If $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ and $q \in \mathbb{Q}$, then $\alpha + q \in \mathbb{R} \setminus \mathbb{Q}$. That is, the sum of an irrational number and a rational number is irrational.

Proof. Suppose $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ and $q \in \mathbb{Q}$, and suppose for contradiction that $\alpha - q \notin \mathbb{R} \setminus \mathbb{Q}$. Then $\alpha - q \in \mathbb{Q}$, so $\alpha - q = \frac{m}{n}$ for $m \in \mathbb{Z}$ and $n \in \mathbb{N}$. Because $q \in Q$, we can write $q = \frac{a}{b}$ for $a \in \mathbb{Z}$ and $b \in \mathbb{N}$. This gives us the following:

$$\alpha - \frac{a}{b} = \frac{m}{n} \implies \alpha = \frac{a}{b} + \frac{m}{n} = \frac{an + bm}{bn}.$$

This says that $\alpha \in \mathbb{Q}$, which is a contradiction. Thus $\alpha - q \in \mathbb{R} \setminus \mathbb{Q}$. $\square$