

# Crypto Background

Blockchains and Cryptocurrencies (Fall 2020)

# Course logistics

- This coming Monday (Sept. 7) is a holiday  
**NO CLASS** (I've updated the syllabus)
- Assignment 1 comes out today:  
It will be posted on the Github page, and Gijs  
(the TA) will make a Piazza announcement
- Due 9/24
- If you want to add the course, ask at EOC

# News?

Story from **Markets** →

# Ethereum Classic Hit by Third 51% Attack in a Month

Aug 29, 2020 at 23:00 UTC ▪ Updated Aug 31, 2020 at 16:59 UTC

⋮



Zack Voell  
   

The Ethereum Classic blockchain suffered a 51% attack Saturday evening, its third such [attack](#) this month, noticed by mining company Bitfly, which also spotted the first attack on



- The attack reorganized over 7,000 blocks, or two days' worth of mining, according to a [tweet](#) shared by Bitfly. The first two attacks reorganized 3,693 and 4,000 blocks respectively.
- Notably, a leading organization behind the Ethereum Classic network, ETC Labs, announced its strategy to protect the network from additional attacks last week, including defensive mining that is intended to stabilize the network's plummeting hashrate and resist future 51% attacks.
- Stevan Lohja, technology coordinator at ETC Labs, in a private message with CoinDesk, said he finds the timing of the attack "very suspicious" as it came just a day after a meeting of Ethereum Classic core developers regarding "aggressive innovation" in the blockchain's proof of work.
- ETC Cooperative, another prominent foundation supporting the network's development, [took to Twitter](#) following Saturday's attack saying, "We are aware of today's attack and are working with others to test and evaluate proposed solutions as quickly as possible."
- After the first two attacks, exchange OKEx responded by [saying](#) it will consider delisting the asset due to the network's severe lack of security. Coinbase also took drastic [measures](#) by extending deposit and withdrawal confirmation times for [ETC](#) to roughly two weeks.
- Following the latest attack, leading cryptocurrency derivatives exchange FTX will reconsider its ETC perpetual futures contracts, according to CEO Sam Bankman-Fried in a private message to CoinDesk. He said this is so even though FTX doesn't support ~~spot trading and the cryptocurrency network's insecurity has less of a direct effect on~~

# This lecture

Unfinished business from last time

Crypto background

hash functions

random oracle model

digital signatures

... and applications

# What we'll also talk about

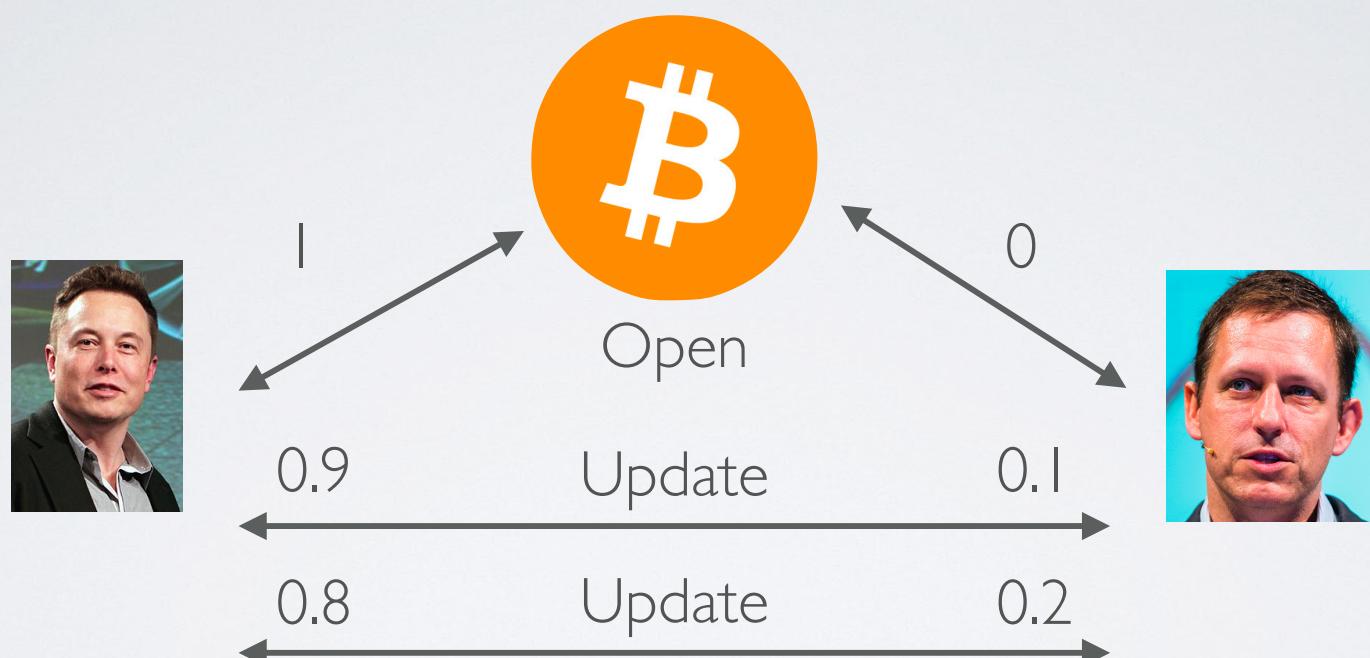
- Scaling, including payment channels
- Replacing PoW
- Other advanced applications (not related to currency)
- DeFi

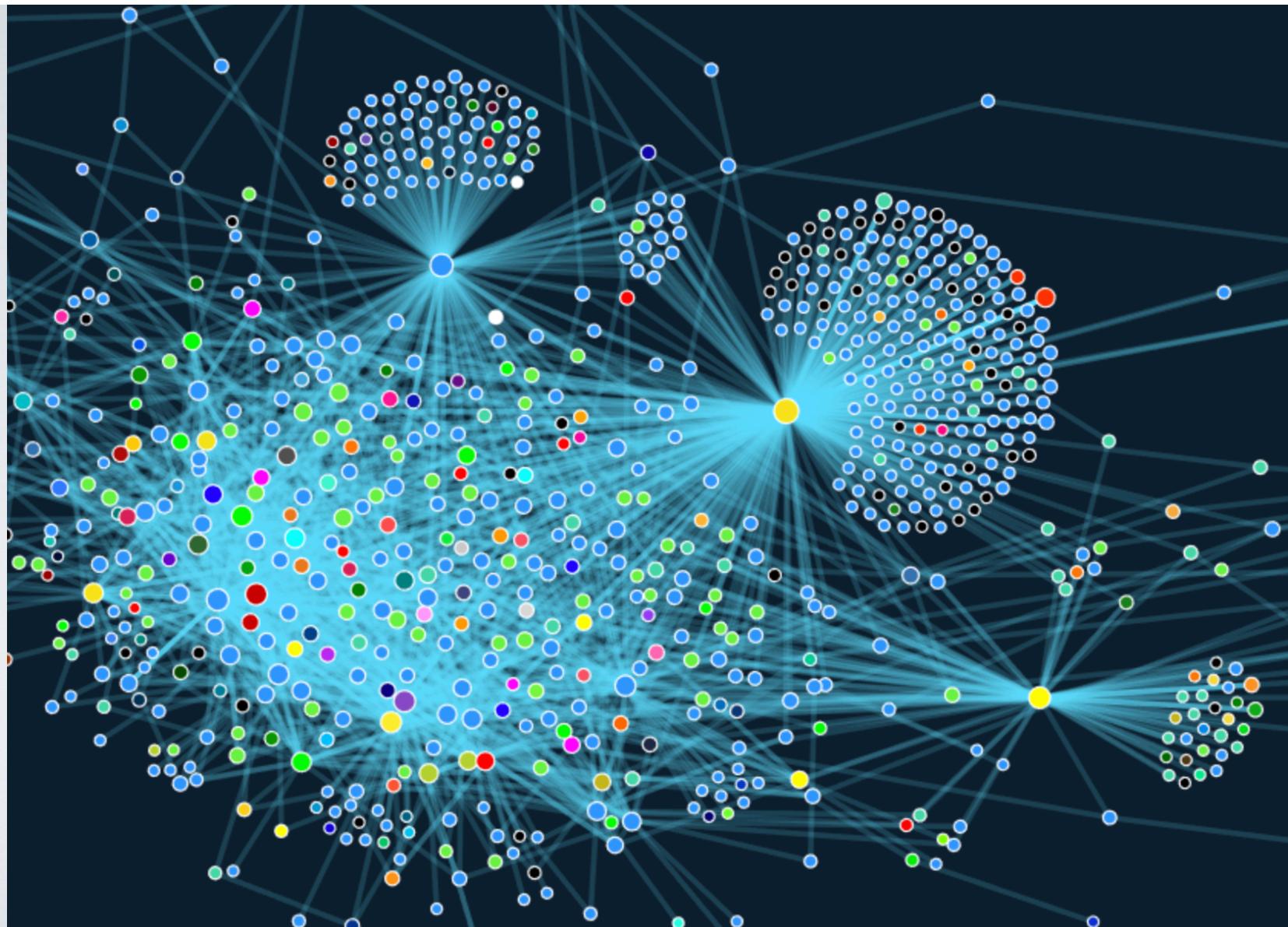


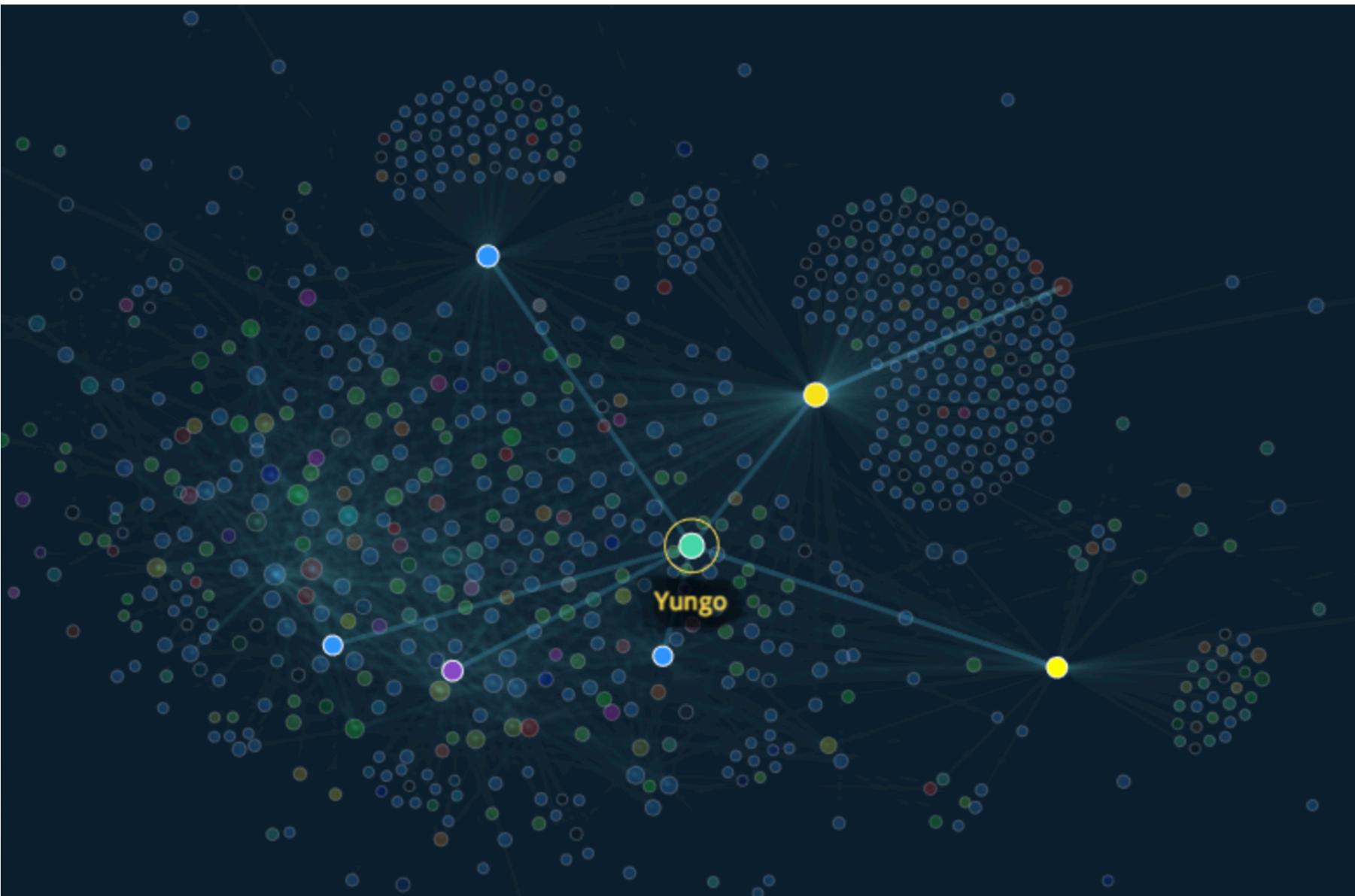
# Scaling

- Current Bitcoin/Ethereum transaction rate is ~7TX/s
- Compare with Visa at 10,000-40,000+ TX.s globally
- This gets worse as transaction complexity increases
- Problems are storage/throughput/validation bandwidth

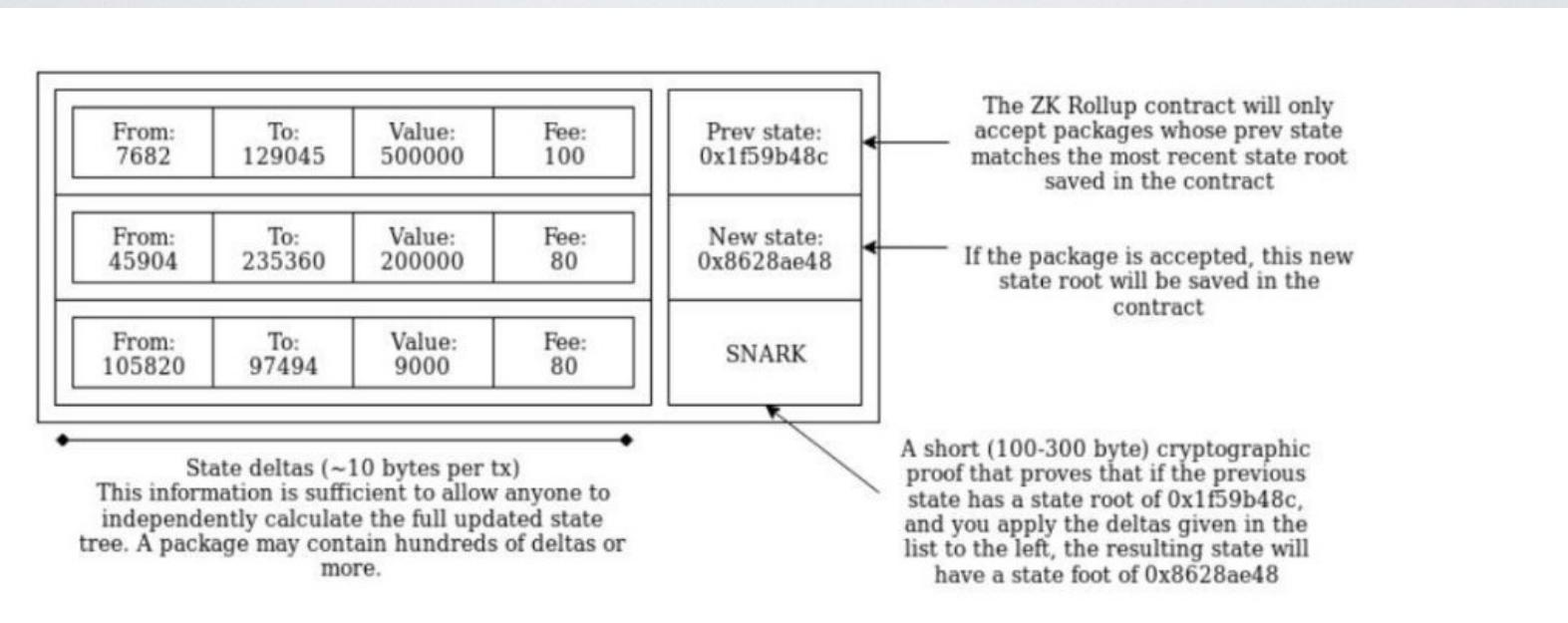
## L2 (Channels)



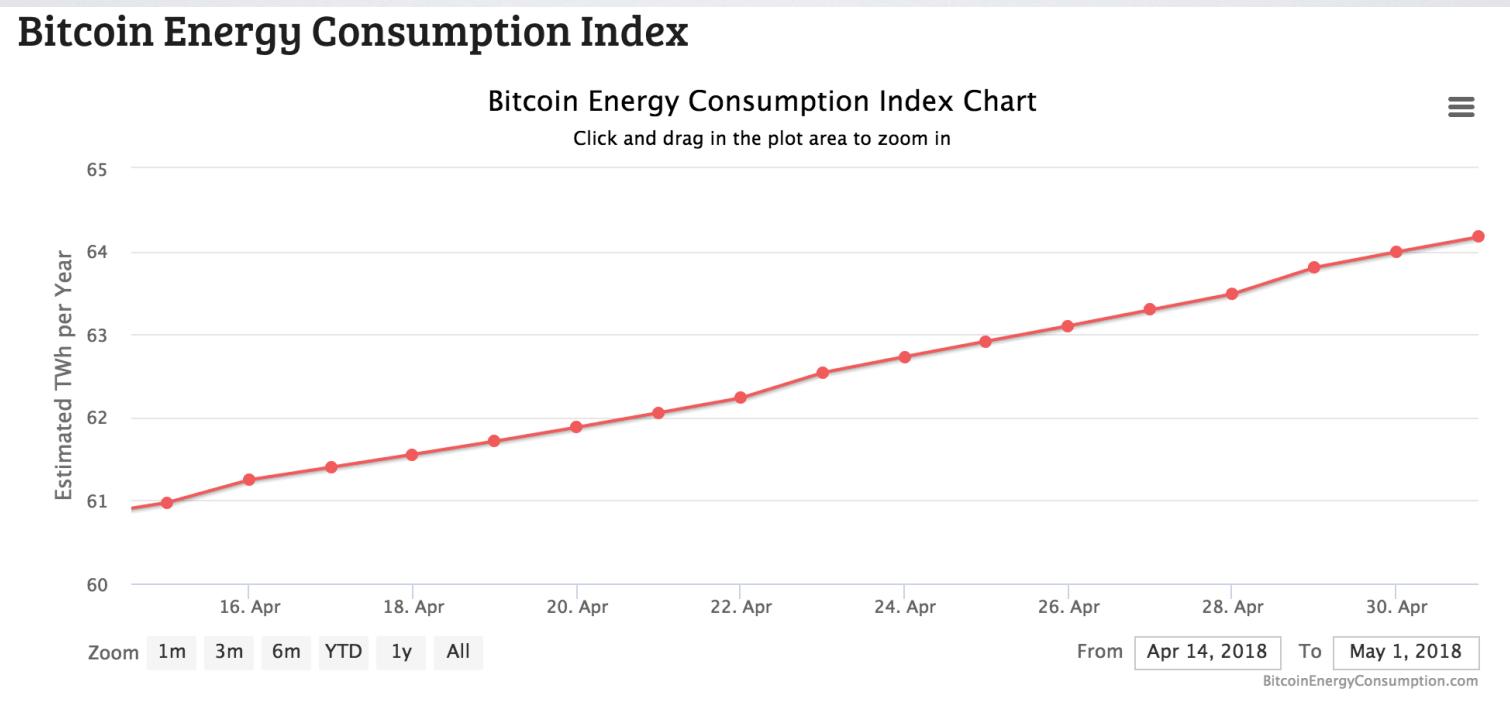




# ZK and Optimistic Rollup



# Replacing PoW



# Proof of Stake

- Current PoW design is obviously unsustainable
- Most common solution (in permissionless) chains is “Proof of Stake”
- Rough summary: enumerate all stakeholders of the coin, scaled by their stake — and then sample one to construct the next block

DeFi



# Moving forward

- Weds: Abhishek gives crypto background
- Next week: consensus networks, Bitcoin
- AI out Weds afternoon
- Do the reading!

# Cryptographic Hash Functions

# Hash function

- takes a string of arbitrary length as input
- fixed-size output (i.e., hash function “compresses” the input)
- efficiently computable

## Security properties:

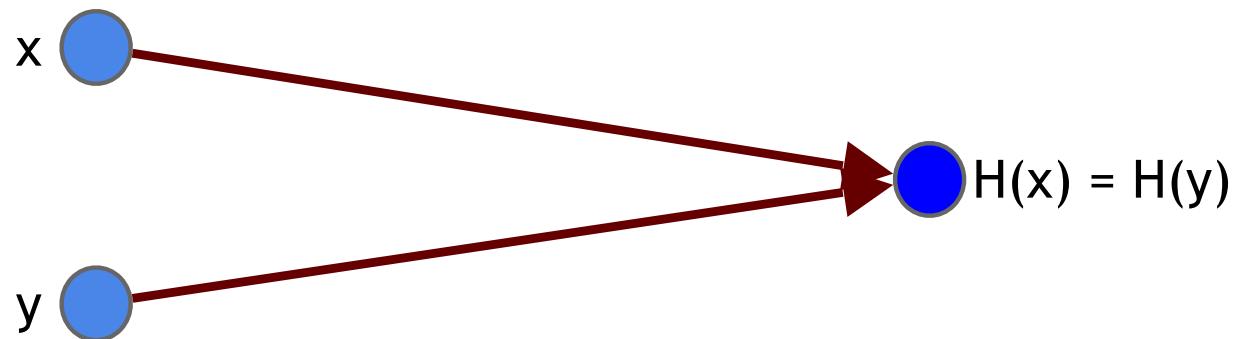
- Collision resistance
- Preimage resistance (one-way)

# **Property 1: Collision resistance**

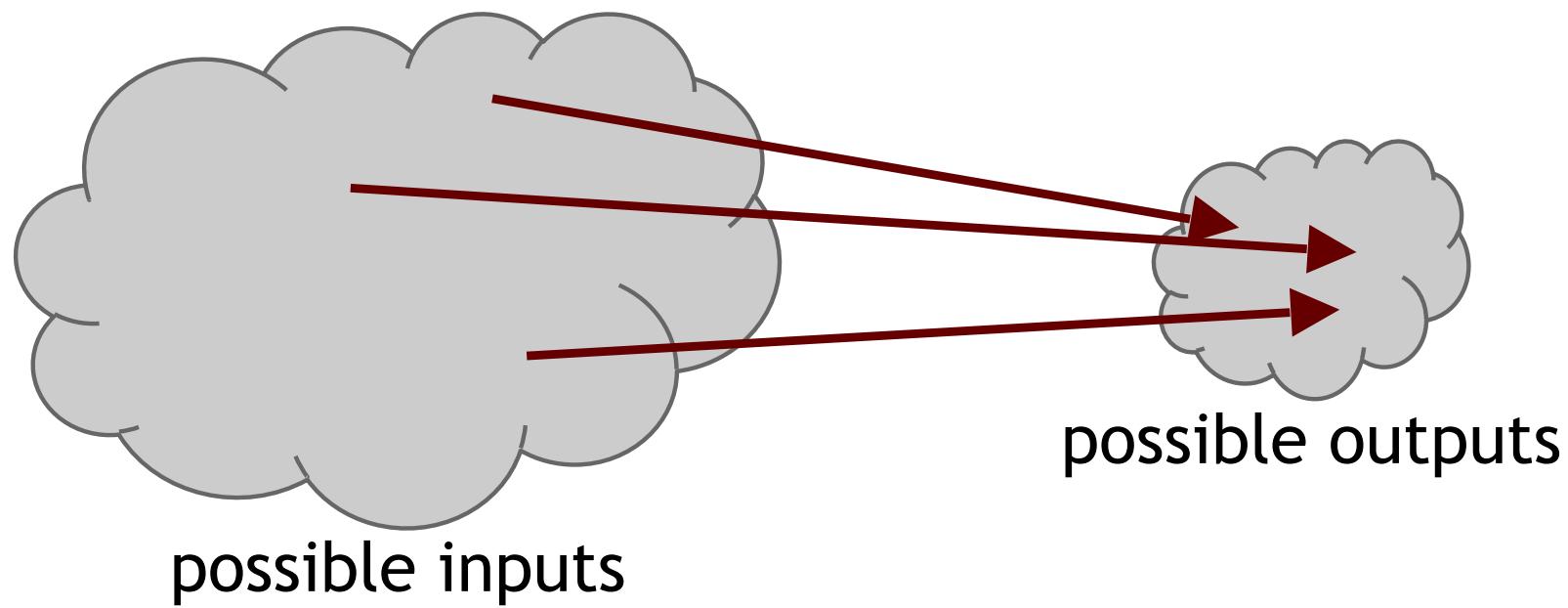
**What's a collision?**

# Property 1: Collision resistance

Do collisions exist in common hash functions?



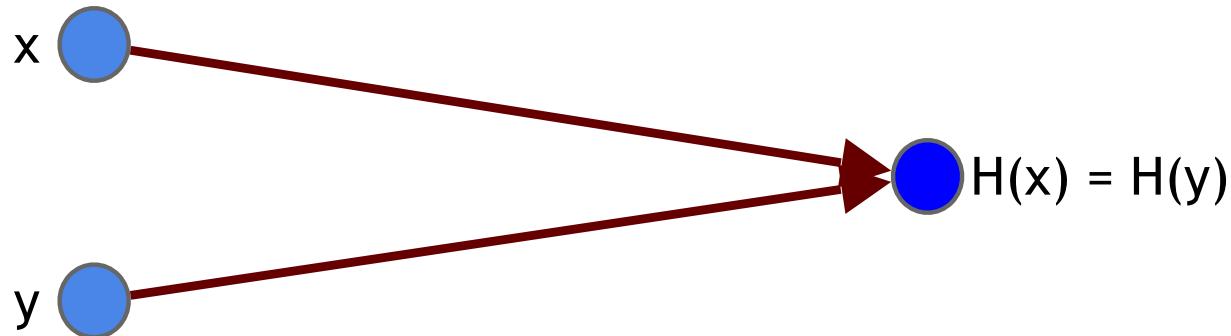
# Collisions do exist ...



... but can a real-world adversary find them?

# Property 1: Collision resistance

No efficient adversary can find  $x$  and  $y$  such that  $x \neq y$  and  $H(x)=H(y)$



## How to find a collision (for 256 bit output)

- try  $2^{130}$  randomly chosen inputs
- 99.8% chance that two of them will collide

This works no matter what H is, but it takes too long to matter

- If a computer calculates 10,000 hashes/sec, it would take  $10^{27}$  years to compute  $2^{128}$  hashes

## How to find a collision (for 256 bit output)

- try  $2^{130}$  randomly chosen inputs
- 99.8% chance that two of them will collide

This work takes  
too long

**Q: How many hashes/sec does the Bitcoin network compute?**

- If a computer calculates 10,000 hashes/sec, it would take  $10^{27}$  years to compute  $2^{128}$  hashes

Is there a faster way to find collisions?

- For some possible  $H$ 's, yes.
- For others (like SHA-256), we don't know of one.

Provably secure collision-resistant hash functions can be constructed based on “hard” number-theoretic problems.

# Defining Collision Resistance

- Real-world adversaries
  - In practice, everyone has bounded resources
  - Therefore, reasonable to model a real-world adversary as such an entity
  - However, we do not make any assumptions about the adversarial strategy. He can use its (bounded) resources in any possible way

Cryptographic adversary: A probabilistic polynomial-time (PPT) algorithm

# Defining Collision Resistance...

- Collision Resistance (informal): A hash function  $H$  is collision-resistant if for all PPT adversaries  $A$ ,

$\Pr[A \text{ outputs } x, y \text{ s.t. } x \neq y \text{ and } H(x) = H(y)]$   
= “very small”

# Defining Collision Resistance...

- Collision Resistance (informal): A hash function  $H$  is collision-resistant if for all PPT adversaries  $A$ ,  
$$\Pr[A \text{ outputs } x, y \text{ s.t. } x \neq y \text{ and } H(x) = H(y)]$$
$$= \text{“very small”}$$
- “Very small” captured via a function that tends to 0.  
Formal definition: Modern Cryptography

# Application: Hash as message digest

If we know  $H(x) = H(y)$ , and  $H$  is collision resistant  
it's safe to assume that  $x = y$ .

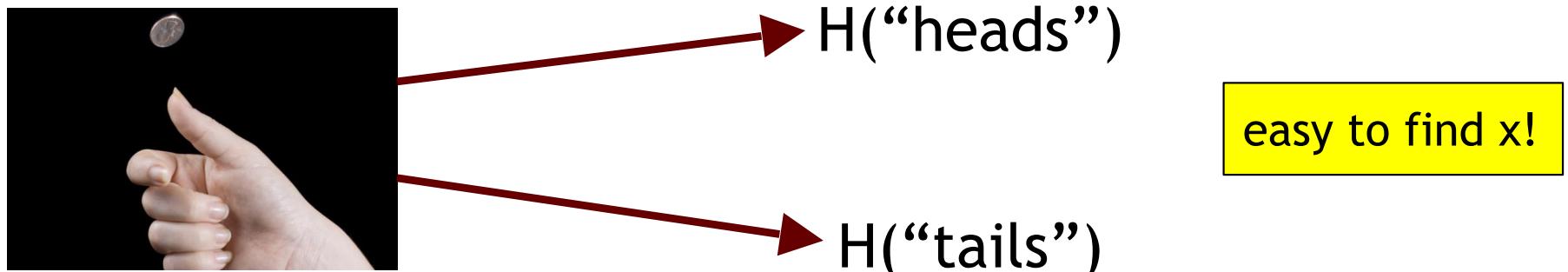
To recognize a file that we saw before,  
just remember its hash.

Useful because the hash is small.

# Property 2: Pre-image Resistance

Intuition: Given  $H(x)$ , no efficient adversary can find  $x$ , except with very small probability

Problem: What if input space of  $x$  is very small, or some inputs are much more likely than others?



## Property 2: Preimage Resistance

This definition is useless in this setting. How can we specify a meaningful version of the definition?

Intuition: Given  $H(x)$ , an efficient adversary can find  $x$ , except with very small probability

Problem: What if input space of  $x$  is very small, or some inputs are much more likely than others?



$H(\text{"heads"})$

$H(\text{"tails"})$

easy to find  $x$ !

# Defining Preimage Resistance

- **Preimage Resistance**: A hash function  $H$  is preimage-resistant if for all PPT adversaries  $A$ ,

$$\Pr[x \leftarrow \{0,1\}^k, A(H(x)) \text{ outputs } x' \text{ s.t. } H(x') = H(x)] = \text{small}$$

x is drawn from uniform distribution over  $\{0,1\}^k$  for some sufficiently large k

# Preimage Resistance (contd.)

- If  $x$  is drawn from the uniform distribution, then inverting  $H(x)$  is hard
- But what if  $x$  is drawn from low-entropy distribution?
- Can append a random string  $r$  to  $x$  and then compute  $H(r \mid x)$  to prevent enumeration attacks

**Theorem:** Collision resistance implies preimage resistance if the hash function is sufficiently compressing

# **Application: Commitment**

Want to “seal a value in an envelope”, and  
“open the envelope” later.

Commit to a value, reveal it later.

# Commitment Schemes

$(com, key) := \text{commit}(msg)$

$\text{match} := \text{verify}(com, key, msg)$

To seal  $msg$  in envelope:

$(com, key) := \text{commit}(msg)$  -- then publish  $com$

To open envelope:

publish  $key, msg$

anyone can use  $\text{verify}()$  to check validity

# Commitment Schemes

$(com, key) \leftarrow \text{commit}(msg)$

$match \leftarrow \text{verify}(com, key, msg)$

Security properties:

- Hiding: Given  $com$ , no PPT adversary can find\*  $msg$
- Binding: No PPT adversary can find\*  $msg \neq msg'$  such that  $\text{verify}(\text{commit}(msg), msg') == \text{true}$

\* Except with very small probability

# Commitment Schemes

$\text{commit}(\text{msg}) \rightarrow (\text{H}(\text{key} \mid \text{msg}), \text{key})$

where  $\text{key}$  is a random 256-bit value

$\text{verify}(\text{com}, \text{key}, \text{msg}) \rightarrow (\text{H}(\text{key} \mid \text{msg}) == \text{com})$

Security properties:

- Hiding: If  $\text{H}$  is a *random oracle*, given  $\text{H}(\text{key} \mid \text{msg})$ , hard to find  $\text{msg}$ .
- Binding: Collision-resistance  $\rightarrow$  Hard to find  $\text{msg} \neq \text{msg}'$  such that  $\text{H}(\text{key} \mid \text{msg}) == \text{H}(\text{key} \mid \text{msg}')$

# Random Oracle (RO)

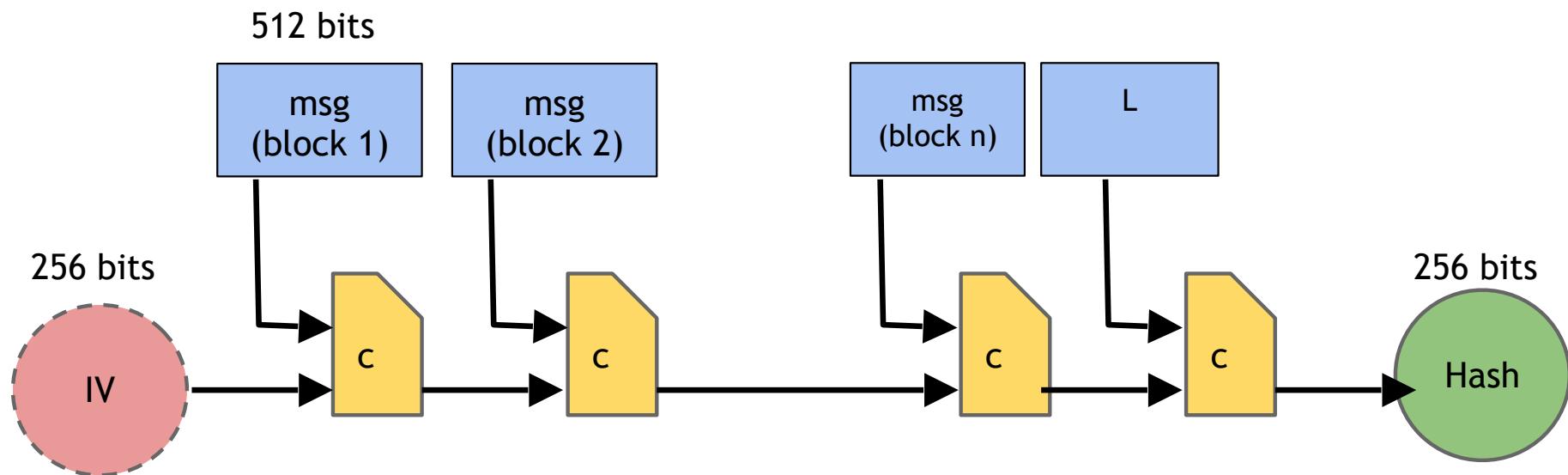
- Imagine an elf in a box with an infinite writing scroll
- Upon receiving an input  $x$ , the elf checks the scroll if there is an entry  $y$  corresponding to  $x$ . If yes, it returns  $y$ .
- Otherwise, elf chooses a random value  $y$  (from the output space) and returns it. It adds an entry  $(x,y)$  to the scroll.

# Random Oracle (RO)

- In practice-oriented provable security, hash functions are often modeled as a random oracle
- Each party (including adversary) is given black-box access to the random oracle. They can query the random oracle any polynomial number of times
- By definition, the answers of random oracle answers are unpredictable
- Random oracle captures many security properties such as one-wayness, collision-resistance .

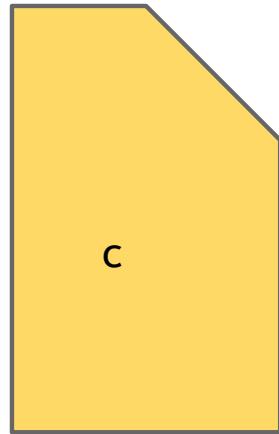
# SHA-256 hash function

Suppose msg is of length L s.t. L is a multiple of 512 (pad with 0s otherwise)



**Theorem [Merkle-Damgard]:** If  $c$  is collision-resistant, then SHA-256 is collision-resistant.

# SHA-256 hash function



Q: What the heck is inside of c?

**Theorem [Merkle-Damgard]:** If c is collision-resistant, then SHA-256 is collision-resistant.

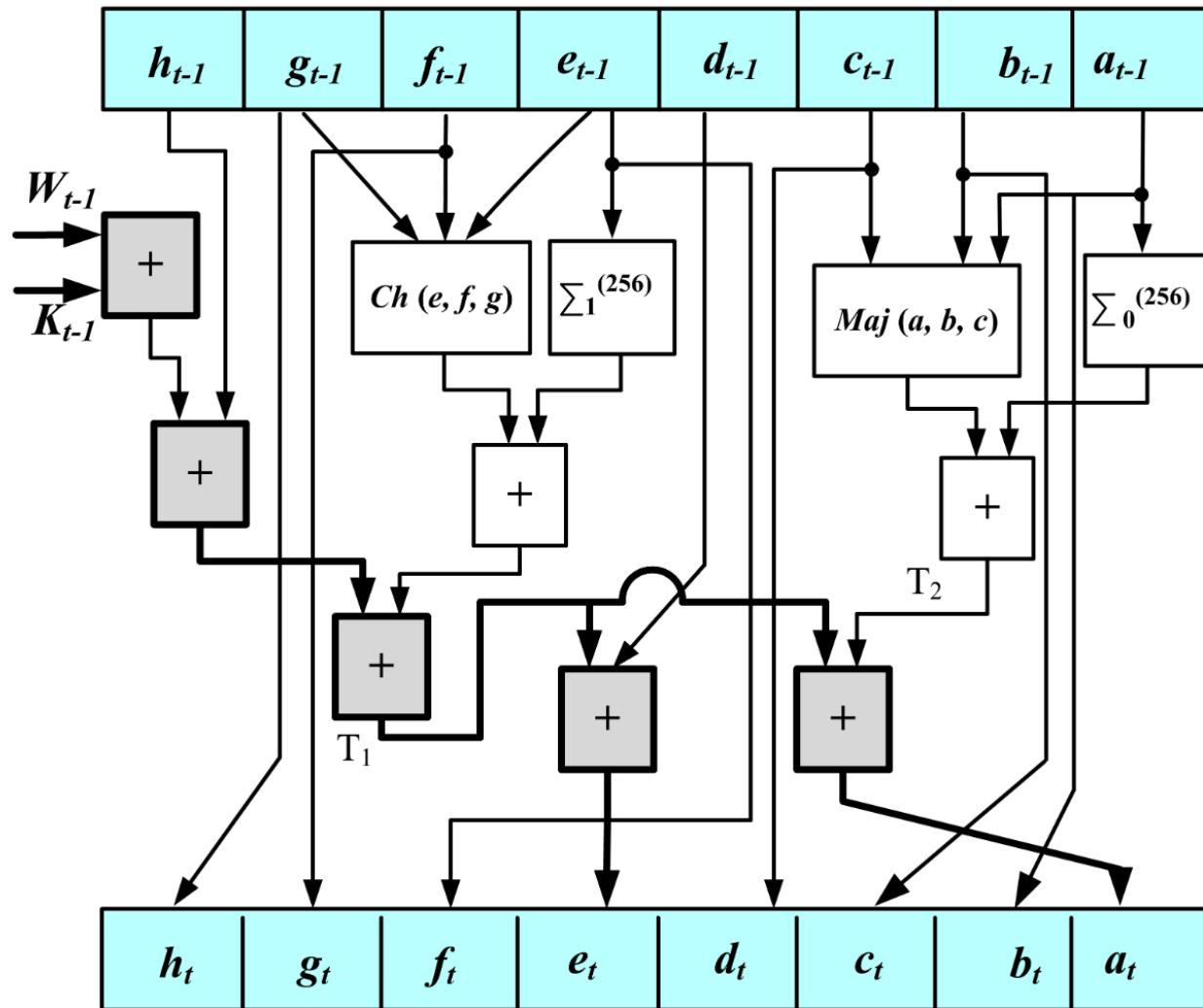


Fig. 2 SHA-256 hash function. Basic transformation round

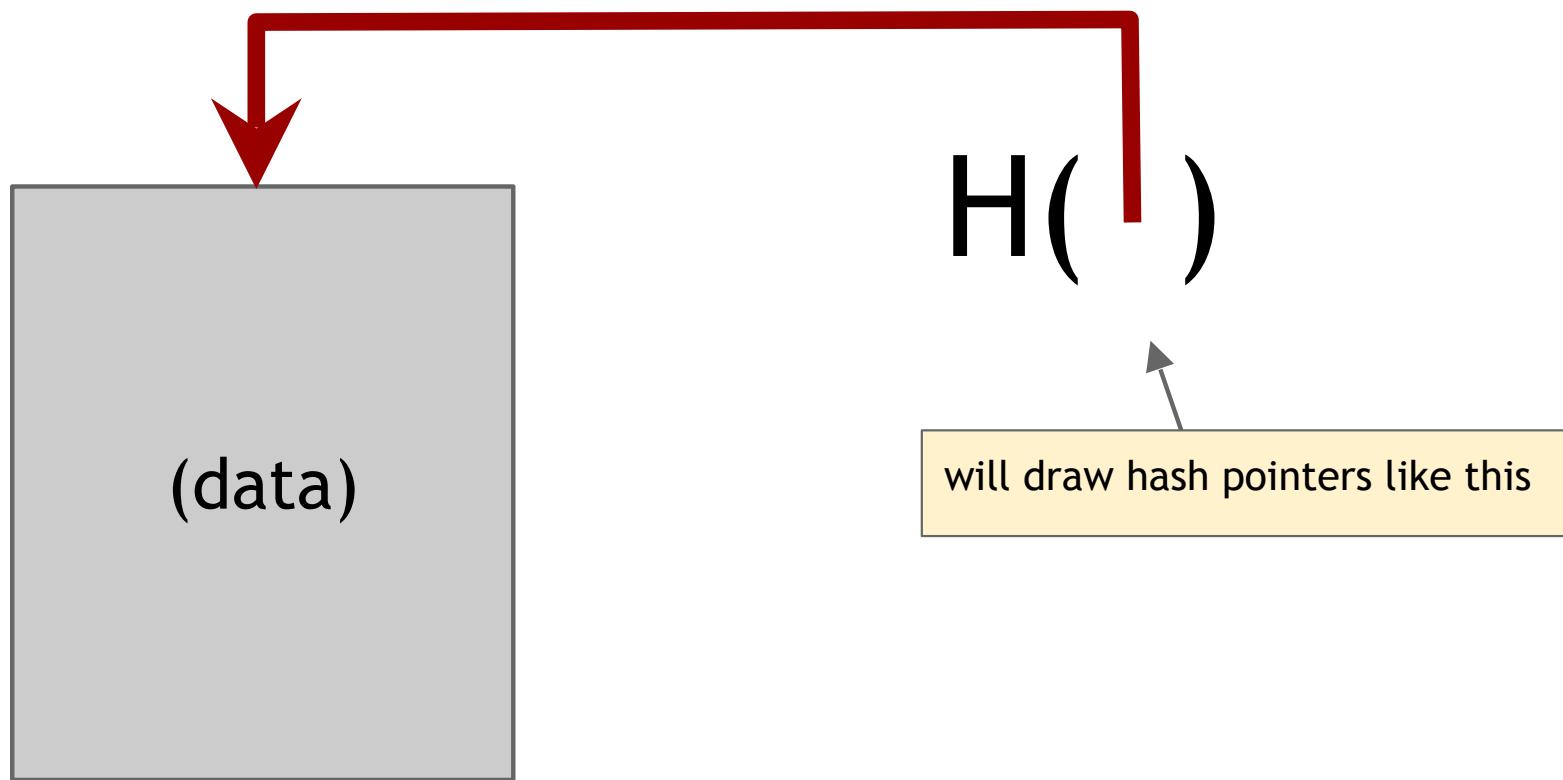
# Hash Pointers and Data Structures

## Hash pointer

- pointer to where some info is stored, *and*
- cryptographic hash of the info

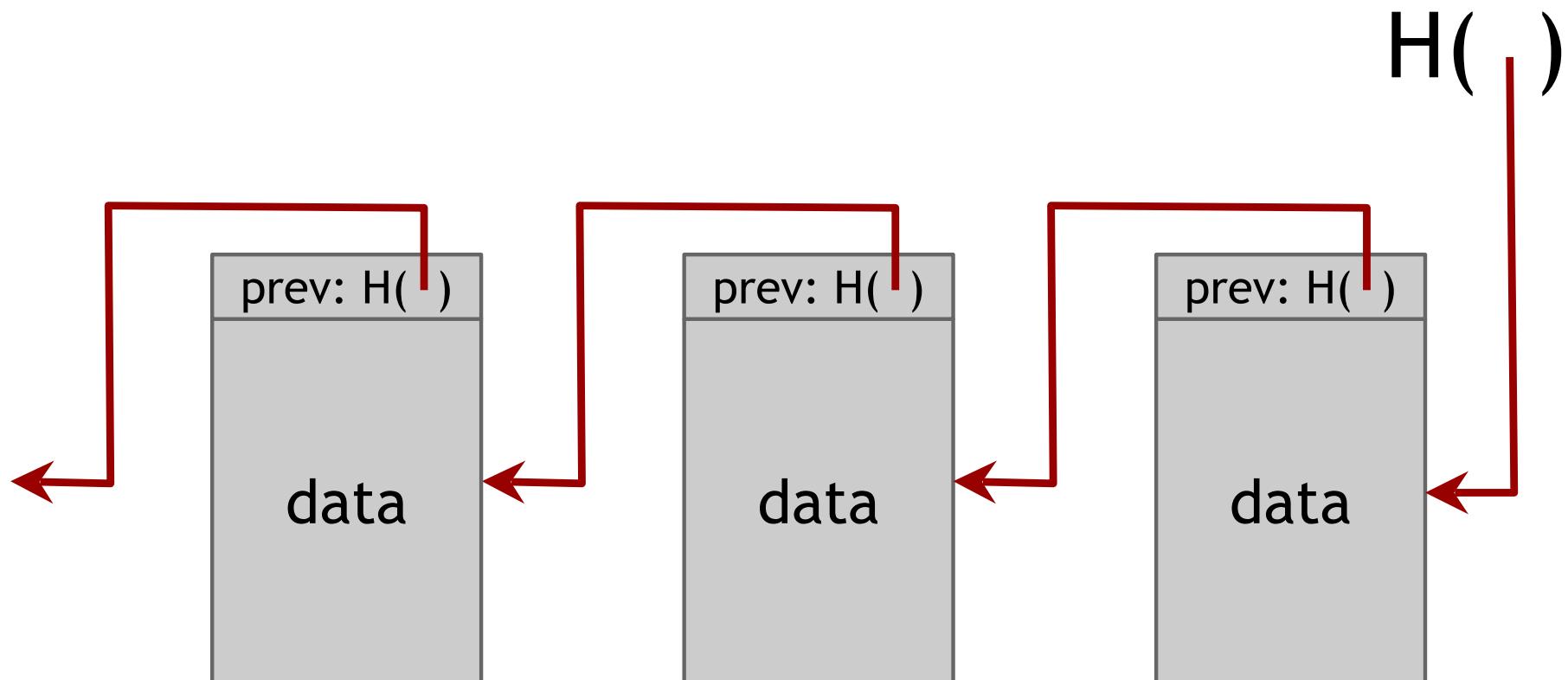
If we have a hash pointer, we can

- ask to get the info back, *and*
- verify that it hasn't changed



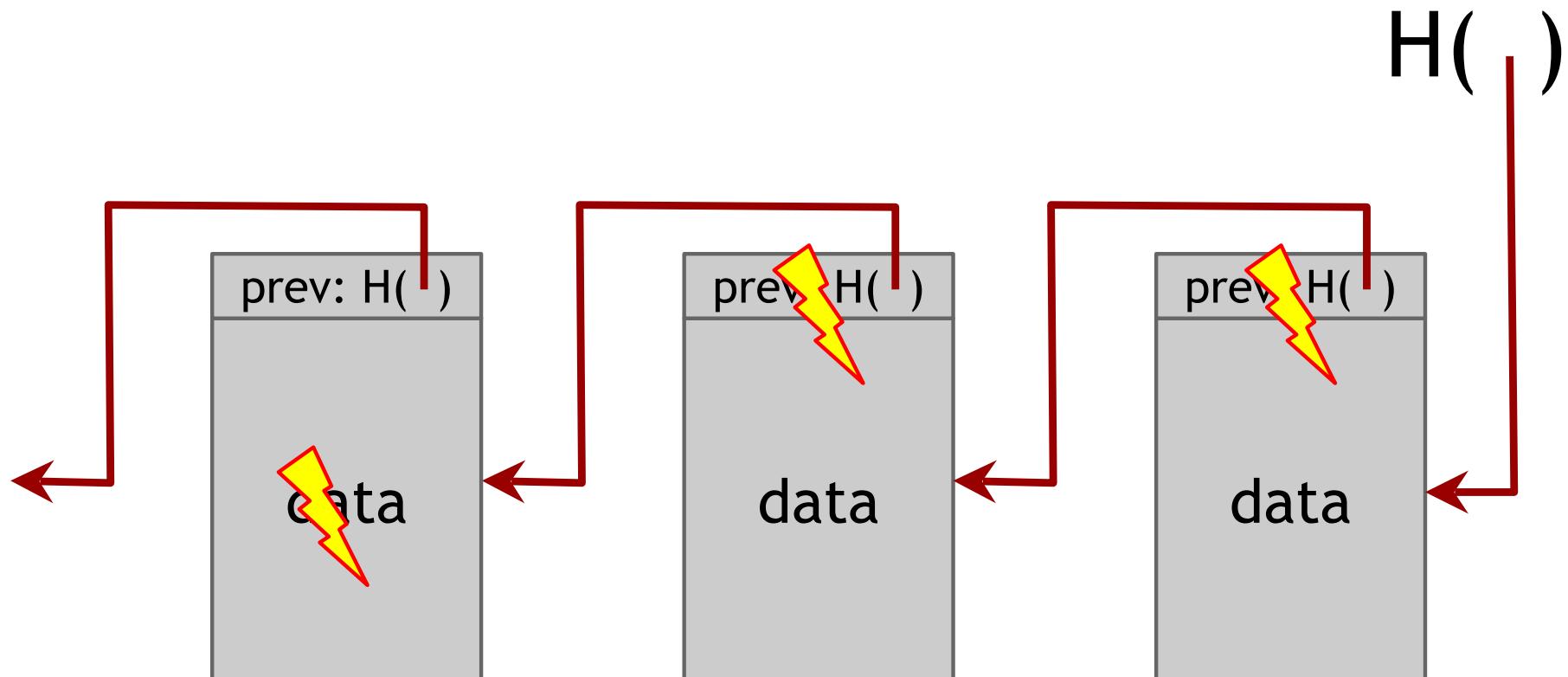
# Building data structures with hash pointers

# Linked list with hash pointers = “Blockchain”



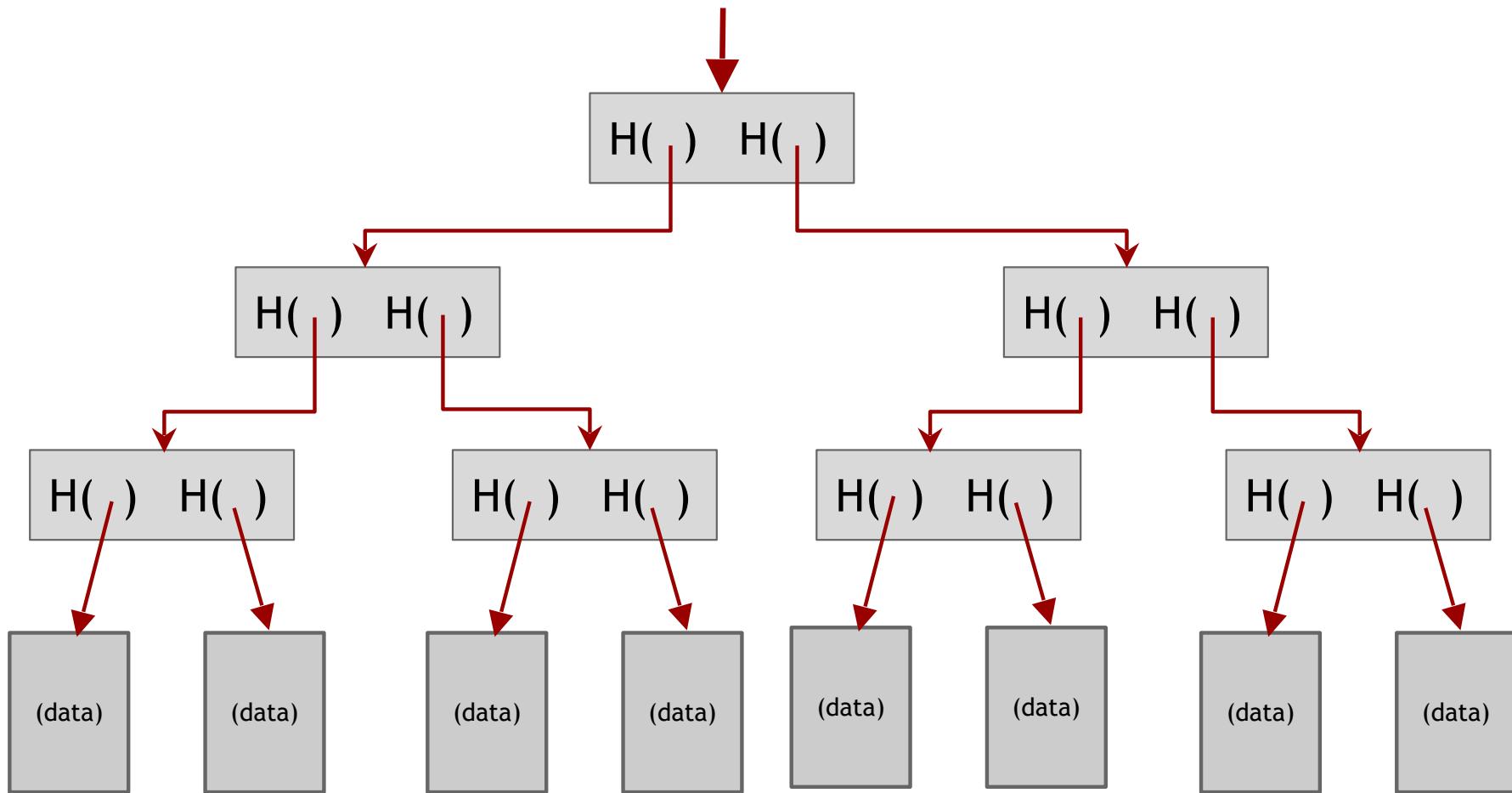
use case: tamper-evident log

# detecting tampering

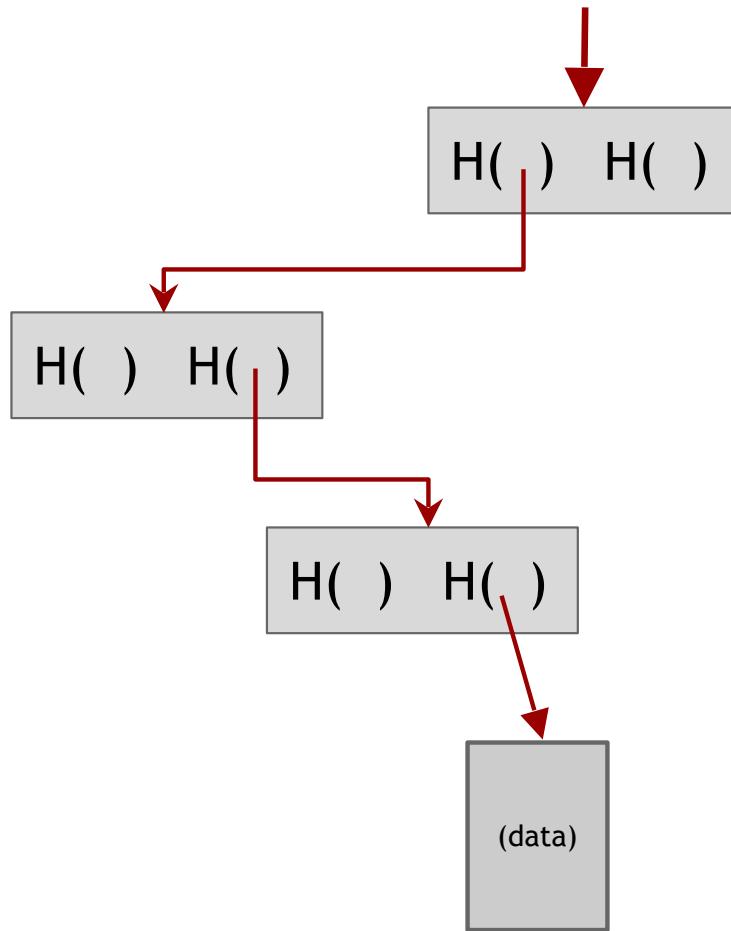


use case: tamper-evident log

# binary tree with hash pointers = “Merkle tree”



# proving membership in a Merkle tree



show  $O(\log n)$  items

# Advantages of Merkle trees

- Tree holds many items, but just need to remember the root hash
- Can verify membership in  $O(\log n)$  time/space

## Variant: sorted Merkle tree

- can verify non-membership in  $O(\log n)$
- show items before, after the missing one

## More generally ...

Can use hash pointers in any pointer-based data structure that has no cycles

# Digital Signatures

# What we want from signatures

- Only you can sign, but anyone can verify
- Signature is tied to a particular document  
*(can't be cut-and-pasted to another doc)*
- Even if one can see your signature on some documents, he cannot “forge” it

# Digital signatures

- $(sk, pk) \leftarrow \text{keygen}(r)$

sk: secret signing key

pk: public verification key

randomness



} randomized algorithm

- $\text{sig} \leftarrow \text{sign}(sk, \text{message})$

} Typically randomized

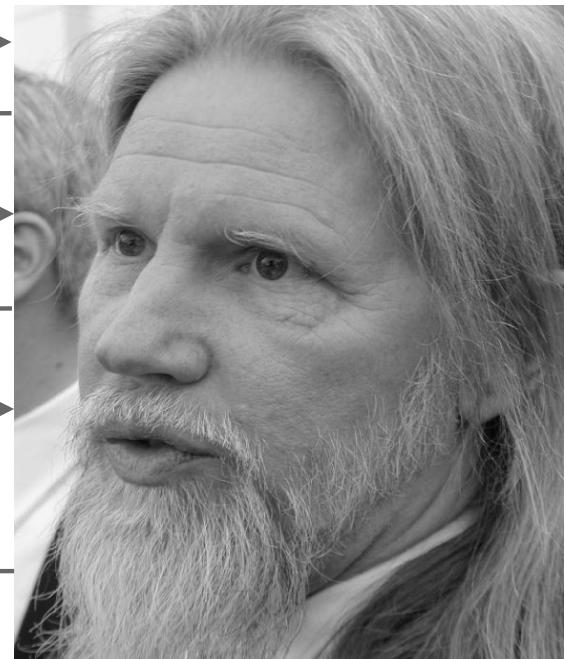
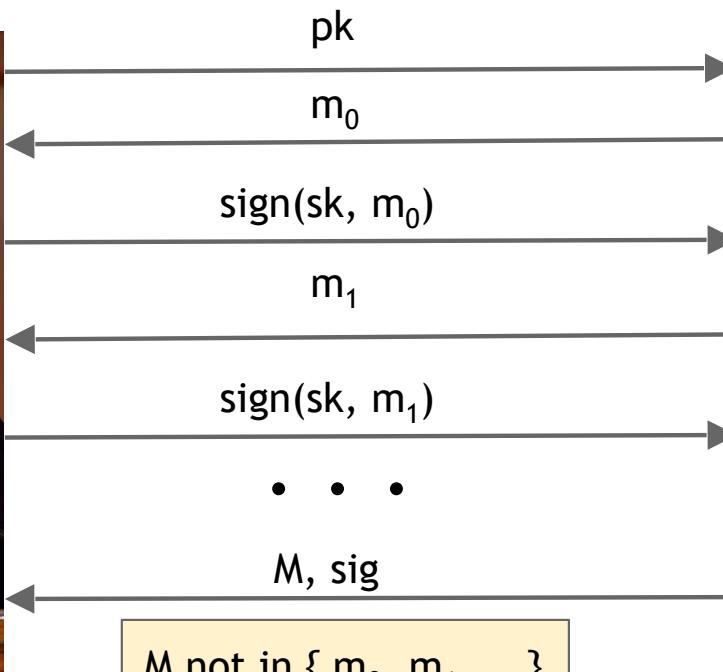
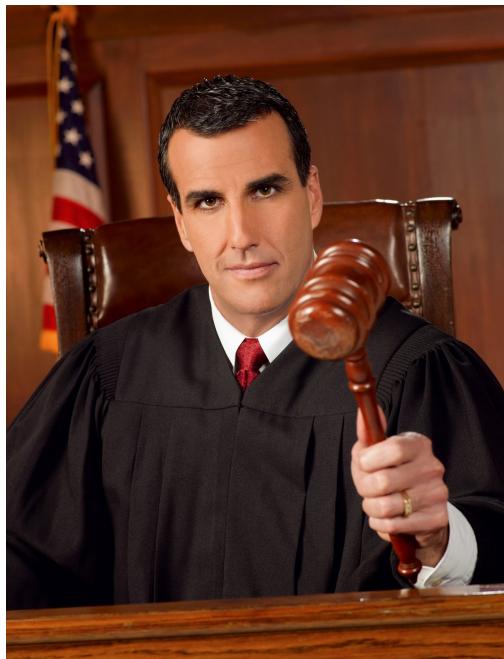
- $\text{isValid} \leftarrow \text{verify}(pk, \text{message}, \text{sig})$

# Requirements for signatures

- Correctness: “valid signatures verify”
  - $\text{verify}(\text{pk}, \text{message}, \text{sign}(\text{sk}, \text{message})) == \text{true}$
- Unforgeability under chosen-message attacks (UF-CMA): “can’t forge signatures”
  - adversary who knows  $\text{pk}$ , and gets to see signatures on messages of his choice, can’t produce a verifiable signature on another message

# UF-CMA Security

$(\text{sk}, \text{pk}) \leftarrow \text{keygen}(1^k)$



Challenger

↓  
 $\text{verify}(\text{pk}, M, \text{sig})$

ifValid, attacker wins

Adversary

**Definition:** A signature scheme  $(\text{keygen}, \text{sign}, \text{verify})$  is UF-CMA secure if for every PPT adversary  $A$ ,  $\Pr[A \text{ wins in above game}] = \text{very small}$

# Notes

- Algorithms are randomized: need good source of randomness. Bad randomness may reveal the secret key
- fun trick: sign a hash pointer. signature “covers” the whole structure
- Bitcoin uses Elliptic Curve Digital Signature Algorithm (ECDSA), a close variant of Schnorr over Elliptic curves