

# L<sup>A</sup>T<sub>E</sub>X 2<sub>ε</sub> Template

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$$u_p^l(r, \phi) \propto r^l L_p^l(2r^2/w^2) e^{-r^2/w^2} e^{-il\phi} \quad (1)$$

1. Low gain.

2.

$$H \sim \int d^3\mathbf{r} \chi^{(2)}(\mathbf{r}) E_p^{(+)}(\mathbf{r}, t) E_s^{(-)}(\mathbf{r}, t) E_i^{(-)}(\mathbf{r}, t) + H.c.$$

3.

$$H=i\hbar\Gamma\int d\mathbf{q}_sd\mathbf{q}_iF(\mathbf{q}_s,\mathbf{q}_i)a_{\mathbf{q}_s}^\dagger a_{\mathbf{q}_i}^\dagger+H.c.$$

4.

$$F(\mathbf{q}_s,\mathbf{q}_i)=C\exp\left\{-\sigma^2\frac{(\mathbf{q}_s+\mathbf{q}_i)^2}{2}\right\}\mathrm{sinc}\left(\frac{L(\mathbf{q}_s-\mathbf{q}_i)^2}{4k_p}\right)\exp\left\{i\frac{L(\mathbf{q}_s-\mathbf{q}_i)^2}{4k_p}\right\}$$

5.

$$|\psi\rangle = \exp\big\{-\frac{i}{\hbar}\int_{-\infty}^{\infty} dt H(t)\big\}|0\rangle \approx -\frac{i}{\hbar}\int_{-\infty}^{\infty} dt H|0\rangle$$

6. Shmidt decomposition

7.

$$F(q_s,q_i,\phi_s-\phi_i)=\sum_n\chi_n(q_s,q_i)e^{in(\phi_s-\phi_i)}$$

8.

$$\chi_n(q_s,q_i)=\sum_m\sqrt{\lambda_{mn}}\frac{u_{mn}(q_s)}{\sqrt{q_s}}\frac{v_{mn}(q_i)}{\sqrt{q_i}}$$

9.

$$F(\mathbf{q}_s, \mathbf{q}_i) = \sum_{m,n} \sqrt{\lambda_{mn}} \frac{u_{mn}(q_s)}{\sqrt{q_s}} \frac{v_{mn}(q_i)}{\sqrt{q_i}} e^{in(\phi_s - \phi_i)}$$

10. broadband modes and high gain

11.

$$A_{mn}^\dagger = \int d\mathbf{q}_s \frac{u_{mn}(q_s)}{\sqrt{q_s}} e^{in\phi_s} a_{\mathbf{q}_s}^\dagger$$

12.

$$B_{mn}^\dagger = \int d\mathbf{q}_i \frac{v_{mn}(q_i)}{\sqrt{q_i}} e^{-in\phi_i} a_{\mathbf{q}_i}^\dagger$$

13.

$$H = i\hbar\Gamma \sum_{m,n} \sqrt{\lambda_{mn}} (A_{mn}^\dagger B_{mn}^\dagger - A_{mn} B_{mn})$$

14.

$$\frac{dA_{mn}}{dt} = \frac{i}{\hbar} [H, A_{mn}]$$

15.

$$A_{mn}^{out} = A_{mn}^{in} \cosh[G\sqrt{\lambda_{mn}}] + [B_{mn}^{in}]^\dagger \sinh[G\sqrt{\lambda_{mn}}]$$

16.

$$B_{mn}^{out} = B_{mn}^{in} \cosh[G\sqrt{\lambda_{mn}}] + [A_{mn}^{in}]^\dagger \sinh[G\sqrt{\lambda_{mn}}]$$

17.

$$\frac{da_{\mathbf{q}_s,i}}{dt} = \Gamma \sum_{m,n} \sqrt{\lambda_{mn}} \frac{u_{mn}(q_s)}{\sqrt{q_s}} [A_{mn}^\dagger e^{-in\phi_{s,i}} + B_{mn}^\dagger e^{in\phi_{s,i}}]$$

18.

$$\langle N_s(\mathbf{q}_s) \rangle = \sum_{m,n} \frac{|u_{mn}(q_s)|^2}{q_s} (\sinh[G\sqrt{\lambda_{mn}}])^2$$

19. three crystals

20.

$$F(\mathbf{q}_s, \mathbf{q}_i) = C \exp \left\{ -\sigma^2 \frac{(\mathbf{q}_s + \mathbf{q}_i)^2}{2} \right\} \text{sinc} \left( \frac{\Delta \tilde{q} L}{2} \right) \\ \times \left( \exp \left\{ \frac{i \Delta \tilde{q} L}{2} \right\} + \exp \left\{ i(\Delta \tilde{q}^{air} d_1 + \frac{3}{2} \Delta \tilde{q} L) \right\} + \exp \left\{ i(\Delta \tilde{q}^{air} (d_1 + d_2) + \frac{5}{2} \Delta \tilde{q} L) \right\} \right) \quad (2)$$

21.

$$\Delta \tilde{q} = \frac{(\mathbf{q}_s - \mathbf{q}_i)^2}{2k_p}$$

22.

$$\Delta \tilde{q}^{air} = \frac{(\mathbf{q}_s - \mathbf{q}_i)^2}{2k_p} n_s + \frac{2\delta n^{air} k_s}{n_s}$$

23. splitters

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n(a_1^\dagger)^n|0\rangle \\ |\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n(a_2^\dagger)^n|0\rangle \\ |\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2} \\ \sum_{m,n} |\alpha_{m,n}|^2 = 1$$

24. loseless BS

25.

$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

26.

$$\sum_{m,n} |\alpha_{m,n}^{(2)}|^2 = 1$$

$$r_j^2 + t_j^2 + a_j^2 = 1 \\ a_1^\dagger \rightarrow r_j a_1^\dagger + i t_j a_2^\dagger \\ a_2^\dagger \rightarrow r_j a_2^\dagger + i t_j a_1^\dagger$$

27. state in area 2 after first BS

28.

$$|\psi_2\rangle = \sum_{m,n} \alpha_{m,n}^{(2)} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

29. 4 channels

30.

$$|\psi_3\rangle = \sum_{p_1, p_2, p_3, p_4} \beta_{p_1, p_2, p_3, p_4} (a_1^\dagger)^{p_1} (a_2^\dagger)^{p_2} (a_3^\dagger)^{p_3} (a_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

31. First and only one ideal detector was clicked

32.

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_4 \\ p_1 > 0, p_3 = 0}} |\beta_{p_1, p_2, 0, p_4} \sqrt{p_1! p_2! p_4!}|^2 |p_2, p_4\rangle$$

33. First and third detectors were clicked

34.

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_3, p_4 \\ p_1 > 0, p_3 > 0}} |\beta_{p_1, p_2, p_3, p_4} \sqrt{p_1! p_2! p_3! p_4!}|^2 |p_2, p_4\rangle$$

35. No detection

36.

$$|\psi_4\rangle = \sum_{\substack{p_2, p_4 \\ p_1 = 0, p_3 = 0}} |\beta_{0, p_2, 0, p_4} \sqrt{p_2! p_4!}|^2 |p_2, p_4\rangle$$

37. final state

38.

$$|\psi_{out}\rangle = \sum_{m,n} \alpha_{m,n}^{out} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

39. example with two coherent states - alpha=1

40.

$$|\psi\rangle = |\alpha\rangle \otimes |\alpha\rangle, \quad \alpha = 1$$

41. two coherent after BS

42.

$$|\psi_2\rangle \approx e^{-1} \left( 1 + \frac{1+i}{\sqrt{2}} a_1^\dagger + \frac{1+i}{\sqrt{2}} a_2^\dagger + i a_1^\dagger a_2^\dagger + \frac{i}{2} (a_1^\dagger)^2 + \frac{i}{2} (a_2^\dagger)^2 + \frac{i-1}{2\sqrt{2}} a_1^\dagger (a_2^\dagger)^2 + \frac{i-1}{2\sqrt{2}} (a_1^\dagger)^2 a_2^\dagger + \dots \right)$$

43. auto correlation

$$\begin{aligned} Autocorr_s(q, q') &= \langle N_s(q) N_s(q') \rangle - \langle N_s(q) \rangle \langle N_s(q') \rangle = \\ &= \langle a_s^\dagger(q) a_s(q) a_s^\dagger(q') a_s(q') \rangle - \langle a_s^\dagger(q) a_s(q) \rangle \langle a_s^\dagger(q') a_s(q') \rangle \end{aligned}$$

44. cross correlation

$$\begin{aligned} Crosscorr(q, q') &= \langle N_s(q) N_i(q') \rangle - \langle N_s(q) \rangle \langle N_i(q') \rangle = \\ &= \langle a_s^\dagger(q) a_s(q) a_i^\dagger(q') a_i(q') \rangle - \langle a_s^\dagger(q) a_s(q) \rangle \langle a_i^\dagger(q') a_i(q') \rangle \end{aligned}$$

45. POVM operators.

$$\begin{aligned} \Pi_{no-click} &= \sum_{n=0}^{\infty} (1 - \eta_{SPD})^n |n\rangle \langle n| \\ \Pi_{click} &= 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1 - (1 - \eta_{SPD})^n] |n\rangle \langle n| \end{aligned}$$

46. After projection

47. Only first clicked

48.

$$|\psi_{out}\rangle = \Pi_{click}^{(1)} |\psi_{in}\rangle$$

49. Two clicked 1st and 3rd

50.

$$|\psi_{out}\rangle = \Pi_{click}^{(1)} \Pi_{click}^{(3)} |\psi_{in}\rangle$$

51. First and only one ideal detector was clicked

52.

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_4 \\ p_1 \neq 0, p_3 = 0}} \beta_{p_1, p_2, 0, p_4} \sqrt{p_1!} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

53. First and third detectors were clicked

54.

$$|\psi_4\rangle = \sum_{\substack{p_1, p_2, p_3, p_4 \\ p_1 \neq 0, p_3 \neq 0}} \beta_{p_1, p_2, p_3, p_4} \sqrt{p_1! p_3!} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

55. No detection

56.

$$|\psi_4\rangle = \sum_{\substack{p_2, p_4 \\ p_1=0, p_3=0}} \beta_{0, p_2, 0, p_4} (b_2^\dagger)^{p_2} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 2}$$

57. Projection operators

58. It starts here

59. States.

$$\begin{aligned} |\psi_{in}\rangle &= g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle \\ |\psi_{aux}\rangle &= f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle \\ |\psi\rangle &= |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2} \\ \sum_{m,n} |\alpha_{m,n}|^2 &= 1 \end{aligned}$$

60. Beam splitter.

$$\begin{aligned} r_j^2 + t_j^2 + a_j^2 &= 1 \\ a_1^\dagger &\rightarrow r_j a_1^\dagger + i t_j a_2^\dagger \\ a_2^\dagger &\rightarrow r_j a_2^\dagger + i t_j a_1^\dagger \end{aligned}$$

61. POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1 - \eta_{SPD})^n |n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1 - (1 - \eta_{SPD})^n] |n\rangle\langle n|$$

$$\hat{\Pi} = \Pi_{click}^{(1)} \otimes I^{(2)} \otimes \Pi_{click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{click}^{(1)} \otimes I^{(2)} \otimes \Pi_{no-click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{no-click}^{(1)} \otimes I^{(2)} \otimes \Pi_{click}^{(3)} \otimes I^{(4)}$$

$$\hat{\Pi} = \Pi_{no-click}^{(1)} \otimes I^{(2)} \otimes \Pi_{no-click}^{(3)} \otimes I^{(4)}$$

62. State before detection.

63.

$$|\psi_4\rangle = \sum_{p_1, p_2, p_3, p_4} \beta_{p_1, p_2, p_3, p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

64. Detection.

65.

$$\rho = Tr_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^\dagger)$$

66. dens matrix after detection

67.

$$\rho = \sum_{\substack{p_2, p_4 \\ p'_2, p'_4}} \rho_{p_2, p_4, p'_2, p'_4} |p_2\rangle\langle p'_2| \otimes |p_4\rangle\langle p'_4|$$

68. Last BS:

69.

$$|p_2, p_4\rangle\langle p'_2, p'_4| = \frac{1}{\sqrt{p_2! p_4! p'_2! p'_4!}} (a_2^\dagger)^{p_2} (a_4^\dagger)^{p_4} |0, 0\rangle\langle 0, 0| (a_2)^{p'_2} (a_4)^{p'_4}$$

70. Density matrix after detection.

71.

$$\rho^{out} = \sum_{\substack{p_1, p_2 \\ p'_1, p'_2}} \rho_{p_1, p_2, p'_1, p'_2}^{out} |p_1\rangle\langle p'_1| \otimes |p_2\rangle\langle p'_2|$$

72. Density matrix of subsystem.

73.

$$\rho_1 = Tr_2(\rho_{12})$$

74. Von Neumann entropy.

$$S(\rho_{12}) = - \sum_n \lambda_n \ln(\lambda_n)$$

$$\rho_1 \rightarrow \{\lambda_n\}, \text{ eigenvalues}$$

75. Linear entropy.

76.

$$S_L(\rho_{12}) = 1 - Tr(\rho_1^2)$$

77. Negativity.

$$N(\rho) = \frac{\|\rho^{T_1}\|_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

$$\rho_{12} \rightarrow \{\lambda_n\}, \text{ eigenvalues}$$

$$LN(\rho) := \log_2(2N + 1)$$