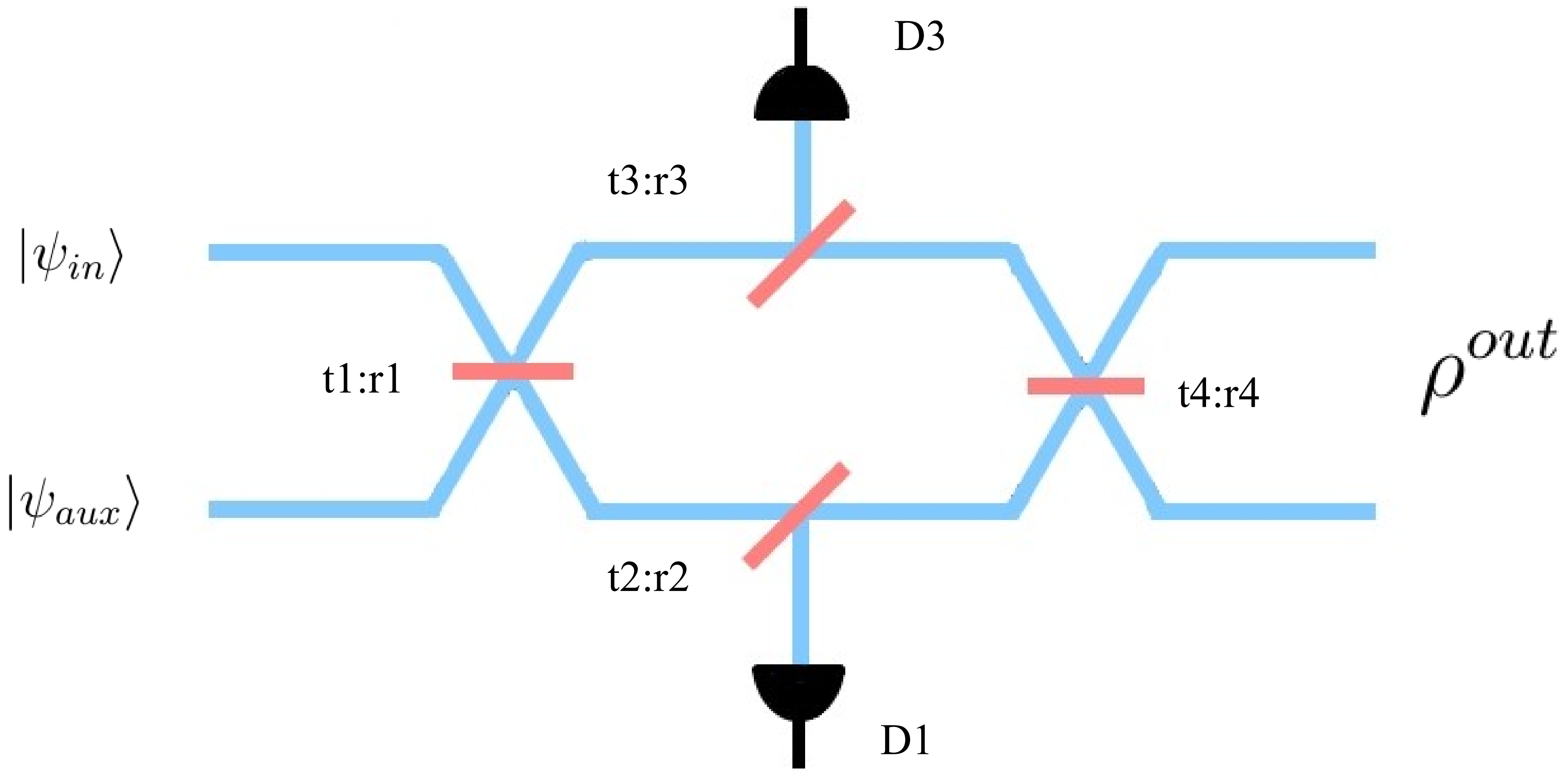
# Setup.



# Theoretical approach.

- As an input for system there could be sent different quantum light states in two channels.

$$|\psi_{in}\rangle = g(a_1^\dagger)|0\rangle = \sum_{n=0} g_n (a_1^\dagger)^n |0\rangle$$

$$|\psi_{aux}\rangle = f(a_2^\dagger)|0\rangle = \sum_{n=0} f_n (a_2^\dagger)^n |0\rangle$$

$$|\psi\rangle = |\psi_{in}\rangle \otimes |\psi_{aux}\rangle = \sum_{m,n} \alpha_{m,n} (a_1^\dagger)^m (a_2^\dagger)^n |0\rangle^{\otimes 2}$$

$$\sum_{m,n} |\alpha_{m,n}|^2 = 1$$

- Mutual state is transformed at beam splitter:

$$r_j^2 + t_j^2 + a_j^2 = 1$$
$$a_1^\dagger \to r_j a_1^\dagger + i t_j a_2^\dagger$$
$$a_2^\dagger \to r_j a_2^\dagger + i t_j a_1^\dagger$$

- After transformation at first, second and third BS, the state in 4 channels will look like this:

$$|\psi_4\rangle = \sum_{p_1,p_2,p_3,p_4} \beta_{p_1,p_2,p_3,p_4} (b_1^\dagger)^{p_1} (b_2^\dagger)^{p_2} (b_3^\dagger)^{p_3} (b_4^\dagger)^{p_4} |0\rangle^{\otimes 4}$$

# Detection.

- Detection is described in terms of POVM operators.

$$\Pi_{no-click} = \sum_{n=0}^{\infty} (1-\eta_{SPD})^n |n\rangle\langle n|$$

$$\Pi_{click} = 1 - \Pi_{no-click} = \sum_{n=0}^{\infty} [1-(1-\eta_{SPD})^n] |n\rangle\langle n|$$

- Four possibilities of detectors behaviour in total.

$\hat{\Pi} = \Pi^{(1)}_{click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{click} \otimes \mathrm{I}^{(4)}$ - both are clicked.

$\hat{\Pi} = \Pi^{(1)}_{click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{no-click} \otimes \mathrm{I}^{(4)}$ - first is clicked, third is silent.

$\hat{\Pi} = \Pi^{(1)}_{no-click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{click} \otimes \mathrm{I}^{(4)}$ - first is silent, third is clicked.

$\hat{\Pi} = \Pi^{(1)}_{no-click} \otimes \mathrm{I}^{(2)} \otimes \Pi^{(3)}_{no-click} \otimes \mathrm{I}^{(4)}$ - both are silent.

- After detection state is collapsed into density matrix:

$$\boxed{\rho = Tr_{1,3}(\hat{\Pi}|\psi_{in}\rangle\langle\psi_{in}|\hat{\Pi}^{\dagger})}$$

$$\rho = \sum_{\substack{p_2,p_4\\ p_2',p_4'}} \rho_{p_2,p_4,p_2',p_4'} |p_2\rangle\langle p_2'| \otimes |p_4\rangle\langle p_4'|$$

- Density matrix is transformed at last beam splitter:

$$|p_2,p_4\rangle\langle p_2',p_4'| = \frac{1}{\sqrt{p_2!p_4!p_2'!p_4'!}}(a_2^{\dagger})^{p_2}(a_4^{\dagger})^{p_4}|0,0\rangle\langle 0,0|(a_2)^{p_2'}(a_4)^{p_4'}$$

# Entanglement.

- Output is found in terms of density matrix for two channels:

$$\rho^{out} = \sum_{\substack{p_1,p_2 \\ p'_1,p'_2}} \rho^{out}_{p_1,p_2,p'_1,p'_2} |p_1\rangle\langle p'_1| \otimes |p_2\rangle\langle p'_2|$$

- Information about subsystem represented by reduced density matrix:

$$\rho_1 = Tr_2(\rho_{12})$$

# Von Neumann entropy.

$$S(\rho_{12}) = -\sum_n \lambda_n \ln(\lambda_n)$$

$$\rho_1 \rightarrow \{\lambda_n\},\ eigenvalues$$

# Linear entropy.

$$S_L(\rho_{12}) = 1 - Tr(\rho_1^2)$$

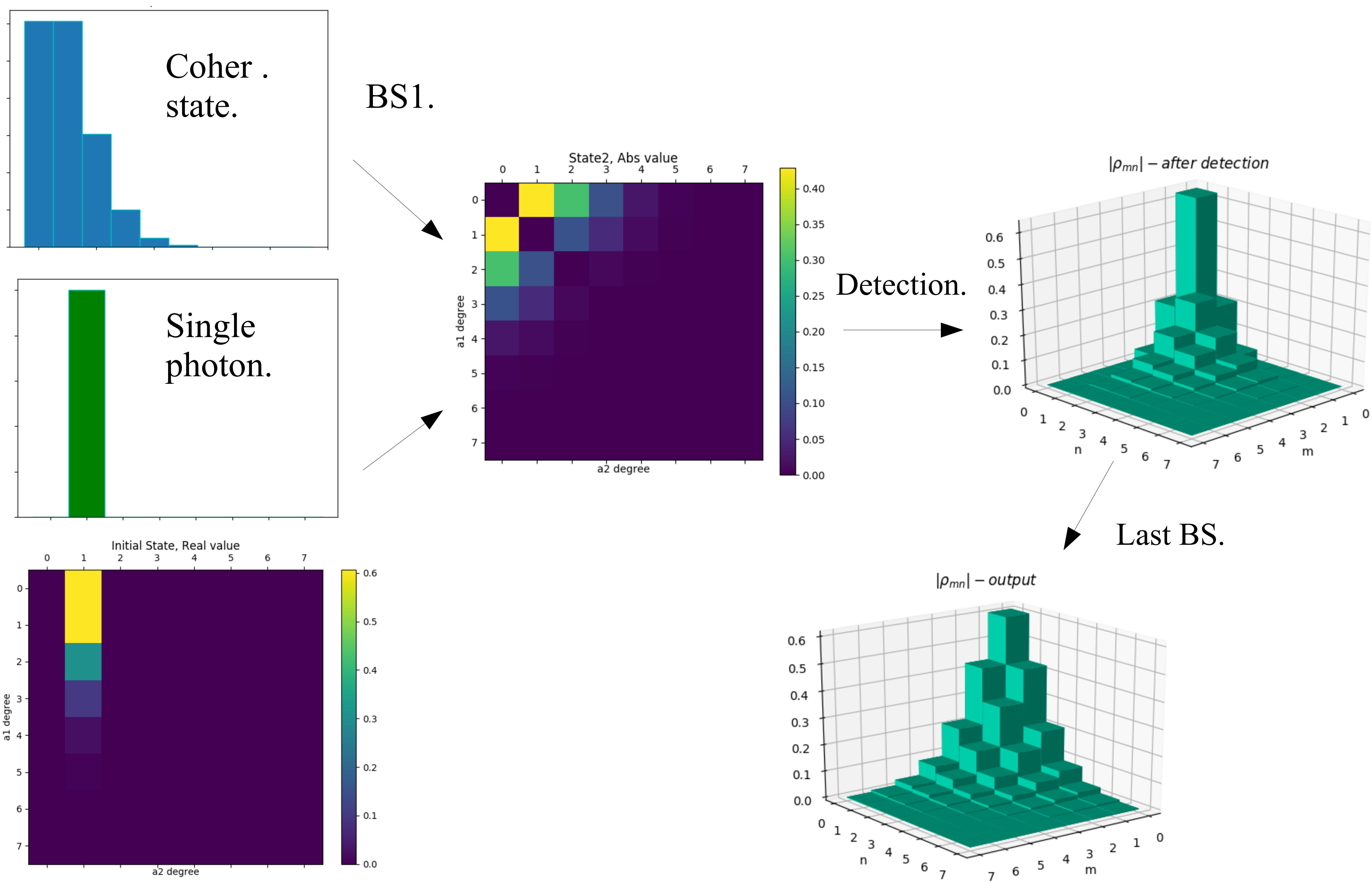
# Negativity.

$$N(\rho) = \frac{||\rho^{T_1}||_1 - 1}{2} = \sum_n \frac{|\lambda_n| - \lambda_n}{2}$$

$$\rho_{12} \rightarrow \{\lambda_n\},\ eigenvalues$$

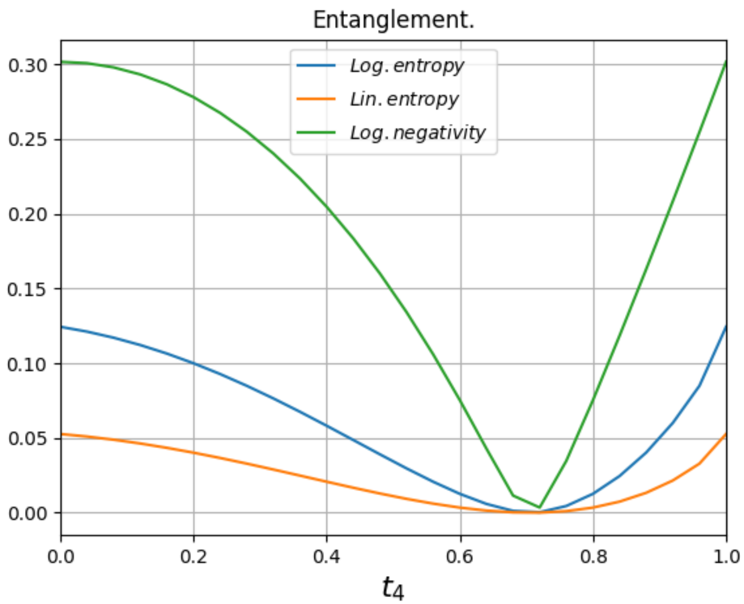
$$LN(\rho) := \log_2(2N + 1)$$

# Example.

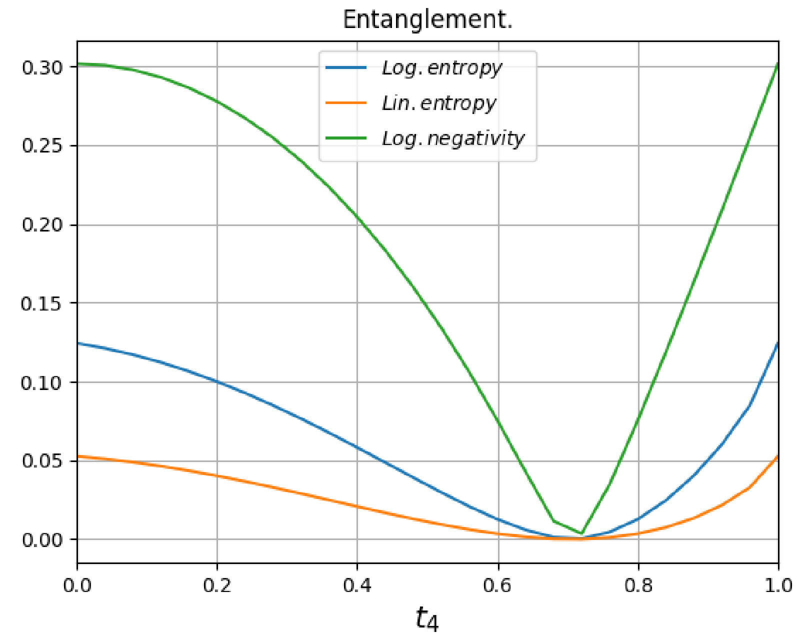

Coher . state.
Single photon.
Initial State, Real value
a1 degree
a2 degree
BS1.
State2, Abs value
a1 degree
a2 degree
Detection.
|ρmn| − after detection
n
m
Last BS.
|ρmn| − output
n
m

# Entanglement.

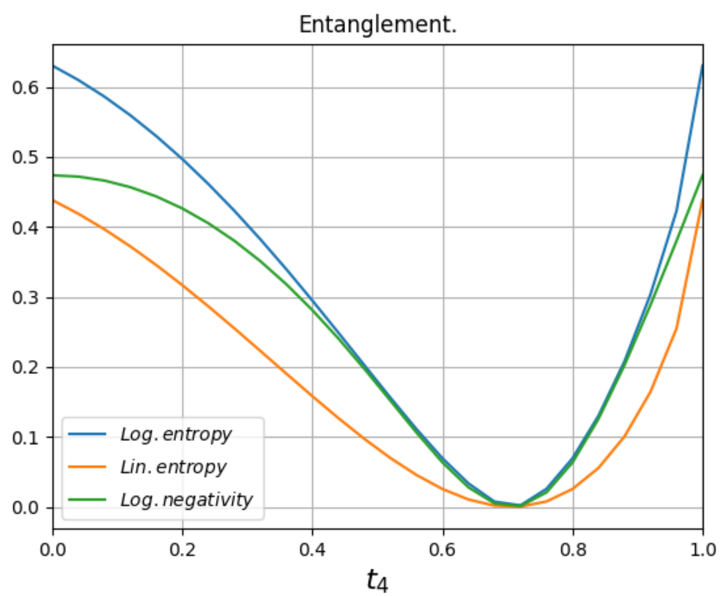
## - Only first detector.




## - Only third detector.




## - None of detectors.




## - Both detectors.