Let n be an odd natural number and k a negative scalar. Then there does not exist a matrix $M \in \mathbb{R}^{n\times n}$ such that $M^2 = kI_n$.

Proof. Suppose for contradiction that there exists some $M \in \mathbb{R}^{2\times 2}$ such that $M^2 = kI_n$. Then $\det M^2 = \det(kI_n) = k^n \det I_n = k^n$ (by either the homogeneity of the determinant or the fact that the determinant of a triangular matrix is the product of its diagonal elements). Since k is odd, $k^n < 0$. Further the multiplicative property of determinants, $\det M^2 = (\det M)^2$, which we know to be greater than zero as the square of a nonnegative number is nonnegative. Thus we have encountered a contradiction, namely that $\det M^2 < 0$ and $\det M^2 \geq 0$. Therefore, there does not exist any $M \in \mathbb{R}^{2\times 2}$ such that $M^2 = kI_n$ for odd n and negative k. $\square$