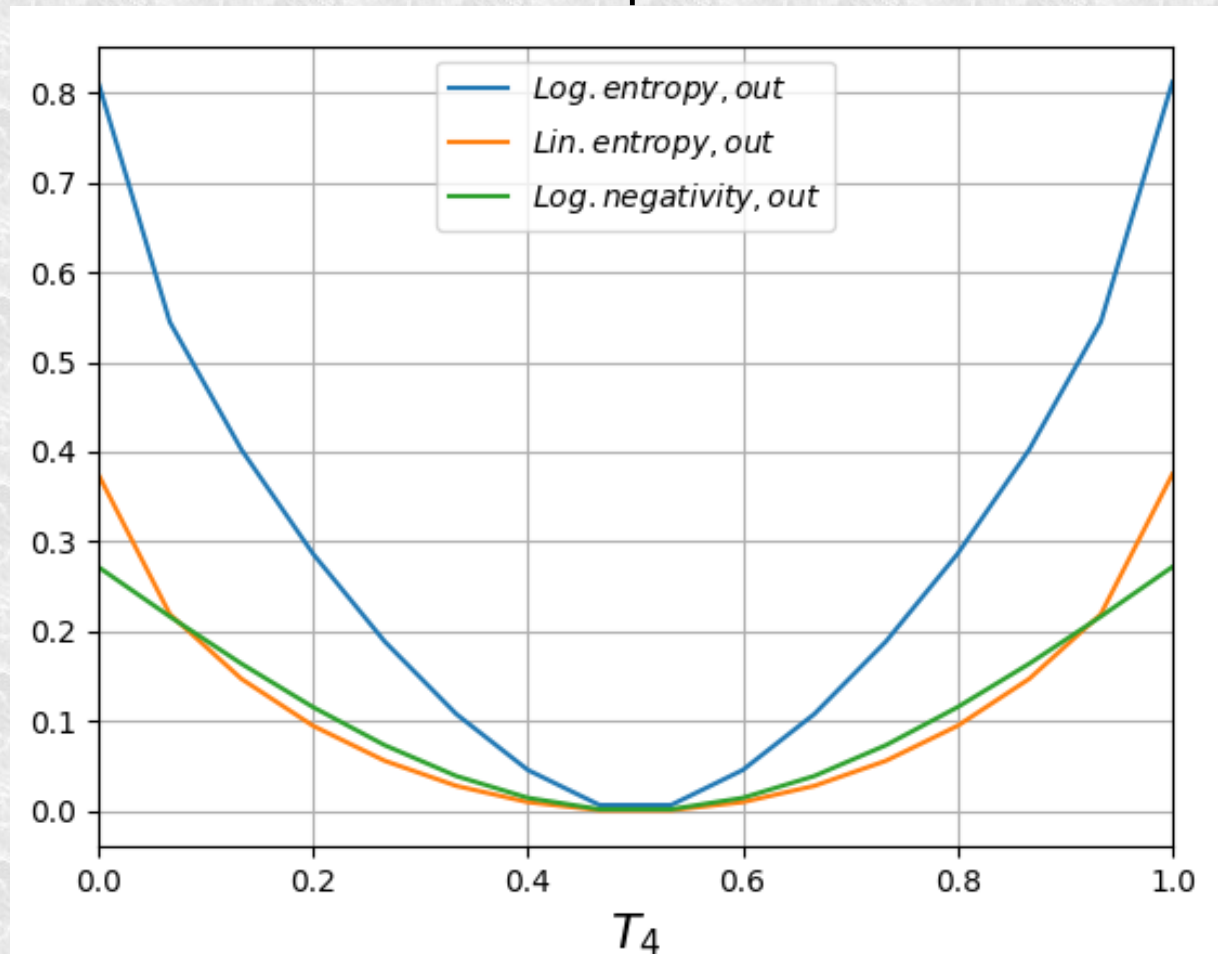
calculate  
entanglement



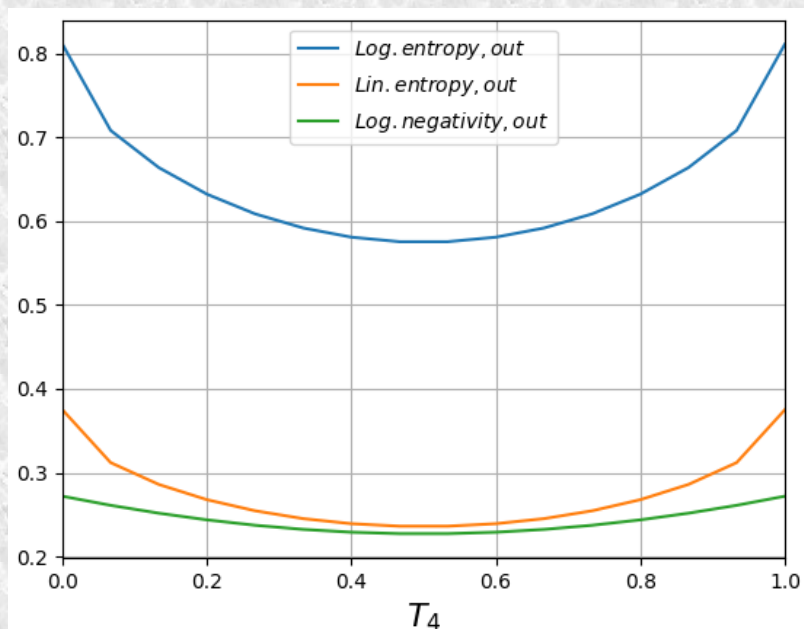
# Entanglement.

As input there are coherent state with phot. number equals 1 and single photon. Considered that **both** detectors is clicked.

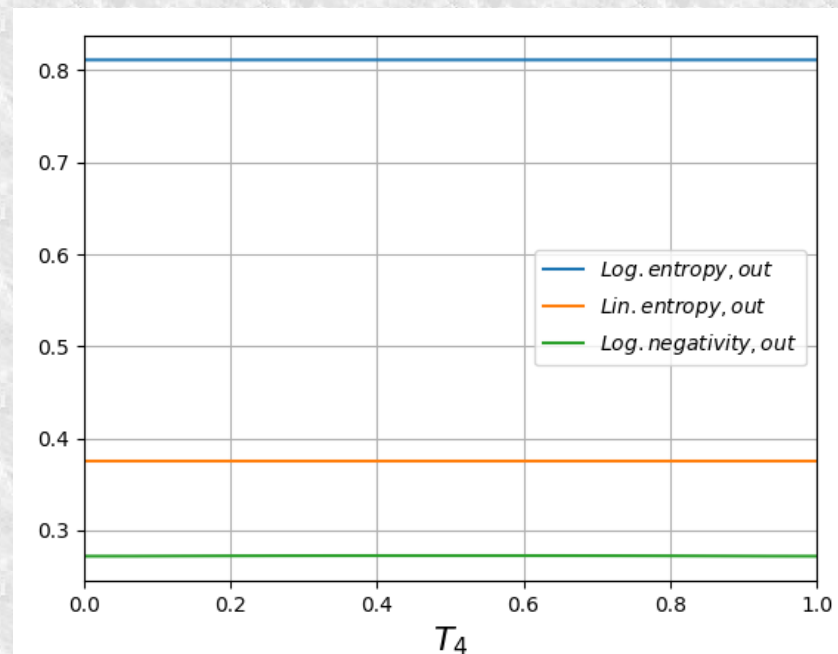
$$\Delta\varphi = 0$$



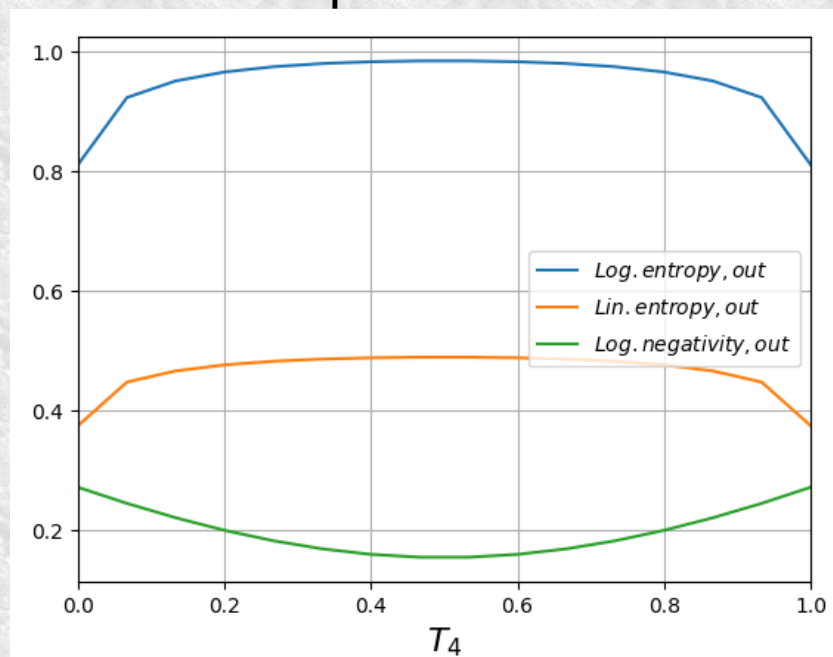
$$\Delta\varphi = 0.35\pi$$



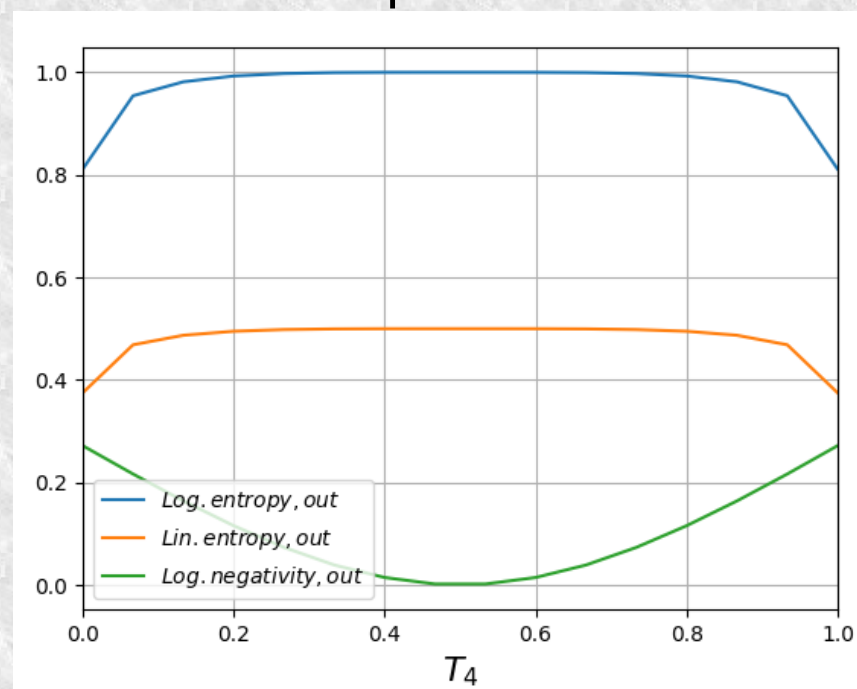
$$\Delta\varphi = 0.5\pi$$



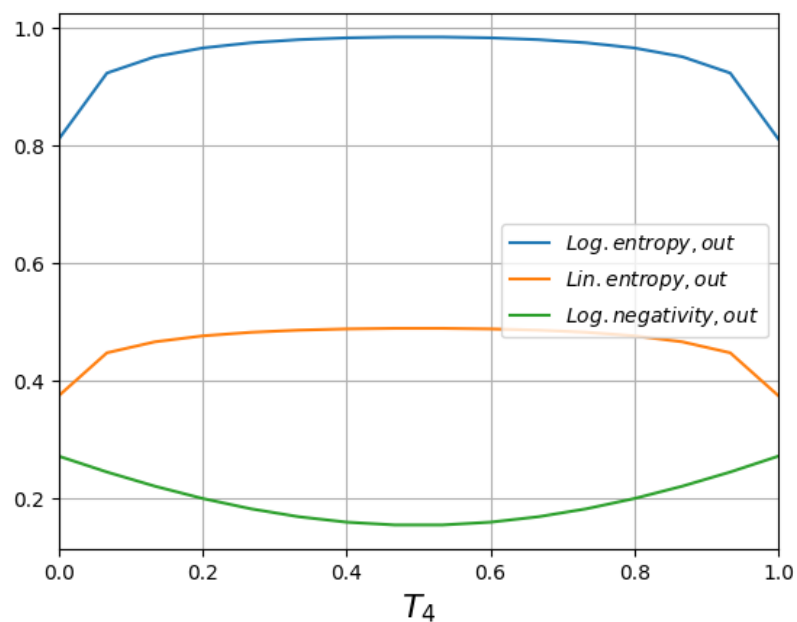
$$\Delta\varphi = 0.75\pi$$



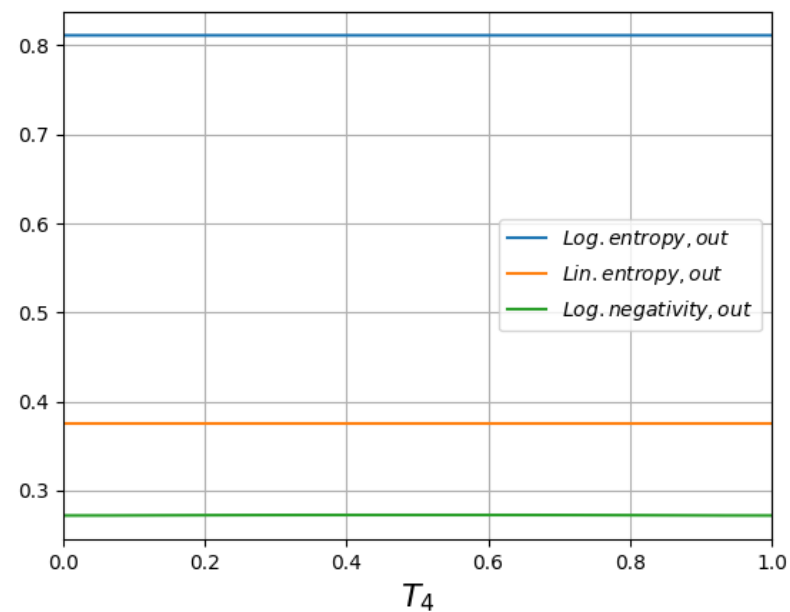
$$\Delta\varphi = \pi$$



$$\Delta\varphi = 1.25\pi$$



$$\Delta\varphi = 1.5\pi$$



$$\Delta\varphi = 1.75\pi$$

