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Swing Up and Stabilization Control of a Rotary Inverted Pendulum

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Abstract: The control of a Rotary Inverted Pendulum (RIP) is a well-known and a challenging problem that serves as a popular benchmark in modern control system studies. The task is to design controllers which drives the pendulum from its hanging-down position to the upright position and then hold it there. The swing up is achieved using an energy based controller. In energy based control the pendulum is controlled in such a way that its energy is driven towards a value equal to the steady-state upright position. Then a mode controller switches between the swing-up controller and stabilizing controller near the upright position. For stabilization control, two control techniques are analyzed. Firstly, a sliding mode controller (SMC) is designed to stabilize the pendulum. Secondly, a state feedback controller is designed that would maintain the pendulum upright and handle disturbances up to a certain point. The state feedback controller is designed using the linear quadratic regulator (LQR). The responses of the LQR controller and SMC controller are compared in simulation.

Keywords: swing up, energy control, mode controller, linear quadratic regulator, sliding mode control

1. INTRODUCTION

Early studies of the Rotary Inverted Pendulum were motivated by the need to design controllers to balance the rockets during vertical take-off. At the instant of launching the rocket is extremely unstable. Similar to rocket launch, the inverted pendulum requires a continuous correction mechanism to stay upright as the open loop configuration is extremely unstable. Thus, the problem can be compared to a rocket during launching. It is also a very good model for an automatic aircraft landing system, aircraft stabilization in the turbulent air-flow, stabilization of a cabin in a ship etc.

In this paper, the objective is to design controllers for swinging up the pendulum from its stable equilibrium position to the unstable equilibrium point and balancing it there. The swing-up can be attained using various strategies like pid control, iterative impulsive control as studied by Wang et al. (2004), energy control as shown by Astrom and Furuta (1996), Barbosa et al. (2011). Here a robust energy based controller is used which swings the pendulum to the upright position by utilizing the total energy of the system as a feedback quantity. In the upright position a stabilization controller is used to balance the link. Stabilization controllers based on sliding mode control (SMC) approach and linear quadratic regulator (LQR) are designed.

The SMC approach is recognized as an efficient tool to design robust controllers for complex higher order nonlinear dynamic plants operating under uncertain conditions. It is a nonlinear control method that alters the dynamics of a nonlinear system by application of a high-frequency switching control. Anvar et al. (2010), Kurode et al. (2011), Khanesar et al. (2007) proposed sliding mode control for the stabilization of the rotary inverted pendulum.

The LQR method is a powerful one for the control of linear systems in the state-space domain. The LQR technique generates controllers with guaranteed closed-loop stability and robustness property even in the face of certain gain and phase variations at the plant input/output. It was studied by many researchers including Akhtaruzzaman and Shafie (2010), Ozbek and Efe (2010) and found much superior to various classical techniques. Since the dynamics of inverted pendulum systems are inherently nonlinear, the equations of motion are linearized about the operating point and a domain of attraction is defined within which the constant gain controller results in local asymptotic stability.

A mode-switching controller is designed to integrate swing up and stabilization control, which means that each time the pendulum reaches a certain location after it is swung up, the stabilizing controller is activated and 'catches' the pendulum allowing it to be balanced at the upright position.

The paper is organized as follows. The mathematical model of the rotary inverted pendulum is presented in the Section II. Section III deals with the controller designs. Simulation results are presented in Section IV. Section V concludes the work.

2. MODEL OF ROTARY INVERTED PENDULUM

2.1. Description of the System

The Rotary inverted pendulum system is shown in Fig. 1. The system consists of two modules – a servo module and a rotary module. The servo module shown in Fig. 2 consists of a DC servomotor with built in gearbox ratio 70:1. The DC servomotor, whose input is ± 5 V, is mounted in a solid aluminium frame. The motor drives a built-in Swiss-made 14:1 gearbox whose output drives an external gear.

Fig. 1 Rotary Inverted Pendulum

The motor gear drives a gear attached to an independent output shaft that rotates in a precisely machined aluminium ball bearing block. The output shaft is equipped with a 1024 count quadrature encoder. This gives the motor shaft position. A second gear on the output shaft drives an anti-backlash gear connected to a precision potentiometer. The potentiometer is used to measure the output angle.

Fig. 2 Servo Module

The Rotary module shown in Fig. 3 consists of two links: a horizontal link called the rotating arm and a vertical link called the pendulum. The DC motor rotates the stiff arm at one end of the horizontal plane. The opposite end of the arm is instrumented with a joint whose axis is along the radial direction of the motor. The pendulum is attached to the joint. The flat arm is instrumented with an encoder at one end such that the encoder shaft is aligned with the longitudinal axis of the arm. This encoder measures the pendulum angle.

Fig. 3 Rotary Module

2.2. Mathematical Model

The schematic representation of the system is shown in Fig. 4. The Lagrangian method is used to obtain the equations of motion of the rotary inverted pendulum system. The generalized co-ordinates for the system are the angular displacements of the rotating arm (θ) and the pendulum angle (α).

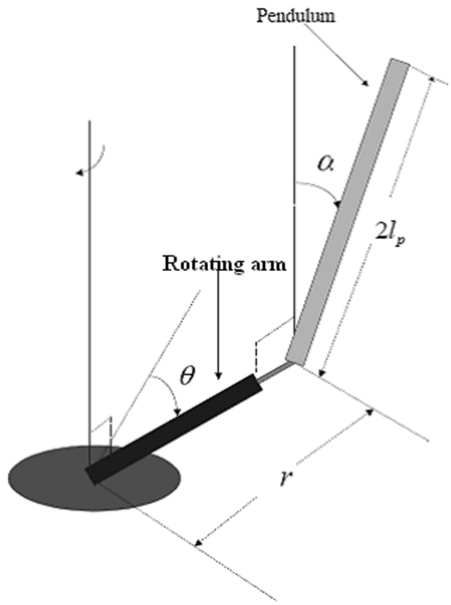

Fig. 4 Schematic representation of the plant

The general form of Lagrange function L of the system is given as L = Total Kinetic Energy (T) – Potential Energy (V)
Taking the horizontal plane where the arm lies as the datum plane, the only potential energy in the mechanical system is gravity, i.e.

$$V = mgl\cos(\alpha) \tag{1}$$

The kinetic energies in the system arise from the moving hub, the velocity of the point mass in the x- direction, the velocity of the point mass in the y-direction and the rotating pendulum about its centre of mass

$$T = \frac{1}{2}J_{eq}\dot{\theta}^2 + \frac{1}{2}m[(r\dot{\theta} - L\cos(\alpha)\dot{\alpha})^2 + (-L\sin(\alpha)\dot{\alpha})^2] + \frac{1}{2}J_B\dot{\alpha}^2$$
$$= \frac{1}{2}(J_{eq} + mr^2)\dot{\theta}^2 + \frac{2}{3}mL^2\dot{\alpha}^2 - mLr\cos(\alpha)\dot{\theta}\dot{\alpha} \tag{2}$$

The Lagrangian can be formulated as $L = T - V$

$$L = \frac{1}{2}(J_{eq} + mr^2)\dot{\theta}^2 + \frac{2}{3}mL^2\dot{\alpha}^2 - mLr\cos(\alpha)\dot{\theta}\dot{\alpha} - mgL\cos(\alpha) \tag{3}$$

Once the Lagrange function of the system is known, the mathematical model of the system is found in the form

$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\theta}}\right) - \frac{\delta L}{\delta\theta} = T_{output} - B_{eq}\dot{\theta}$$
$$\frac{d}{dt}\left(\frac{\delta L}{\delta\dot{\alpha}}\right) - \frac{\delta L}{\delta\alpha} = 0 \tag{4}$$

Substituting (3) into (4), we obtain the equations of motion of the system as

$$-mLr\cos(\alpha)\ddot{\theta}+\frac{4}{3}mL^2\ddot{\alpha}-mgL\sin(\alpha)=0$$
$$(J_{eq}+mr^2)\ddot{\theta}+mLr\sin\alpha(\dot{\alpha}^2)-mLr\cos\alpha(\ddot{\alpha})=T_{output}-B_{eq}\dot{\theta} \quad (5)$$

The output torque (T_{output}) of the driving unit on the load shaft is

$$T_{output}=\eta_m\eta_g K_t K_g(\frac{V_m-K_m K_g\theta}{R_m}) \quad (6)$$

Substituting (6) into (5), we obtain the nonlinear model of the system as follows

$$a\ddot{\theta}-b\cos(\alpha)\ddot{\alpha}+b\sin(\alpha)\dot{\alpha}^2+e\dot{\theta}=fV_m$$
$$-b\cos(\alpha)\ddot{\theta}+c\ddot{\alpha}-d\sin(\alpha)=0 \quad (7)$$

Where

$$a=J_{eq}+mr^2+\eta_g K_g{}^2 J_m$$
$$b=mLr$$
$$c=\frac{4}{3}mL^2$$
$$d=mgL$$
$$e=B_{eq}+\frac{\eta_m\eta_g K_t K_g{}^2 K_m}{R_m}$$
$$f=\frac{\eta_m\eta_g K_t K_g}{R_m}$$

Equation (7) represents the nonlinear model of the system. Linearizing (7) under the assumption that α =0 and $\dot{\alpha}=0$, we get the linearized model as follows:

$$a\ddot{\theta}-b\ddot{\alpha}+e\dot{\theta}=fV_m$$
$$-b\ddot{\theta}+c\ddot{\alpha}-d\alpha=0 \quad (8)$$

Defining $x=[\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]^T$ $y=[\theta\ \alpha]^T$ $u=V_m$ linearizing about the upright position, that is $\alpha=0$, and substituting the system parameters as given in Table 1, we obtain the state space representation of the system as

$$\dot{x}=Ax+Bu$$
$$y=Cx \quad (9)$$

Where $A=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & 41.68 & -15.47 & 0\\ 0 & 84.05 & -14.89 & 0\end{bmatrix}$

$$B=\begin{bmatrix}0\\ 0\\ 27.12\\ 23.13\end{bmatrix}$$

To get some sense about how well the linearized model represents the original nonlinear system, the dynamics of the system is simulated using both linear and non-linear models. The simulation verifies the linear model and also establishes a threshold on α of the linear model.

Table 1 Parameters of Rotary Inverted Pendulum

Symbol	Description	Value	Unit
m	Mass of the pendulum	.125	Kg
L	Half length of pendulum	16.75	cm
r	Length of the rotating arm	21.5	cm
g	Gravitational acceleration	9.81	m/s^2
R_m	Armature resistance	2.6	Ω
K_m	Motor voltage constant	0.0076	V-s/rad
K_t	Motor torque constant	0.0076	N-m/A
K_g	SRV02 system gear ratio	70	
B_{eq}	Equivalent viscous friction	0.004	Nm/(rad/s)
η_m	Motor efficiency due to rotational loss	0.87	
η_g	Gearbox efficiency	0.85	
V_m	Motor input voltage	6	Volts
J_{eq}	Equivalent inertia	0.0023	Kg m^2

Fig. 5 shows that the linear model quite accurately describes the system for the first 25 degrees and then begins to diverge from the actual motion.

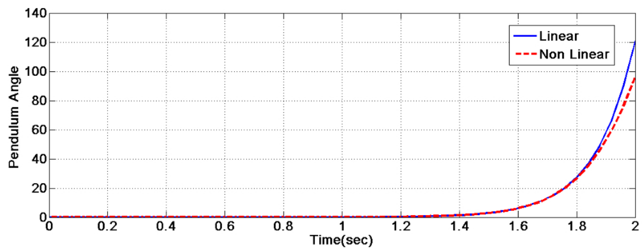

Fig. 5 Overlapped plots from linear and nonlinear models

3. CONTROL SYSTEM DESIGN

The control objective is to bring the pendulum from downward position to vertically upright position and maintain it there. Control strategies for swing up and for stabilization are presented in this paper. The swing-up controller drives the pendulum from its suspended downward position to the vertical upright position, where the balance controller can then be used to balance the link. A mode-switching controller is also designed to switch over from swing up control to stabilization control when the pendulum reaches a certain location after it is swung up.

3.1. Swing-Up Control

Swing-up is attained using an energy controller. The energy based approach attempts to swing the pendulum upright by utilizing the total energy of the system as a feedback quantity. The pendulum is controlled in such a way that its energy is driven towards a value equal to the steady-state upright position. Neglecting friction and assuming pendulum as a rigid body, the equation of motion of the pendulum is

$$J_p\ddot{\alpha} - mgL\sin(\alpha) + muL\cos(\alpha) = 0 \tag{10}$$

where g is acceleration of gravity and *u(=ng)* is the maximum acceleration of the pivot. The parameter *n* is dimension free. Normalized variables are useful to characterize the properties of a system. We introduce $\omega_0 = \sqrt{\frac{mgL}{J_p}}$ which is the frequency of small oscillations around the downward position. The equation of motion is then characterized by two parameters only. We choose the energy of the system as zero in the upright position, and normalize it by mgL, which is the energy required to raise the pendulum from the hanging down position to the horizontal position. The normalized energy can be then written as

$$E = mgL\{\frac{1}{2}(\frac{\dot{\alpha}}{\omega_0})^2 + \cos(\alpha) - 1\} \tag{11}$$

Computing the derivative of *E* with respect to time

$$\frac{dE}{dx}t = J_p\dot{\alpha}\ddot{\alpha} - mgL\dot{\alpha}\sin(\alpha) = -muL\dot{\alpha}\cos(\alpha) \tag{12}$$

It follows from the above equation that it is easy to control the energy. The system is simply an integrator with varying gain. Now, the controller should drive the system to the desired value. Let the desired energy be E_0. The following control is a simple strategy for achieving the desired energy

$$u = sat_{ng}(k(E - E_0))\,\mathrm{sgn}(\dot{\alpha}\cos\alpha) \tag{13}$$

where *k* is a design parameter. The function sat_{ng} denotes a function which saturates at *ng*. This strategy is essentially a bang-bang strategy for large errors and a proportional control for small errors.

3.2. Mode Control

The mode controller determines when to switch between the two controllers. Once we have attained acceptable amplitude of the pendulum oscillations, we want to start stabilizing the controller at the right time.

The pendulum can be stabilized when it is about 20 degrees from the vertical and not moving faster than 200-deg/sec. Also, the energy of the system should be smaller than a small positive value, $\boldsymbol{\xi}$. To avoid switch bouncing (oscillation), the value of $\boldsymbol{\xi}$ is important, but easily obtainable by trial-and-error method. This simple mode control helps achieve good global stabilization.

3.3. *Stabilization Control*

1) LQR Controller:

A linear quadratic regulator (LQR) was used to regulate the system about the upright equilibrium point. The LQR controller requires a linear system for which it will generate constant gains for full state feedback to make the equilibrium point globally asymptotically stable. So the equations of motion are linearized about the operating point and a domain of attraction is defined within which the constant gain controller results in local asymptotic stability. With the realms of Matlab, a full state feedback LQR controller is developed by solving the Algebraic Ricatti Equation based upon an effort weighting matrix and a state penalty matrix. The nonlinear dynamical equations written in the linear state space format are used for this.

In the Matlab, the program for LQR is executed and the gain values are obtained as K_1 = -2.2361 V/rad for θ . K_2 = 20.774 V/rad for α. K_3 = -2.0004 V/(rad/sec) for $\dot{\theta}$ K_4 = 2.8137 V/(rad/sec) for $\dot{\alpha}$.

2) Sliding Mode Control

The design approach using sliding mode control comprises of two steps: i) Design of a sliding surface in the state space ii) Synthesis of the control law such that the trajectories of the closed loop motion are directed towards the surface. The state space equation of the system is given by:

$$\dot{x} = Ax + Bu \tag{14}$$

Where $x = [\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]$, A is the system matrix and B is the input matrix. The sliding surface is defined in state space as:

$$s = C^T x_e \tag{15}$$

Where C^T is a [1 x 4] matrix and x_e is the error state vector i.e. error in $[\theta\ \alpha\ \dot{\theta}\ \dot{\alpha}]$. If x_d is the desired state vector and *x* is the actual state vector then

$$x_e = x - x_d \tag{16}$$

Gao's power rate reaching law which guarantees finite time reaching is

$$\dot{s} = -k|s|^{\alpha}\,\mathrm{sgn}(s) \tag{17}$$

Differentiating the equation of the sliding surface we get:

$$\dot{s} = C^T\dot{x}_e \tag{18}$$

Substituting for $\dot{x}_e$ we get,

$$\dot{s} = C^T\dot{x} - C^T\dot{x}_d \tag{19}$$

Putting the value of $\dot{x}$,

$$\dot{s} = C^T Ax + C^T Bu - C^T\dot{x}_d \tag{20}$$

Substituting for $\dot{s}$ we get

$$-k|s|^{\alpha}\,\mathrm{sgn}(s) = C^T Ax + C^T Bu - C^T\dot{x}_d \tag{21}$$

Solving for u we get,

$$u = -(C^T B)^{-1}(C^T Ax + k|s|^{\alpha}\,\mathrm{sgn}(s) - C^T\dot{x}_d) \tag{22}$$

The desired state vector x_d is $\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}$ and its derivative $\dot{x}_d$ is also $\begin{bmatrix}0 & 0 & 0 & 0\end{bmatrix}$. Thus, the synthesized control law becomes

$$u = -(C^T B)^{-1}(C^T Ax + k|s|^{\alpha}\,\mathrm{sgn}(s)) \tag{23}$$

This is the stabilization control for the rotary inverted pendulum.

4. SIMULATION RESULTS

The control strategy of an RIP system is composed of the swing up control, mode switching and stabilizing control of the pendulum. For swing-up, energy based control method is compared with a conventional proportional-derivative control strategy. The energy based approach is found to swing-up the pendulum much faster than the PD controller. The PD gains used are: P = 0.5 D = 0.0001, K_p = 24.7 and K_d = -1.5. The parameters used in energy control method are: E_0 = -0.1, k = 50 and n = 2.6

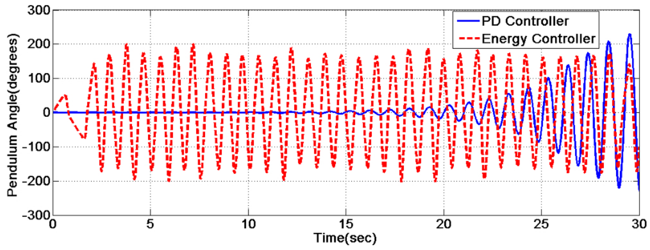

Fig. 6 Swinging-up of the pendulum using PD and Energy controller

For stabilization control, the performance of LQR and SMC controller are compared. The design parameters used for sliding mode control are: α = 0.5, s = [16.95 −80.96 7.29 −12.35] x_e , k = 10. The settling time and overshoots are less for LQR controller. Sliding mode controller provides excellent disturbance rejection capabilities. In case of sliding mode control we are able to use the non-linear system model, whereas in case of LQR we are using the linearized system model

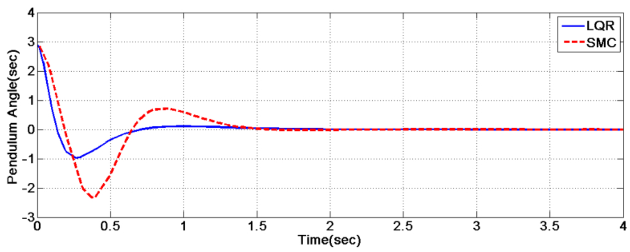

Fig. 7 Stabilization of the pendulum angle α

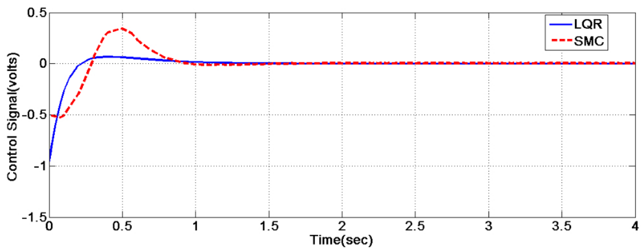

Fig. 8 Evolution of the control signal u.

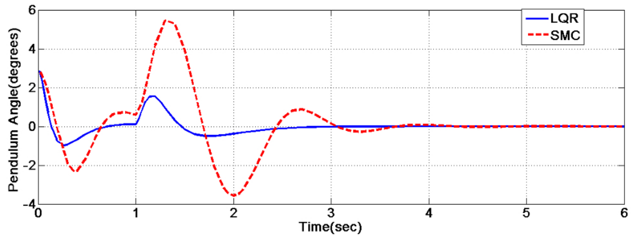

Fig. 9 Stabilization of α in presence of step disturbance

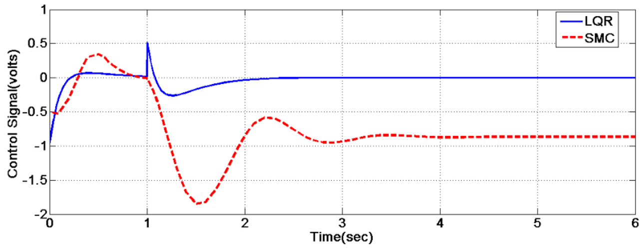

Fig. 10 Evolution of control signal in presence of step disturbance

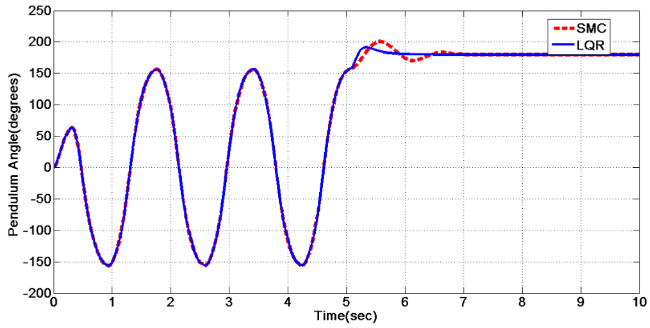

Fig. 11 Swing-up and stabilization of the pendulum

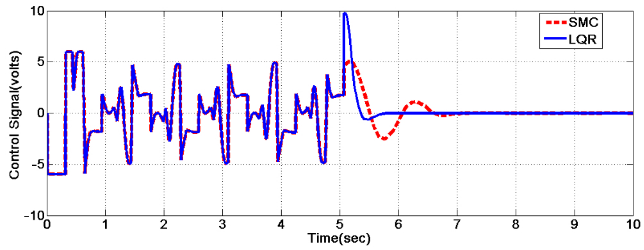

Fig. 12 Evolution of control signal during swing-up and stabilization

5. CONCLUSION

This paper compares the performance of different control techniques on a rotary inverted pendulum system. Both swing up and stabilization problems have been studied. Two control schemes are elaborated for stabilization, namely, linear quadratic control and sliding mode control. Energy based control is used for swinging up the pendulum. According to the results much faster swing up of the pendulum is obtained using energy based control. For stabilization, smoother control signal is produced by the linear quadratic controller. But sliding mode control provides more robustness against parameter uncertainties. Chattering is eliminated to a great extent using fractional order reaching law approach in sliding mode control.

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