

# Problem 2.

## Part I.

### Jacobi

Approximate Solution:

0.9750  
-0.3250  
0.3000

Error: 2.256E-6

### Gauss-Seidel

Approximate Solution:

0.9750  
-0.3250  
0.3000

Error: 1.011E-6

On average, Jacobi Iteration took 2.44 times as many iterations as Gauss-Seidel to converge on an answer with a tolerance of 0.00005.



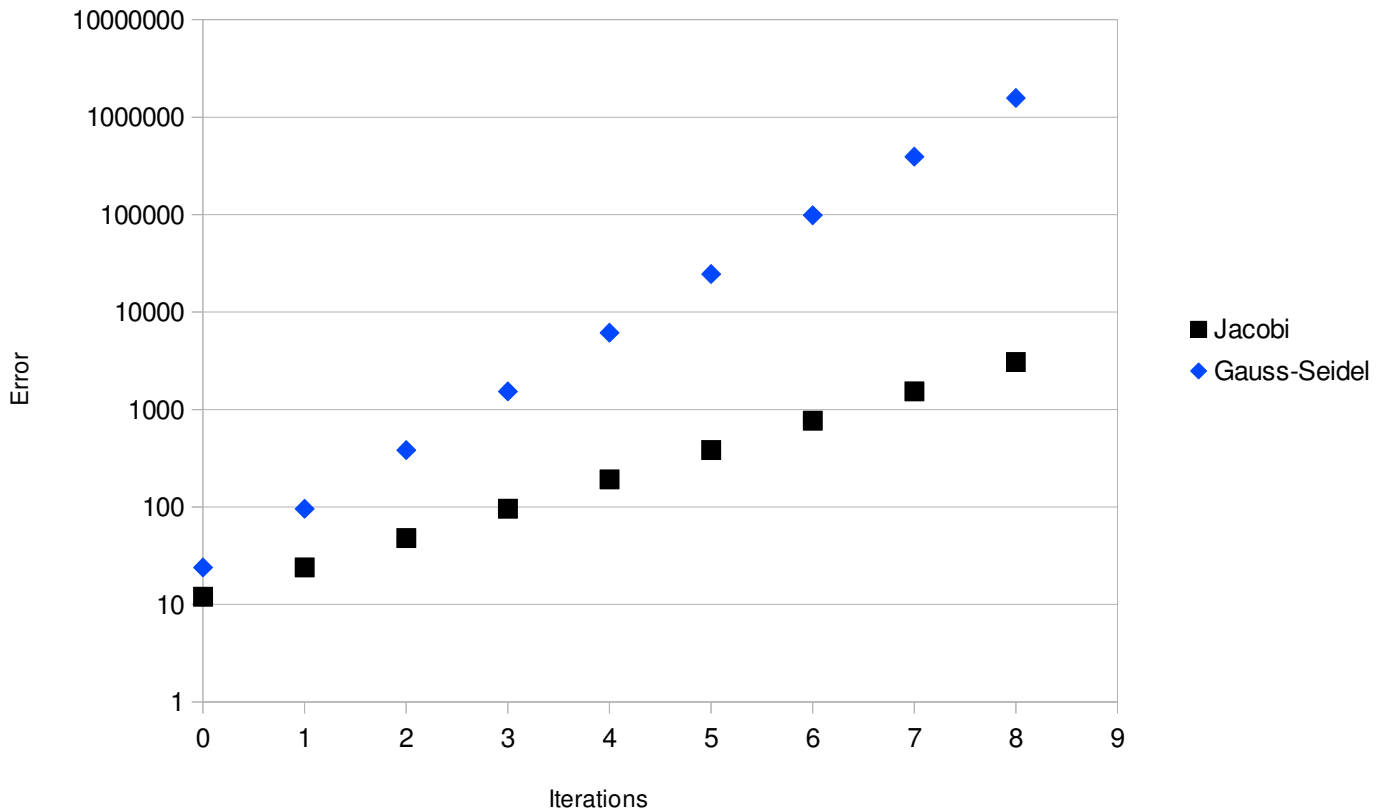
## Discussion

According to the data, the initial error did not seem to have very much effect on the number of iterations for either iterative method. There was no discernible positive or negative correlation between initial error and iterations. The method of iteration, however, had an effect on the number of iterations required. On average, Jacobi Iteration required 2.44 times as many iterations as Gauss-Seidel. This is due to the maximum eigenvalue of the matrix  $S^{-1}T$  also known as the Spectral Radius.

For Jacobi, the Spectral Radius was 0.419 whereas for Gauss-Seidel, the Spectral Radius was 0.111. The lower the Spectral Radius, the more the error is reduced with each successive iteration. Because matrix used in Gauss-Seidel has a lower Spectral Radius, error was decreased more with each iteration which is why it converged about twice as fast as Jacobi Iteration.

## Part II.

Iterations vs. Error



## Discussion

With this matrix, neither method converged, causing the error to increase exponentially with each successive iteration. This can be explained by the Spectral Radius of the  $S^{-1}T$  matrices. For the Jacobi, the Spectral Radius was 2 whereas for Gauss-Seidel, it was 4. This explains why the error was amplified so much faster in the Gauss-Seidel method than in the Jacobi method. Every iteration amplified the error by 4 instead of 2. Both of these Spectral Radii are greater than 1 which is why neither method converged on an answer.