1 Assumptions

- The independent variables $X_1, \ldots, X_m$ are independent of one another.
- The observations $y_1, \ldots, y_n$ are independent of one another.
- There is no measurement error in our observations.
- The independent variables $X_1, \ldots, X_m$ are linearly related to the logit of the probability that $Y = 1$ or, equivalently, the log-odds that $Y = 1$.

2 Derivation

$$
\begin{aligned}
p_i &= \mathbb{P}(Y=1) \\
\Rightarrow p_i &= \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}}{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} \\
\Rightarrow \frac{1}{p_i} &= \frac{1+e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} \\
\Rightarrow \frac{1}{p_i} &= \frac{1}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} + \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} \\
\Rightarrow \frac{1}{p_i} &= \frac{1}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} + 1 \\
\Rightarrow \frac{1}{p_i} - 1 &= \frac{1}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} \\
\Rightarrow \frac{1-p_i}{p_i} &= \frac{1}{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}} \\
\Rightarrow \frac{p_i}{1-p_i} &= \frac{e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}}{1} \\
\Rightarrow \frac{p_i}{1-p_i} &= e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m} \\
\Rightarrow \ln\left(\frac{p_i}{1-p_i}\right) &= \ln\left(e^{\beta_0+\beta_1 X_1+\cdots+\beta_m X_m}\right) \\
\Rightarrow \ln\left(\frac{p_i}{1-p_i}\right) &= \beta_0+\beta_1 X_1+\cdots+\beta_m X_m \\
\Rightarrow logit(p_i) &= \beta_0+\beta_1 X_1+\cdots+\beta_m X_m \\
\Rightarrow logit(\mathbb{P}(Y=1)) &= \beta_0+\beta_1 X_1+\cdots+\beta_m X_m
\end{aligned}
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3 References

- http://www.ats.ucla.edu/stat/stata/webbooks/logistic/chapter3/statalog3.htm
- https://en.wikipedia.org/wiki/Logit