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TCSS 343

Homework3

1.)

$$T(n) = \begin{cases} c, \ if \ n<8 \\\\ 16T(\frac{n}{8})+nlogn, \ if \ n \geq 8 \end{cases}$$

From the recurrence we see that: $a = 16, b = 8, f(n) = nlogn$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$nlogn \ ? \ n^{log_8 16}, \ nlogn < n^{\frac{3}{4}}$$

Based on these conditions we know that the recurrence fits case 1, where: $T(n) = \Theta(n^{log_b(a)-\epsilon})$ or $T(n) = \Theta(n^{log \frac{4}{3}})$.

2.)

$$T(n) = \begin{cases} c, \ if \ n<8 \\\\ 2T(\frac{n}{8})+\sqrt[3]{n}, \ if \ n \geq 8 \end{cases}$$

From the recurrence we see that: $a = 2, \ b = 8, \ f(n) = n^{1/3}$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^{\frac{1}{3}} \ ? \ n^{log_8 2}, \ n^{\frac{1}{3}} = n^{\frac{1}{3}}$$

Based on these conditions we know that the the recurrence fits case 2, where: $T(n) = \Theta(n^{log_b a}logn)$ or $T(n) = \Theta(n^{\frac{1}{3}}logn)$

3.)

$$T(n) = \begin{cases} c, \ if \ n<2 \\\\ 3T(\frac{n}{2})+9^n, \ if \ n \geq 2 \end{cases}$$

From the recurrence we see that: $a = 3, b = 2, f(n) = 9^n$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$9^n \ ? \ n^{log_2 3}, \ 9^n > n^{1.58496...}$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n))$ or $T(n) = \Theta(9^n)$.

4.)

$$T(n) = \begin{cases} c, \; if \; n \leq 1 \\ 3T(\frac{3n}{5}) + n^2, \; if \; n > 1 \end{cases}$$

From the recurrence we see that: $a = 3, b = \frac{5}{3}, f(n) = n^2$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^2 \; ? \; n^{log_{\frac{5}{3}} 3}, \; n^2 < n^{2.15066...}$$

Based on these conditions we know that the recurrence fits case 1, where: $T(n) = \Theta(n^{log_b a})$ or $T(n) = \Theta(n^{2.15066...})$.

5.)

$$T(n) = \begin{cases} c, \; if \; n \leq 1 \\ 3T(\frac{3n}{5}) + n^2 \sqrt{n}, \; if \; n > 1 \end{cases}$$

From the recurrence we see that: $a = 3, b = \frac{5}{3}, f(n) = n^2 \sqrt{n}$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^{2.5} \; ? \; n^{log_{\frac{5}{3}} 3}, \; n^{2.5} > n^{2.15066...}$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n))$ or $T(n) = \Theta(n^{2.5})$.