Notes on 信号与系统

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目录

1 Basics

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2. 线性时不变系统时域分析/Temporal Ana. of Linear Time-Invariant System
3. Fourier Analysis
4. Laplace Transform
5. Z Transform
6. DFT and FFT

Contents: System

1. 线性时不变系统分析
2. 数字滤波器设计
3. 数字滤波器结构
4. 有限字长效应的统计分析

Grades

- 出席 5’ 课堂讨论 5’
- 作业 30’
  - 课后网上发布, 一周后随堂提交.
- 考试 60’

2 Signals

2.1 Common Continous Signals

Exponential

$$f(t) = K\exp(at) \tag{1}$$

$$\text{where } \tau = \frac{1}{|a|}\text{是时间常数} \tag{2}$$

Sine

$$f(t) = K\sin(\omega t + \theta) \tag{3}$$

$$\text{where } T = \frac{2\pi}{\omega}s \tag{4}$$

Complex Exponential

$$f(t) = K\exp(st) \tag{5}$$

Sa/抽样函数

$$f(t) = \frac{\sin t}{t} \tag{6}$$

$$\int_{\mathbb{R}} f(t)dt = \pi, \int_{\mathbb{R}_+} f(t)dt = \pi/2 \tag{7}$$

Gaussian

$$f(t) = E\exp\left(-\frac{t^2}{\tau^2}\right) \tag{8}$$

Rectified Linear/ReLU

$$R(t) = \max(0,t) \qquad R(t) = \min(\tau, \max(0,t))\text{截顶的 ReLU} \tag{9}$$

Unit Leap/Heavside

$$u(t) = \begin{cases} 1, t > 0 \\ 1/2, t = 0 \\ 0, t < 0 \end{cases} \tag{10}$$

Square Impulse

$$G_\tau(t) = \begin{cases} 1, t \in (-\tau/2, \tau/2) \\ 1/2, |t| = \tau/2 \\ 0, \text{otherwise} \end{cases} \tag{11}$$

$$G_\tau(t) = u(t+\tau/2) - u(t-\tau/2) \tag{12}$$

Sign

$$sgn(t) = \begin{cases} 1, t > 0 \\ -1, t < 0 \end{cases} \tag{13}$$

$$sgn(t) = 2u(t) - 1 \tag{14}$$

Dirac Delta 一个广义函数/测度.

$$\delta(t) = \begin{cases} \infty, t = 0 \\ 0, \text{otherwise} \end{cases} \tag{15}$$

有性质

$$\int_{\mathbb{R}} \delta(t)dt = 1 \tag{16}$$

$$\delta(at) = \frac{1}{|a|}\delta(t) \tag{17}$$

$$\int_{\mathbb{R}} \delta_{t_0}(t)f(t)dt = f(t_0) \tag{18}$$

他的导数是一对正负冲激

$$\int_{R} \delta'_{t_0}(t)f(t)dt = -f'(t_0) \tag{19}$$

2.2 Discrete Time Signal

Real/Complex Exponential

$$x[n] = Az^n \tag{20}$$

Unit Sampling/Leap

$$\delta[n] = \begin{cases} 1, n = 0 \\ 0, n \neq 0 \end{cases} \tag{21}$$

$$u[n] = \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases} \tag{22}$$

Rectangular Seq.

$$R_N[n] = \begin{cases} 1, 0 \leq n < N \\ 0, n \geq N \end{cases} \tag{23}$$

2.3 Decompostion of Signals

直流/交流分量记信号的平均值为直流分量

$$f_{DC} = \lim_{T\to\infty} \frac{1}{T} \int_{t\in[-T/2,T/2]} f(t)dt \tag{24}$$

$$f_{AC} = f - f_{DC} \tag{25}$$

$$P = f_{DC}^2 + \lim_{T\to\infty} \frac{1}{T} \int_{t\in[-T/2,T/2]} f_{AC}(t)^2 dt \tag{26}$$

奇偶分量

$$f(t) = g(t) + h(t), \tag{27}$$

$$\text{where } g(t) = \frac{1}{2}(f(t) + f(-t)) \tag{28}$$

$$h(t) = \frac{1}{2}(f(t) - f(-t)) \tag{29}$$

R/C 分量

$$f(t) = \mathrm{Re}(f(t)) + \mathrm{Im}(f(t)) \tag{30}$$

Impulse/Leap Decomp.

$$f(t) = \int_0^t f(\tau)\delta(t-\tau)d\tau \tag{31}$$

$$= f_{[0..t]} * \delta \tag{32}$$

$$f(t) = f(0)u(t) + \int_0^t f'(\tau)u(t-\tau)d\tau \tag{33}$$

Orthogonal Decomp.

2.4 Classifications of Signals

1. 连续/离散
2. 奇偶
3. 确定性/随机性
4. 周期/非周期
5. 因果/反因果/非因果: 仅在 $\mathbb{R}_+/\mathbb{R}_-/\mathbb{R}$ 有信号
6. 实/复信号
7. 能量/功率有限信号