Suppose A is an $n \times n$ matrix with only one eigenvalue. If A is diagonalizable, then A is diagonal.

If A is a diagonalizable matrix with only one eigenvalue, then it is similar to

$$D = \begin{bmatrix} \lambda & & \\ & \ddots & \\ & & \lambda \end{bmatrix} = \lambda I.$$

That is,

$$A = SDS^{=1} = S(\lambda I)S^{-1} = \lambda SS^{-1} = \lambda I.$$

Thus, A is already diagonal.