2014 IEEE International Conference on Robotics & Automation (ICRA)
Hong Kong Convention and Exhibition Center
May 31 - June 7, 2014. Hong Kong, China

Real-Time Distributed Optimal Trajectory Generation for Nonholonomic Vehicles in Formations

Reze Haghighi, Danwei Wang and Changboon Low

***Abstract*— This paper addresses the distributed formation trajectory planning for a group of nonholonomic vehicles. This is realized with a decentralized Model Predictive Control under dynamic virtual structure architecture. A specific limitation of virtual structure based formation method is the necessity of access to the desired reference. To remove this requirement, a distributed estimator is developed so that each vehicle can construct the desired reference based on the local information exchange. In formation trajectory planning, several issues are taken into consideration which includes: (i) distributed formation achievement by a team of nonholonomic vehicles from initial situation. (ii) inter-group collision avoidance. (iii) dynamic formation to obtain flexible manoeuvring during movement in unknown and cluttered environment. (iv) obstacle avoidance. Finally, simulation results are presented to illustrate the performance of the proposed methodology in producing optimal formation trajectory planning for multiple nonholonomic vehicles.**

I. Introduction

The past few decades have witnessed increasing interest in coordination and control of multi-vehicle systems in the robotic and control communities [1]. The attention to multi-vehicle systems has been promoted by advances in communication technologies. Multi-vehicle systems have been utilized in search and rescue, surveillance, cooperative transportation and so on. In majority of these applications, vehicles are required to form a desired formation during movement [2],[3]. In formation control of multiple vehicle systems various scenarios are usually involved such as: (i) vehicles should maintain a desired geometric formation of varied shapes. (ii) Physical constraints imposed by the vehicles (such as mobility constraints or nonholonomic constraints), must be satisfied. (iii) Formations should be flexible to accommodate the environment constraints, such as presence of obstacles and moving objects. (iv) Limitations in sensing and communication ranges must be taken into account.

Trajectory planning is a fundamental problem for robotic applications [4]. Trajectory Planning can be described as generating a feasible trajectory and its corresponding control to guide a robotic system from an initial state to a final state without violating any given constraints on the system and its environment. For nonholonomic vehicles, due to kinematic constraints a collision-free path in the configuration space is not necessarily feasible. Primary works on trajectory generation for nonholonomic systems was done in [5],[6], where steering control was designed based on sinusoidal, polynomial, or piecewise constant functions. Afterwards various methods have been proposed on trajectory planning of nonholonomic vehicles [7]-[9].

Consensus control is one of the important problems in networks of multi-vehicle systems that has gained increasing attention, owing to its wide potential applications [10]. Recently, there has been considerable study on improving formation control techniques to make it more distributed. As a result, distributed leader-follower approach has been proposed which rely on information exchange between neighboring agents [11]. Su et al. [12] extended the multi-agent flocking algorithm proposed by Olfati-Saber [13] to the case that only a fraction of agents are informed of the virtual leader with time-varying velocity. Choi et al. [14] addressed a leader-follower formation control in the absence of leader's velocity information. Dong [15] discussed the formation control of multiple mobile agents under the condition that the desired trajectory is available to only portion of agents. In [16] the distributed shape formation control was proposed for multi-agent systems.

Model Predictive Control (MPC) or Receding Horizon Control (RHC) has gained more and more attention in the control community [17]. Model Predictive Control (MPC) has addressed issues of trajectory tracking, point stabilization and formation control of nonholonomic mobile robots. The increase in popularity of MPC is due to its abilities of constraint handling, real-time prediction and optimizing. Main idea of MPC is to compute the control action by minimizing a finite horizon optimization problem. MPC has been frequently used in trajectory planning. Kim et al. [18] considered nonlinear MPC for path planning of autonomous helicopters. Falcone et al. [19] presented a MPC approach for controlling an active front steering system in an autonomous vehicle. Yoon et al. [20] addressed trajectory generation for unmanned ground vehicles based on nonlinear MPC.

This paper presents the real-time distributed trajectory generation for multiple nonholonomic vehicles based on the virtual structure architecture. Since in the virtual structure architecture, vehicles are required to have knowledge of the desired reference, a distributed estimator is proposed to construct the desired reference for each vehicle based on local information exchange by its neighboring vehicles. Consequently, a decentralized Model Predictive Control is utilized to obtain formation control for various objectives such as inter-collision avoidance and obstacle avoidance.

II. Kinematics Model of Nonholonomic Vehicle

The kinematic model for a differential drive mobile robot under the nonholonomic constraint of pure rolling and non-slipping is expressed in discrete format as follows [21]:

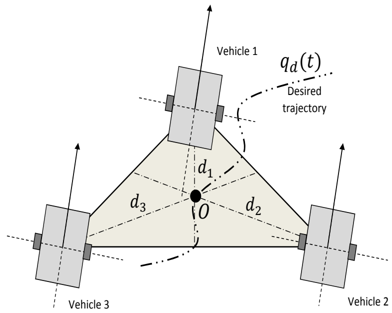

Fig. 1: An illustration of the virtual structure based formation for a group of three vehicles.

$$
\begin{aligned}
&\omega_i(k) \neq 0\\
&\begin{cases}
x_i(k+1) = x_i(k) + \frac{v_i(k)}{\omega_i(k)}\left[\sin(\theta_i(k)+\omega_i(k)T) - \sin(\theta_i(k))\right]\\
y_i(k+1) = y_i(k) - \frac{v_i(k)}{\omega_i(k)}\left[\cos(\theta_i(k)+\omega_i(k)T) - \cos(\theta_i(k))\right]\\
\theta_i(k+1) = \theta_i(k) + \omega_i(k)T
\end{cases}\\
&\omega_i(k) = 0\\
&\begin{cases}
x_i(k+1) = x_i(k) + v_i(k)\cos(\theta_i(k))T\\
y_i(k+1) = y_i(k) + v_i(k)\sin(\theta_i(k))T\\
\theta_i(k+1) = \theta_i(k)
\end{cases}
\end{aligned}
\tag{1}
$$

where $x_i(k)$ and $y_i(k)$ are the position of the center of mass of the wheeled mobile robot in a Cartesian coordinate frame, $\theta_i(k)$ denotes the orientation of the robot, $v_i(k)$ is the linear velocity, $\omega_i(k)$ is the angular velocity of vehicle i, and T is the sampling time.

III. Formation Trajectory Planning

In formation trajectory planning for a group of nonholonomic vehicles, several issues need to be taken into consideration such as decentralized trajectory planning, nonholonomic constraints, formation maintenance, formation deformation, inter-vehicle collision avoidance, obstacle avoidance. A key solution to formation trajectory planning is to obtain a dynamic formation control approach which is adaptable to the changes of the environment. One way of approaching formation control problem is through the use of the concept of the virtual structures. In the virtual structure based formation control method the entire desired formation is treated as a single entity specified by a reference frame. The formation as a whole should follow a predefined trajectory, i.e. the reference frame should follow the desired trajectory and each vehicle should follow a specified and possibly time-varying location relative to the reference frame. An illustration of the virtual structure based formation for a group of three vehicles is depicted in figure 1, such that O represents the reference of the virtual structure, $d_i \in R^2$ is the offset from the reference and $q_d(t) \in R^3$ is the desired trajectory.

IV. Virtual Structure based Formation

In the virtual structure architecture, the specified structure is defined based on a reference which follows the desired trajectory. Vehicles require to have knowledge about the desired trajectory. The desired trajectory can be preplanned which reduces the flexibility of the group in moving in an unknown environments. The other solution is introducing a virtual leader, which in this case the state of the virtual leader have to broadcast to all members of the group. However, limitation in communication range is a problem in global information broadcasting. To deal with these problems, an estimator is introduced so that each vehicle estimates the state of the virtual leader based on the local information exchange. In the following, the distributed algorithm for estimation of the desired trajectory of the virtual structure is presented.

Consider that communication topology among agents is represented by a directed graph $\mathcal{G}$ with the associate laplacian matrix $\mathcal{L} = [l_{ij}]$, where

$$
l_{ij} = \begin{cases} -a_{ij} & i \neq j \\ a_{ii} = \sum\limits_{j=1}^{n} a_{ij} & i = j \end{cases} \tag{2}
$$

such that n denotes the number of vehicles, and if the agent i can obtain information from the agent j, $a_{ij} > 0$, otherwise $a_{ij} = 0$. The proposed estimator to construct the desired reference is expressed as follows:

$$
\begin{aligned}
\hat{q}_d^i(k+1) &= \hat{q}_d^i(k) - Tk_e \sum_{j=1}^{n} a_{ij}\left(\hat{q}_d^i(k) - \hat{q}_d^j(k)\right)\\
&\quad - Tk_e b_i\left(\hat{q}_d^i(k) - q_d(k)\right),
\end{aligned}
\tag{3}
$$

where $\hat{q}_d^i(k)$ is an estimation of the desired reference, $q_d(k) = [x_d(k)\ y_d(k)\ \theta_d(k)]^T$, by vehicle i, k_e is a positive constant, and b_i is equal to 1 if agent i has an access to the desired reference and 0 otherwise. The error dynamic can be expressed as follows:

$$
\begin{aligned}
\tilde{q}_d^i(k+1) &= (1 - Tk_e b_i)\tilde{q}_d^i(k) - Tk_e \sum_{j=1}^{n} a_{ij}\left(\tilde{q}_d^i(k) - \tilde{q}_d^j(k)\right)\\
&\quad - (q_d(k+1) - q_d(k)).
\end{aligned}
\tag{4}
$$

Equation (4) can be written in the closed form as follow:

$$
\tilde{q}_d(k+1) = \mathcal{H}\tilde{q}_d(k) - (q_d(k+1) - q_d(k))\mathbf{1}_n \tag{5}
$$

where $\tilde{q}_d(k) = \left[\tilde{q}_d^1(k), ..., \tilde{q}_d^n(k)\right]^T$, $\mathbf{1}_n = [1, ..., 1]^T$, and $\mathcal{H} = I_n - Tk_e(\mathcal{L} + \mathcal{B})$, such that $\mathcal{B} = \mathbf{diag}\{b_1, ..., b_n\}$. The solution of (5) is expressed as follows:

$$
\tilde{q}_d(k) = \mathcal{H}^k \tilde{q}_d(0) - \sum_{m=0}^{k-1} \mathcal{H}^{k-(m+1)}(q_d(m+1) - q_d(m))\mathbf{1}_n \tag{6}
$$

yields

$$
\begin{aligned}
\|\tilde{q}_d(k)\|_\infty &\leq \left\|\mathcal{H}^k\right\|_\infty \|\tilde{q}_d(0)\|_\infty\\
&\quad + \left\|\sum_{m=0}^{k-1} \mathcal{H}^{k-(m+1)}(q_d(m+1) - q_d(m))\mathbf{1}_n\right\|_\infty .
\end{aligned}
\tag{7}
$$

Assume that the virtual leader has a directed path to all vehicles, and the desired reference has a bounded rate of change i.e. $\frac{\|q_d(k+1) - q_d(k)\|}{T} \leq \bar{q}_d$. Using lemmas A1 and A2, and choosing k_e as follows:

$$k_e < \frac{1}{T \max_i \{a_{ii} + b_i\}}, \tag{8}$$

the eigenvalues of $\mathcal{H}$ lie in the interior of the unit circle. Therefore, when k goes to infinity inequality (7) is simplified as follows:

$$\|\tilde{q}_d(k)\|_\infty \leq T\bar{q}_d \left\| \sum_{m=0}^{k-1} \mathcal{H}^m \right\|_\infty. \tag{9}$$

Since the spectral radius of $\mathcal{H}$ is less than 1, using the Taylor series expansion of the matrix inverse, yields

$$\begin{aligned} \|\tilde{q}_d(k)\|_\infty &\leq T\bar{q}_d \left\| (I_n - \mathcal{H})^{-1} \right\|_\infty \leq T\bar{q}_d \left\| (T k_e (\mathcal{L} + \mathcal{B}))^{-1} \right\|_\infty \\ &\leq \frac{\bar{q}_d}{k_e} \left\| (\mathcal{L} + \mathcal{B})^{-1} \right\|_\infty. \end{aligned} \tag{10}$$

Remark 1: Equation (10) shows that, by increasing k_e and decreasing T such that inequality (8) is satisfied, the reference tracking error is going to zero.
To obtain the virtual structure based formation, the Model Predictive Control (MPC) is applied. In the following, the formulation of decentralized MPC for virtual structure based formation trajectory planning is presented.

V. MPC Formulation for Formation Trajectory Planning

In Model Predictive Control the current control action is obtained for a system by solving, at each sampling time, a finite horizon optimization problem using the current state of the system as the initial state. Then the optimization yields an optimal input sequence and the first element of the optimal input sequence is applied to the plant. An illustration of the decentralized MPC-based formation trajectory planning is depicted in figure 2.
In the proposed formulation of MPC for formation trajectory planning, each vehicle formulates an optimization problem over a finite time horizon. In the optimization formulation, each vehicle takes its current position, $q_i = \begin{bmatrix} x_i & y_i & \theta_i \end{bmatrix}^T$, and the last known positions of neighboring vehicle within its detection region, $q_j = \begin{bmatrix} x_j & y_j & \theta_j \end{bmatrix}^T$, as initial condition. During the optimization process each vehicle optimizes independently its predicted trajectory to follow as close as possible the set of N samples of its future desired trajectory. In the following, the cost function utilized for MPC is presented.

A. Cost function formulation

Given the current state of vehicle i, i.e. $q_i(k)$, MPC is to compute the optimal input sequence which minimizes the following cost function:

$$J(q_i(k), u_i(k)) = \phi(\tilde{q}_i(N)) + \sum_{k=0}^{N-1} L(\tilde{q}_i(k), u_i(k)) \tag{11}$$

where k is the time step, $q_i(0)$ refers to the current state of vehicle i, $u_i(0)$ is the current input, N is the horizon length. $\tilde{q}_i(k) = \hat{q}_d^i(k) - q_i(k) - [d_i \ 0]^T$ where $\hat{q}_d^i(k) \in R^3$ is the

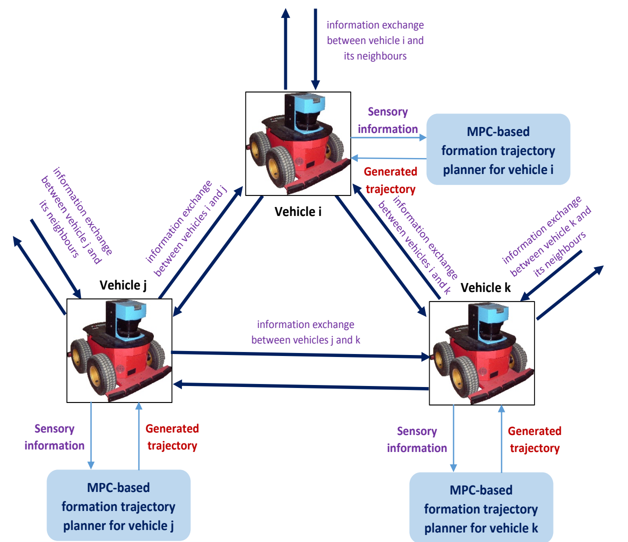

Fig. 2: Decentralized MPC-based formation trajectory planning.

estimation of the desired reference in the virtual structure, $d_i \in R^2$ is the offset from the reference, $\phi(\tilde{q}_i(N))$ and $L(\tilde{q}_i(k), u_i(k))$ is defined as follows:

$$\phi(\tilde{q}_i(N)) = \frac{1}{2} \tilde{q}_i^T(N) P \tilde{q}_i(N) \tag{12}$$

$$L(\tilde{q}_i(k), u_i(k)) = \frac{1}{2} \tilde{q}_i^T(k) Q \tilde{q}_i(k) + \frac{1}{2} u_i^T(k) R u_i(k) \tag{13}$$

such that P, Q and R are constant weighting matrices. To obtain the desired performance, the cost function (11) is solved subject to the following constraints:

1) Kinematic model equality constraints: Each vehicle must satisfy the discrete kinematic model presented in (1). This requirement is incorporated into the finite time horizon optimization problem as equality constraint as follows:

$$f(q_i(k), u_i(k)) - q_i(k+1) = 0 \tag{14}$$

2) Input constraints: For nonholonomic vehicles, input constraints are the bounded values for linear and angular velocities which are formulated as the following inequality:

$$S_u(u_{i,l}(k)) = |u_{i,l}(k)| - u_{\max}^{i,l} \leq 0 \tag{15}$$

where $u_{i,l}(k)$ is the l^{th} input of input vector $u_i(k)$, $u_{\max}^{i,l}$ is the bounded value for l^{th} input of the input vectors.

3) Inter-vehicle collision avoidance: To achieve collision avoidance, the following potential-like cost function is considered using the information from neighboring vehicles:

$$P_i^C(d_{ij}(k)) = \frac{\gamma_i^C}{\max\left(|d_{ij}(k)|^2 - r_c^2, \varepsilon\right)} \tag{16}$$

where d_{ij} is the distance between vehicle i and its neighbouring vehicle j, max is the maximum function, r_c is the safe inter-vehicle distance, and ε is a small positive constant for non-singularity.

4) Obstacle avoidance: To achieve obstacle avoidance, the following potential-like cost function is considered:

$$P_i^O\left(d_{io}(k)\right) = \frac{\gamma_i^O}{\max\left(\left|\min\left(d_{io}(k)\right)\right|^2 - r_o^2, \varepsilon\right)} \tag{17}$$

where d_{io} is the vector of distances between vehicle i and the detected obstacles, min is the minimum function, r_o is the safe distance to the obstacle.

B. Augmented cost function

The constraints can be incorporated into the original cost function (11) using Lagrange multipliers and Karush-Kuhn-Tucker (KKT) conditions, as follows:

$$\begin{aligned} J_{aug}^i = \phi\left(\tilde{q}_i(N)\right) + \sum_{k=0}^{N-1} \Big[& L\left(\tilde{q}_i(k), u_i(k)\right) \\ & + \lambda_i^T(k+1)\left(f\left(q_i(k), u_i(k)\right) - q_i(k+1)\right) \\ & + \tfrac{1}{2}\boldsymbol{\mathcal{S}}_u^T(u_i(k))\Gamma_i^U \boldsymbol{\mathcal{S}}_u(u_i(k)) + P_i^C\left(d_{ij}(k)\right) + P_i^O\left(d_{io}(k)\right) \Big] \end{aligned} \tag{18}$$

where λ_i is Lagrange multiplier vectors, Γ_i^U is a positive diagonal matrix, $P_i^C\left(d_{ij}(k)\right)$ and $P_i^O\left(d_{io}(k)\right)$ are defined by (16) and (17), respectively. $\boldsymbol{\mathcal{S}}_u(u_i(k))$ is defined as,

$$\boldsymbol{\mathcal{S}}_u(u_i(k)) = \left[S_u\left(u_{i,1}(k)\right), \ldots, S_u\left(u_{i,m}(k)\right)\right]^T \tag{19}$$

such that $S_u\left(u_{i,l}(k)\right)$ for $l = 1, \ldots, m$ is defined by (15). The Hamiltonian function is defined as follows:

$$\begin{aligned} H_k^i = \; & L\left(\tilde{q}_i(k), u_i(k)\right) + \lambda_i^T(k+1) f\left(q_i(k), u_i(k)\right) \\ & + \frac{1}{2}\boldsymbol{\mathcal{S}}_u^T(u_i(k))\Gamma_i^U \boldsymbol{\mathcal{S}}_u(u_i(k)) \\ & + P_i^C\left(d_{ij}(k)\right) + P_i^O\left(d_{io}(k)\right) \end{aligned} \tag{20}$$

Therefore, (18) can be rewritten as,

$$\begin{aligned} J_{aug}^i = \; & \phi\left(\tilde{q}_i(N)\right) - \lambda_i^T(N) q_i(N) + \sum_{k=0}^{N-1}\left[H_k^i - \lambda_i^T(k) q_i(k)\right] \\ & + \lambda_i^T(0) q_i(0) \end{aligned} \tag{21}$$

In order to find the desired input that minimizes the augmented cost function (21), the derivative of (21) is taken as follows:

$$\begin{aligned} dJ_{aug}^i = \; & \left[\frac{\partial \phi\left(\tilde{q}_i(N)\right)}{\partial \tilde{q}_i(N)} \frac{\partial \tilde{q}_i(N)}{\partial q_i(N)} - \lambda_i^T(N)\right] dq_i(N) \\ & + \lambda_i^T(0) dq_i(0) + \sum_{k=0}^{N-1}\Bigg[\Bigg\{\frac{\partial H_k^i}{\partial q_i(k)} + \frac{\partial H_k^i}{\partial \tilde{q}_i(k)} \frac{\partial \tilde{q}_i(k)}{\partial q_i(k)} \\ & + \frac{\partial H_k^i}{\partial d_{ij}(k)} \frac{\partial d_{ij}(k)}{\partial q_i(k)} + \frac{\partial H_k^i}{\partial d_{io}(k)} \frac{\partial d_{io}(k)}{\partial q_i(k)} \\ & - \lambda_i^T(k)\Bigg\} dq_i(k) + \frac{\partial H_k^i}{\partial u_i(k)} du_i(k)\Bigg] \end{aligned} \tag{22}$$

Hence, λ_k for $k = 0, \ldots, N$ is computed as follows:

$$\lambda_i^T(N) = \frac{\partial \phi\left(\tilde{q}_i(N)\right)}{\partial \tilde{q}_i(N)} \frac{\partial \tilde{q}_i(N)}{\partial q_i(N)} \tag{23}$$

$$\begin{aligned} \lambda_i^T(k) = \; & \frac{\partial H_k^i}{\partial q_i(k)} + \frac{\partial H_k^i}{\partial \tilde{q}_i(k)} \frac{\partial \tilde{q}_i(k)}{\partial q_i(k)} + \frac{\partial H_k^i}{\partial d_{ij}(k)} \frac{\partial d_{ij}(k)}{\partial q_i(k)} \\ & + \frac{\partial H_k^i}{\partial d_{io}(k)} \frac{\partial d_{io}(k)}{\partial q_i(k)} \end{aligned} \tag{24}$$

Therefore, (22) is simplified as follows:

$$dJ_{aug}^i = \sum_{k=0}^{N-1}\left[\frac{\partial H_k^i}{\partial u_i(k)} du_i(k)\right] + \lambda_i^T(0) dq_i(0) \tag{25}$$

where $\frac{\partial H_k^i}{\partial u_i(k)}$ is obtained by differentiation of (20) with respect to input vector as follows:

$$\begin{aligned} \frac{\partial H_k^i}{\partial u_i(k)} = \; & u_i^T(k) R + \lambda_i^T(k+1) \frac{\partial f\left(q_i(k), u_i(k)\right)}{\partial u_i(k)} \\ & + \boldsymbol{\mathcal{S}}_u^T(u_i(k))\Gamma_i^U \frac{\partial \boldsymbol{\mathcal{S}}_u(u_i(k))}{\partial u_i(k)} \end{aligned} \tag{26}$$

Using the gradient method, the following iterative formula can be utilized to obtain the optimal input sequence:

$$u_i^{New}(k) = u_i(k) - \Delta_i \frac{\partial H_k^i}{\partial u_i(k)} \tag{27}$$

where Δ_i is the step length. Algorithm 1 is presented to solve the constraint nonlinear optimization problem. Algorithm 1 produces the desired input sequence at each time instant for a future finite time horizon. Consequently, the first element of each sequence is utilized to generate the desired trajectory for each vehicle using the kinematic model (1).

VI. Simulations

A numerical simulation was performed to illustrate the effectiveness of the proposed MPC-based formation trajectory planning methodology for multiple nonholonomic vehicles. For the simulation, Matlab software was employed. The values of parameters used for simulation is presented in table I. For the first case, formation tracking was considered for 3 nonholonomic vehicles such that vehicles assigned to form a triangular formation while tracking the following time-varying reference:

TABLE I: The values of parameters used for simulation.

P	Q	R	N	Δ_i	k_e
I_3	$1e4.*I_3$	$5.*I_3$	30	0.0002	5

$$q_d(k) = \begin{bmatrix} 1 + 0.25k - 2\sin(0.1k) \\ 1 + 0.25k + 2\sin(0.1k) \end{bmatrix} \tag{28}$$

The communication topology between the vehicles is shown in figure 3. The total simulation time was 100s. The norm of estimation error of the desired reference for each vehicle is depicted in figure 4. The generated trajectories for vehicles are depicted in figure 5. For cases 2-4, formation tracking and obstacle avoidance was considered. As illustrated in figures 6-8 vehicles are assigned to form a triangular formation. The total simulation time was 60s.

To examine the proposed method in more severe condition, the desired reference is assigned to pass through an obstacle,

Algorithm 1 Algorithm for solving distributed MPC-based formation for multiple nonholonomic vehicles.

# n: the number of vehicles;
# ℓ_i: the iteration number;
# $J_{aug}^{i,\ell}$: the augmented cost function at iteration ℓ;
# N: the horizon length;
for $i = 1,\ldots,n$ **do**
  for $k = 1,\ldots,N$ **do**
    compute $\hat{q}_d^i(k)$ using (3);
  end for
  $\ell_i = 1$;
  while $\left|J_{aug}^{i,\ell+1} - J_{aug}^{i,\ell}\right| > \delta$ **do**
    for $k = 1,\ldots,N$ **do**
      compute $q_i(k)$ using (1);
    end for
    for $k = N,\ldots,1$ **do**
      compute $\lambda_i(k)$ using (23) and (24);
    end for
    for $k = 1,\ldots,N$ **do**
      compute $\frac{\partial H_k^i}{\partial u_i(k)}$ using (26);
    end for
    if $\left(J_{aug}^{i,\ell+1} - J_{aug}^{i,\ell}\right) < 0$ **then**
      update $u_i(k)$ for $k = 0,\ldots,N$ using (27);
    else
      reduce Δ_i;
    end if
    $\ell = \ell + 1$;
  end while
end for

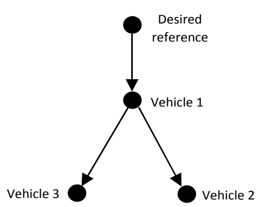

Fig. 3: Communication topology among the vehicles.

yields a local minima for vehicle 1. Unlike many approaches that cannot handle this situation, the proposed method results in an acceptable performance. The desired trajectory was considered as follows:

$$q_d(k) = \begin{bmatrix} 1+0.3k \\ 1+0.3k \end{bmatrix}. \qquad (29)$$

To consider various scenarios, obstacles are positioned in different geometrical patterns. It should be noted that even though some planned trajectories intersect in space (for example trajectories of vehicle 1 and 2 in figure 6), they do not intersect in time. To clarify this issue for the case 2 (i.e. figure 6) trajectories of vehicles are depicted in figure 9 in 3 dimension versus time.

VII. Conclusion

In this paper, the distributed formation trajectory planning for a group of nonholonomic vehicles has been presented.

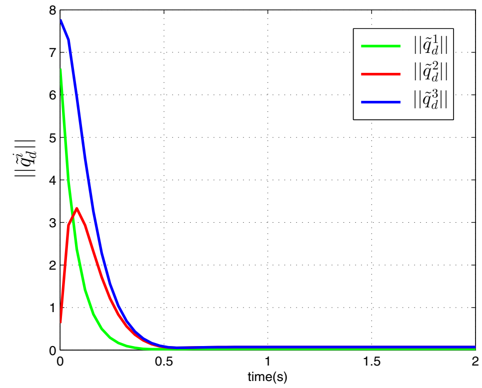

Fig. 4: The norm of estimation error of the desired reference.

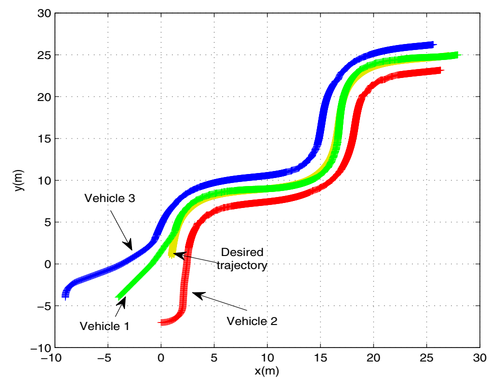

Fig. 5: Formation trajectory planning for the tracking of the time-varying reference.

A decentralized Model Predictive Control has been utilized for trajectory generation. The desired formation has been obtained using the virtual structure architecture. A distributed estimator has been proposed to deal with the limitation of virtual structure based formation method which is the necessity of access to the desired reference. Simulation results have been presented to illustrate the performance of the proposed methodology in producing optimal formation trajectory planning for multiple nonholonomic vehicles in static/moving obstacle based environments.

Appendix

Lemma A1: [22] Consider a digraph $\mathcal{G}$ with the corresponding laplacian matrix $\mathcal{L}$, which has a directed spanning tree; and a diagonal matrix $\mathcal{B}$ with nonnegative diagonal entries. The matrix $\mathcal{L}+\mathcal{B}$ is nonsingular, if and only if, at least one of the diagonal entries of $\mathcal{B}$ is positive.

Lemma A2: [23] Let $W = I - \varepsilon A$, such that $\varepsilon < \frac{1}{\max_i A_{ii}}$. If λ is an eigenvalue of A, then $1-\varepsilon\lambda$ is an eigenvalue of W. Consequently, if A has positive eigenvalues then all the eigenvalues of W lie in the interior of the unit circle.

References

[1] R.M. Murray, "Recent research in cooperative control of multi-vehicle systems", *J. Dyn. Sys., Meas., Contr.*, 129(5), pp. 571-583, 2007.

[2] W.B. Dunbar and R.M. Murray, "Distributed receding horizon control for multi-vehicle formation stabilization," *Automatica*, 42(4), pp. 549-558, 2006.

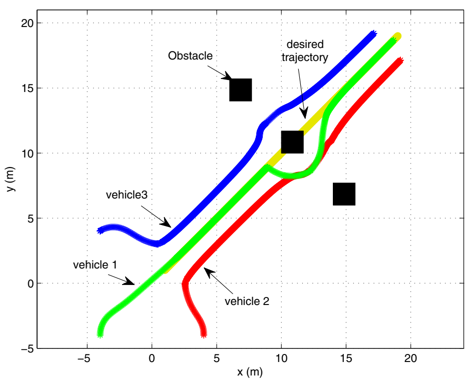

Fig. 6: Formation trajectory planning in the presence of obstacles.

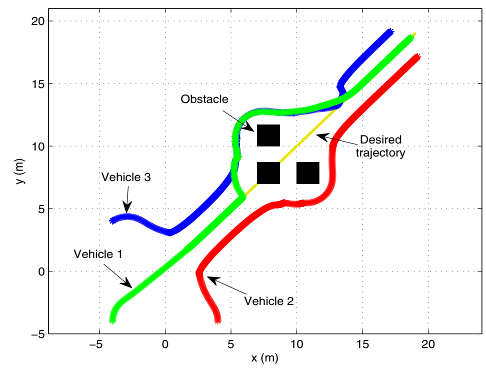

Fig. 8: Formation trajectory planning in the presence of obstacles.

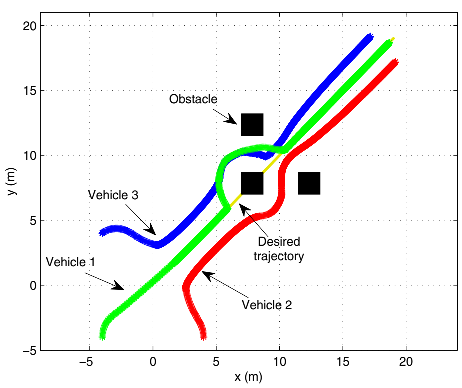

Fig. 7: Formation trajectory planning in the presence of obstacles.

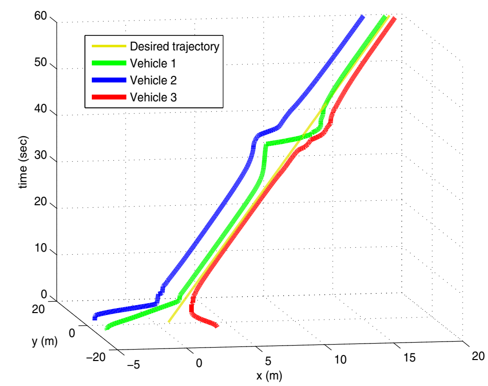

Fig. 9: Trajectories of vehicles versus time.

[3] R. Haghighi, C.C. Cheah, "Multi-group coordination control for robot swarms," *Automatica*, 48(10) pp. 2526-2534, 2012.

[4] A. Gasparetto, P. Boscariol, A. Lanzutti, and R. Vidoni, "Trajectory planning in robotics," *Math. Comput. Sci.*, 6, pp. 269-279, 2012.

[5] R.M. Murray, and S.S. Sastry, "Nonholonomic motion planning: steering using sinusoids," *IEEE Trans. Automat. Contr.*, 38, pp. 700-716, 1993.

[6] D. Tilbury, R.M. Murray, and S.S. Sastry, "Trajectory generation for the N-trailer problem using goursat normal form," *IEEE Trans. Automat. Contr.*, 40(5), pp. 802-819, 1995.

[7] A. Kelly, and B. Nagy, "Reactive nonholonomic trajectory generation via parametric optimal control" *Int. J. Robot. Res.*, 22(7-8), pp. 583-601, 2003.

[8] P. Ogren, and N. Leonard, "A convergent dynamic window approach to obstacle avoidance" *IEEE Trans. Robot.*, 21(2), pp. 188-195, 2005.

[9] A. Widyotriatmo, B. Hong, and K.-S. Hong , "Predictive navigation of an autonomous vehicle with nonholonomic and minimum turning radius constraints" *J. Mech. Sci. Tech.*, 23(1), pp. 381-388, 2009.

[10] W. Ren, R. W. Beard, and E. Atkins, "Information Consensus in Multivehicle Cooperative Control", *IEEE Contr. Syst. Mag.*, 27(2), pp. 71-82, 2007.

[11] Y. Hong, G. Chen, and L. Bushnellc, "Distributed observers design for leader-following control of multi-agent networks", *Automatica*, 44(3), pp. 846850, 2008.

[12] H. Su, X. Wang, and Z. Lin, "Flocking of multi-agents with a virtual leader", *IEEE Trans. Automat. Contr.*, 54(2), pp. 293-307, 2009.

[13] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: algorithms and theory", *IEEE Trans. Automat. Contr.*, 51(3), pp. 401-420, 2006.

[14] K. Choi, S. J. Yoo, J. B. Park, and Y. H. Choi,"Adaptive formation control in absence of leader's velocity Information", *Automatica*, 4(4), pp. 521-528, 2010.

[15] W. Dong,"Tracking control of multiple-wheeled mobile robots with limited information of a desired trajectory", *IEEE Trans. Robot.*, 286(1), pp. 262268, 2012.

[16] R. Haghighi, C.C. Cheah, "Distributed shape formation of multi-agent systems," *12th Int. Conf. on Contr. Autom. Robot. & Vision*, pp. 1466-1471, 2012.

[17] G. Klancar, and I. Skrjanc, "Tracking-error model-based predictive control for mobile robots in real time," *Robot. Auton. Syst.*, 55(6), pp. 460-469, 2007.

[18] H.J. Kim, D.H. Shim, and S. Sastry, "Nonlinear model predictive tracking control for rotorcraft-base unmanned aerial vehicles," *Proc. of the American Contr. Conf.*, pp. 3576-3581, 2002.

[19] P. Falcone, F. Borrelli, J. Asgari, H.E. Tseng, and D. Hrovat, "Predictive active steering control for autonomous vehicle systems," *IEEE Trans. Contr. Sys. Tech.*, 15(3), pp. 566-580, 2007.

[20] Y. Yoon, J. Shin, H.J. Kim, Y. Park, and S. Sastry, "Model-predictive active steering and obstacle avoidance for autonomous ground vehicles," *Contr. Eng. Pract.*, 17(7), pp. 741-750, 2009.

[21] D. Fox, W. Burgard, and S. Thrun, "The Dynamic Window approach to collision avoidance" *IEEE Robot. and Autom. Mag.*, 4(1), pp. 23-33, 1997.

[22] J. Hu, and Y. Hong, "Leader-follower coordination of multi-agent systems with coupling time delays", *Physica A*, 374(7346), pp. 853-863, 2007.

[23] S.K. Mazumder, "Wireless networking based control", *Springer Science and Business Media*, 2011.