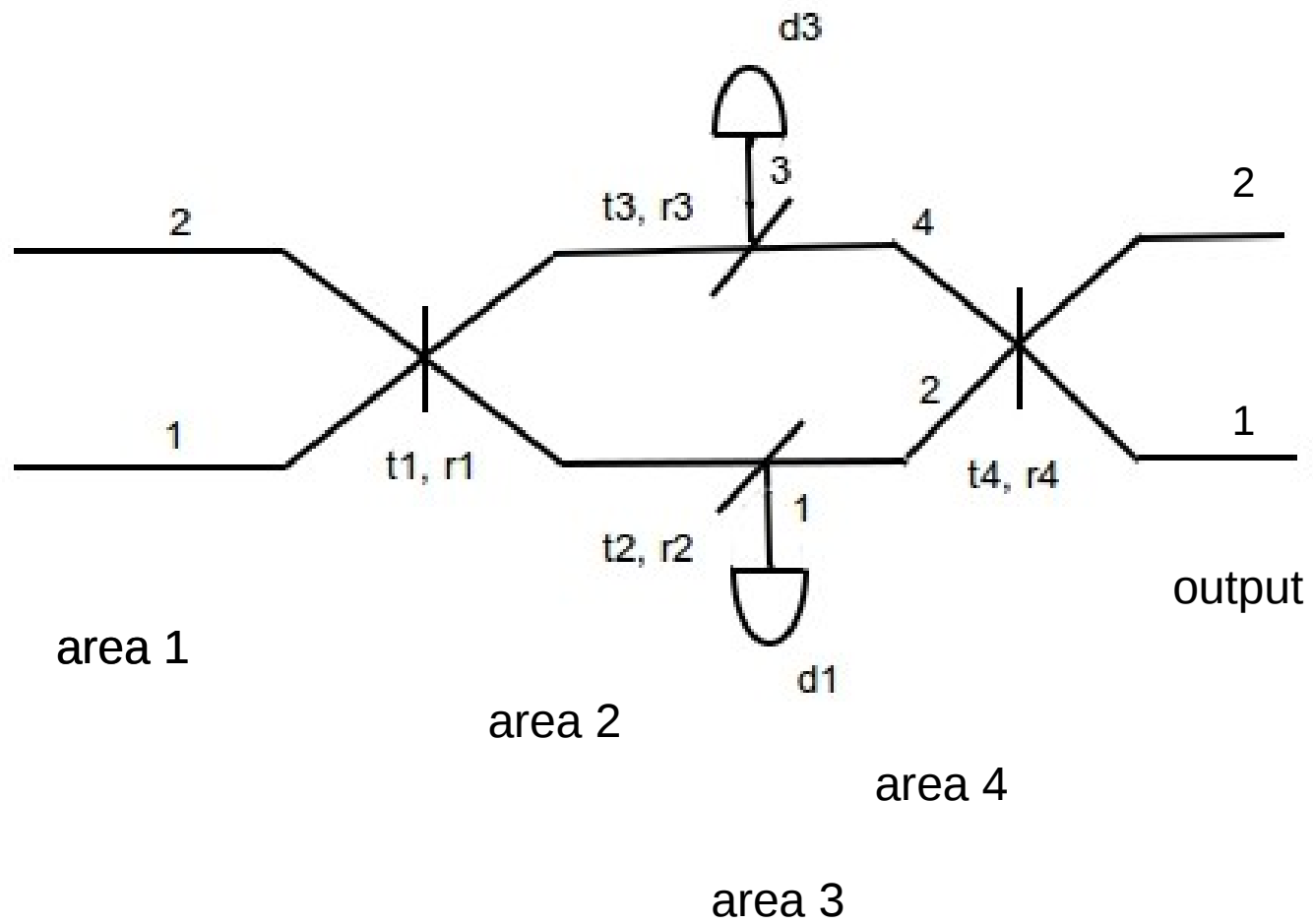


Setup.



Theory.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n(a_1^\dagger)^n|0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n(a_2^\dagger)^n|0\rangle$$

1) State in two channels — area 1:

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

2) Mixed at first BS1:

$$\begin{aligned} a_1^\dagger &\rightarrow r_j a_1^\dagger + it_j a_2^\dagger \\ a_2^\dagger &\rightarrow r_j a_2^\dagger + it_j a_1^\dagger \end{aligned} \quad r_j^2 + t_j^2 + a_j^2 = 1$$

3) State in area 2 — after BS1: $|\psi_2\rangle = \sum_{m,n} \alpha_{m,n}^{(2)} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$

$$\sum_{m,n} |\alpha_{m,n}^{(2)}|^2 = 1 \quad \text{For ideal BS (a = 0)}$$

4) State in area 3 (4 channels) — after BS2 and BS3 but before detection:

$$|\psi_3\rangle = \sum_{p_1, p_2, p_3, p_4} \beta_{p_1, p_2, p_3, p_4} (a_1^\dagger)^{p_1} (a_2^\dagger)^{p_2} (a_3^\dagger)^{p_3} (a_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

5) State in area 4 (4 channels) — after detection:

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1 - \eta_{SPD})^n |n\rangle \langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1 - (1 - \eta_{SPD})^n] |n\rangle \langle n|$$

$$|\psi_{out}\rangle = \Pi_{click} |\psi_{in}\rangle$$

5) State in area 4 (4 channels) — after detection:

5.1) Only first detector was clicked:

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_4 \\ p_1 > 0, p_3 = 0}} \beta_{p_1, p_2, 0, p_4} \sqrt{p_1! p_2! p_4!} |p_2, p_4\rangle$$

Detectors are ideal.

5.2) Both detectors (1st and 3rd) were clicked:

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_3, p_4 \\ p_1 > 0, p_3 > 0}} \beta_{p_1, p_2, p_3, p_4} \sqrt{p_1! p_2! p_3! p_4!} |p_2, p_4\rangle$$

5.3) No detection:

$$|\psi_4\rangle = \sum_{\substack{p_2, p_4 \\ p_1 = 0, p_3 = 0}} \beta_{0, p_2, 0, p_4} \sqrt{p_2! p_4!} |p_2, p_4\rangle$$

6) State after having been mixed at BS4 (final state):

$$|\psi_{out}\rangle = \sum_{m, n} \alpha_{m, n}^{out} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

Example with two coherent states:

1) State before BS1:

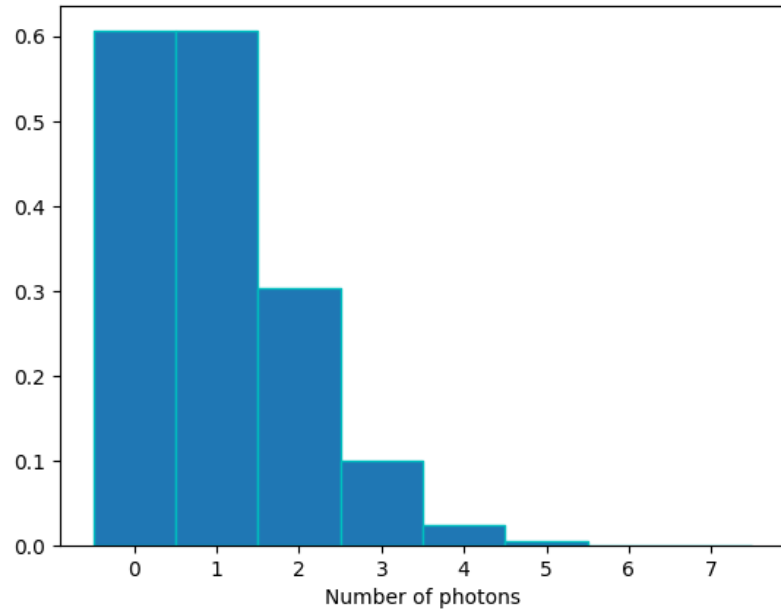
$$|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle, \quad \alpha = 1$$

2) State after BS1:

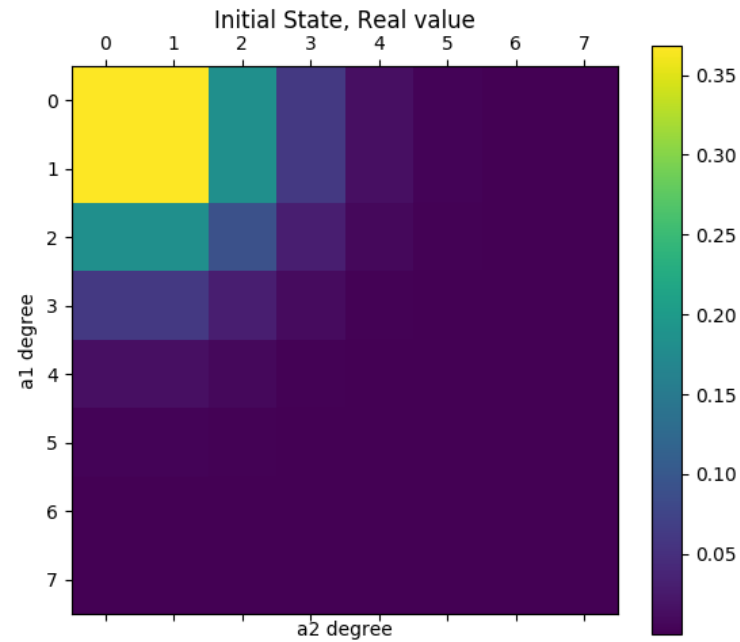
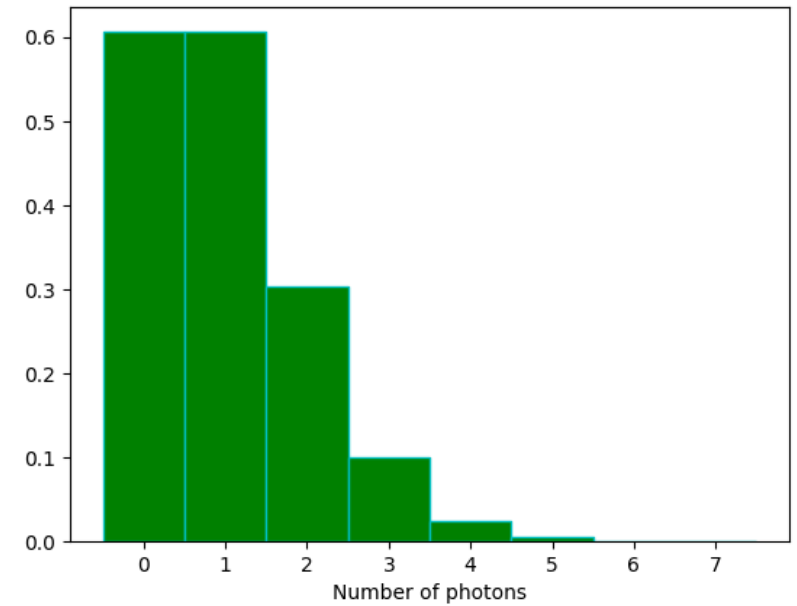
$$|\psi_2\rangle \approx e^{-1} \left(1 + \frac{1+i}{\sqrt{2}} a_1^\dagger + \frac{1+i}{\sqrt{2}} a_2^\dagger + i a_1^\dagger a_2^\dagger + \frac{i}{2} (a_1^\dagger)^2 + \frac{i}{2} (a_2^\dagger)^2 + \frac{i-1}{2\sqrt{2}} a_1^\dagger (a_2^\dagger)^2 + \frac{i-1}{2\sqrt{2}} (a_1^\dagger)^2 a_2^\dagger + \dots \right)$$

1) Input — coherent state. Aux — coherent state.

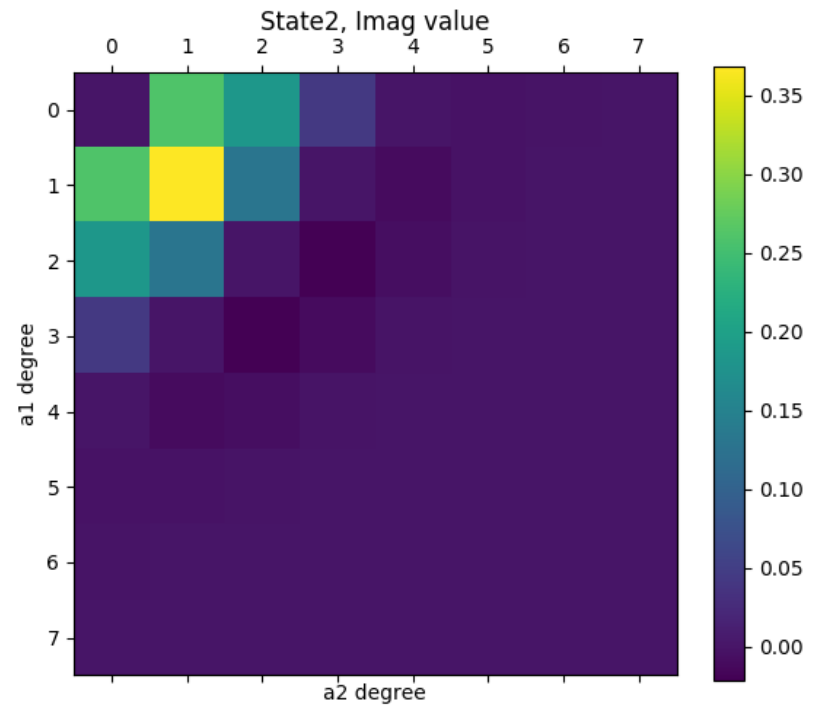
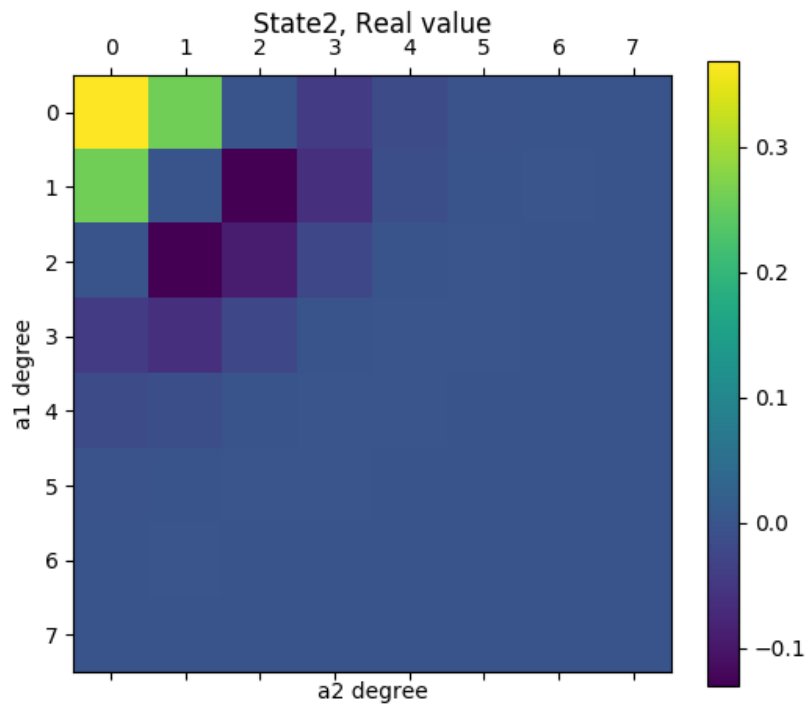
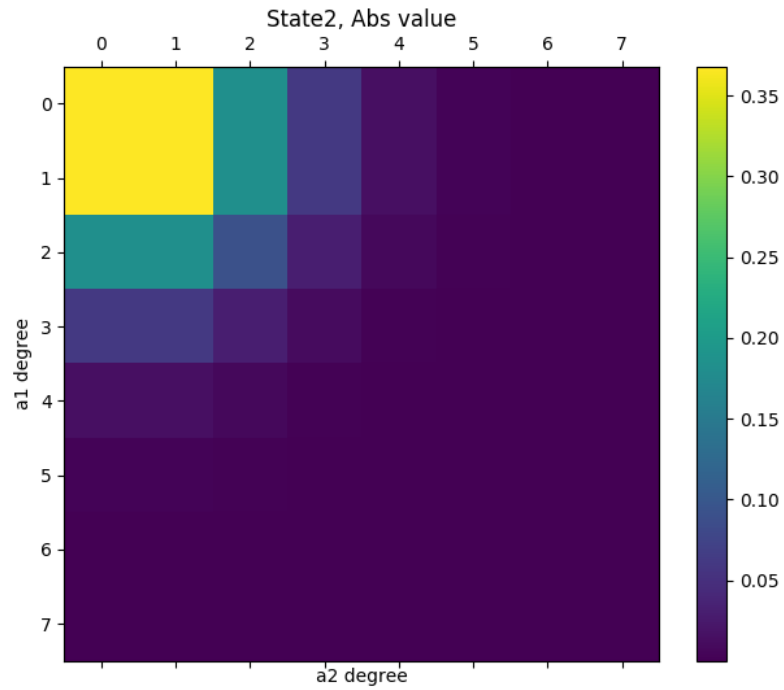
Input state



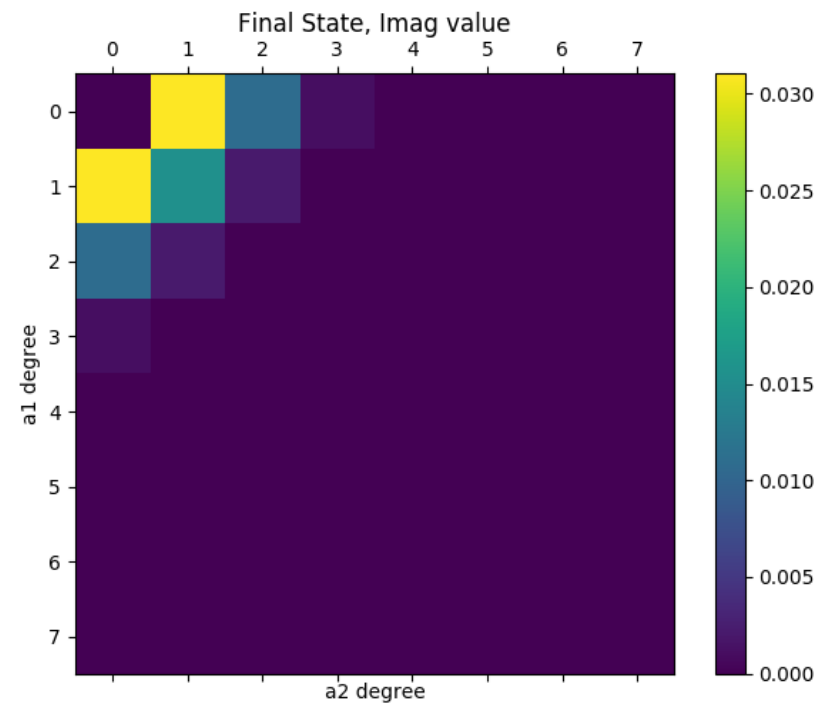
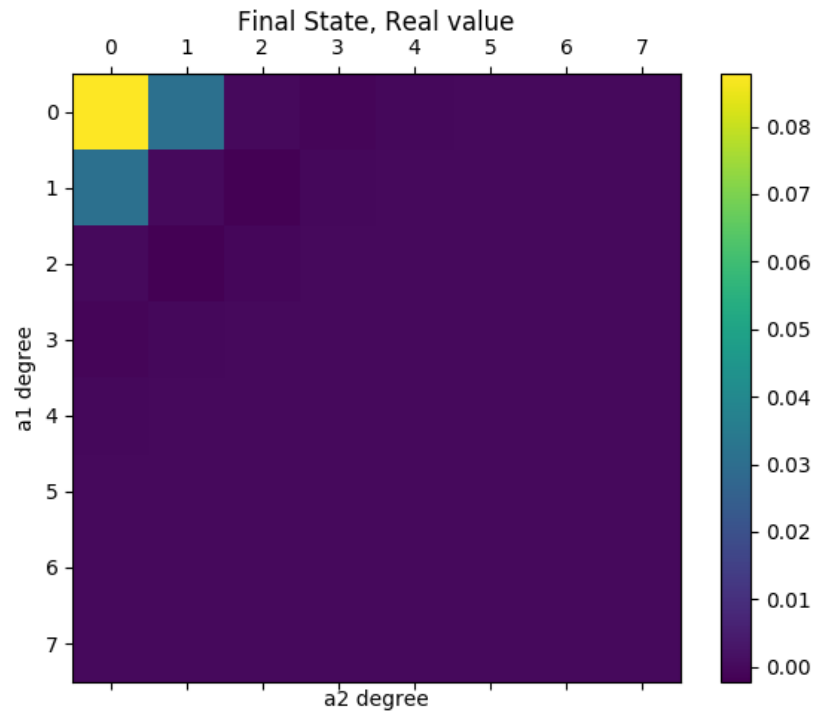
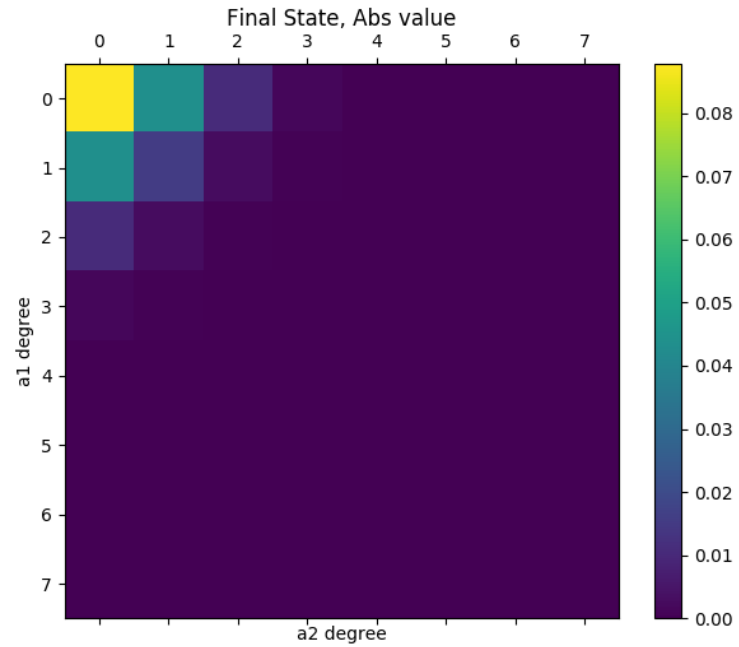
Auxiliary state



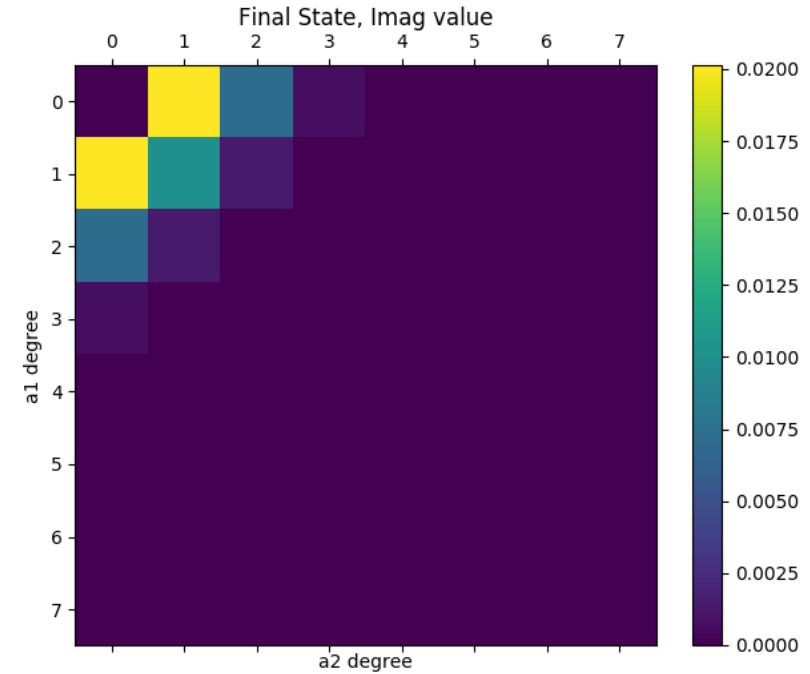
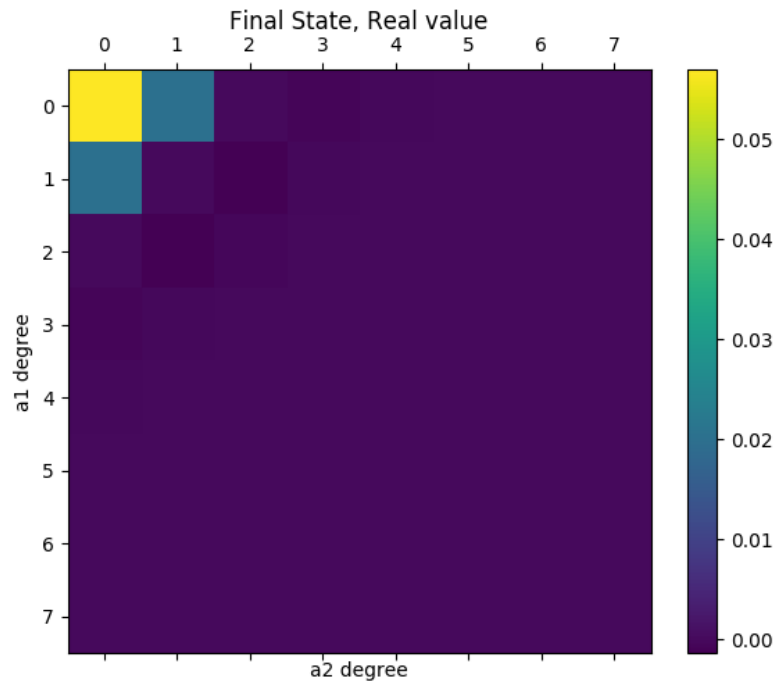
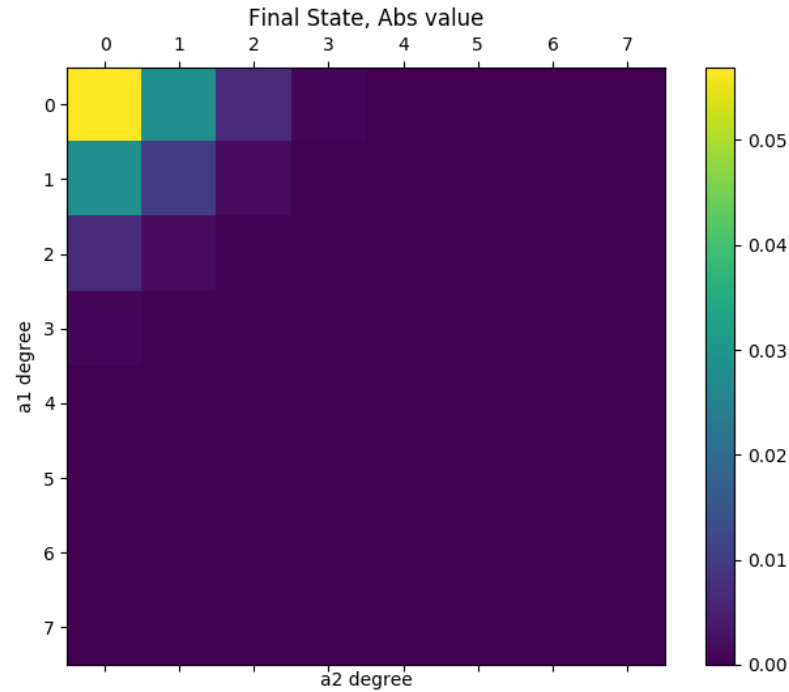
State in area2.



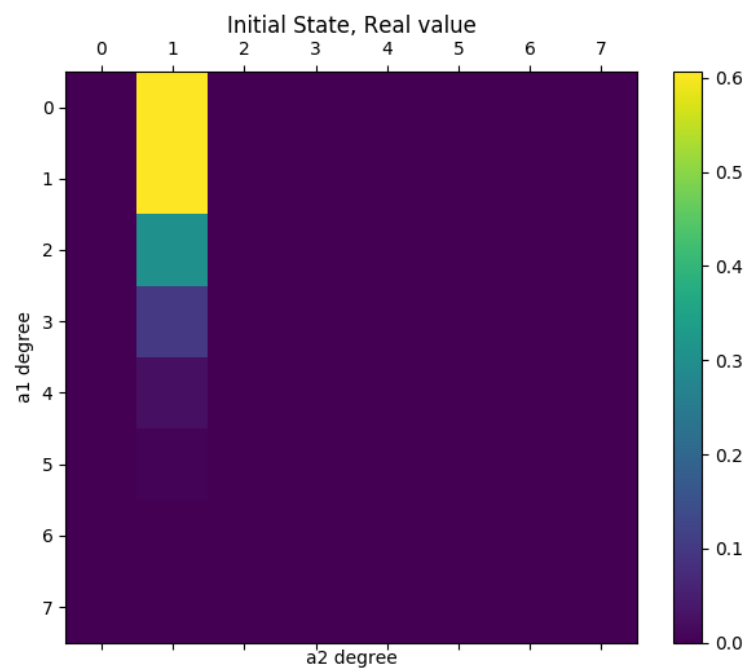
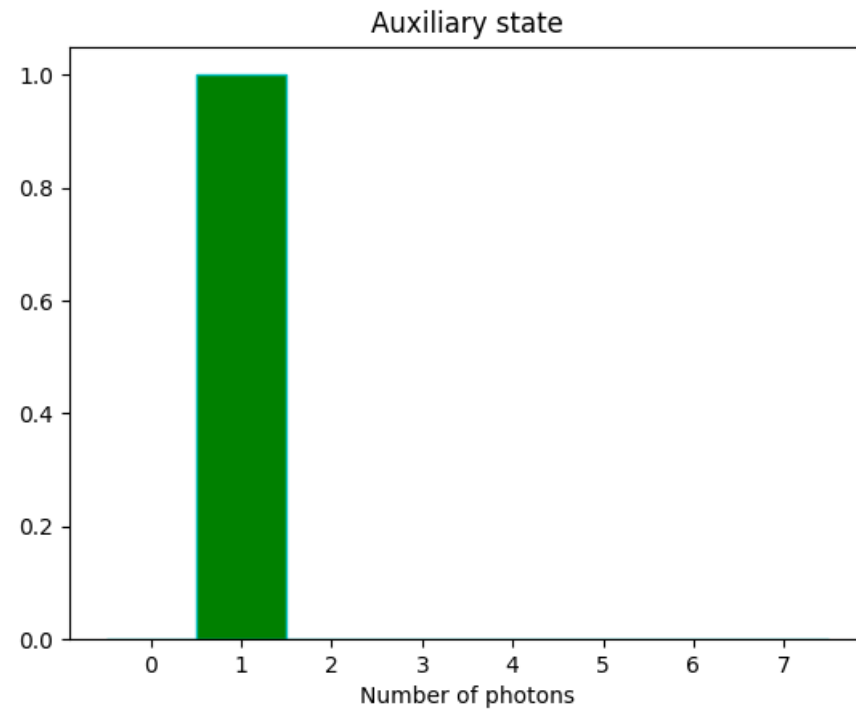
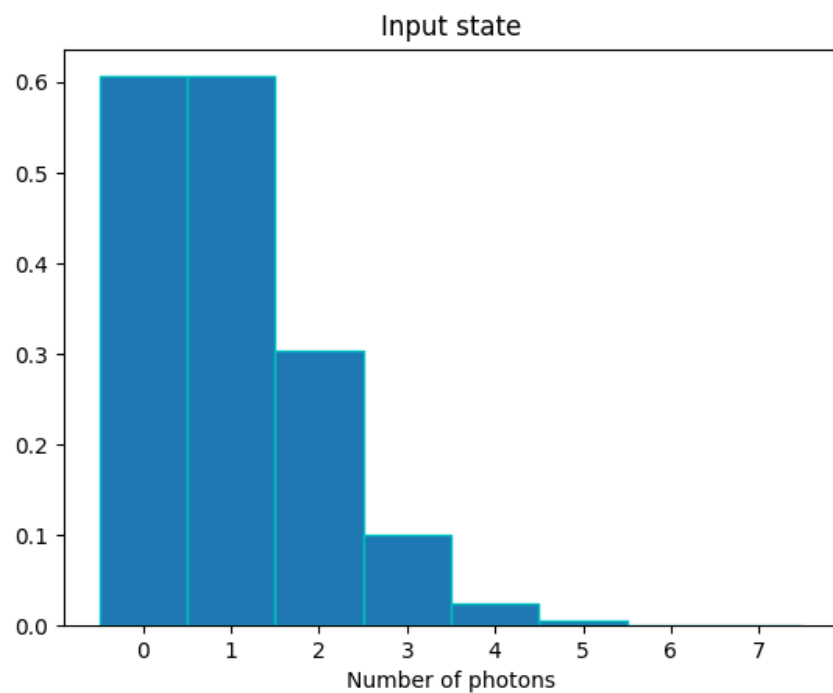
Output(final) state. First detector was clicked.



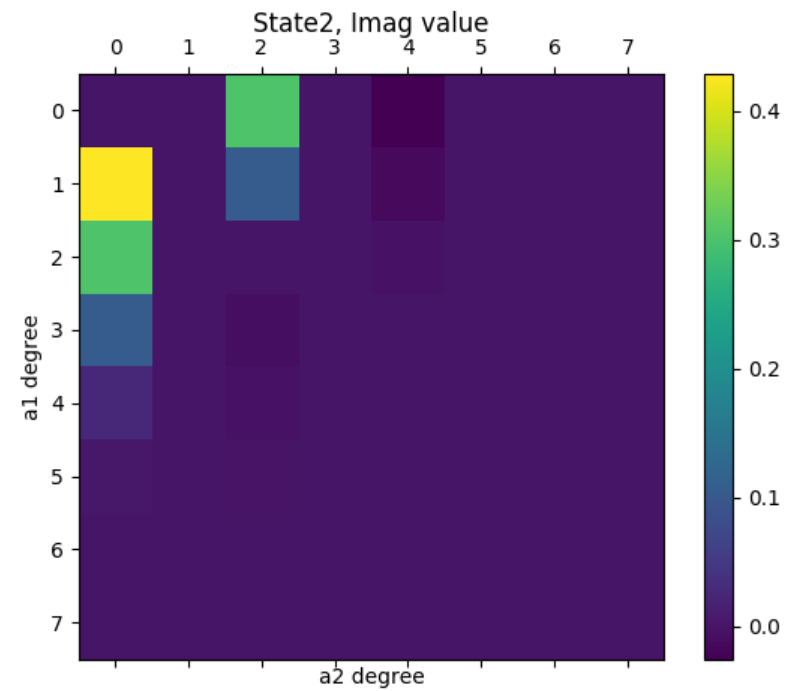
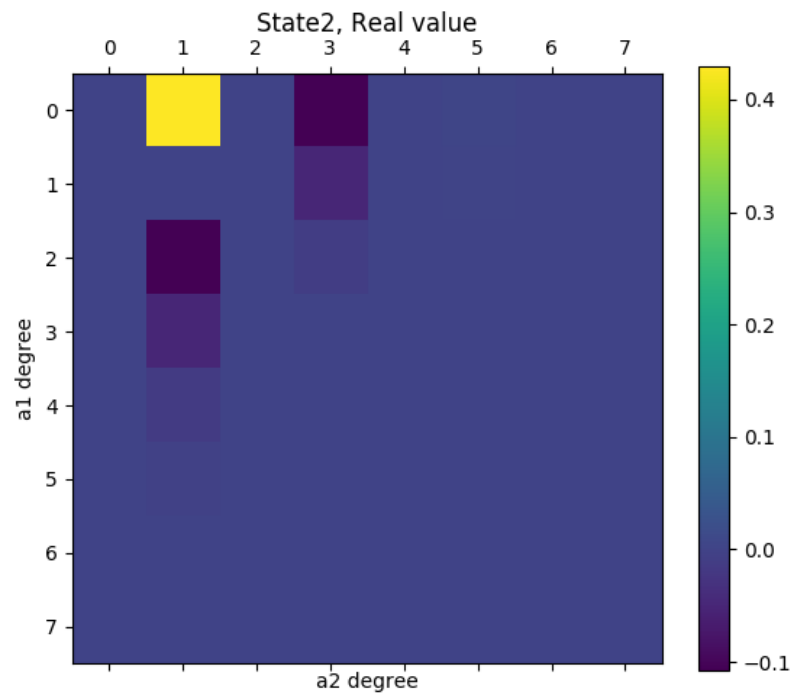
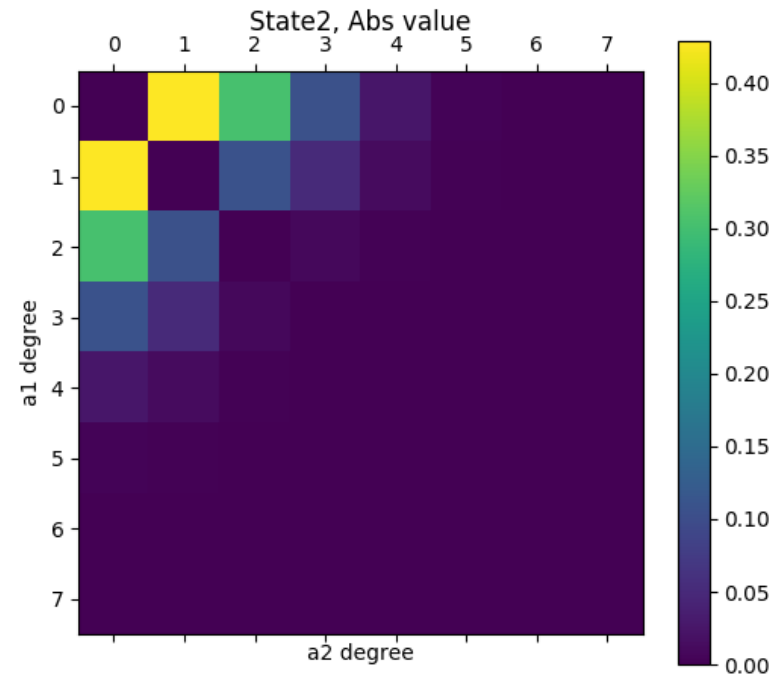
Output(final) state. Both (1st and 3rd) detectors were clicked.



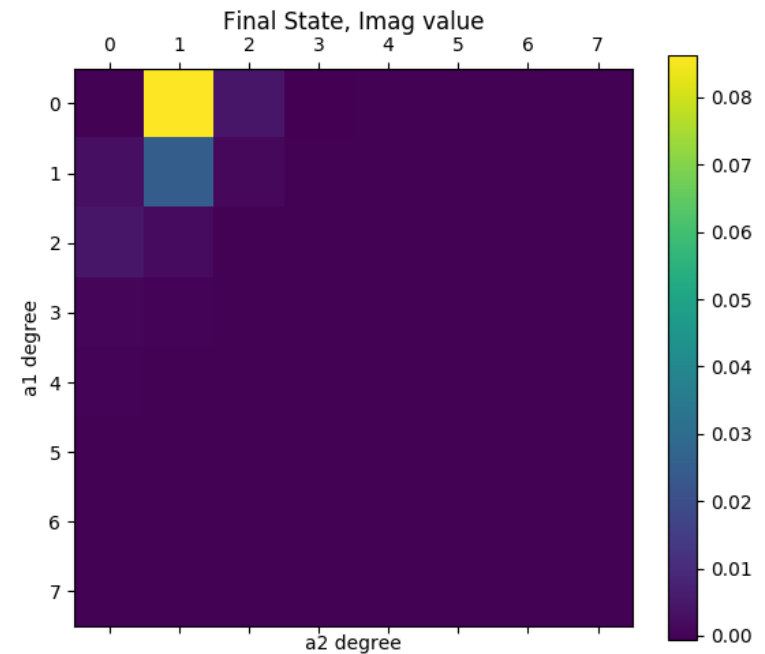
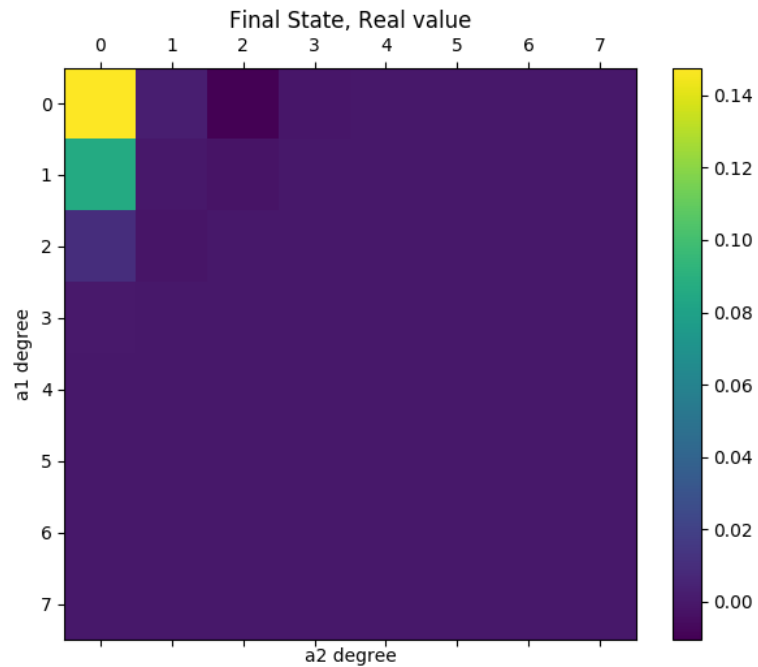
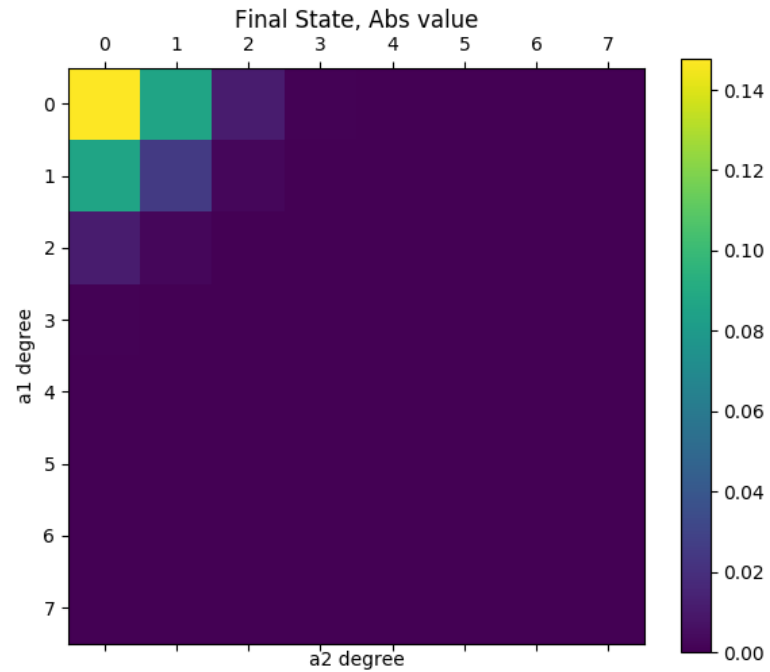
2) Input — coherent state. Aux — single photon.



State in area2.



Output(final) state. First detector was clicked.



Output(final) state. Both (1st and 3rd) detectors were clicked.

