2: The Hilbert Matrix

(a) For each n = 2, 3, . . . , 20, solve the system H~x = ~b, where ~b = 0.1 n/3 . For each n, using solve_lu, solve_qr_house, and solve_qr_givens to obtain the solution, your program should output the solution ~xsol, and the error kLU − Hk∞, kQR − Hk∞, and kH~xsol −~bk∞. The output should be easily readable on the screen or a text file.

The output can be viewed from the command line by starting problem 1. To view, first, all of the classes must be compiled using:

```
javac Main.java
javac Problems.java
javac Matrix.java
javac IO.java
javac SolveUtil.java
javac FactorUtil.java
```

and then Main.java must be run using

```
java Main
```

Upon running Main.java from the command line, the terminal will read:

```
Hello and welcome to our Calc 3 Project
Authors: Anush Mattapalli, Matthew Cuevas, Prerak Upadhyaya

Please input:
1: Problem 1
2: Problem 2
3: Problem 3
```

Enter 1 and hit enter on your keyboard to printout the output of the solutions for each of the 20 iterations of the Hilbert matrix and their errors for LU Factorization, Givens QR, and Householders QR factorizations.

(b) Summarize your findings by plotting the errors obtained as a function of n, for each of the methods. The plot can be done using your own code, Excel, or any graphing program. The plots should be included in the written component.

Givens Error of Hilberts Matrices
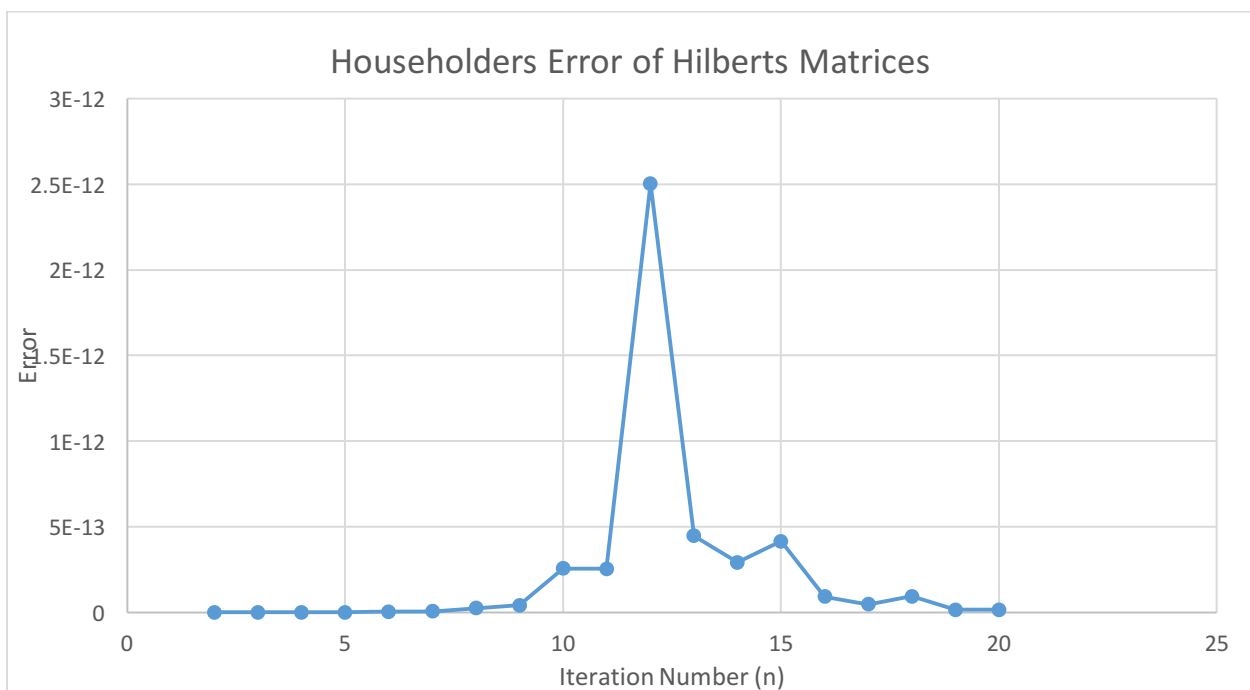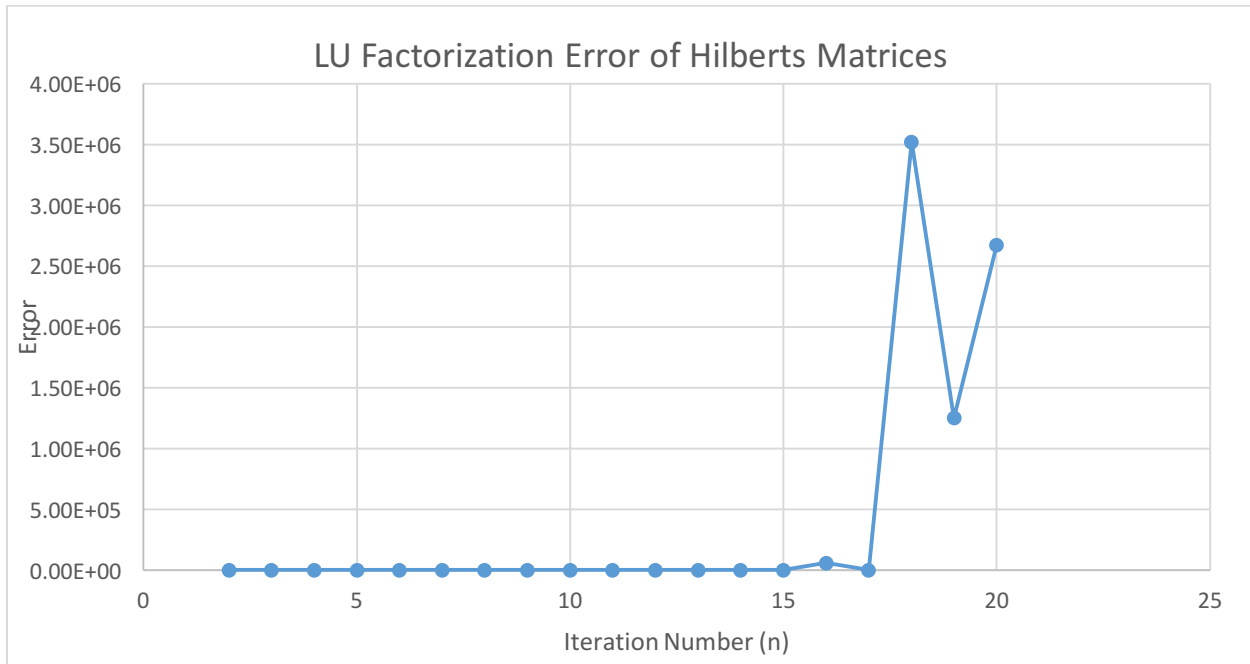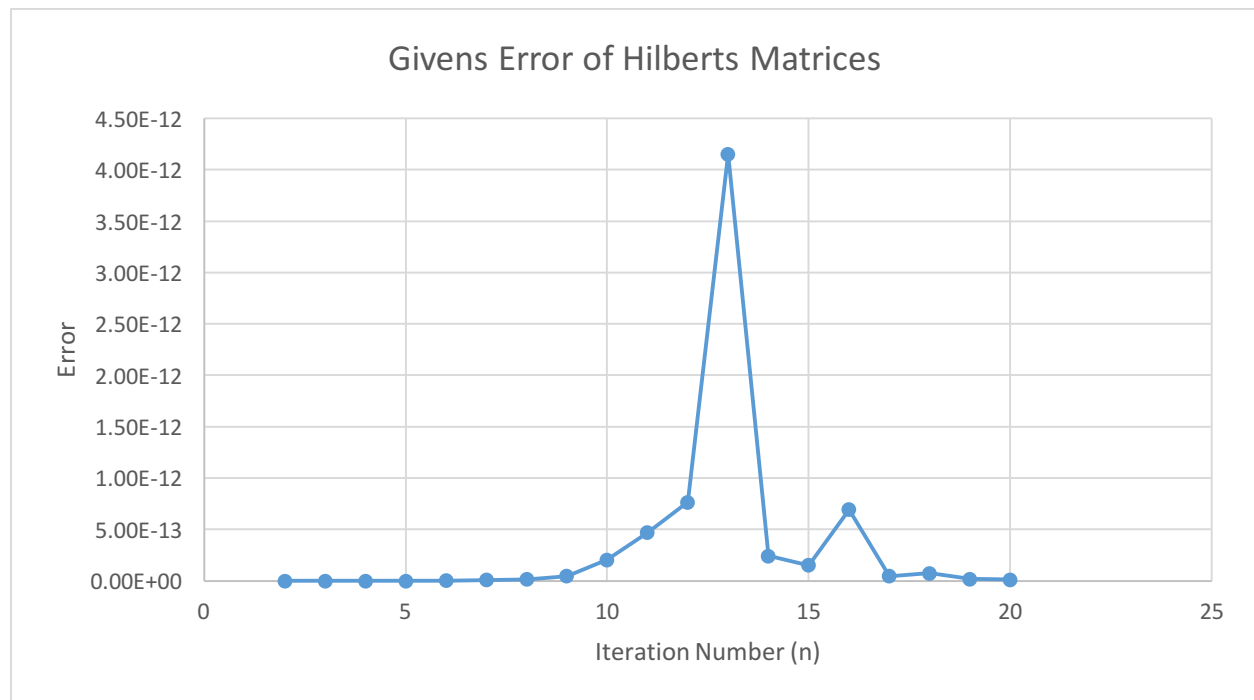
(c) Answer the following questions in the associated written component for this part of the project:

(i) Why is it justified to use the LU or QR−factorizations as opposed of calculating an inverse matrix?

Calculating the solution, x, to a matrix, A, via either LU or QR factorization Is preferable to computing an inverse matrix for A to find the solution because finding the typically takes a significantly amount of computational value and is very time-expensive. Using the QR method, given the matrices Q and R, solving for x is computationally quick in comparison. Additionally, calculating the inverse matrix of a matrix A can result in a error that is larger in matrices with larger condition numbers.

Additionally, not all matrices have exact solutions within them. And, as the charts above show, the LU factorization, the Givens QR factorization, and the Householders QR factorization have relatively small errors.

(ii) What is the benefit of using LU or QR−factorizations in this way? (Your answer should consider the benefit in terms of conditioning error.)

LU or QR factorization methods of solving matrices for their solution result in a lower computationally intensive task. Instead of solving for n number of systems of equations, both LU and QR methods create triangular matrices which can be more easily solved via the methods of substitution. Additionally, as can be seen in the charts above, the conditioning error is not too large or significant except in a couple of instances. It is important to note, however, that the Hilbert matrices generally have very high condition numbers, and as a result, any method will not be as stable as it could be with a lower conditioned matrix.