601.445/645
Practical Cryptographic Systems

Asymmetric Cryptography III

Instructor: Matthew Green

Housekeeping

- A2
  - Due 23rd February, 11:59pm
- Quiz moved to Weds
  - All reading material before RSA reading

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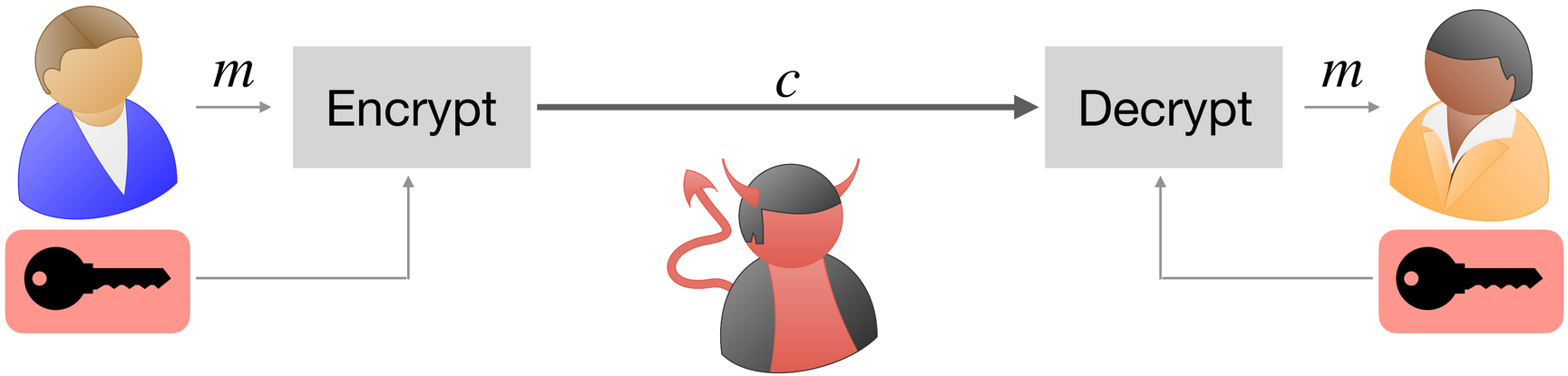
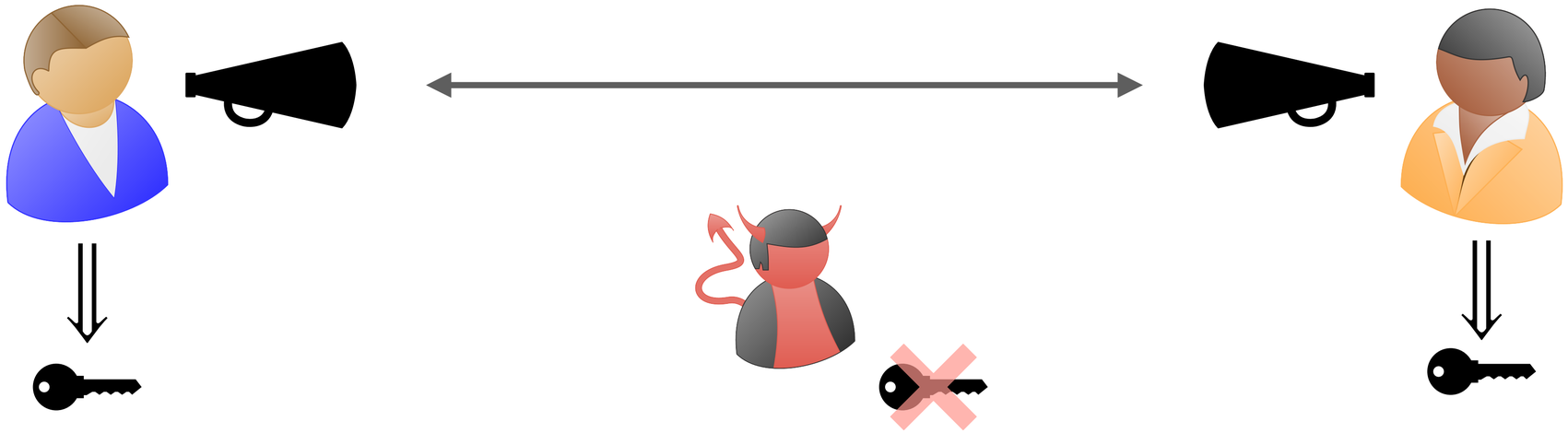
Review



How do parties agree on a common key?

Key Exchange

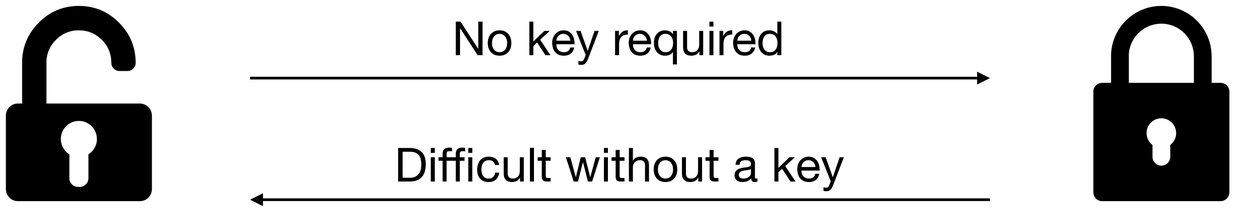


Diffie-Hellman Key Exchange

Agreeing on a common secret over an untrusted/public channel

Key Idea: Exploiting asymmetry

Often present in the real world!



Discrete Logarithm problem

- **Discrete logarithm problem**

  Given: $x \in_R 0, \ldots, p-2$

  $$\langle g \rangle = \mathbb{G} \qquad order(g) = p - 1$$

  $$h = g^x$$

  Find: x

  This problem is hard if for all p.p.t. adversaries, all attackers find x with "small" probability

See Boneh-Shoup, Appendix A

Discrete Logarithm problem

This means that “reversing” exponentiation is assumed to have super-polynomial running time.

How about the exponentiation itself?

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Note that for this to hold, the size of *p* must be pretty large!

In practice, we typically assume *p* is at least 1024 bits. And 3072 bits is the minimum in modern protocols!

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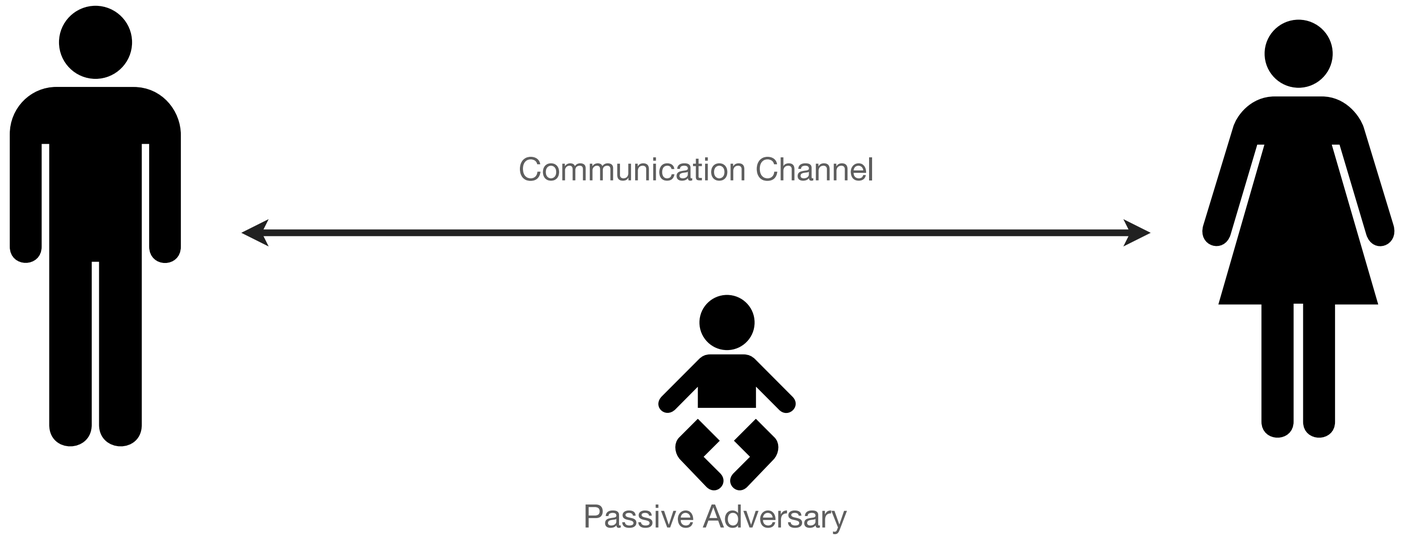
Key Agreement

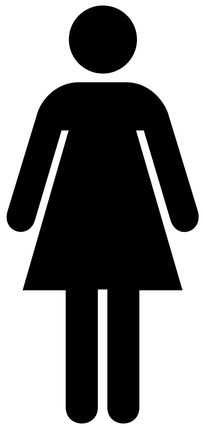
- Establish a shared key in the presence of a passive adversary

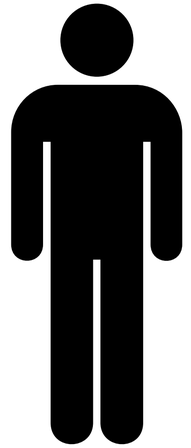


D-H Protocol

$$p, \langle g \rangle = \mathbb{Z}_p^*$$

$$a \in \mathbb{Z}_{\phi(p)}$$





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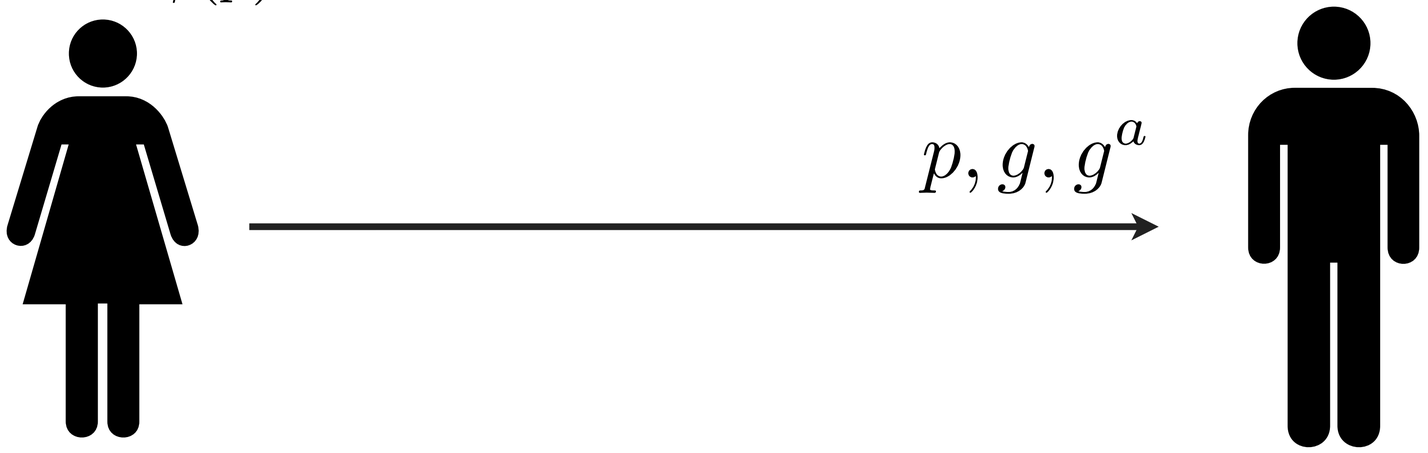

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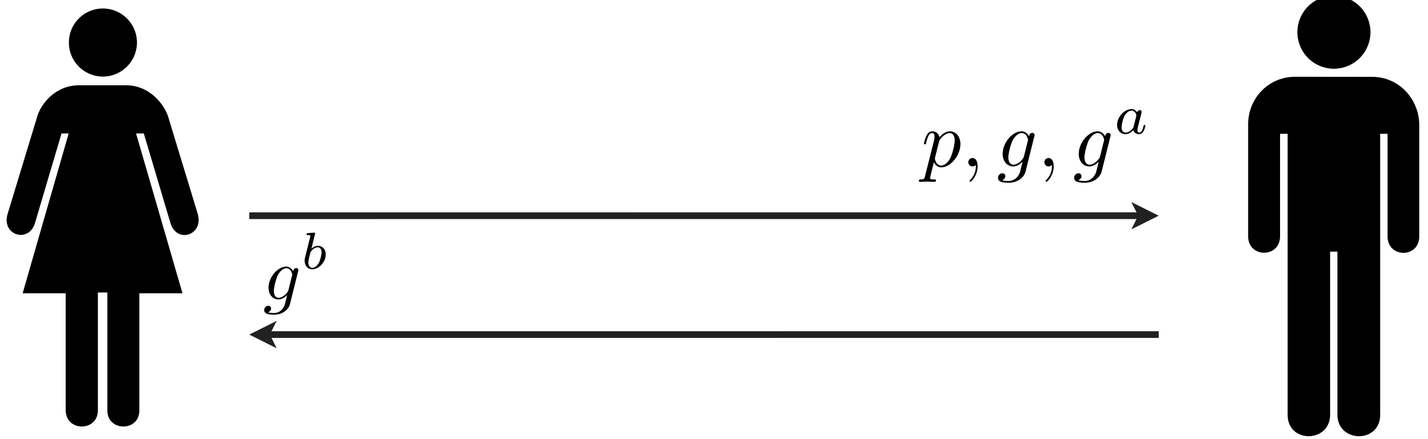

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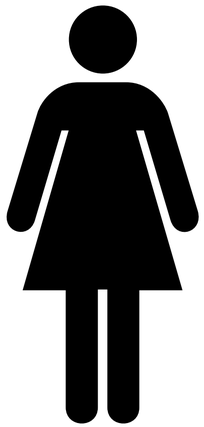

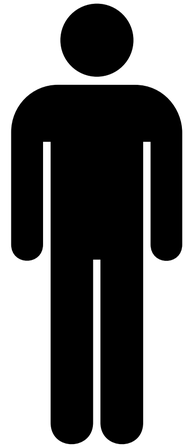

p, g, g^a

g^b

g^{ba}

g^{ab}

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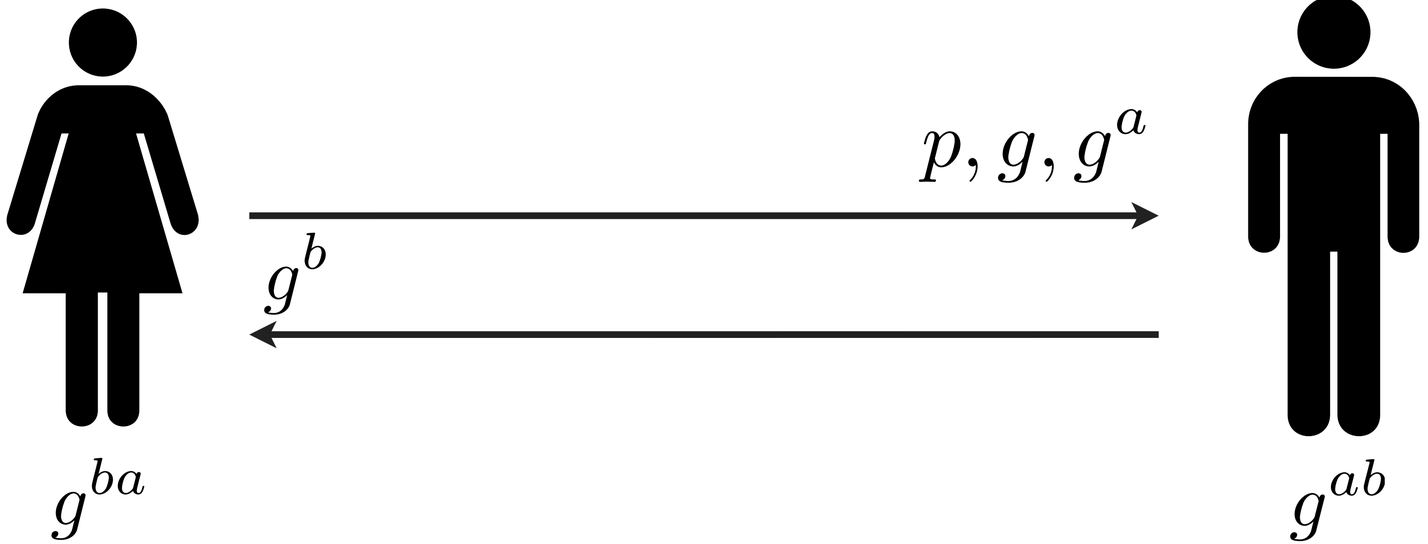

Usually we “hash” the shared secret value to make a secret encryption key, and then encrypt using a fast symmetric encryption scheme!

Hard problems (2)

- **Diffie-Hellman problem**

  Given: $a, b \in_R 0, \ldots, p-2$

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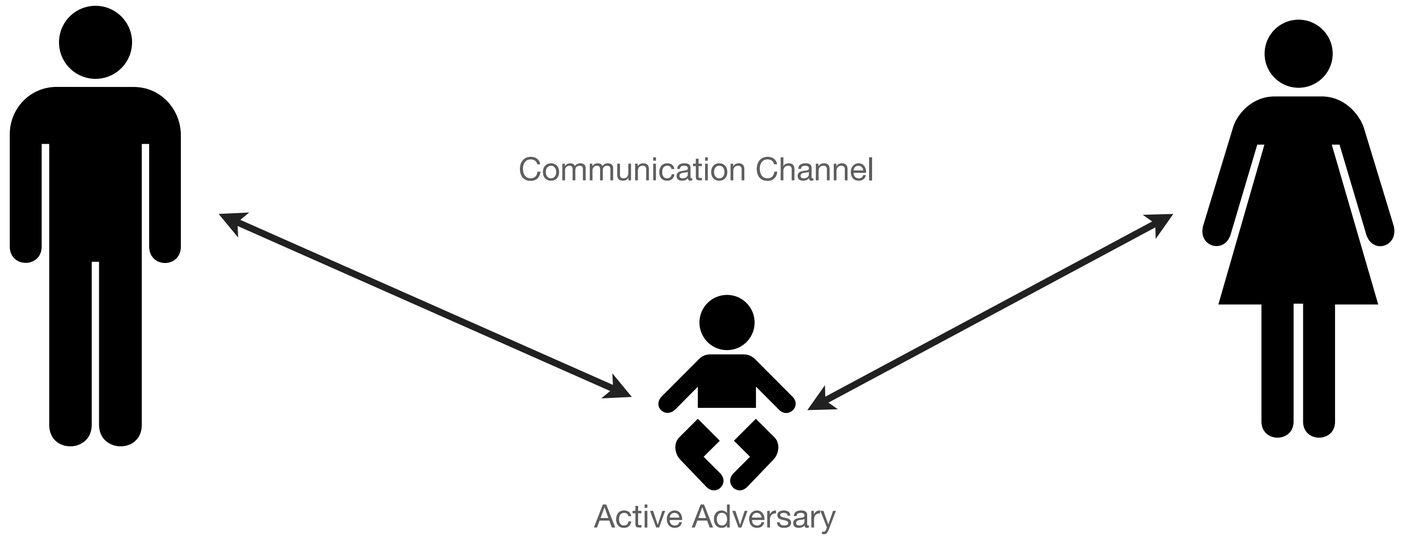
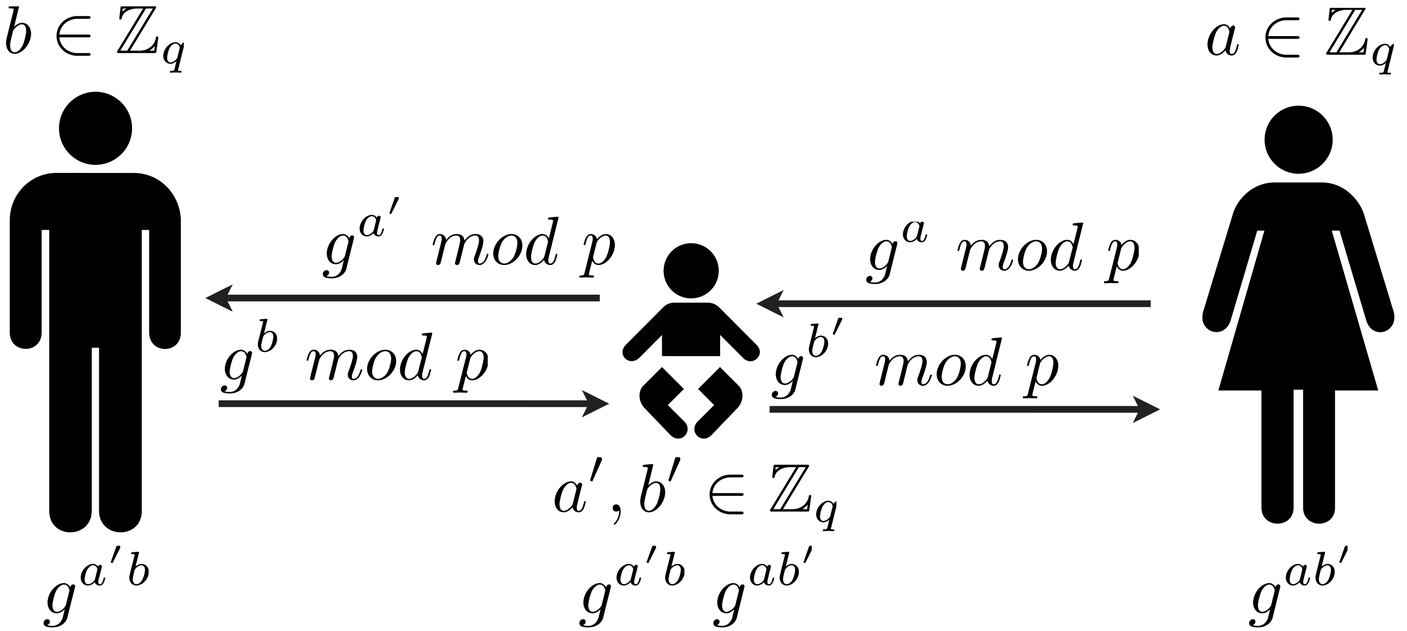
What if we have an active adversary?



Man in the Middle

- Assume an active adversary:



Man in the Middle

- Caused by lack of authentication
  - D-H lets us establish a shared key with anyone...
    but that's the problem…
  - We don't know if the person we're talking to is the right person
- Solution?

Preventing MITM

- Verify key via separate channel
- Password-based authentication
- Authentication via PKI

Digital Signatures

- Similar to MACs, with public keys
  - Secret key used to sign data
  - Public key can verify signature
  - Advantages over MACs?

Digital Signatures

- Three algorithms:

  Keygen() -> (vk, sk)

  Sign(sk, message) -> sig

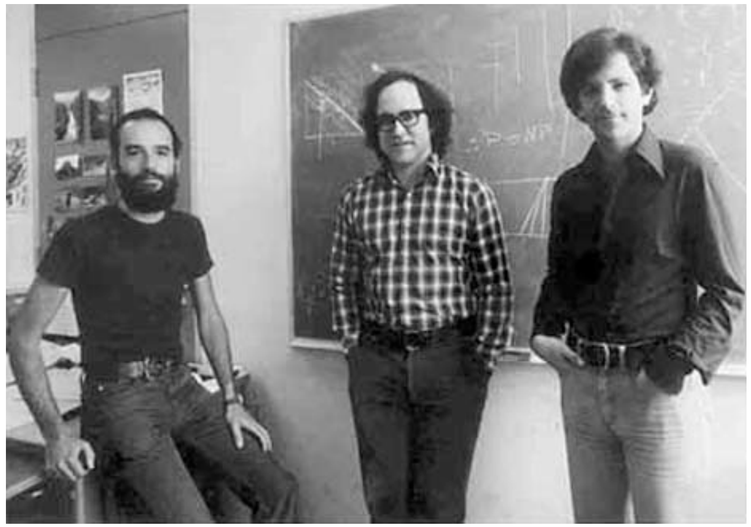
  Verify(pk, message, sig) -> True/False

Signature: definitions

- Existential Unforgeability under Chosen Message Attack
  - No efficient adversary can “forge” a signature on any new message (that it hasn’t received a signature for)
  - Even if it has access to an “oracle” that signs any message it wants
  - “Strong unforgeability” <- can’t even make a new signature for an existing message

Public Key Encryption

- What if our recipient is offline?
  - Key agreement protocols are interactive
  - e.g., want to send an email



Public Key Encryption