Statistics with SpaRRows

Lecture 15

Julia Schroeder

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Outline

- Linear models – going big

Multiple linear models

```
lm(response~explanatory)
```

Multiple linear models

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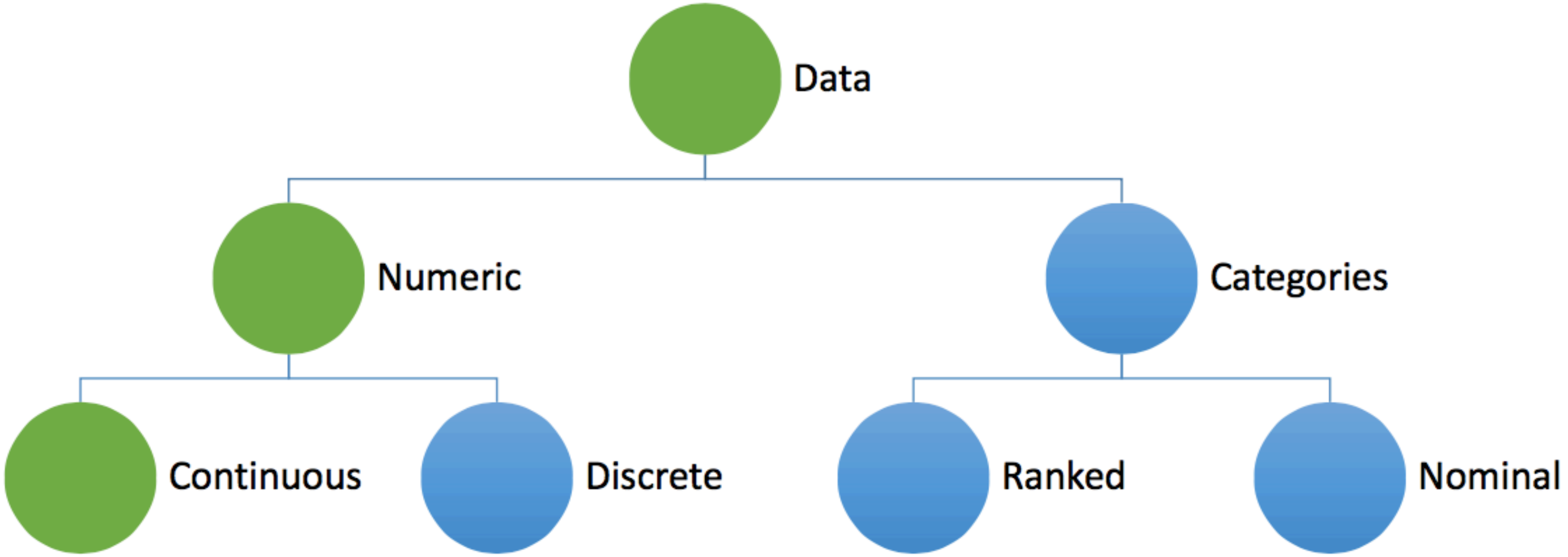
lm(response~explanatory)
```

Data types



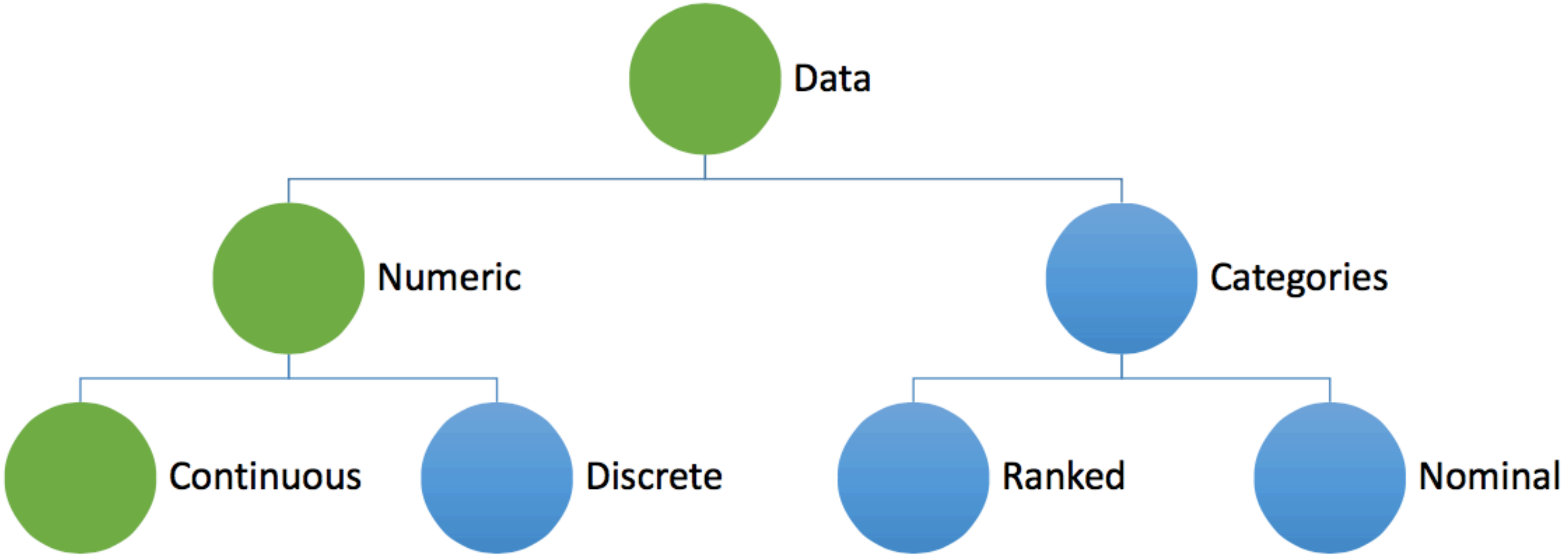
Multiple linear models

`lm(response~explanatory)`

Response y:
    Continuous

Explanatory x:
    Continuous (tarsus, wing, mass)
    Categorical (Sex, Year, Observer, BirdID)

Data types



Multiple linear models

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lm(response~explanatory)
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- Continuous

Explanatory x:
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$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

Multiple linear models

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```
lm(response~explanatory)
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$$y_i = b_0 + b_1 x_i + \varepsilon_i$$

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i2} + \varepsilon_i$$

Multiple linear models

Response y:
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Explanatory x:
Continuous (tarsus, wing, mass)
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```
lm(response~explanatory)
```

We can have more than one explanatory variable!

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i2} + \varepsilon_i$$

Multiple linear models

Response y:

Continuous

Explanatory x:

Continuous (tarsus, wing, mass)

Categorical (Sex, Year, Observer, BirdID)

```
lm(response~explanatory)
```

We can have more than one explanatory variable!

We can even mix continuous and factorial explanatory variables!

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i2} + \varepsilon_i$$

Let’s try this

- b_0 = intercept

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

- b_1 = estimates effect of continuous variable x_0
- b_2 = estimates effect of 2-level factor x_1

Let’s try this

- b_0 = intercept

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

- b_1 = estimates effect of continuous variable x_0 (tarsus)
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Let’s try this

- b_0 = intercept

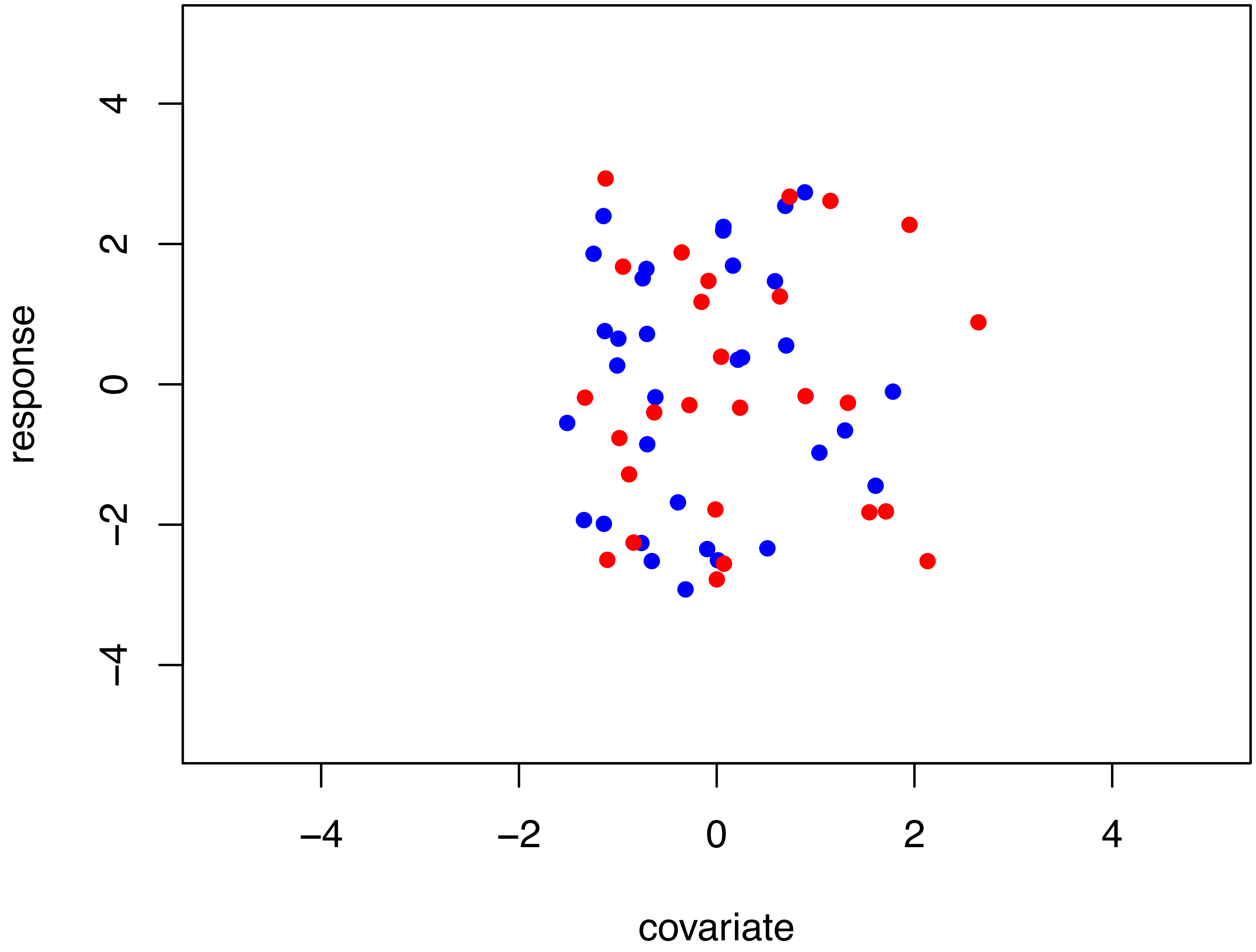
$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

- b_1 = estimates effect of continuous variable x_0 (tarsus)
- b_2 = estimates effect of 2-level factor x_1 (sex)

- Sex will be re-coded internally. Females are 0.

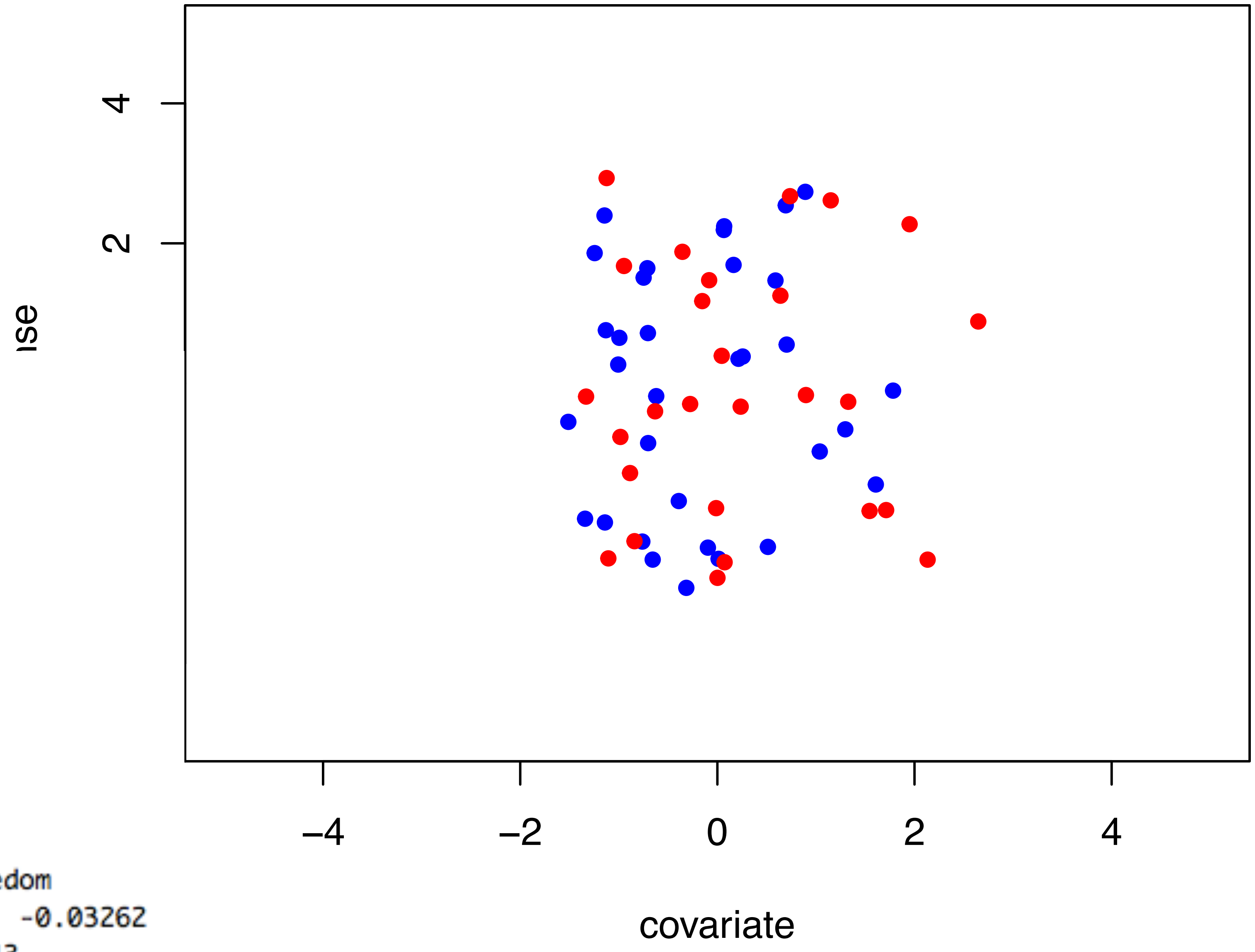
$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let's try this



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

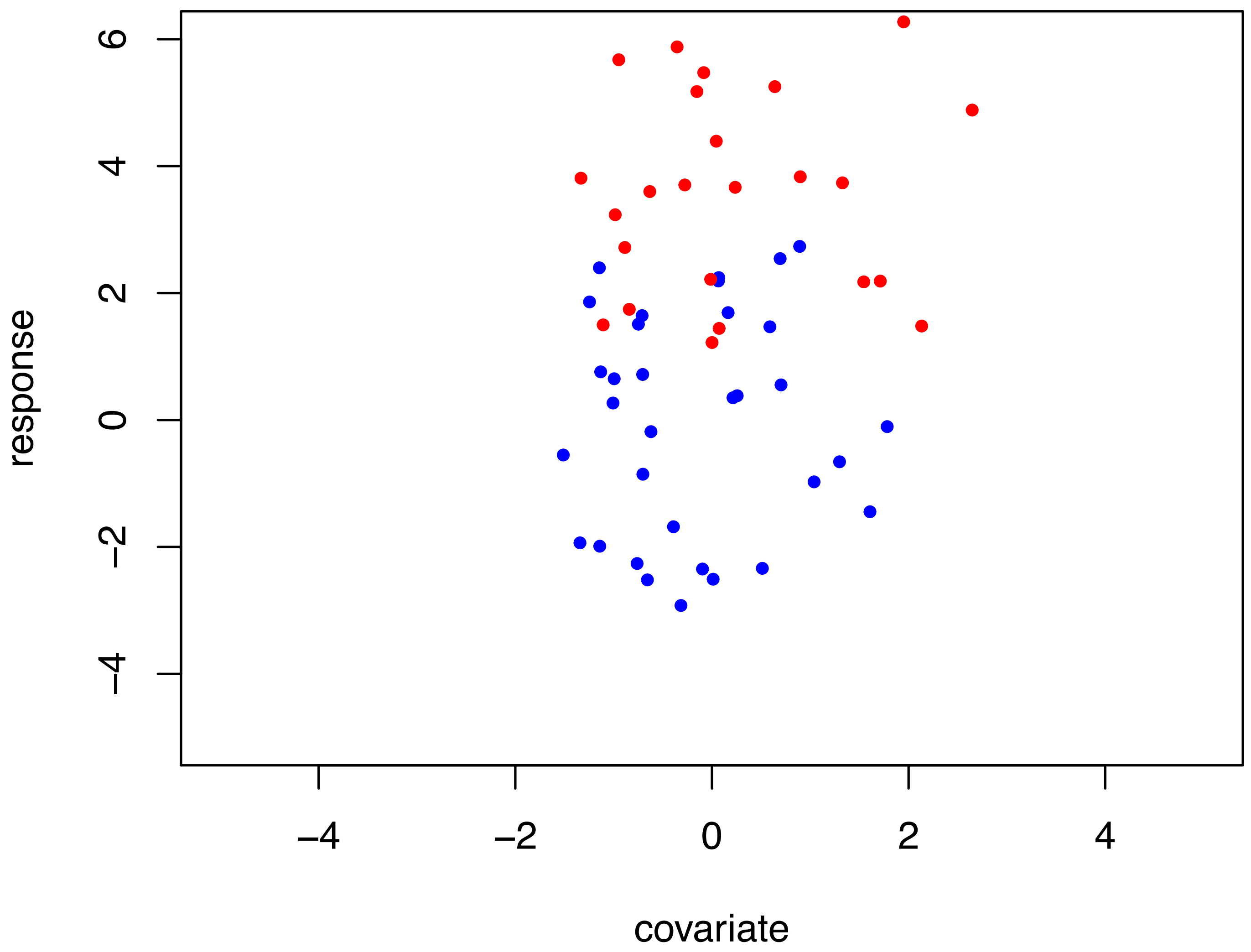
Let’s try this




```
Call:
lm(formula = y ~ x + sx)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.8702 -1.7023 -0.1178  1.5936  3.1370 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.02613    0.31286  -0.084    0.934
x            0.08222    0.23471   0.350    0.727
sx          -0.08598    0.47224  -0.182    0.856

Residual standard error: 1.784 on 57 degrees of freedom
Multiple R-squared:  0.00238,	Adjusted R-squared:  -0.03262 
F-statistic: 0.06799 on 2 and 57 DF,  p-value: 0.9343
```

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let's try this



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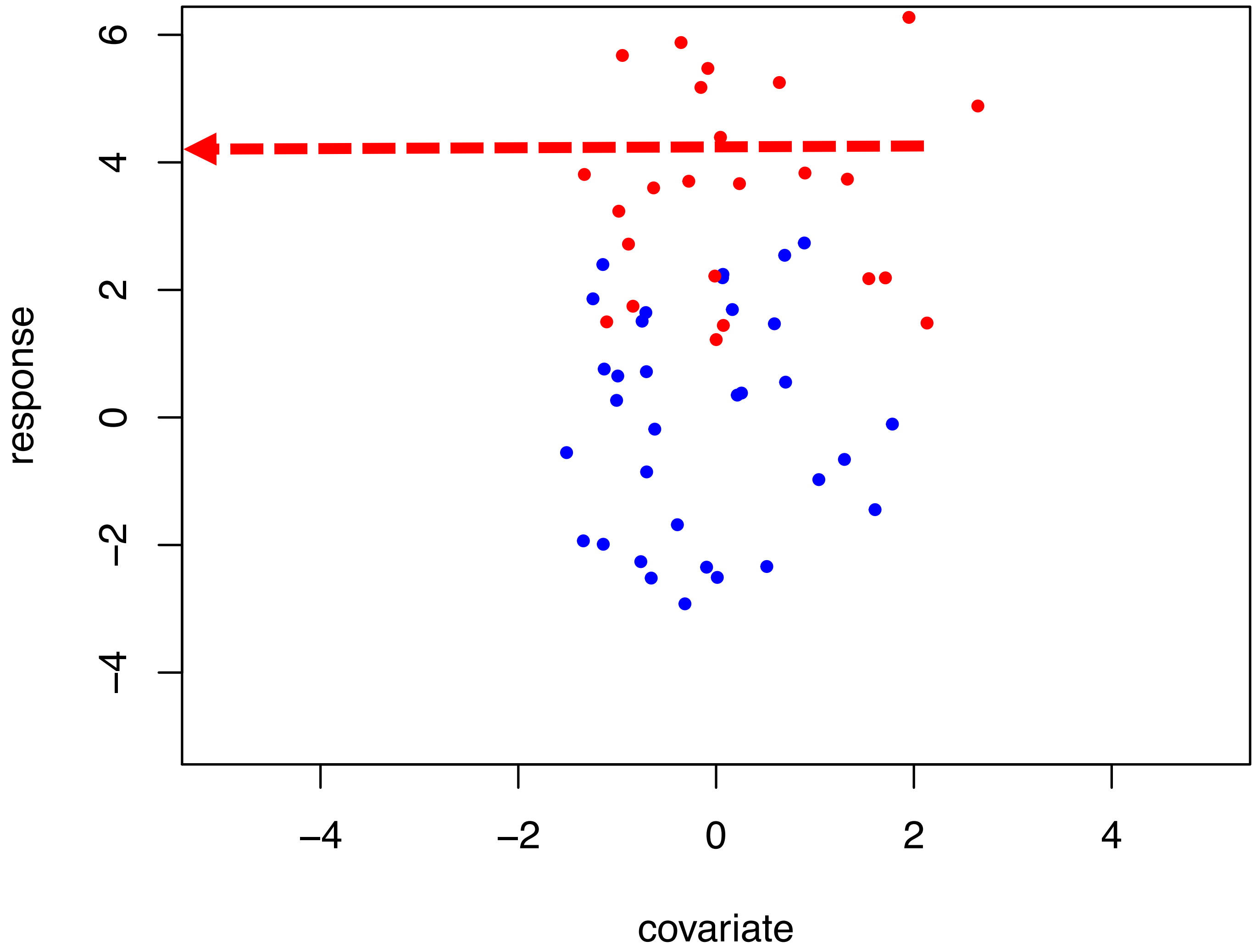

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Let’s try this

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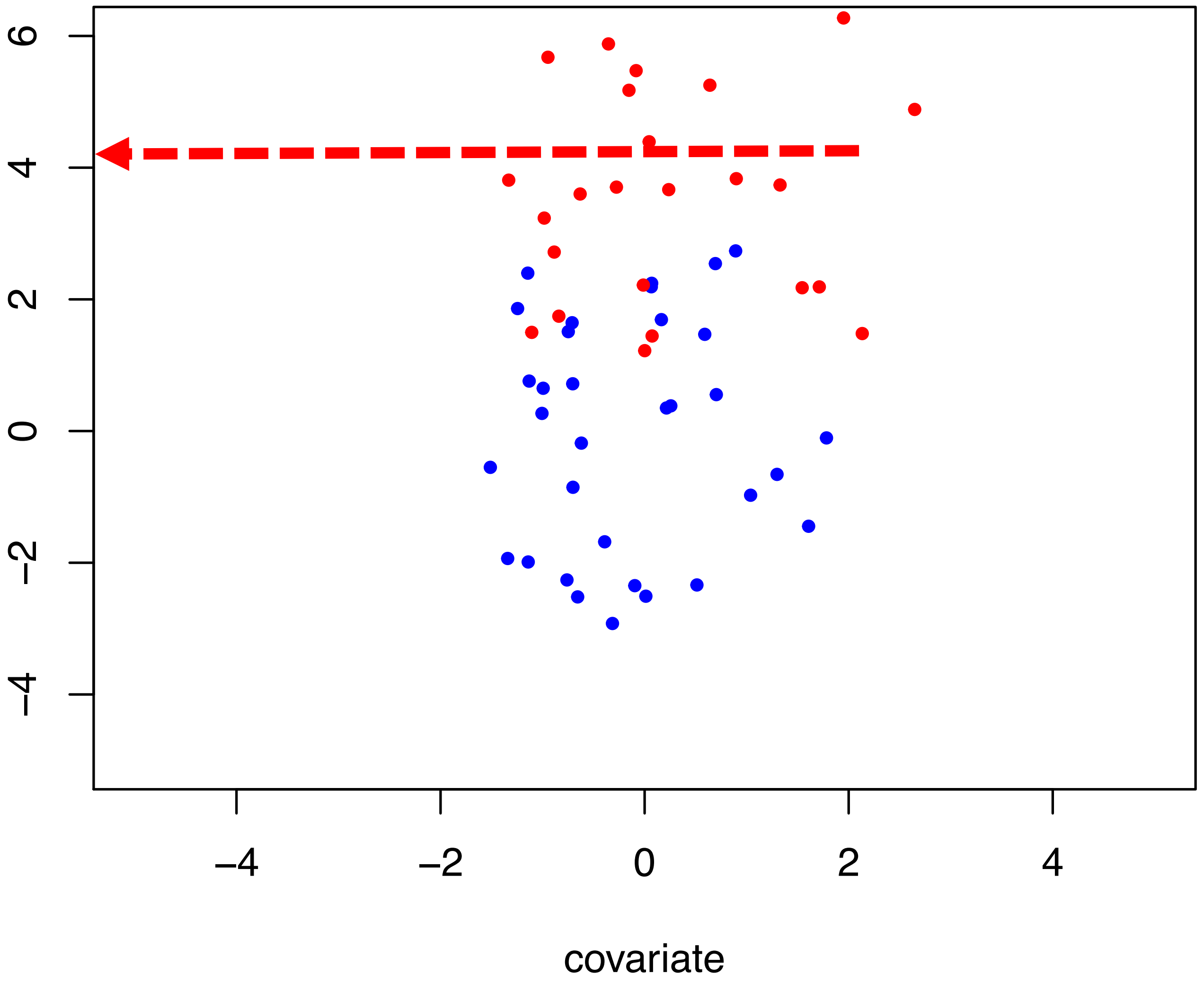
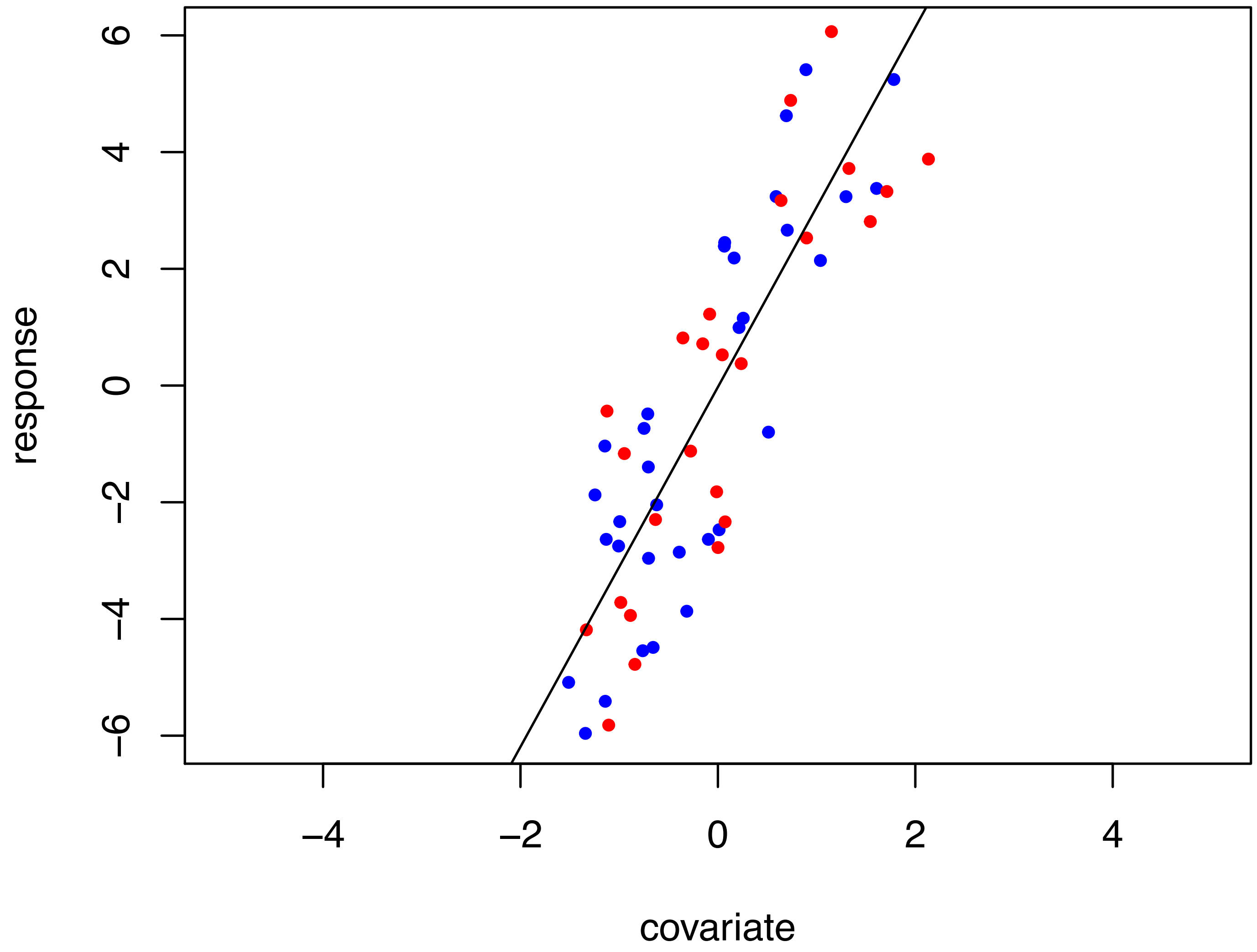
(Intercept) -0.02613    0.31286  -0.084    0.934    
x            0.08222    0.23471   0.350    0.727    
sx           3.91402    0.47224   8.288 2.29e-11 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.784 on 57 degrees of freedom
Multiple R-squared:  0.5608,	Adjusted R-squared:  0.5454 
F-statistic:  36.4 on 2 and 57 DF,  p-value: 6.524e-11
```



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let’s try this



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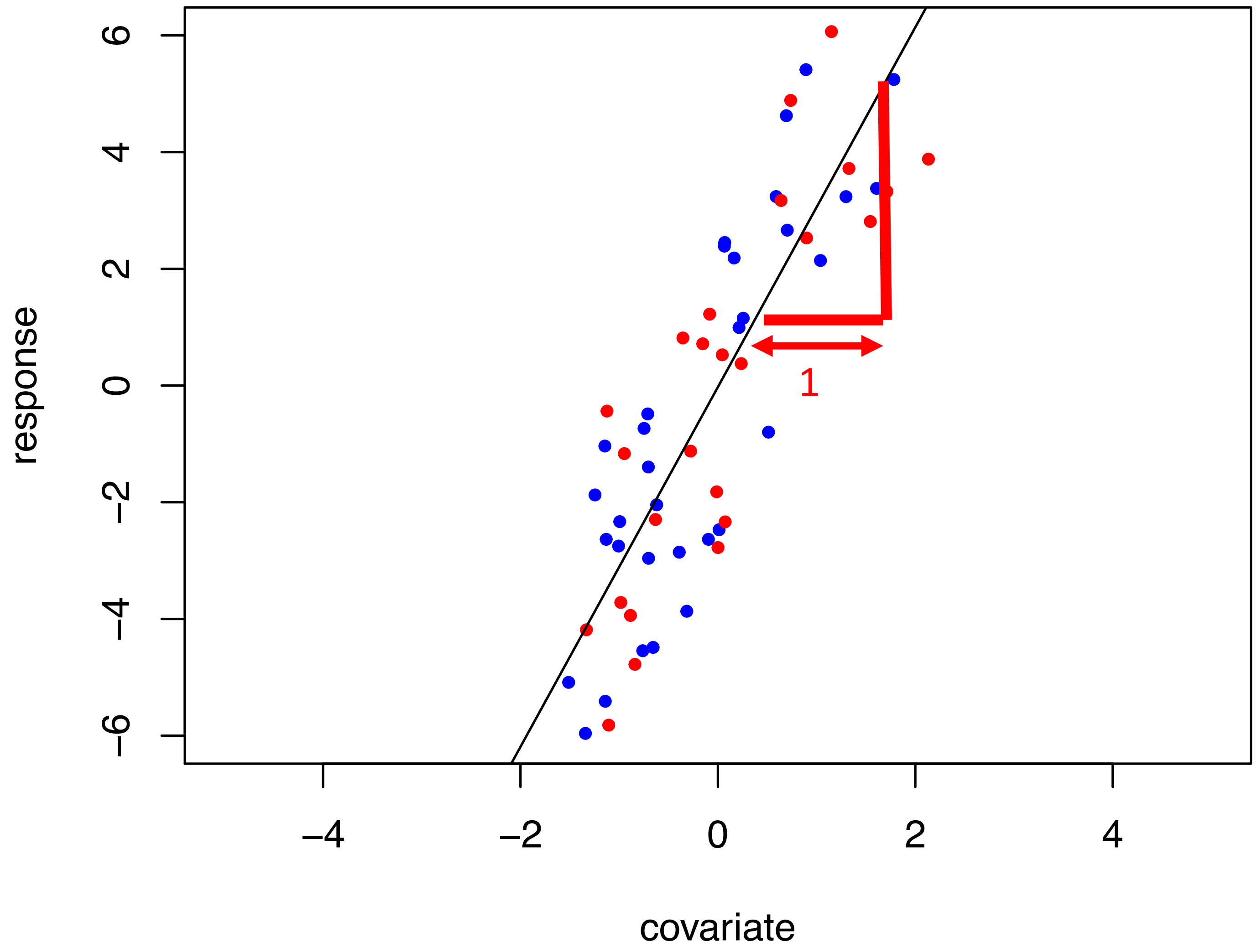

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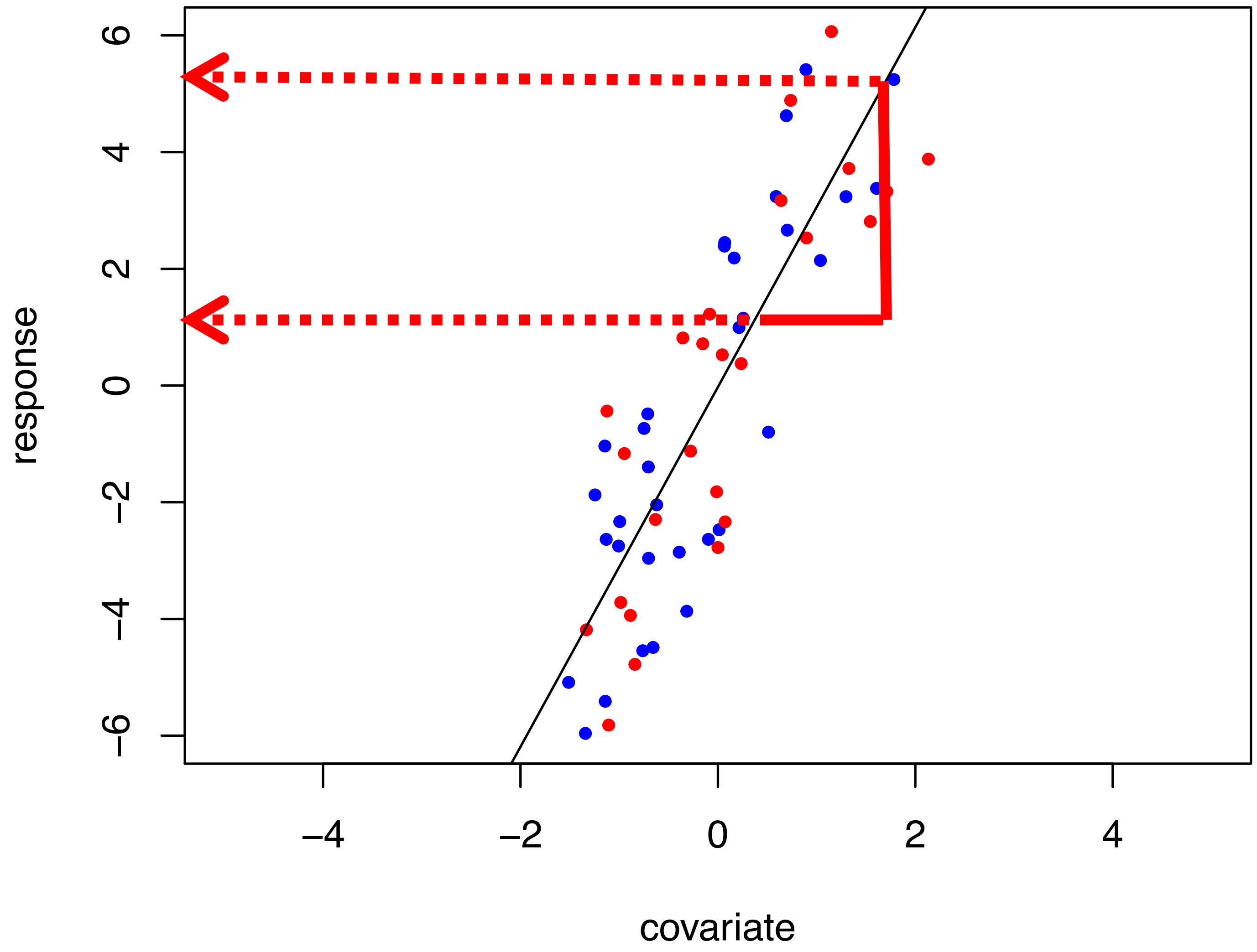

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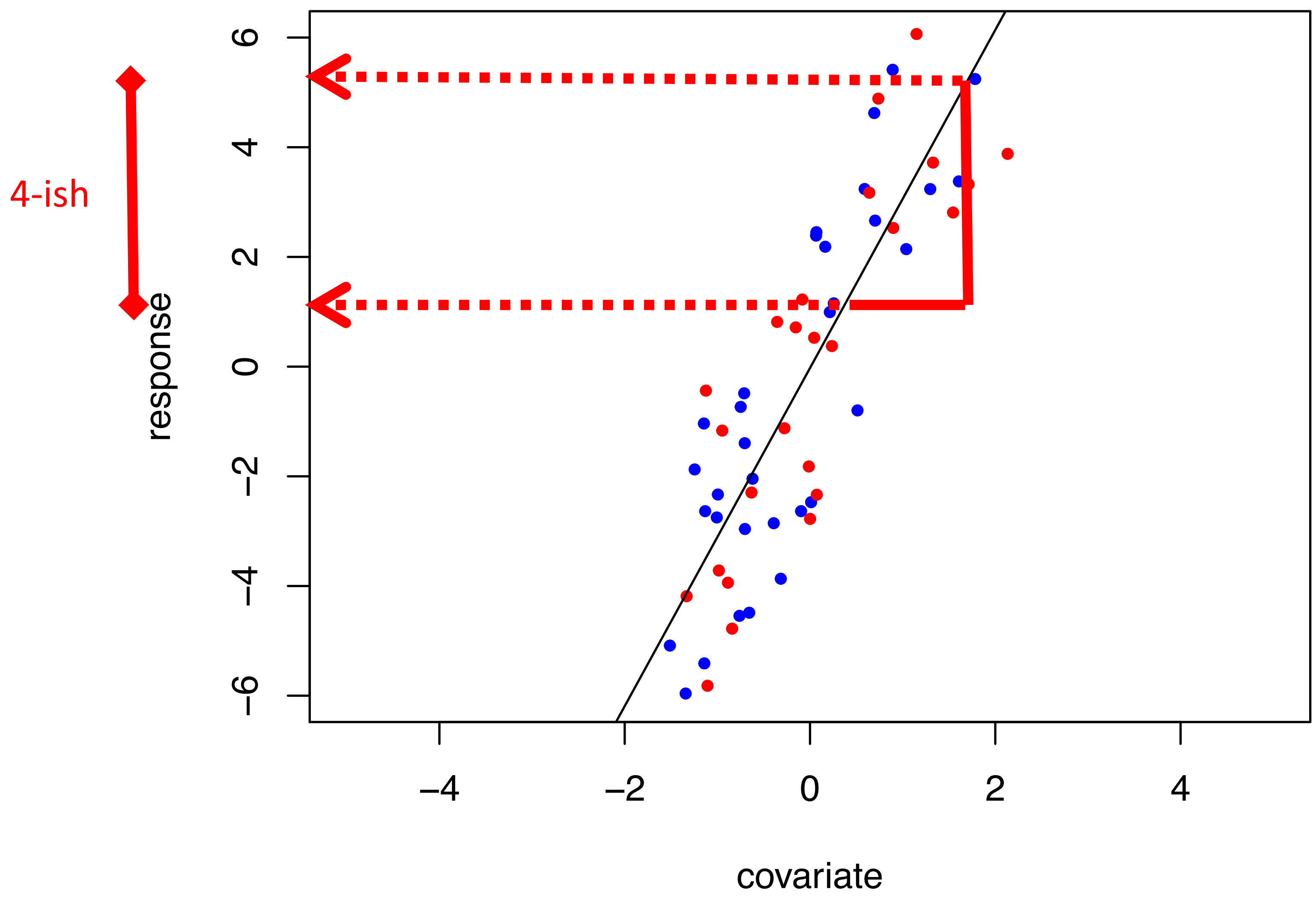

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let’s try this

4-ish

```
Call:
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Residuals:
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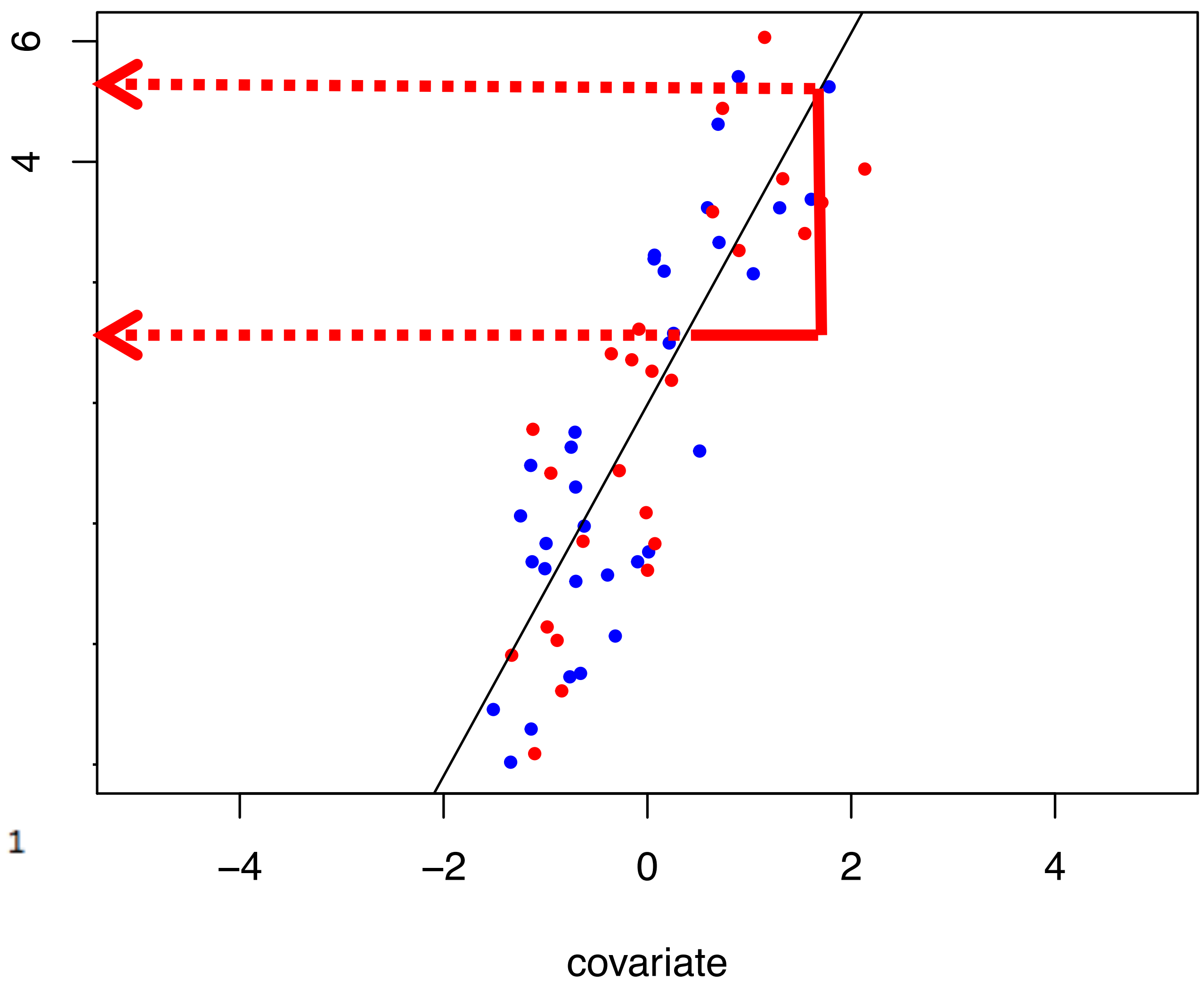
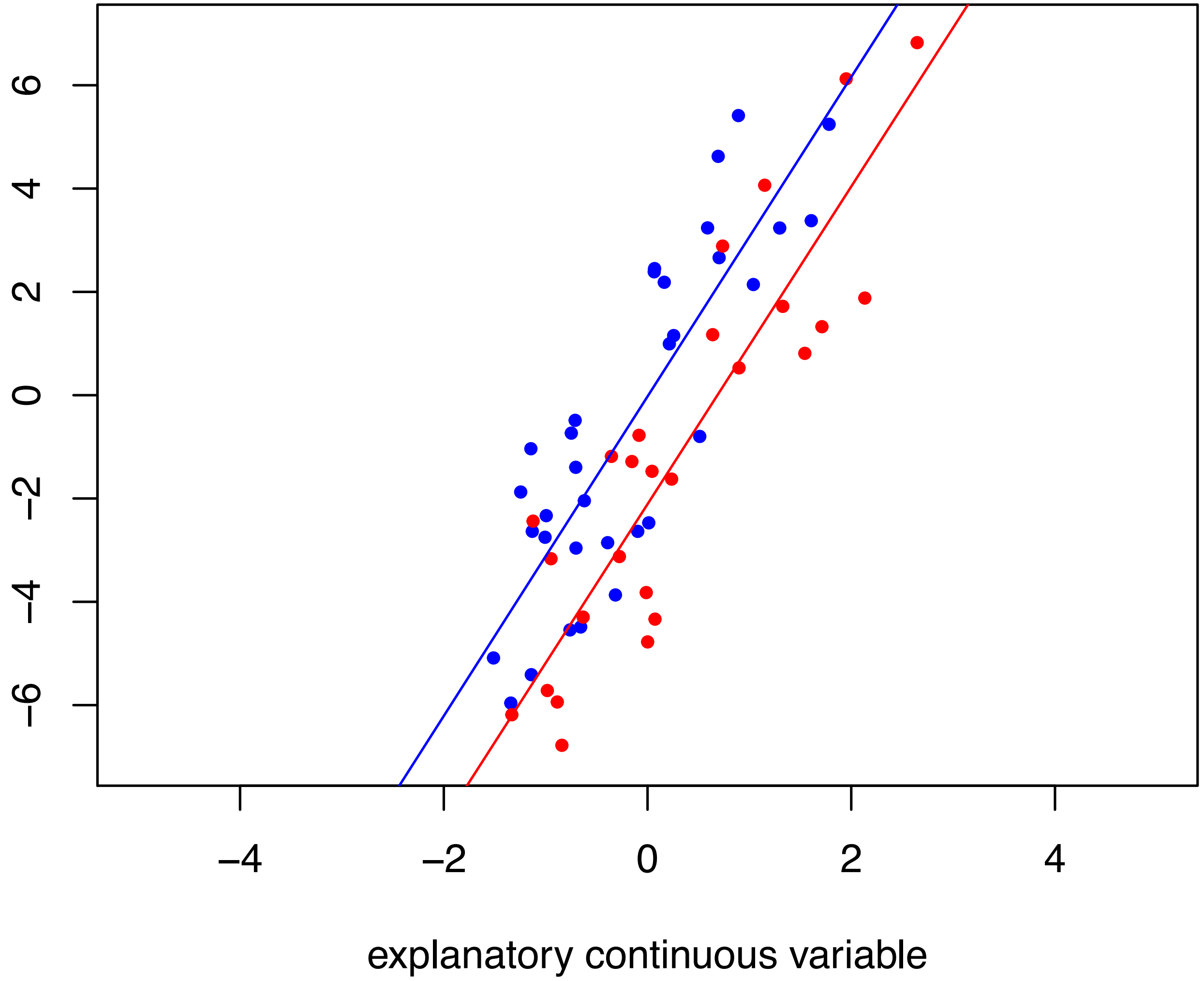
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.02613    0.31286  -0.084    0.934    
x            3.08222    0.23471  13.132   <2e-16 ***
sx          -0.08598    0.47224  -0.182    0.856    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.784 on 57 degrees of freedom
Multiple R-squared:  0.7579,	Adjusted R-squared:  0.7494 
F-statistic: 89.24 on 2 and 57 DF,  p-value: < 2.2e-16
```



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

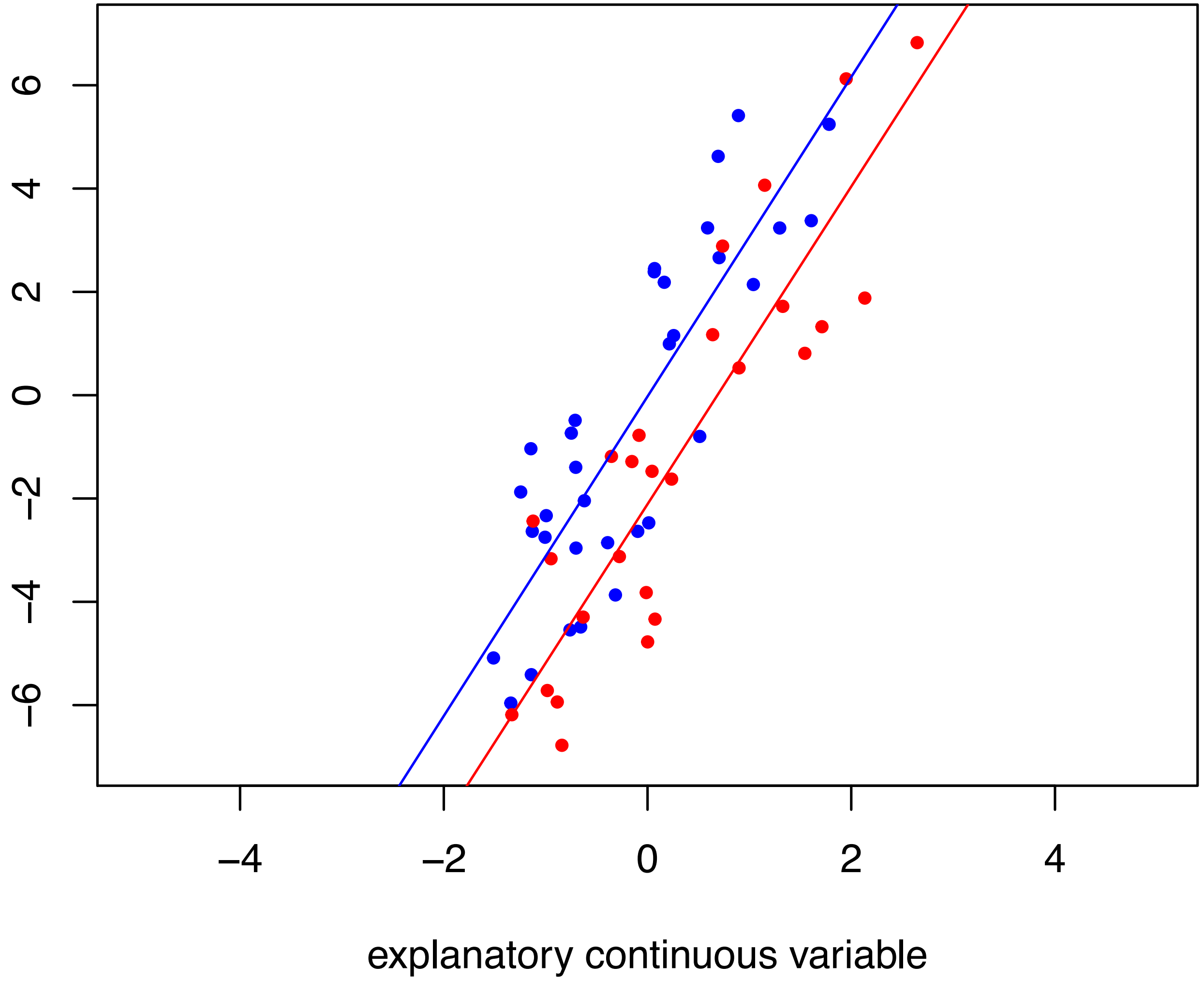
Let’s try this



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let's try this

- We really need two slopes and two intercepts, one for each sex…



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Let's try this

```
Call:
lm(formula = y ~ x + sx)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.8702 -1.7023 -0.1178  1.5936  3.1370 

Coefficients:
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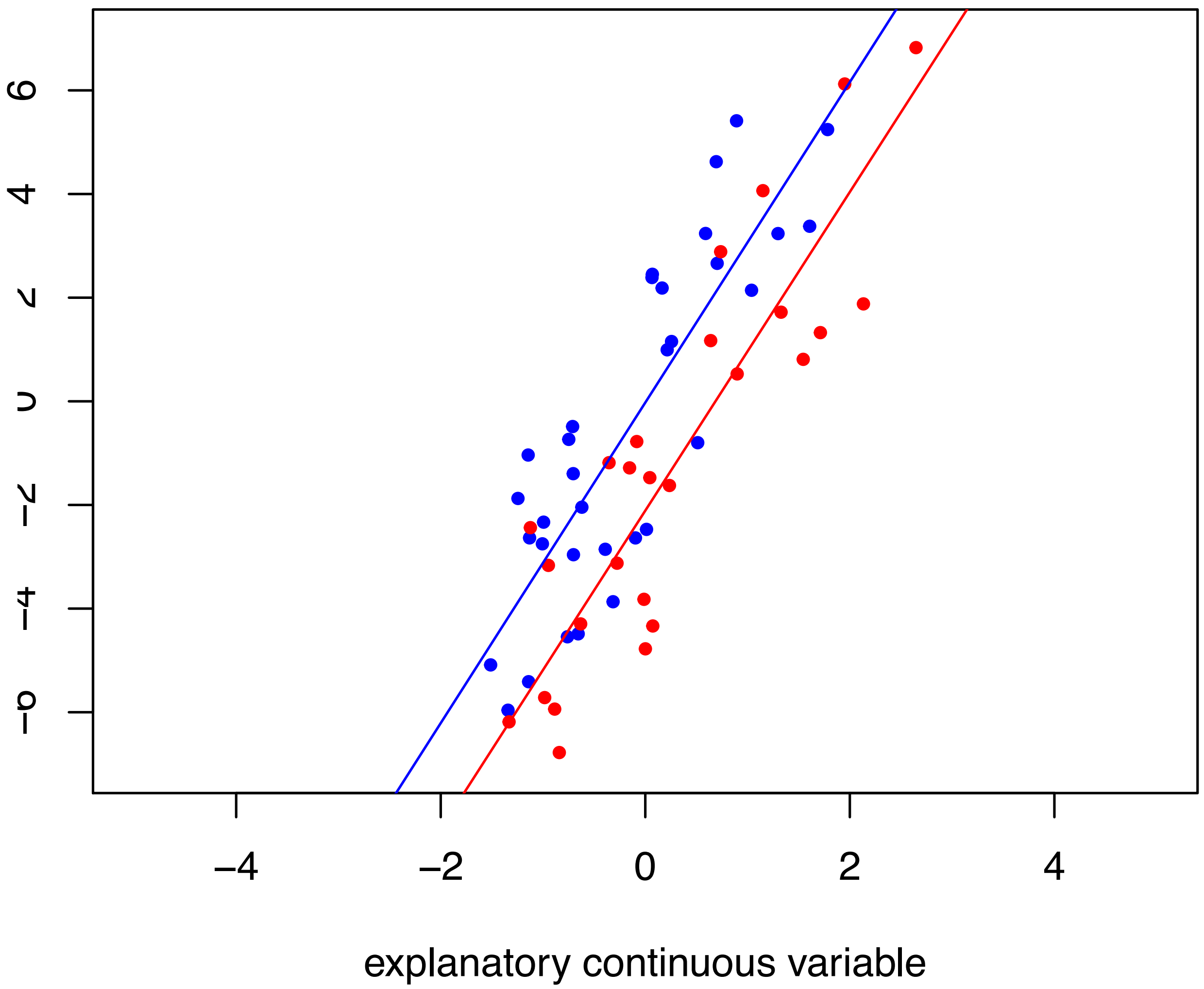
(Intercept) -0.02613    0.31286  -0.084    0.934    
x            3.08222    0.23471  13.132  < 2e-16 ***
sx          -2.08598    0.47224  -4.417 4.53e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.784 on 57 degrees of freedom
Multiple R-squared:  0.7553,	Adjusted R-squared:  0.7467 
F-statistic: 87.96 on 2 and 57 DF,  p-value: < 2.2e-16
```



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let’s try this

- Intercept: reference group

```
Call:
lm(formula = y ~ x + sx)

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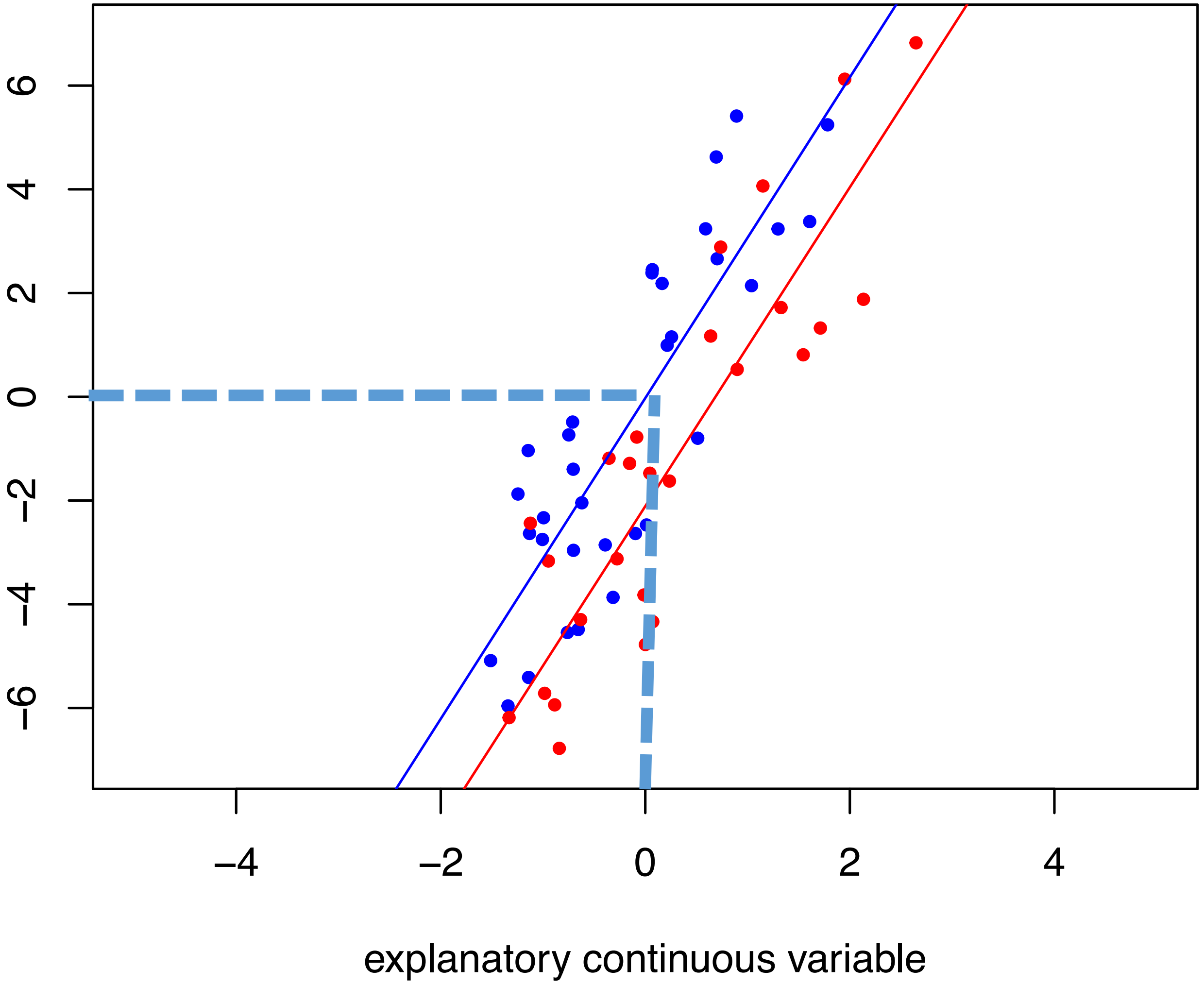

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let’s try this

- Intercept: reference group
- b_2 (sx): *absolute difference* of group 1 to reference

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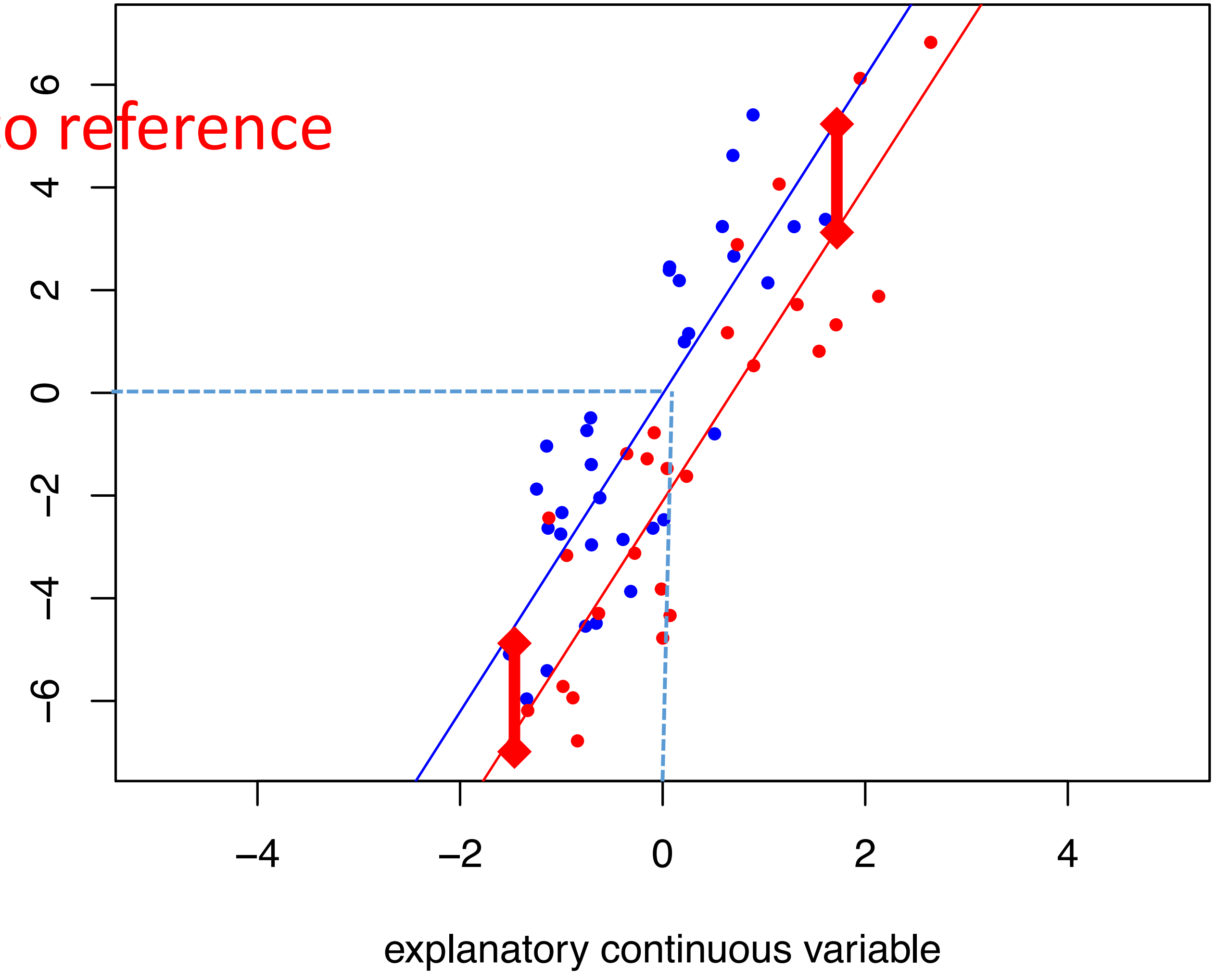

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let's try this

- Intercept: reference group
- b_2 (sx): *absolute difference* of group 1 to reference
- b_1 (x) slope, equal for both groups

```
Call:
lm(formula = y ~ x + sx)

Residuals:
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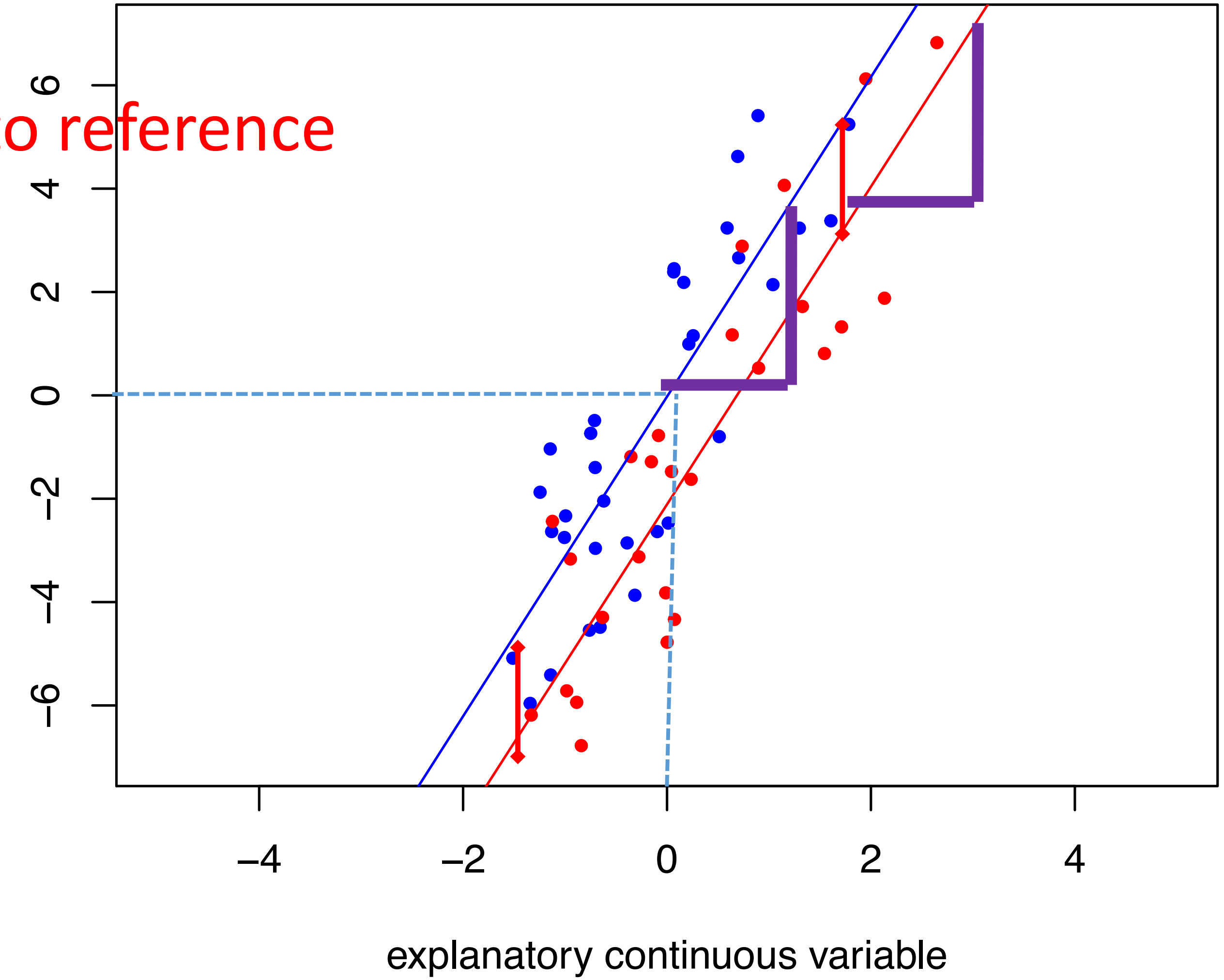

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Let's try this

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

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Let's try this

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

- interaction between sex and tarsus

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let’s try this

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

- interaction between sex and tarsus
- one more parameter estimate

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + \varepsilon_i$$

Let's try this

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

- interaction between sex and tarsus
- one more parameter estimate
- but not more variables

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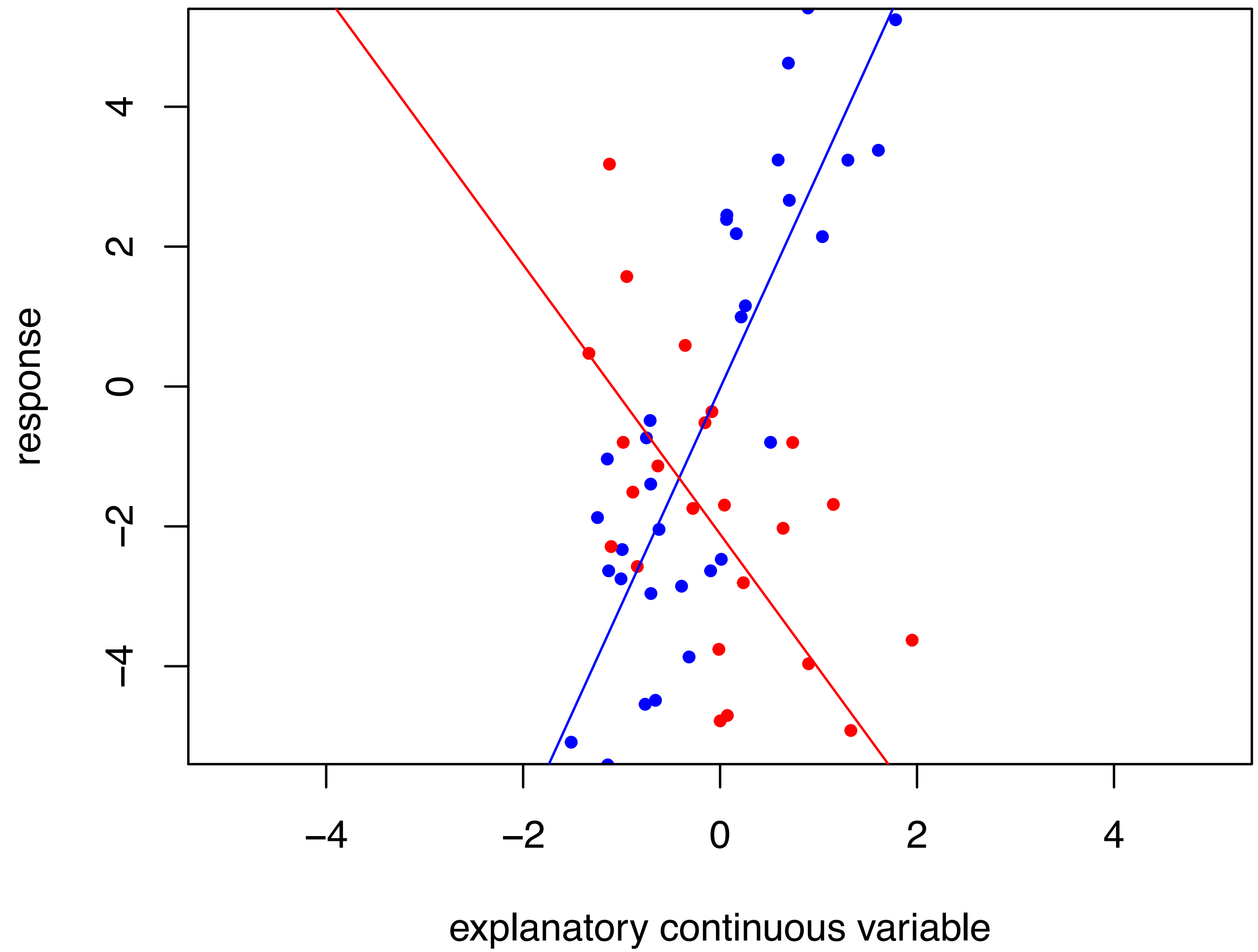
Let’s try this



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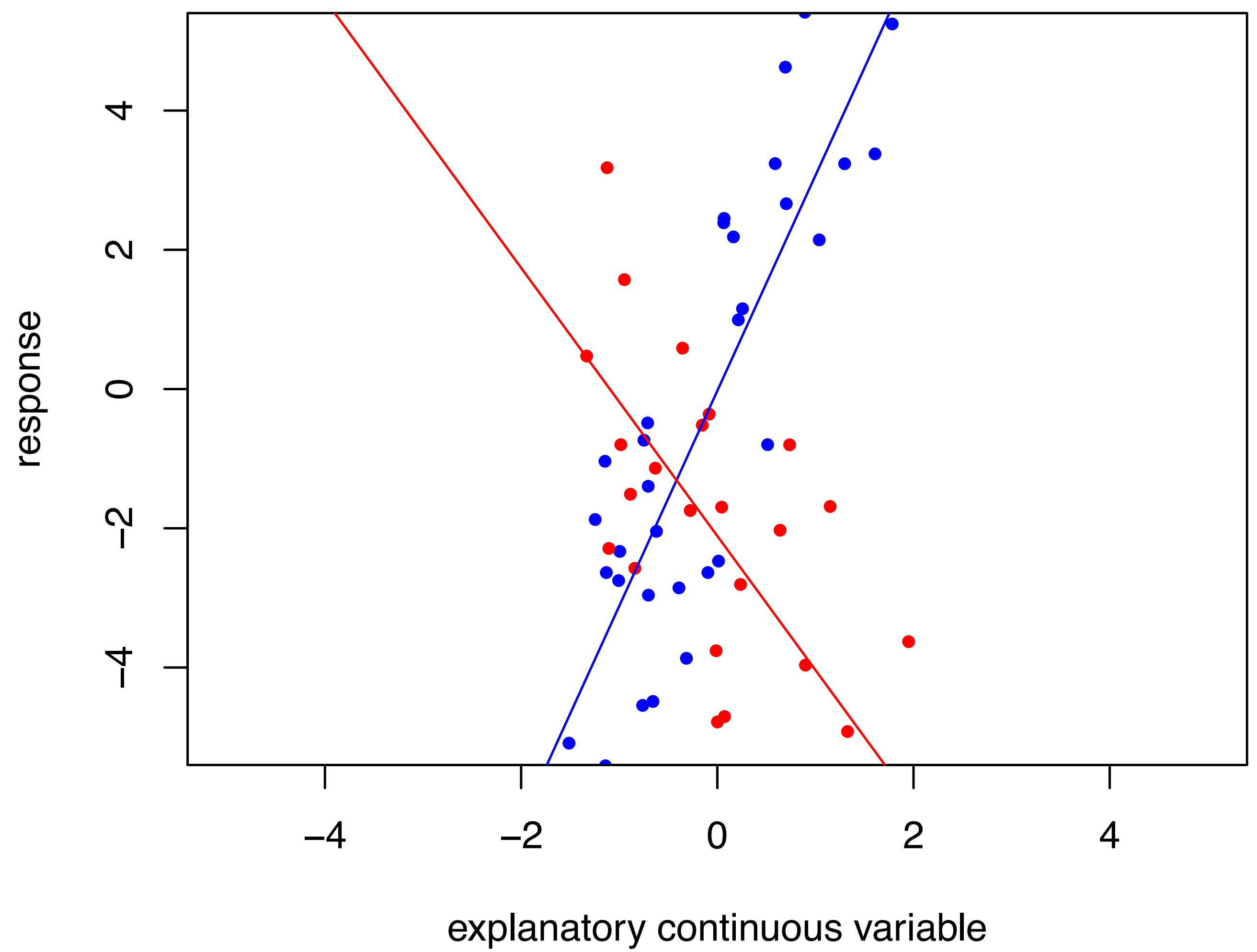
Let's try this

```
Call:
lm(formula = y ~ x * sx)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.8687 -1.7008 -0.1129  1.5931  3.1264 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.02453    0.31856  -0.077    0.939    
x            3.09216    0.35685   8.665 6.30e-12 ***
sx          -2.08574    0.47648  -4.377 5.30e-05 ***
x:sx        -5.01776    0.47700 -10.519 7.07e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.8 on 56 degrees of freedom
Multiple R-squared:  0.7008,	Adjusted R-squared:  0.6848 
F-statistic: 43.73 on 3 and 56 DF,  p-value: 1.079e-14
```



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

Let's try this

- Intercept: reference group, b_2: *absolute difference* of group 1 to reference

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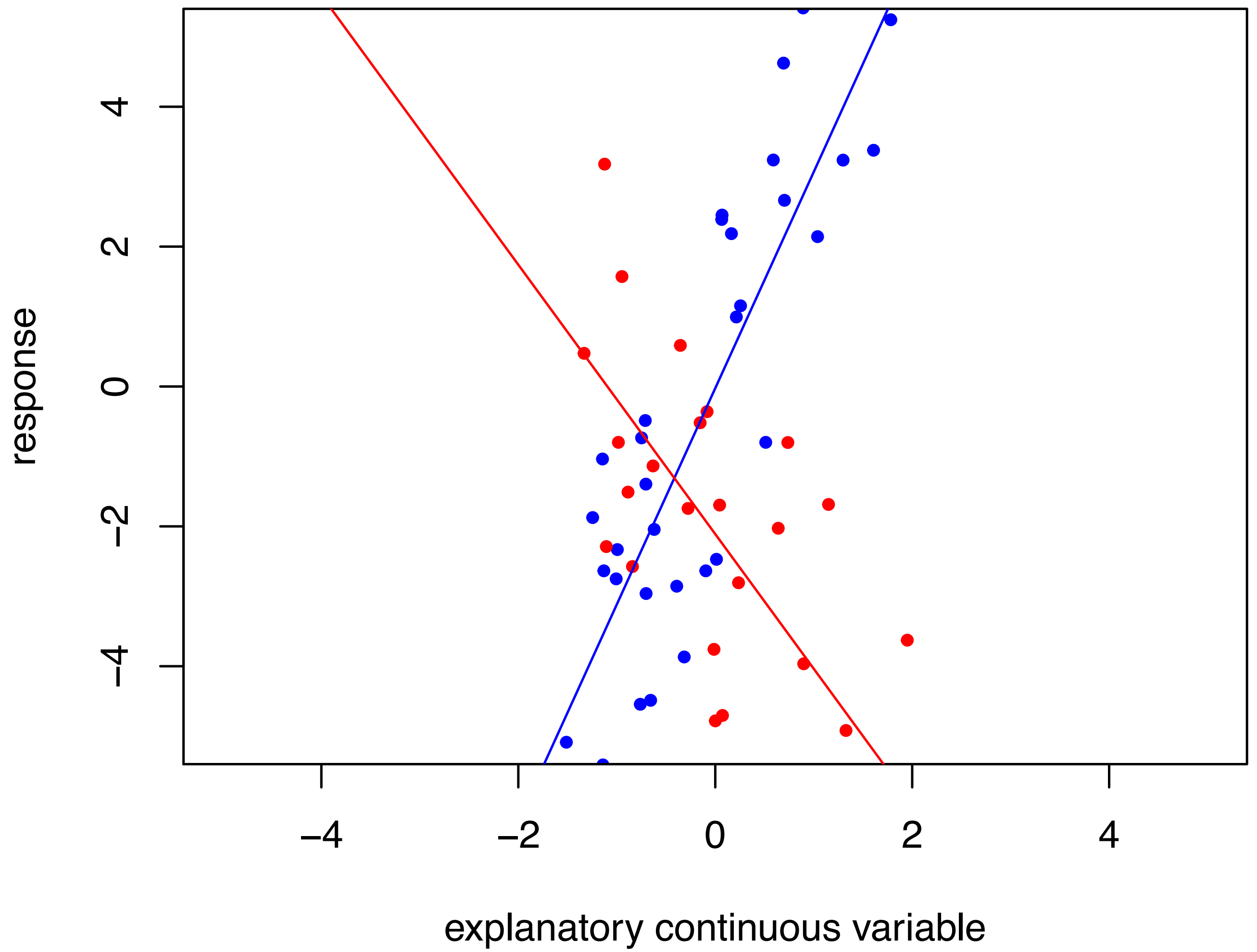

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

Let's try this

- Intercept: reference group, b_2: *absolute difference* of group 1 to reference
- b_1 slope of reference group

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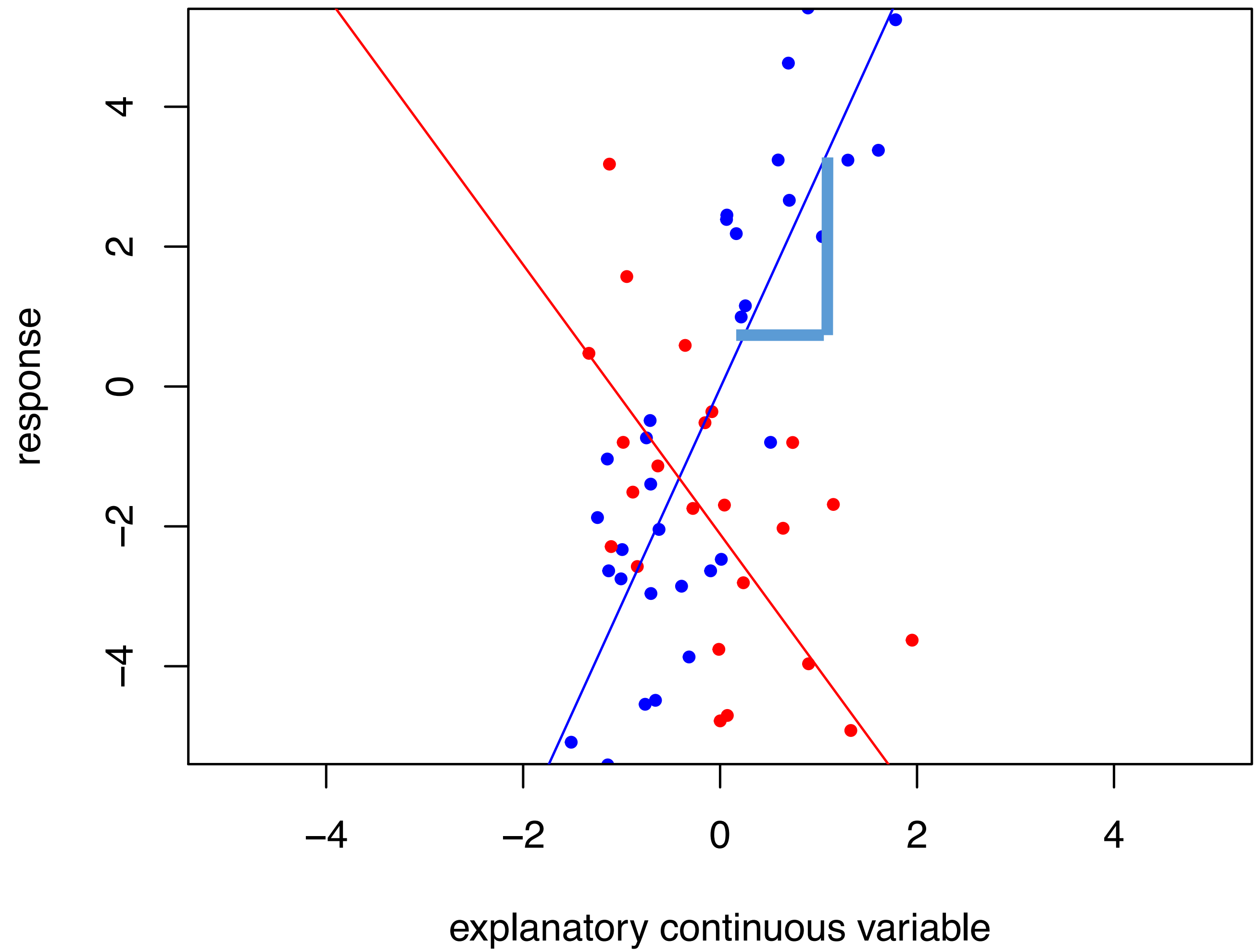

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

Let's try this

- Intercept: reference group, b_2: *absolute difference* of group 1 to reference
- b_1 slope of reference group
- b3 *absolute difference* of slope of group 1 to reference

```
Call:
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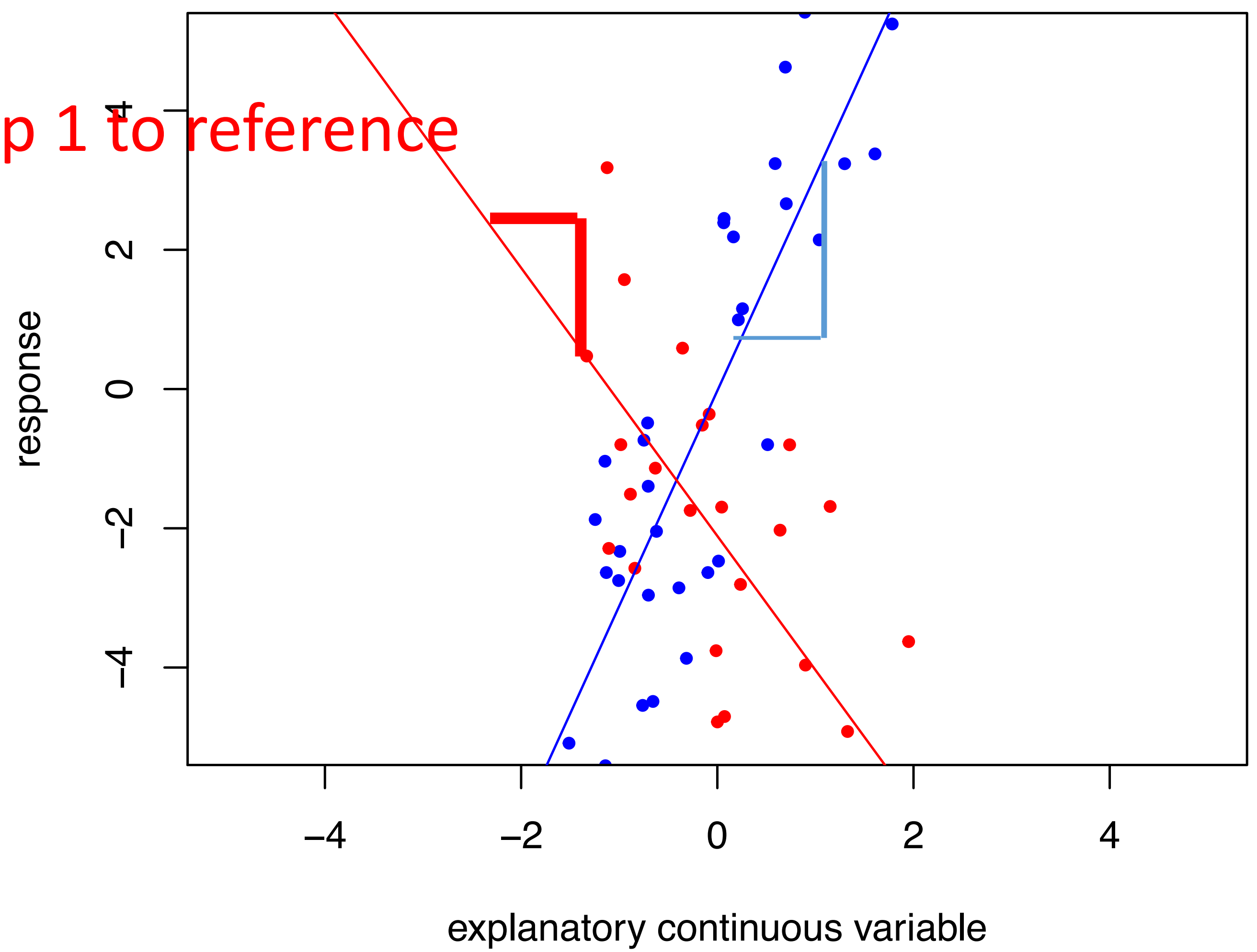

$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

Let's try this

- Intercept: reference group, b_2: *absolute difference* of group 1 to reference
- b_1 slope of reference group
- b3 *absolute difference* of slope of group 1 to reference

```
Call:
lm(formula = y ~ x * sx)

Residuals:
    Min      1Q  Median      3Q     Max
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Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.02453    0.31856  -0.077    0.939
x            3.09216    0.35685   8.665 6.30e-12 ***
sx          -2.08574    0.47648  -4.377 5.30e-05 ***
x:sx        -5.01776    0.47700 -10.519 7.07e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.8 on 56 degrees of freedom
Multiple R-squared:  0.7008,    Adjusted R-squared:  0.6848
F-statistic: 43.73 on 3 and 56 DF,  p-value: 1.079e-14
```
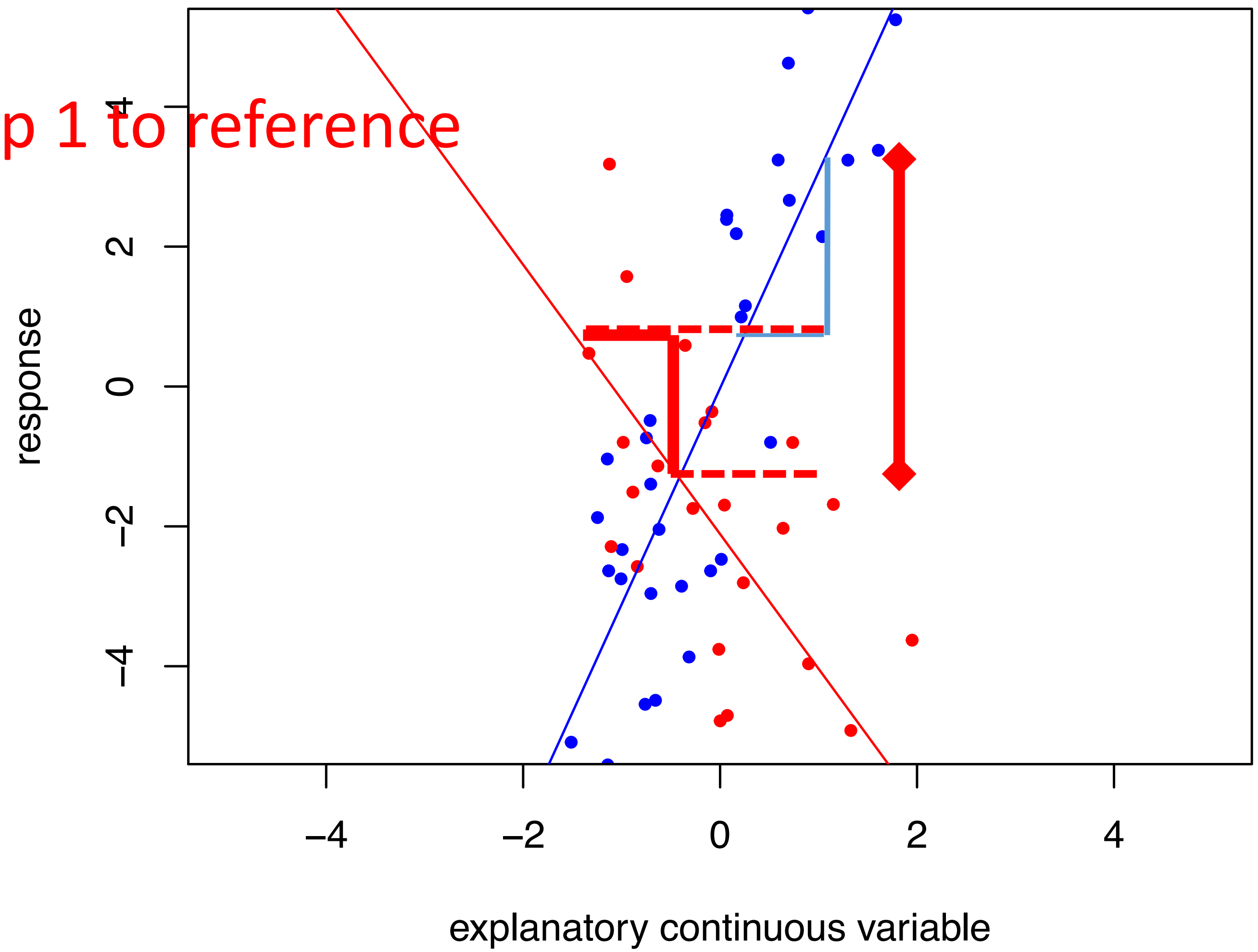
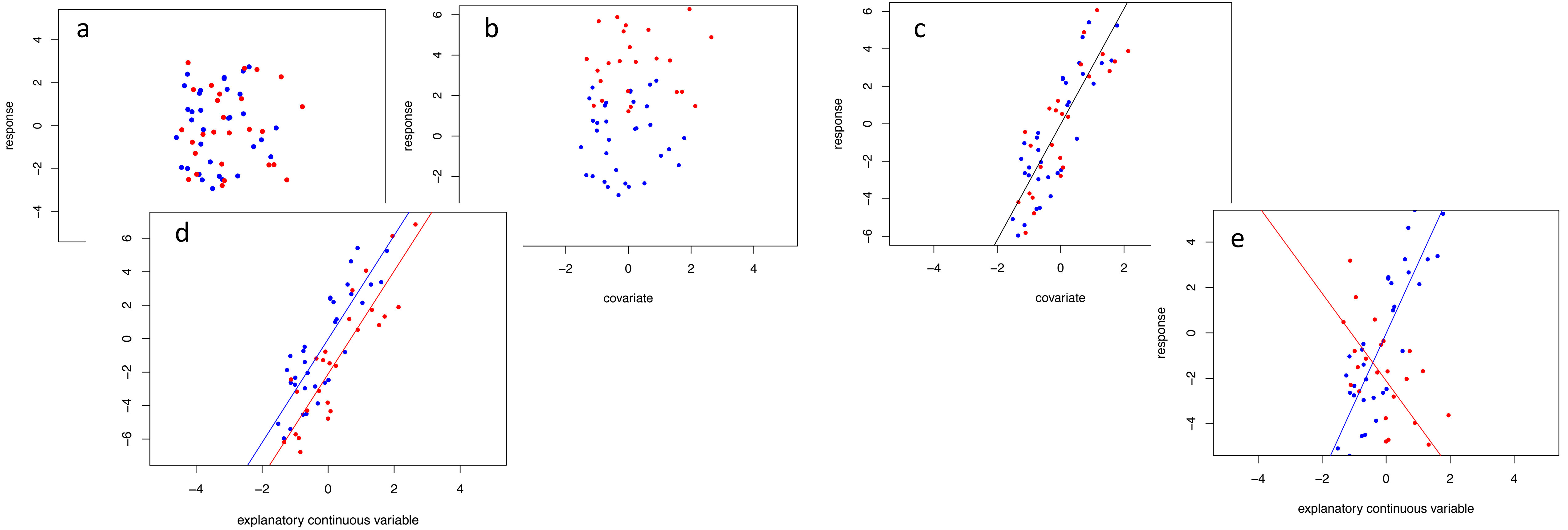



$$y_i = b_0 + b_1 x_{i0} + b_2 x_{i1} + b_3 x_{i0} x_{i1} + \varepsilon_i$$

plot	a	b	c	d	e
Intercept					
b_1 (sex)	0	+	0	+	+
b_2 (tarsus)	0	0	+	+	+
b_3 (tarsus x sex)	0	0	0	0	-



Interpreting

- Interpret effect as “*if all other predictors are kept constant*”

Interpreting

- Interpret effect as "*if all other predictors are kept constant*"
- This is impossible for interaction effects

Linear models

- multivariate models
- two way analysis of variance
- multiple regression
- analysis of covariance (ANCOVA)
- interaction terms
- factorial analysis of variance

Linear models

- Response:
- continuous

- Explanatory (fixed)
- Covariates (continuous)
- Factors (categorical)

Learning aim

- Running linear models with more than one explanatory variable
- Interactions
- Interpretation

Do it now – HO 15!

- Run a model where you test whether the relationship between bill length (response) and tarsus differs between sexes in sparrows

- Interpret the results
- Write down model results on whiteboard
- Sketch model, including regression line(s) on whiteboard