GENERAL ASSEMBLY

BAYES' RULE AND INTRODUCTION TO STATISTICS

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LEARNING OBJECTIVES

- Derive and apply Bayes' Rule.
- Describe the relationship between probability and statistics.
- Understand the difference between descriptive statistics and inferential statistics.
- Identify the two fields of inferential statistics and the two most popular methods of parameter inference.

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- **<u>This will be very important later.</u>**

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- $P(B)$ is what we scale $P(B|A)P(A)$ by to ensure we are only looking at A within the context of B occurring.

BREAKING DOWN BAYES' RULE

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- $P(A)$ is the probability that A occurs given no supplemental information.
  - "Prior"
- $P(B|A)$ is the likelihood of seeing evidence (data) B assuming that A is true.
  - "Likelihood"
- $P(B)$ is what we scale $P(B|A)P(A)$ by to ensure we are only looking at A within the context of B occurring.
  - "Marginal Likelihood of B"

BAYES RULE APPLICATION

- Incorporating Context
  - iPhone, you text ‘radom.’
  - iPhone might correct to ‘random’ or ‘radon’ or leave as ‘radom,’ but which?

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  - iPhone, you text ‘radom.’
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- In Bayesian statistics, often we let the data (or what we have observed) be y and our unknown or parameter of interest be θ.
  - Let ‘radom’ = y and we want to figure out the “truth,” or what you intended to text, labeled θ.

BAYES RULE APPLICATION

- $y =$ 'radom,' and suppose for simplicity that the three possibilities are $\theta =$ 'random,' 'radon,' or 'radom.'

- Let's find:
  - $P(\theta = random | y = radom)$
  - $P(\theta = radon | y = radom)$
  - $P(\theta = radom | y = radom)$

- Our thought process is that we'll find all three of these probabilities and then whichever probability is highest is the best θ and thus the one to which our iPhone should autocorrect.

BAYES RULE APPLICATION

- Recall:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)}$$

- In order to get $P(\theta|y)$, we need $P(y|\theta)$, $P(\theta)$, and $P(y)$.

BAYES RULE APPLICATION

- Let's find:
  - $P(\theta_1 = random|y = radom)$
  - $P(\theta_2 = radon|y = radom)$
  - $P(\theta_3 = radom|y = radom)$

- We need:
  - $P(y|\theta_1)$, $P(y|\theta_2)$, and $P(y|\theta_3)$.
  - $P(\theta_1)$ $P(y|\theta_2)$, and $P(y|\theta_3)$.
  - $P(y)$.

- Brainstorm: how might we estimate these?

BAYES RULE APPLICATION

- From Google:

$\boldsymbol{\theta}$	$\boldsymbol{p(\theta)}$
random	7.60×10^{-5}
radon	6.05×10^{-6}
radom	3.12×10^{-7}

$\boldsymbol{\theta}$	$\boldsymbol{p(y = \text{“}radom\text{”}|\theta)}$
random	0.00193
radon	0.000143
radom	0.975

BAYES RULE APPLICATION

θ	$Prior: p(\theta)$	$Likelihood: p(y = \text{“radom”}\mid\theta)$	$Marginal\ Likelihood: p(y)$	$Posterior: P(\theta\mid y)$
random	7.60×10^{-5}	0.00193	$p(y)$	
radon	6.05×10^{-6}	0.000143	$p(y)$	
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radon	6.05×10^{-6}	0.000143	$p(y)$	$8.65 \times 10^{-10}/p(y)$
radom	3.12×10^{-7}	0.975	$p(y)$	$3.04 \times 10^{-7}/p(y)$

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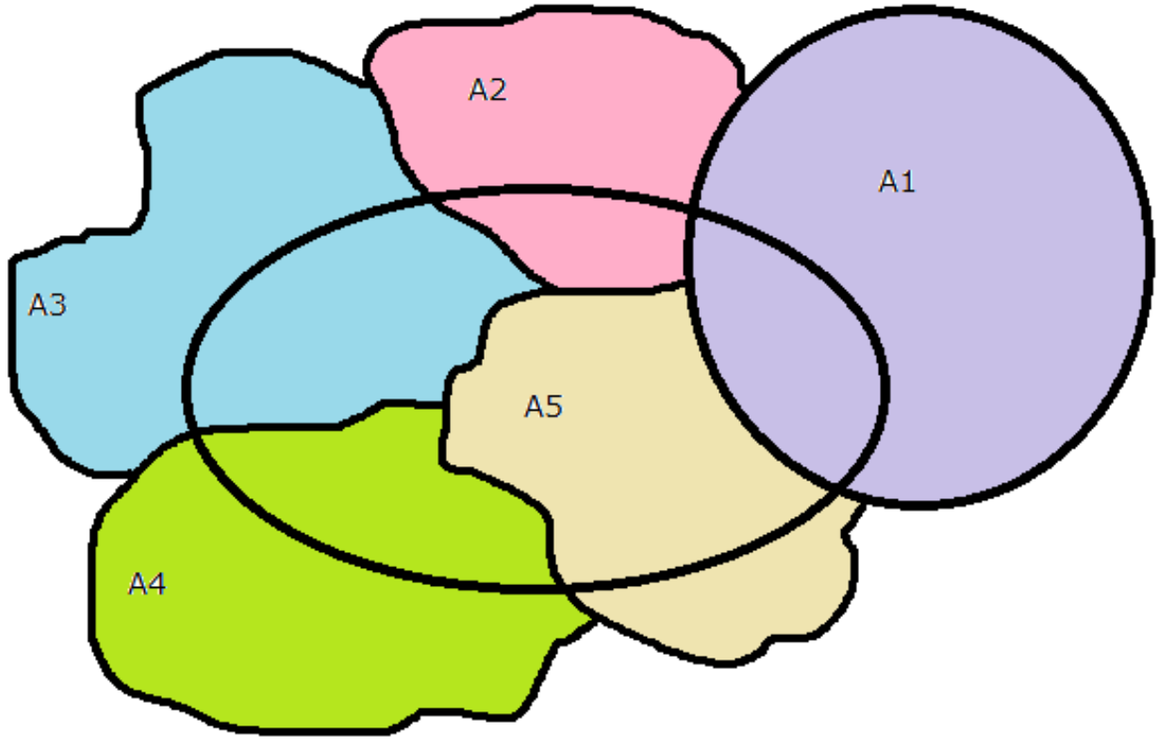
PROBABILITY RULES

- $P(B) = \sum_{i=1}^{n} P(B \cap A_i)$
  - "Law of Total Probability"



- $P(y) = \sum_{\{i=1\}}^{n} P(y \cap \theta_i) = \sum_{\{i=1\}}^{3} P(y|\theta_i)P(\theta_i)$

BAYES RULE APPLICATION

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radon	6.05×10^{-6}	0.000143	$p(y) \approx 4.52\times10^{-7}$	$0.002 = 0.2\%$
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BAYES RULE APPLICATION

- Goal: Find posterior probability of parameter θ given our data or evidence y.
  - This is written as $P(\theta|y)$.

- Needed:
  - Prior probability of parameter θ.
  - Likelihood of data y given parameter θ.
  - Marginal likelihood of data y with no knowledge of parameter.*

BAYES RULE APPLICATION

- If your hypotheses are mutually exclusive and collectively exhaustive, the marginal likelihood is not necessary.

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WRAPPING UP FROM LAST TIME

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- For example, rather than finding the probability that someone has an IQ of exactly 100, we might be interested in looking at all possible IQ scores and how frequently we observe each IQ value.

- Recall: a distribution is the set of all possible values of a variable and how frequently the variable takes on each value.

AN ASIDE: PROBABILITY AND STATISTICS

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- Let X = IQ score.
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- Now let Y = time it takes all American workers to get to work.
  - What do we do here?

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- This gets to the relationship between probability and statistics.
- In probability, we know the values of these parameters (measures of a population) and can thus completely define the probability distribution.
- In statistics, we don't know the values of these parameters, so we have to estimate them.
- We gather a sample to learn about the population.
- We calculate statistics to learn about parameters.

RECAP

- "Tree of statistics."

- Probability is a building block to learning about statistics.
  - Samples help us to learn about populations.
  - Statistics help us to learn about parameters.

- Independence is a huge consideration that will depend on your use-case.

- Probability is complicated, but being familiar with the basics will go a long way in understanding how pieces fit together.