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TCSS 343
Homework3

2.1

1.)

$$T(n) = \begin{cases} c,\ if\ n<8 \\ \\ 16T(\frac{n}{8})+nlogn,\ if\ n\ \geq 8 \end{cases}$$

From the recurrence we see that: $a = 16, b = 8, f(n) = nlogn$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$nlogn\ ?\ n^{log_8 16},\ nlogn < n^{\frac{3}{4}}$$

Based on these conditions we know that the recurrence fits case 1, where: $T(n) = \Theta(n^{log_b(a)-\epsilon})$ or $T(n) = \Theta(n^{log_{\frac{4}{3}}})$.

2.)

$$T(n) = \begin{cases} c,\ if\ n<8 \\ \\ 2T(\frac{n}{8})+\sqrt[3]{n},\ if\ n\ \geq 8 \end{cases}$$

From the recurrence we see that: $a = 2,\ b = 8,\ f(n) = n^{1/3}$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^{\frac{1}{3}}\ ?\ n^{log_8 2},\ n^{\frac{1}{3}} = n^{\frac{1}{3}}$$

Based on these conditions we know that the the recurrence fits case 2, where: $T(n) = \Theta(n^{log_b a} logn)$ or $T(n) = \Theta(n^{\frac{1}{3}} logn)$

3.)

$$T(n) = \begin{cases} c,\ if\ n<2 \\ \\ 3T(\frac{n}{2})+9^n,\ if\ n\ \geq 2 \end{cases}$$

From the recurrence we see that: $a = 3, b = 2, f(n) = 9^n$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$9^n\ ?\ n^{log_2 3},\ 9^n > n^{1.58496...}$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n))$ or $T(n) = \Theta(9^n)$.

4.)

$$T(n) = \begin{cases} c, \ if \ n \leq 1 \\ \\ 3T(\frac{3n}{5}) + n^2, \ if \ n > 1 \end{cases}$$

From the recurrence we see that: $a = 3, b = \frac{5}{3}, f(n) = n^2$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^2 \ ? \ n^{log_{\frac{5}{3}} 3}, \ n^2 < n^{2.15066...}$$

Based on these conditions we know that the recurrence fits case 1, where: $T(n) = \Theta(n^{log_b a})$ or $T(n) = \Theta(n^{2.15066...})$.

5.)

$$T(n) = \begin{cases} c, \ if \ n \leq 1 \\ \\ 3T(\frac{3n}{5}) + n^2\sqrt{n}, \ if \ n > 1 \end{cases}$$

From the recurrence we see that: $a = 3, b = \frac{5}{3}, f(n) = n^2\sqrt{n}$. We then plug the values into the function $n^{log_b a}$ and check it against $f(n)$ for equality.

$$n^{2.5} \ ? \ n^{log_{\frac{5}{3}} 3}, \ n^{2.5} > n^{2.15066...}$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n))$ or $T(n) = \Theta(n^{2.5})$.

2.2

1.)

$$T(n) = \begin{cases} c \ for \ n \leq 1 \\ \\ 3T(\frac{n}{2}) + n^3 \ for \ n > 1 \end{cases}$$

2.) From the recurrence we see that $a = 3, b = 2, f(n) = n^3$. We then plug the values into the function $n^{log_b a}$ and check it against f(n) for equality.

$$n^3 \ ? \ n^{log_2 3}, \ n^3 > n^{1.58496...}$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n)) = \Theta(n^3)$.

3.)

$$T(n) = \begin{cases} c \ for \ n \leq 1 \\ \\ T(\frac{n}{2}) + nlog(logn) \ for \ n > 1 \end{cases}$$

4.) From the recurrence we see that $a = 1, b = 2, f(n) = nlog(logn)$. We then plug the values into the function $n^{log_b a}$ and check it against f(n) for equality.

$$nlog(logn) \ ? \ n^{log_2 1}, \ nlog(logn) > n^0$$

Based on these conditions we know that the recurrence fits case 3, where: $T(n) = \Theta(f(n)) = \Theta(nlog(logn))$.