Suppose we have r eigenspaces $E_{\lambda_1}, \ldots, E_{\lambda_r}$ of a linear transformation $T : V \to V$. Let $\mathfrak{B}_k$ be a basis for the eigenspace E_{λ_k}. Then the intersection of the bases $\mathfrak{B}_1, \ldots, \mathfrak{B}_r$ is linearly independent.

Proof. Write $\mathfrak{B}_k = \{v_{k1}, v_{k2}, \ldots, v_{kn_k}\}$. Then consider the relation

$$\sum_{k=1}^{r} \sum_{j=1}^{n_k} c_{kj} v_{kj} = 0.$$

Now let

$$w_k = \sum_{j=1}^{n_k} c_{kj} v_{kj}.$$

Observe that w, as a linear combination of the vectors in $\mathfrak{B}_k$, is an eigenvector with the eigenvalue λ_k. Returning to our relation with w we have,

$$\sum_{k=1}^{r} w_k = 0.$$

Note that eigenvectors with distinct eigenvalues are linearly independent, so there is no nontrivial relation on the set $\{w_1, \ldots, w_r\}$. Therefore, $w_1 = \ldots = w_r = 0$. Now, because $\mathfrak{B}_k$ is linearly independent, and $c_{kj} = 0$ for all k, j. Therefore the relation is trivial, and the theorem follows. $\square$