

# **650.445: Practical Cryptographic Systems**

## **Symmetric Cryptography II & Asymmetric Cryptography**

**Instructor: Matthew Green**

# Housekeeping

- Office hours:
  - Weds: 2pm & by appointment
  - My email: [mgreen@cs.jhu.edu](mailto:mgreen@cs.jhu.edu)
  - Readings!

# Review

- Last time:
  - Historical tour of Cryptography
  - Classical Crypto, Mechanical ciphers
  - One Time Pads
  - Block ciphers
    - Modes of Operation (ECB)

# Review

- Review: Block Ciphers

- Keyed primitive
- Operate on fixed-size input blocks
- Invertible (pseudo-random permutation)
- Examples: DES, AES...



# ECB Mode

- ECB is deterministic
  - Leads to problems, e.g.,:



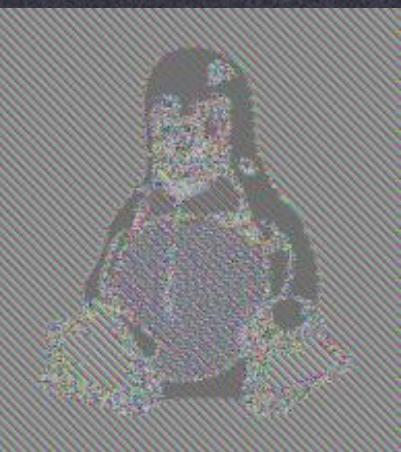
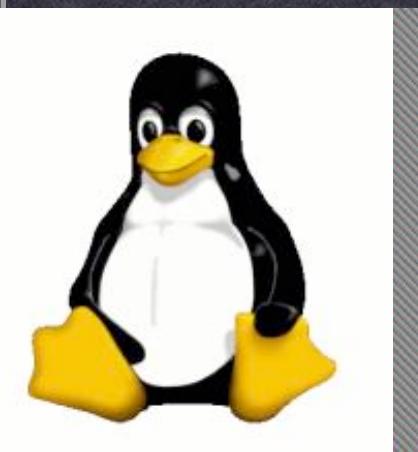
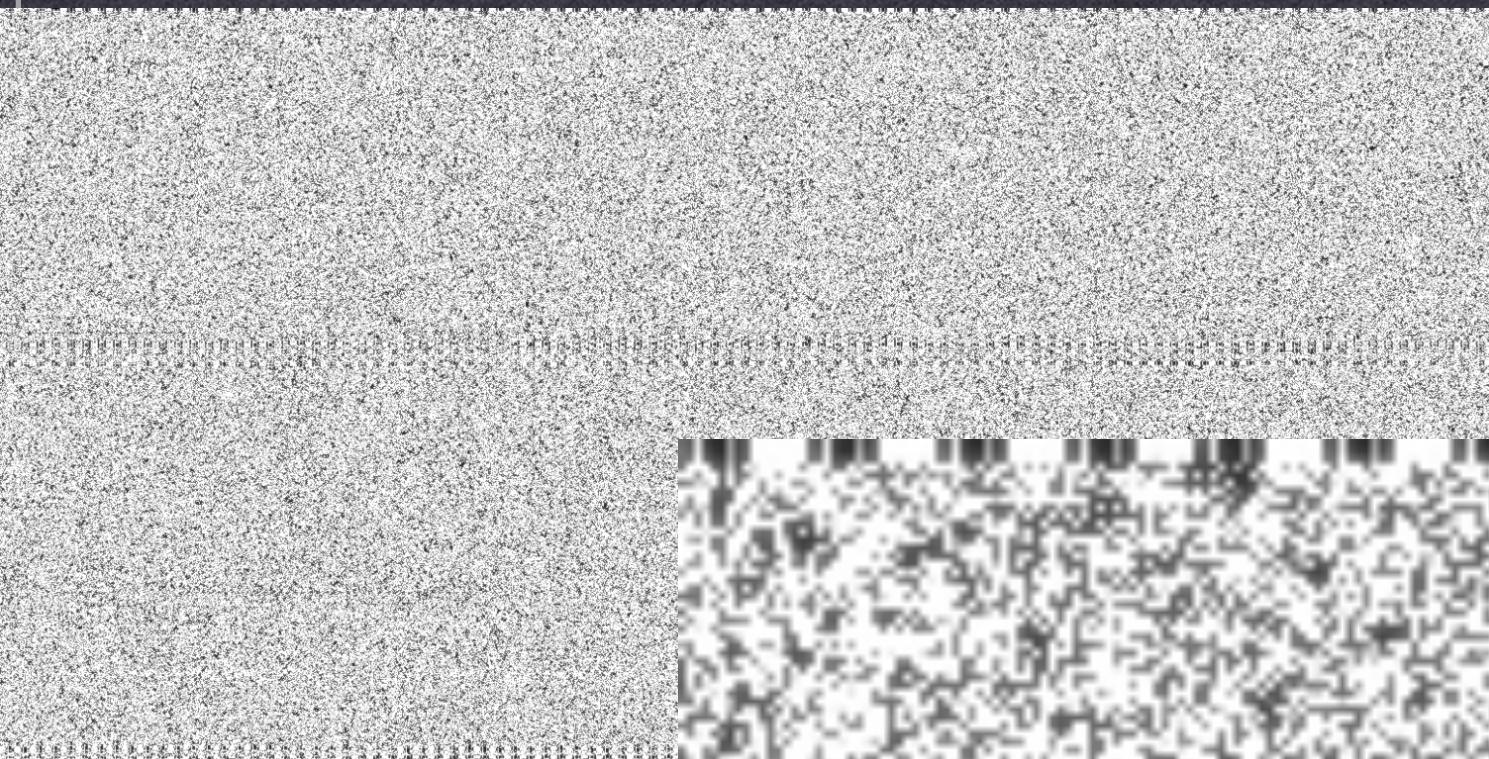
Game server



Game client

# Defining Security

- What we'd like from our encryption scheme?
  - Ciphertext doesn't leak any info about plaintext
  - Even if Adversary knows a lot about the plaintext distribution
  - Even if Adversary can choose the plaintext distribution
  - Even if Adversary can obtain encryptions of chosen plaintexts



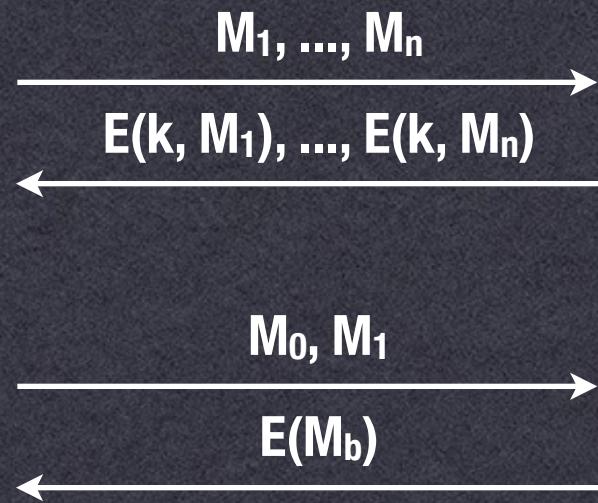
# Review

- Semantic Security (IND-CPA)



Adversary

b?



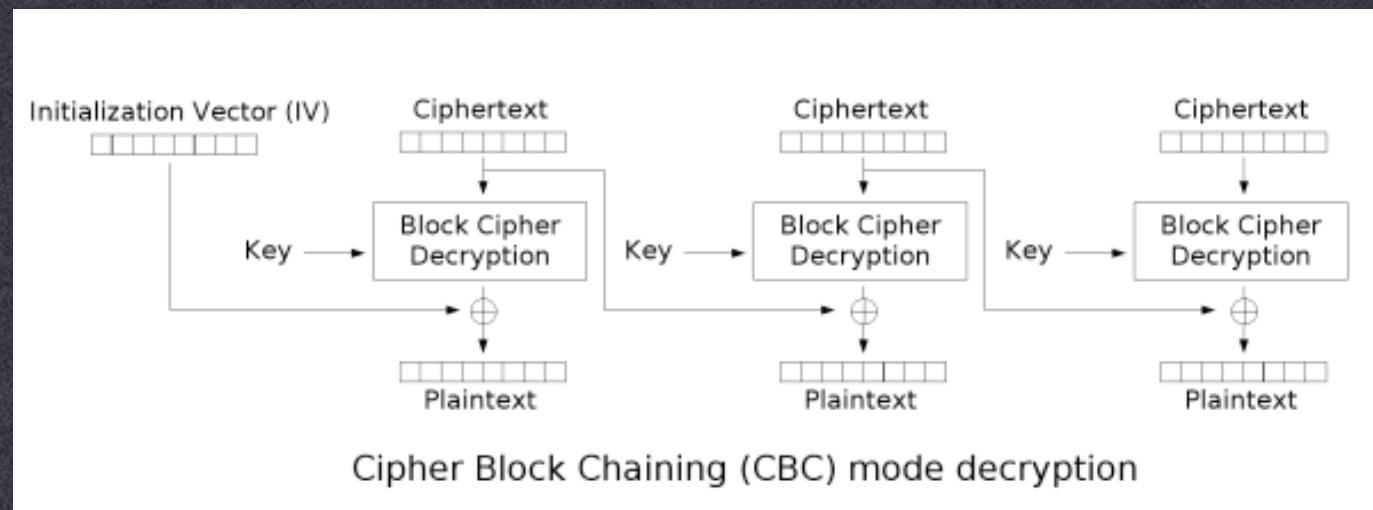
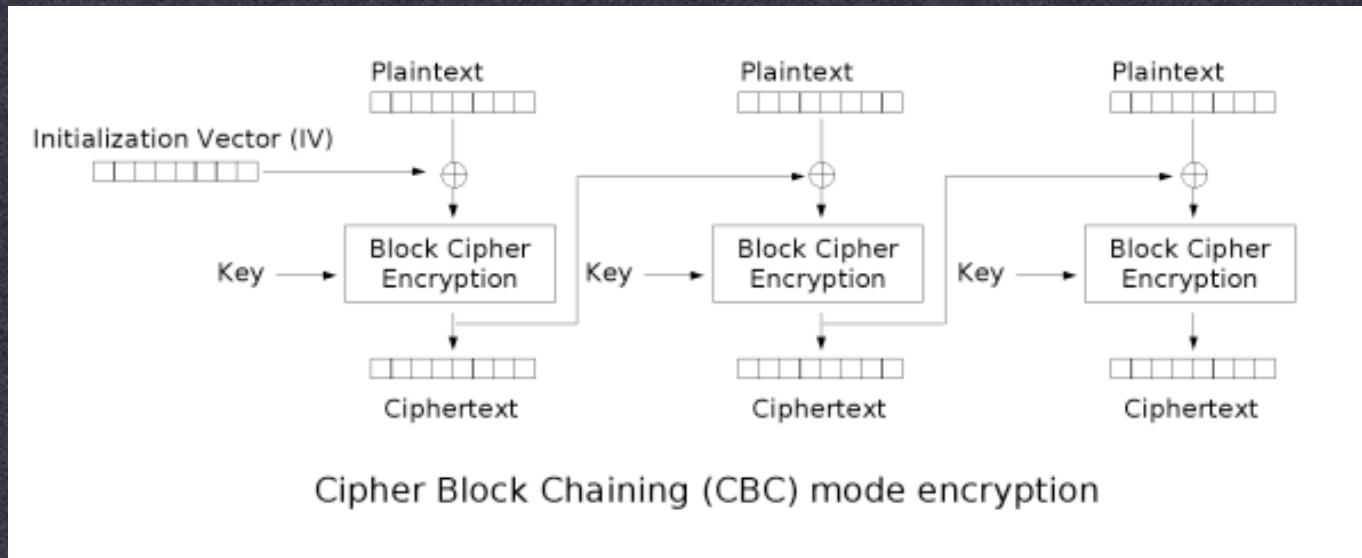
k

$$b \xleftarrow{\$} \{0, 1\}$$

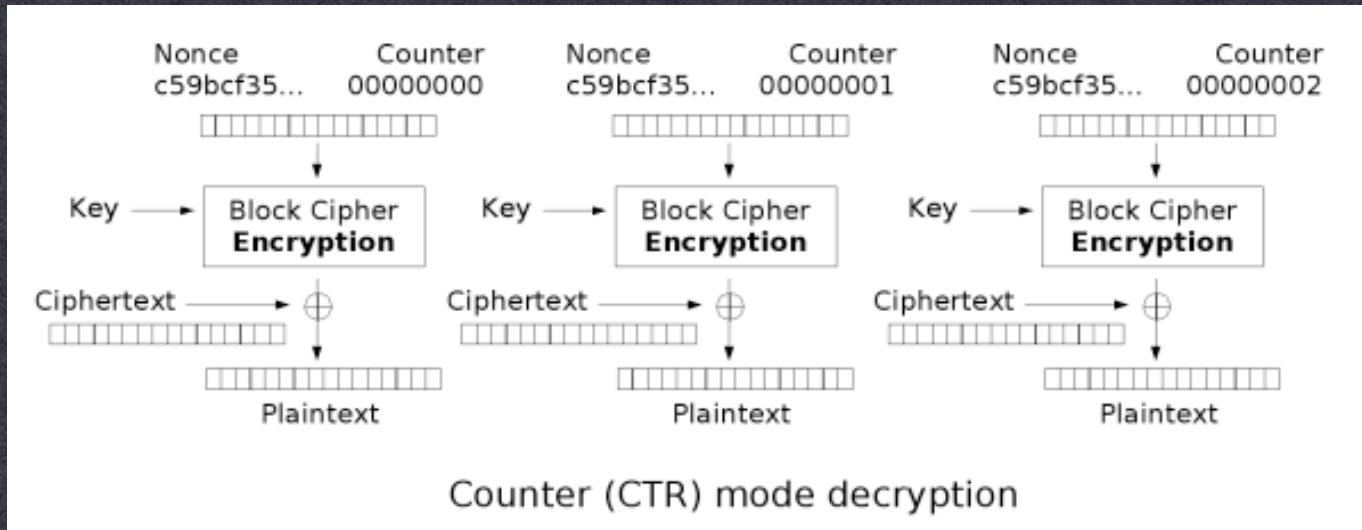
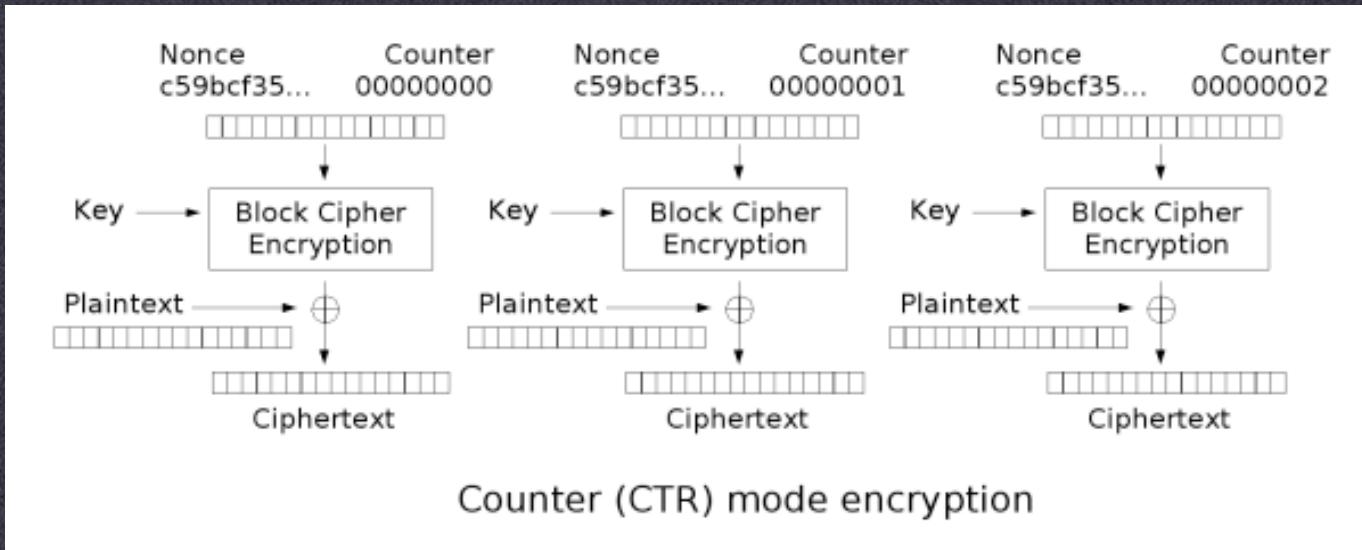
# Using Block Ciphers

- ECB is not semantically secure, hence we use a “mode of operation”
  - e.g., CBC, CTR, CFB, OFB (and others)
- These provide:
  - Security for multi-block messages
  - Randomization (through an Initialization Vector)

# CBC Mode



# CTR Mode



# CTR Mode

- Intuition:
  - We're using the block cipher to generate a stream of “randomish” bits
  - XOR that stream of bits with the message
  - This should look familiar

# CTR Mode

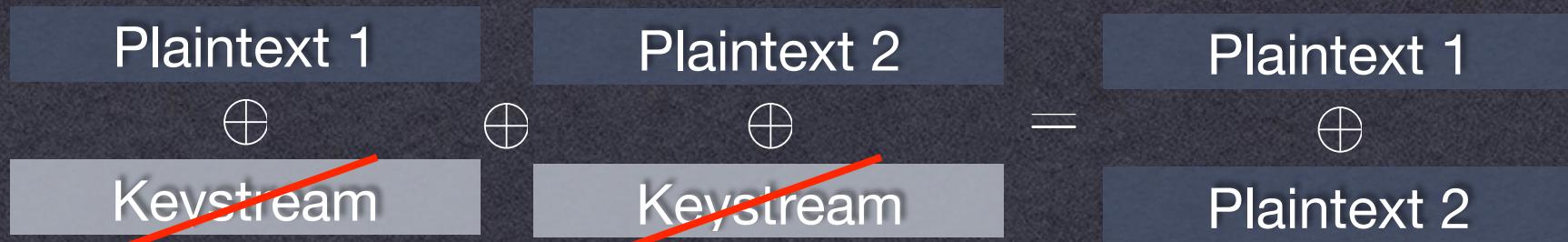
- CTR vs CBC
  - Unlike CBC, CTR can be parallelized (each block of the message encrypted separately by a different processor)
  - CTR keystream can be pre-computed
  - IV is much simpler

# Security of CTR

- IND-CPA assuming secure block cipher (PRP)

# Security of CTR

- IND-CPA assuming secure block cipher (PRP)
- However, counter range must never be re-used



- Similar example: MS Word 2003
  - (they used RC4, but same problem)

# “Two-time pad”

- Re-using CTR IV range gives “two time pad”

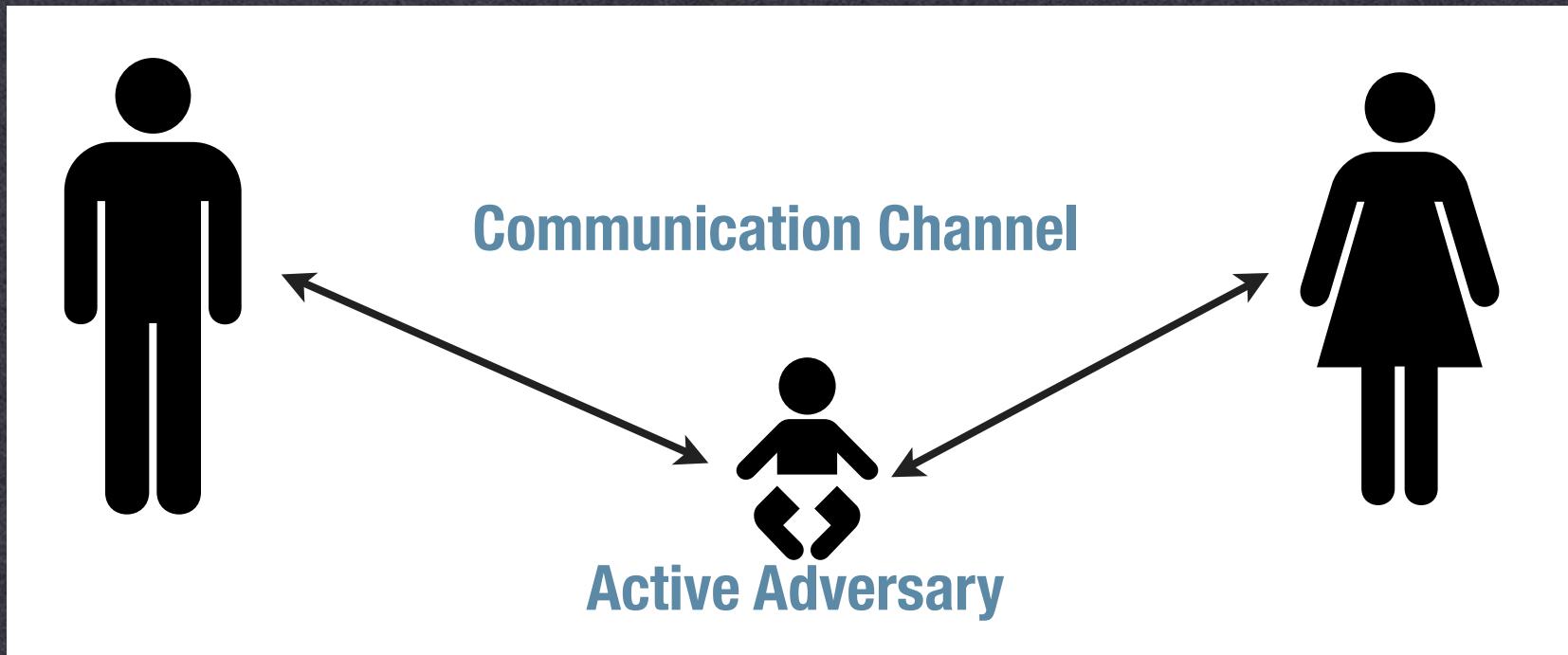
$$\begin{array}{c} \text{Plaintext 1} \\ = \\ \oplus \\ \text{Plaintext 2} \end{array}$$

- But can we tear these messages apart?
  - Answer: depends on the plaintexts
  - Venona project (hand-decode)
  - Modern NLP techniques (auto-decode)

# Malleability

- The ability to modify a ciphertext
  - Such that the plaintext is meaningfully altered
  - CTR Mode (bad)
  - CBC Mode (somewhat bad)
- The solution:
  - Authenticated Encryption

# Authenticated Encryption



# MACs

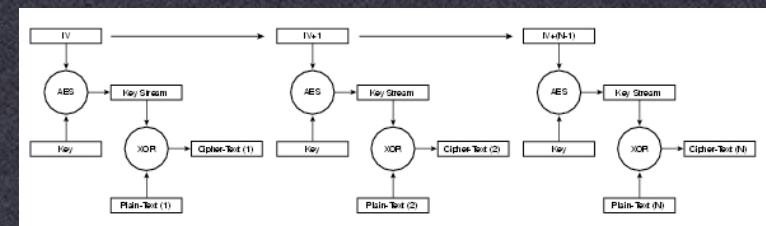
- **Symmetric-key primitive**
  - Given a key and a message, compute a “tag”
  - Tag can be verified using the same key
  - Any changes to the message detectable
- **To prevent malleability:**
  - Encrypt then MAC
  - Under separate keys

# MACs

- **Definitions of Security**
  - Existential Unforgeability under Chosen Message Attack (EU-CMA)
- **Examples:**
  - **HMAC (based on hash functions)**
  - **CMAC/CBC-MAC (block ciphers)**

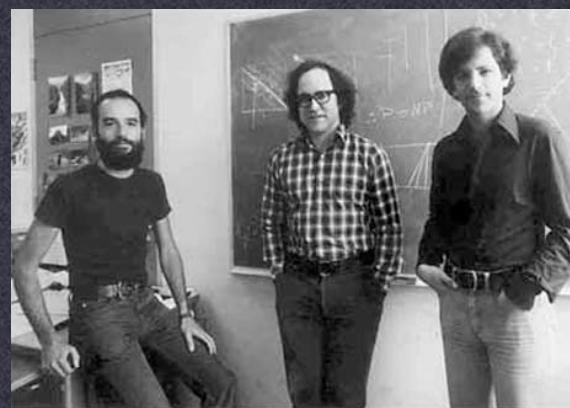
# Authenticated Encryption

- Two ways to get there:
  - Generic composition  
Encrypt (e.g., CBC mode) then MAC
    - two different keys, multiple primitives
  - Authenticated mode of operation
    - Integrates both encryption & authentication
    - Single key, typically uses only one primitive (e.g., block cipher)
  - Ex: CCM, OCB, GCM modes



# Asymmetric Crypto

- So far we've discussed symmetric crypto
  - Requires both parties to share a key
  - Key distribution is a hard problem!



# Key Agreement

- Establish a shared key in the presence of a passive adversary



# D-H Protocol

Malcolm Williamson in 72

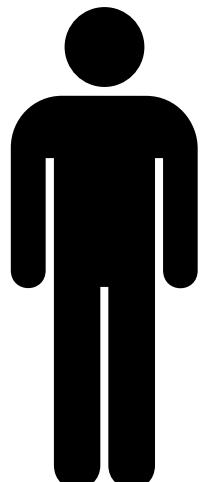
Diffie-Hellman in 76



$$b \in \mathbb{Z}_q$$

$$p, q : p = 2q + 1$$

$$a \in \mathbb{Z}_q$$



$$g^{ab}$$

$$\xrightarrow{g^b \text{ mod } p}$$

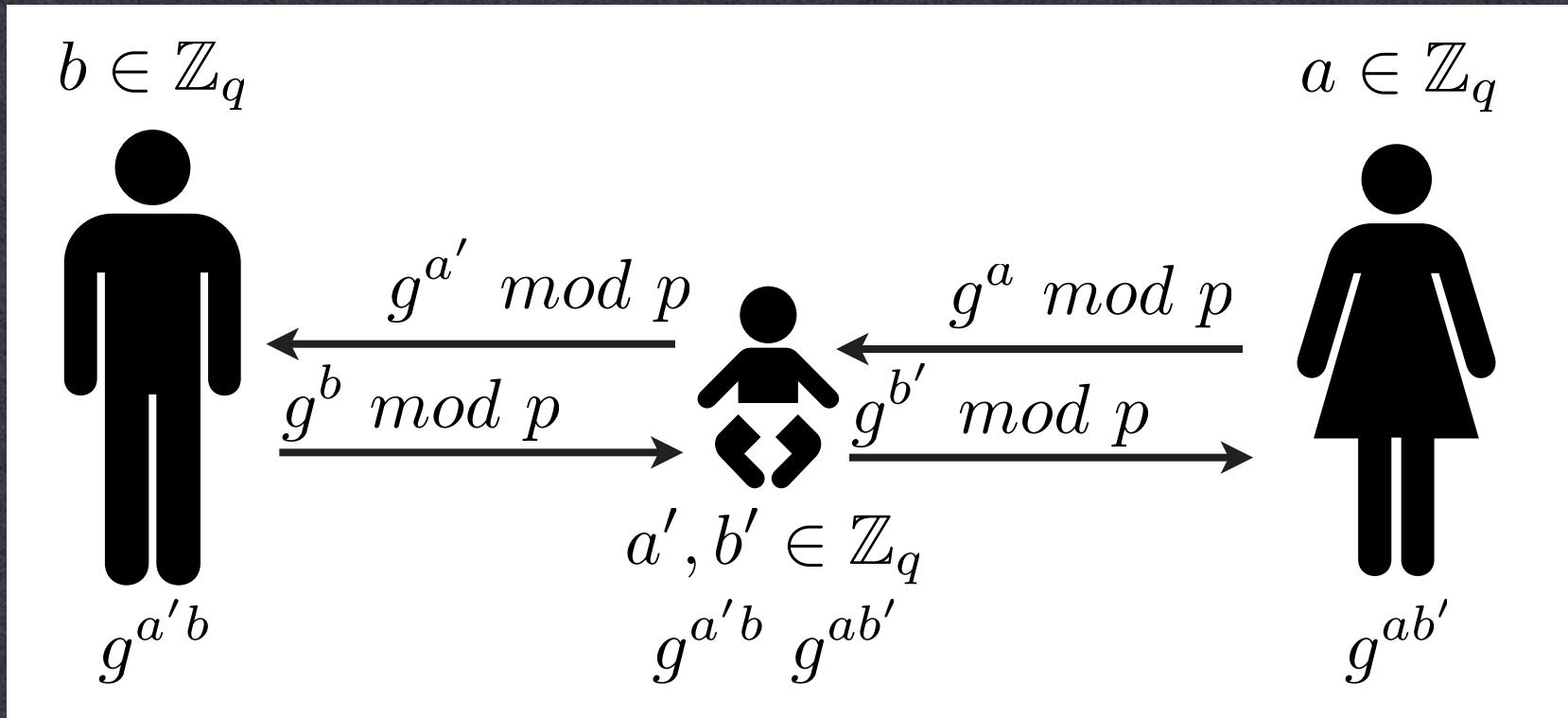
$$\xleftarrow{g^a \text{ mod } p}$$



$$g^{ab}$$

# Man in the Middle

- Assume an active adversary:

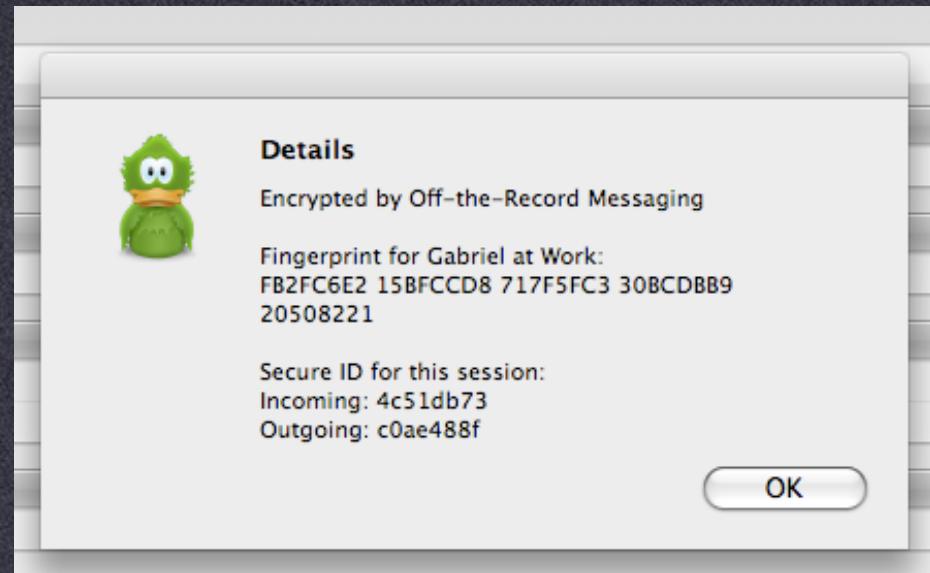


# Man in the Middle

- Caused by lack of authentication
  - D-H lets us establish a shared key with anyone... but that's the problem...
- Solution: Authenticate the remote party

# Preventing MITM

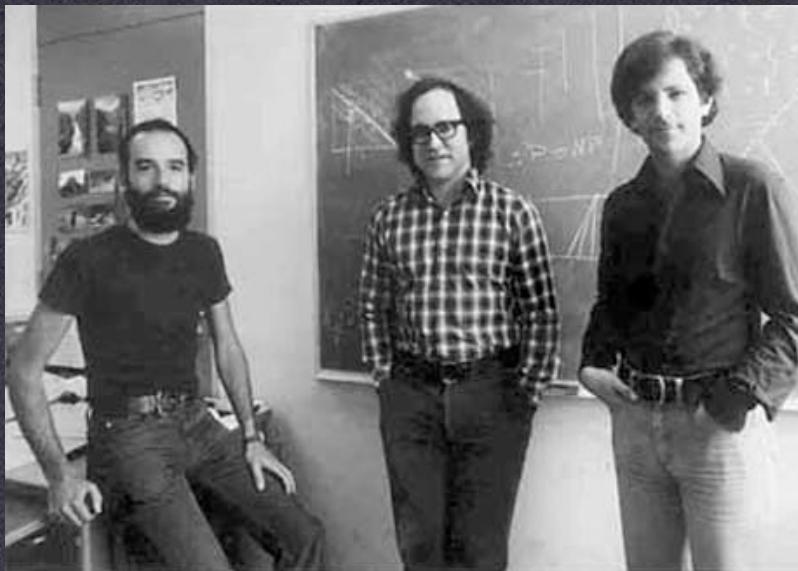
- Verify key via separate channel
- Password-based authentication
- Authentication via PKI



# Public Key Encryption

- What if our recipient is offline?
  - Key agreement protocols are interactive
  - e.g., want to send an email

Ellis in 72, Cocks a few months later



# Public Key Encryption



# RSA Cryptosystem

## Key Generation

**Choose large primes:**  $p, q$

$$N = p \cdot q$$

$$\phi(N) = (p - 1)(q - 1)$$

**Choose:**

$$e : \gcd(e, \phi(N)) = 1$$

$$d : ed \bmod \phi(N) = 1$$

**Output:**

$$pk = (e, N)$$

$$sk = d$$

## Encryption

$$c = m^e \bmod N$$

## Decryption

$$m = c^d \bmod N$$

# “Textbook RSA”

- In practice, we don't use Textbook RSA
  - Fully deterministic (not semantically secure)
  - Malleable

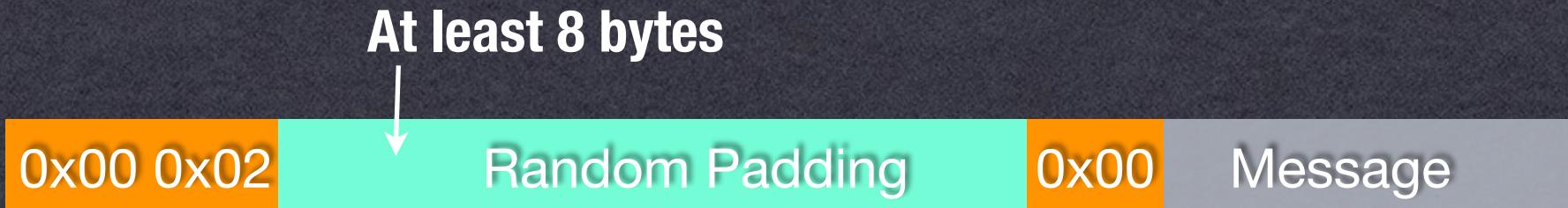
$$c' = c \cdot x^e \bmod N$$

$$c'^d = (m^e \cdot x^e)^d = m \cdot x \bmod N$$

- Might be partially invertible
  - Coppersmith's attack: recover part of plaintext (when m and e are small)

# RSA Padding

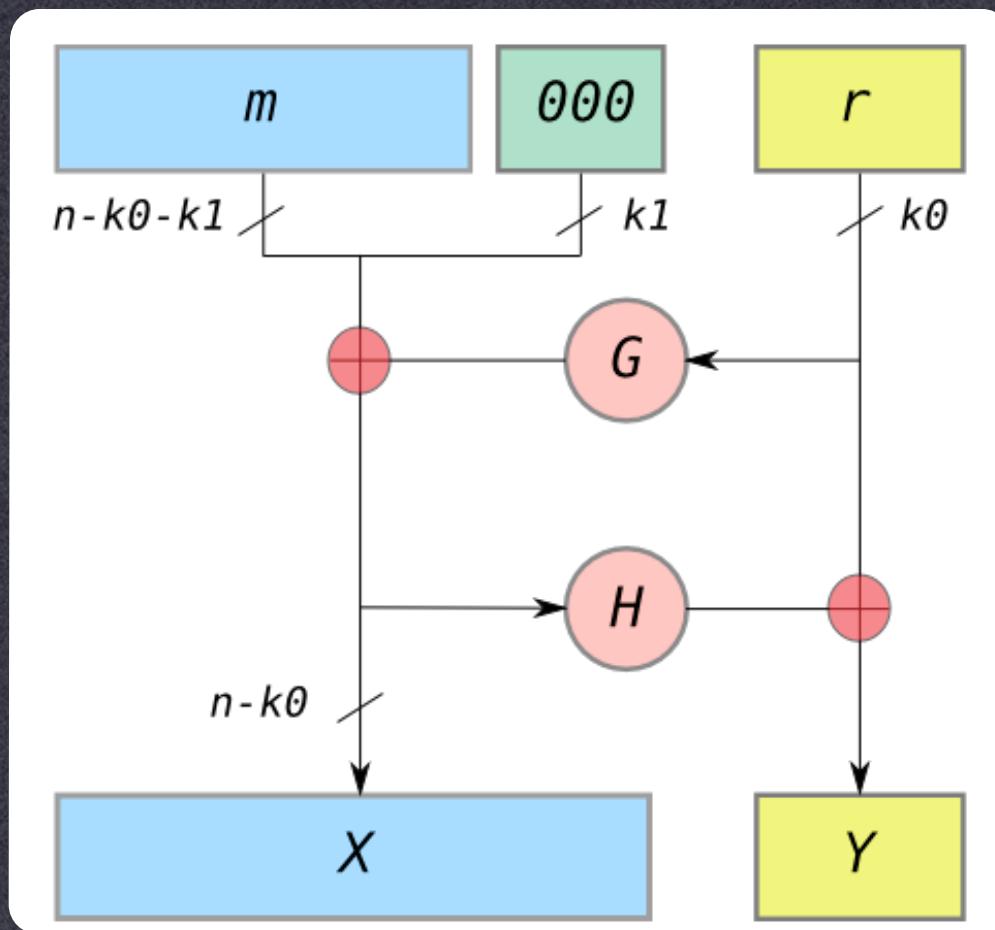
- Early solution (RSA PKCS #1 v1.5):
  - Add “padding” to the message before encryption
  - Includes randomness
  - Defined structure to mitigate malleability
  - PKCS #1 v1.5 badly broken (Bleichenbacher)



~ 1024 bits (128 bytes)

# RSA Padding

- Better solution (RSA-OAEP):
  - G and H are hash functions



# Efficiency

$m^e \bmod N$   
 $e = 65,537$

$m^d \bmod N$

	Cycles/Byte
AES (128 bit key)	18
DES (56 bit key)	51
RSA (1024 bit key) <u>Encryption</u>	1,016
RSA (1024 bit key) <u>Decryption</u>	21,719

Benchmarks from: <http://www.cryptopp.com/benchmarks.html>  
Microsoft Visual C++, Windows XP, Intel Core 2 1.83Mhz in 32-bit mode

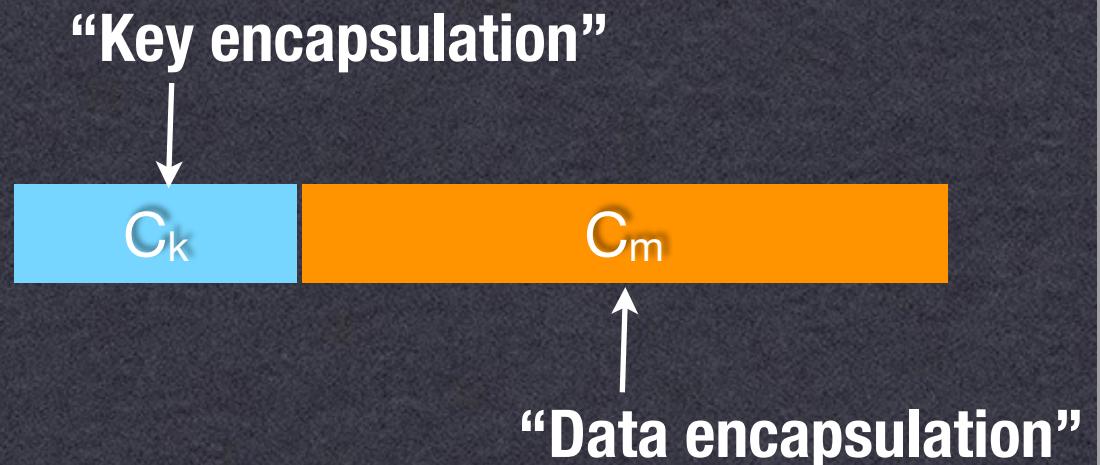
# Hybrid Encryption

- Mixed Approach
  - Use PK encryption to encrypt a symmetric key
  - Use (fast) symmetric encryption on data

$$k \xleftarrow{\$} \{0, 1\}^k$$

$$C_k \leftarrow RSA.Encrypt_{pk}(k)$$

$$C_m \leftarrow AES.Encrypt_k(message)$$



# Key Strength

Level	Protection	Discrete Logarithm Key Group						Elliptic Curve Hash
		Symmetric	Asymmetric	Logarithm				
1	Attacks in "real-time" by individuals <i>Only acceptable for authentication tag size</i>	32	-	-	-	-	-	-
2	Very short-term protection against small organizations <i>Should not be used for confidentiality in new systems</i>	64	816	128	816	128	128	128
3	Short-term protection against medium organizations, medium-term protection against small organizations	72	1008	144	1008	144	144	144
4	Very short-term protection against agencies, long-term protection against small organizations <i>Smallest general-purpose level, Use of 2-key 3DES restricted to 2<sup>40</sup> plaintext/ciphertexts, protection from 2009 to 2011</i>	80	1248	160	1248	160	160	160
5	Legacy standard level <i>Use of 2-key 3DES restricted to 10<sup>6</sup> plaintext/ciphertexts, protection from 2009 to 2018</i>	96	1776	192	1776	192	192	192
6	Medium-term protection <i>Use of 3-key 3DES, protection from 2009 to 2028</i>	112	2432	224	2432	224	224	224
7	Long-term protection <i>Generic application-independent recommendation, protection from 2009 to 2038</i>	128	3248	256	3248	256	256	256
8	"Foreseeable future" <i>Good protection against quantum computers</i>	256	15424	512	15424	512	512	512

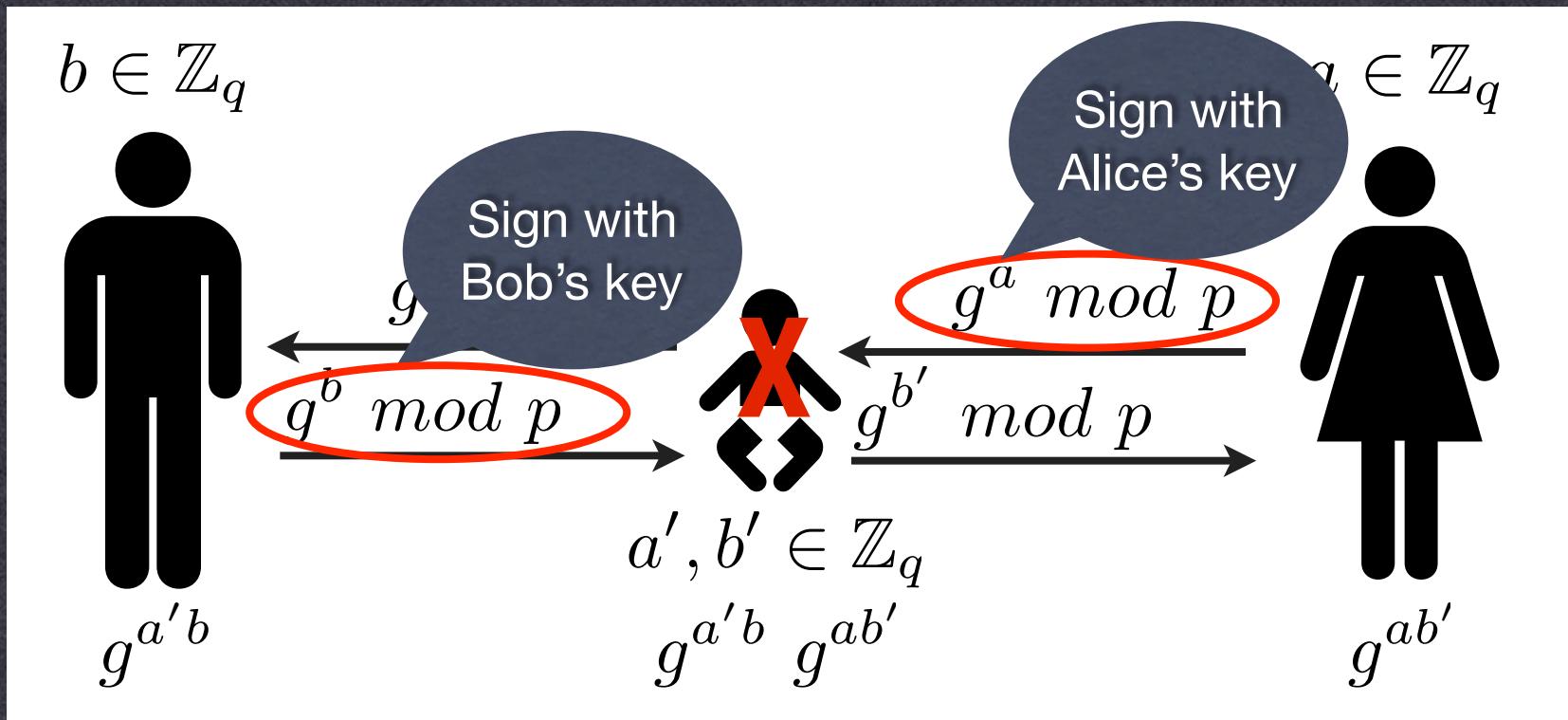
Source: [www.keystrength.com](http://www.keystrength.com) (BlueKrypt). Based on ECrypt recommendations.

# Digital Signatures

- Similar to MACs, with public keys
  - Secret key used to sign data
  - Public key can verify signature
  - Advantages over MACs?

# Preventing MitM

- Assume an active adversary:



# PKI & Certificates

- How do I know to trust your public key?
  - Put it into a file with some other info, and get someone else to sign it!

