

The intersection of two eigenspaces is a zero subspace.

Proof. Suppose we have two distinct E_λ and $E_{\lambda'}$ with eigenvalues λ and λ' respectively. And suppose $v \in E_\lambda \cap E_{\lambda'}$, so $v \in E_\lambda$ and $v \in E_{\lambda'}$. Then $T(v) = \lambda v = \lambda' v$, so $(\lambda' - \lambda)v = 0$, which holds precisely when $v = 0$ (as $\lambda \neq \lambda'$.) Thus $E_\lambda \cap E_{\lambda'} = \{0\}$. \square