

Notes on 信号与系统

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1 Basics

Contents: Signal Proc.

1. Intro to Signal Processing
2. 线性时不变系统时域分析/Temporal Ana. of Linear Time-Invariant System
3. Fourier Analysis
4. Laplace Transform
5. Z Transform
6. DFT and FFT

Contents: System

1. 线性时不变系统分析
2. 数字滤波器设计
3. 数字滤波器结构
4. 有限字长效应的统计分析

Grades

- 出席 5' 课堂讨论 5'
- 作业 30'
 - 课后网上发布, 一周后随堂提交.
- 考试 60'

2 Signals

2.1 Common Continuous Signals

Exponential

$$f(t) = K \exp(at) \quad (1)$$

$$\text{where } \tau = \frac{1}{|a|} \text{ 是时间常数} \quad (2)$$

Sine

$$f(t) = K \sin(\omega t + \theta) \quad (3)$$

$$\text{where } T = \frac{2\pi}{\omega} s \quad (4)$$

Complex Exponential

$$f(t) = K \exp(st) \quad (5)$$

Sa/抽样函数

$$f(t) = \frac{\sin t}{t} \quad (6)$$

$$\int_{\mathbb{R}} f(t) dt = \pi, \int_{\mathbb{R}_+} f(t) dt = \pi/2 \quad (7)$$

Gaussian

$$f(t) = E \exp\left(-\frac{t^2}{\tau^2}\right) \quad (8)$$

Rectified Linear/ReLU

$$R(t) = \max(0, t) \quad R(t) = \min(\tau, \max(0, t)) \text{ 截顶的 ReLU} \quad (9)$$

Unit Leap/Heavside

$$u(t) = \begin{cases} 1, & t > 0 \\ 1/2, & t = 0 \\ 0, & t < 0 \end{cases} \quad (10)$$

Square Impulse

$$G_\tau(t) = \begin{cases} 1, t \in (-\tau/2, \tau/2) \\ 1/2, |t| = \tau/2 \\ 0, \text{otherwise} \end{cases} \quad (11)$$

$$G_\tau(t) = u(t + \tau/2) - u(t - \tau/2) \quad (12)$$

Sign

$$\text{sgn}(t) = \begin{cases} 1, t > 0 \\ -1, t < 0 \end{cases} \quad (13)$$

$$\text{sgn}(t) = 2u(t) - 1 \quad (14)$$

Dirac Delta 一个广义函数/测度.

$$\delta(t) = \begin{cases} \infty, t = 0 \\ 0, \text{otherwise} \end{cases} \quad (15)$$

有性质

$$\int_{\mathbb{R}} \delta(t) dt = 1 \quad (16)$$

$$\delta(at) = \frac{1}{|a|} \delta(t) \quad (17)$$

$$\int_{\mathbb{R}} \delta_{t_0}(t) f(t) dt = f(t_0) \quad (18)$$

他的导数是一对正负冲激

$$\int_R \delta'_{t_0}(t) f(t) dt = -f'(t_0) \quad (19)$$

2.2 Discrete Time Signal

Real/Complex Exponential

$$x[n] = Az^n \quad (20)$$

Unit Sampling/Leap

$$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases} \quad (21)$$

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases} \quad (22)$$

Rectangular Seq.

$$R_N[n] = \begin{cases} 1, 0 \leq n < N \\ 0, n \geq N \end{cases} \quad (23)$$

2.3 Decompostion of Signals

直流/交流分量记信号的平均值为直流分量

$$f_{DC} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t \in [-T/2, T/2]} f(t) dt \quad (24)$$

$$f_{AC} = f - f_{DC} \quad (25)$$

$$P = f_{DC}^2 + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{t \in [-T/2, T/2]} f_{AC}(t)^2 dt \quad (26)$$

奇偶分量

$$f(t) = g(t) + h(t), \quad (27)$$

$$\text{where } g(t) = \frac{1}{2}(f(t) + f(-t)) \quad (28)$$

$$h(t) = \frac{1}{2}(f(t) - f(-t)) \quad (29)$$

R/C 分量

$$f(t) = \text{Re}(f(t)) + \text{Im}(f(t)) \quad (30)$$

Impulse/Leap Decomp.

$$f(t) = \int_0^t f(\tau) \delta(t - \tau) d\tau \quad (31)$$

$$= f_{[0..t]} * \delta \quad (32)$$

$$f(t) = f(0)u(t) + \int_0^t f'(\tau)u(t - \tau) d\tau \quad (33)$$

Orthogonal Decomp.

2.4 Classifications of Signals

1. 连续/离散
2. 奇偶
3. 确定性/随机性
4. 周期/非周期
5. 因果/反因果/非因果: 仅在 $\mathbb{R}_+/\mathbb{R}_-/\mathbb{R}$ 有信号
6. 实/复信号
7. 能量/功率有限信号