Suppose $n \in \mathbb{Z}$. n^3 is even if and only if n is even.

Proof. ($\implies$) Suppose that a^3 is even and suppose for contradiction that a is not even. Then $(a^3/a) \in \mathbb{Q} \setminus \mathbb{Z}$. Then $a^2 \in \mathbb{Q} \setminus \mathbb{Z}$. This is a contradiction with $a \in \mathbb{Z}$ because if $a \in \mathbb{Z}$, $a = 2k$ for some integer k so $a^2 = 4k^2 = 2(2k) \in \mathbb{N} \subseteq \mathbb{Z}$. Thus a is even.

($\impliedby$) If a is even, then it can be written in the form $2k$ for some integer k. Then $a^3 = 2^3k^3 = 8k^3 = 2(4k^3)$. Thus a^3 is even. $\square$