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Real-Time Damping Estimation for Variable Impedance Actuators

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Abstract— **Recently-developed variable damping mechanisms have been exploited as a complement to compliant actuators. While accurate knowledge and control of generated damping is essential for achieving the desired performance, no physical sensor measuring the damping exists. This work introduces a novel non-model-based approach for the estimation of time-variant damping for variable impedance actuation systems. The approach is based only on torque and position/velocity measurements; without the knowledge of system's inputs, to ensure the estimation of both intentional and unintentional changes. Hence, a recursive least square estimator, modified for achieving a proper convergence for the estimation of time-variant parameters, is exploited. Experiments on a variable physical damping actuator are also presented to validate the performance of proposed approach.**

I. Introduction

The inclusion of variable damping mechanisms in elastic systems has recently attracted increasing attention among robotic researchers . This is particularly true for compliant actuators, which have been identified as having unique potential for the development of the next generation of robots. It is believed that these future robots will have greater contact robustness [1], improved energy efficiency [2], [3], and will facilitate safer human-robot interaction across a range of different environments [4], [5]. However, introducing elastic elements does create problems [6], particularly due to undesirable vibrations resulting from the under-damped dynamics. These unintentional oscillations limit the stability margin and decrease the tracking accuracy [7] but this can be countered by integrating dissipative elements into compliant actuation system [8].

Although the difficulties caused by passive elasticity can be considerably reduced by using variable stiffness actuators (VSAs) [9], active damping control schemes are required to achieve desirable damped robot behavior [10]. However, these schemes requires accurate dynamic model and proper velocity feedback to show the desirable dissipative operation.

Researchers, have integrated dissipative elements into compliant actuators using a variety of approaches. Fig. 1 displays different layouts of adding dissipative elements to compliant actuators [9]. Laffranchi *et al.* [11] embedded a piezoelectric based friction damping mechanism in parallel to the passive elasticity, Fig. 1a. This technique regulated the dissipation by controlling the normal force applied to the friction plates. Radulescu *et al.* [12] employed electrical inductance effects in a DC motor to create a mechanical damping effect on the link that was independent of the motor motion, Fig. 1b. Catalano *et al.* [13] also added a variable viscous damping module that is connected between the link and the ground.

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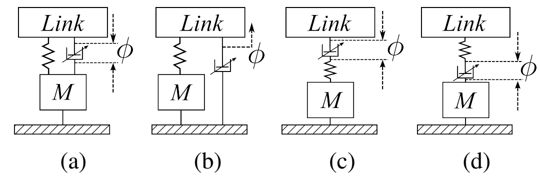

Fig. 1: different mechanical models for the inclusion of damping in compliant actuators: (a) damping in parallel with spring (b) damping acting on the link (c) and (d) damping in series with spring.

For each of the variable stiffness/damping systems above, a major issue is how to control the variable stiffness/damping although there is no physical sensor to directly measure the transmission stiffness/damping; hence open-loop model-based controllers are typically used to control these physical properties with the transmission stiffness/damping being estimated from available readings. However, although there have been several studies on stiffness/damping estimation in contact situations [14], [15] that focus on obtaining time-invariant parameters, there has been limited investigation of the real-time estimation of time-variant transmission stiffness/damping in compliant joints. Grioli and Bicchi [16] proposed a real-time parametric method for measuring variable stiffness based on measured deflection angle and elastic torque. Flacco *et al.* [17] presented an on-line stiffness estimation approach based on a recursive least square method that uses measured deflection angle and elasticity torque measured from the sensor or estimated from a residual-based method using motor position angle and motor-side dynamic parameters. However, the estimation of transmission damping has rarely been studied.

This paper studies damping estimation in variable damping actuators by employing a robust recursive least square algorithm. The method is based only on the measured dissipation torque and the transmission deflection velocity, which is derived from the measured deflection angle. The performance and accuracy of the proposed algorithm is validated in real systems, consisting of a variable physical damping actuator (VPDA) that modulates the damping by regulating the normal force applied to friction plates. Experimental results showing the accuracy of estimation are presented.

The paper is organized as follows: in Section II the damping estimation algorithm is described. The performance of the proposed algorithm is evaluated using a VPDA system in Section III, first describing the experimental set-up, modeling and construction, and then showing the results. Finally in Section IV the conclusion is discussed, together with future work.

II. Damping Estimation

A. Problem Statement

Consider a generic damping element embedded in a mechanical system, generating a dissipation torque τ_d which depends on its displacement ϕ and velocity $\dot{\phi}$, and the damping element's control input variable u. The corresponding damping value $D > 0$ of this element can be expressed as follows

$$D(\phi, \dot{\phi}, u) = \frac{\partial \tau_d(\phi, \dot{\phi}, u)}{\partial \dot{\phi}}, \tag{1}$$

in which the dissipation torque is assumed to be an odd function with respect to the velocity

$$\tau_d(\phi, -\dot{\phi}, u) = -\tau_d(\phi, \dot{\phi}, u). \tag{2}$$

Using the dissipation torque and velocity measured or computed from the system's readings, the aim is to obtain the damping value using a recursive least square algorithm (RLS). Consider an estimation problem in which the output signal, which is the dissipation torque τ_d, is recursively estimated by a linear finite impulse response adaptive filter with n unknown coefficients $\boldsymbol{w} = [w_1, ..., w_n]^T$ and n input signals $\boldsymbol{v} = [v_1, ..., v_n]^T$ which can be functions of the other readings of the system such as displacement ϕ, velocity $\dot{\phi}$ and the damping element's control input variable u. The output and input signals can be expressed by the linear model

$$\tau_d(i) = \boldsymbol{v}^T(i)\boldsymbol{w}^* + \xi(i), \tag{3}$$

where $\boldsymbol{w}^*$ is the optimal solution in the mean-square sense, ξ is a zero-mean white Gaussian noise sequence with variance σ^2, and i is a non-negative integer denoting the sample instances.

Defining the dissipation torque as a polynomial function of the damper's velocity that satisfies condition (2), the elements of the input signal vector are expressed by

$$v_j = \dot{\phi}^{2j-1} \quad \text{for} \quad j = 1, ..., n. \tag{4}$$

The damping value can then be derived from (1) as follows

$$D = \boldsymbol{v}'^T(i)\boldsymbol{w}, \tag{5}$$

where $\boldsymbol{v}' = [v'_1, ..., v'_n]^T$ and its elements are derived from

$$v'_j = (2j-1)\dot{\phi}^{2j-2} \quad \text{for} \quad j = 1, ..., n. \tag{6}$$

Given m data points, $i = 1, ..., m$ where $m > n$, the relation (3) can be presented as an over-constrained linear system

$$\boldsymbol{\tau}_d = \mathbf{V}\boldsymbol{w}^* + \boldsymbol{\xi}, \tag{7}$$

where $\boldsymbol{\tau}_d = [\tau_d(1), ..., \tau_d(m)]^T$, $\boldsymbol{\xi} = [\xi(1), ..., \xi(m)]^T$ and $\mathbf{V} = [\boldsymbol{v}^T(1), ..., \boldsymbol{v}^T(m)]^T \in \Re^{m \times n}$.

B. Recursive Least Square Algorithm

Given an estimate of coefficients $\hat{\boldsymbol{w}}$, the prediction error vector $\boldsymbol{\epsilon} \in \Re^m$ is defined by

$$\boldsymbol{\epsilon} = \boldsymbol{\tau}_d - \mathbf{V}\hat{\boldsymbol{w}}. \tag{8}$$

Provided that $\mathbf{V}$ is non-singular, the best value of $\hat{\boldsymbol{w}}$ is to minimize the following cost function

$$J(\hat{\boldsymbol{w}}) = \boldsymbol{\epsilon}^T\boldsymbol{\epsilon}, \tag{9}$$

with the least square (LS) principle as follows

$$\hat{\boldsymbol{w}} = (\mathbf{V}^T\mathbf{V})^{-1}\mathbf{V}^T\boldsymbol{\tau}_d. \tag{10}$$

To perform the estimation on-line, the least square method should be described in a recursive manner. The RLS algorithm is given by [18]

$$\epsilon(i) = \tau_d(i) - \boldsymbol{v}^T(i)\hat{\boldsymbol{w}}(i-1), \tag{11}$$

$$r(i) = \boldsymbol{v}^T(i)\mathbf{P}(i-1)\boldsymbol{v}(i), \tag{12}$$

$$\boldsymbol{k}(i) = \frac{\mathbf{P}(i-1)\boldsymbol{v}(i)}{1 + r(i)}, \tag{13}$$

$$\hat{\boldsymbol{w}}(i) = \hat{\boldsymbol{w}}(i-1) + \boldsymbol{k}(i)\epsilon(i), \tag{14}$$

$$\mathbf{P}(i) = \Big[\mathbf{I} - \boldsymbol{k}(i)\boldsymbol{v}^T(i)\Big]\mathbf{P}(i-1), \tag{15}$$

where r is the modified Kalman residual covariance, $\boldsymbol{k} \in \Re^n$ is the modified Kalman gain vector, $\mathbf{P} \in \Re^{m \times m}$ denotes the covariance matrix of the prediction error, and $\mathbf{I} \in \Re^{m \times m}$ is the identity matrix.

C. Proposed Damping Estimation Algorithm

The algorithm in this work comes from a combination of the RLS method. The form of estimation algorithm presented in (11)-(15), resulting from objective function (9), is typically suitable for constant linear systems, and a weighting factor needs to be incorporated in the RLS algorithm to obtain faster convergence in the identification of time-variant systems [19]. The exponential weighting factor, named forgetting factor, is included to discount older signals and emphasis more recent ones, thereby improving the adaptation quality for time-variant plants. By including the forgetting factor, the cost function is rewritten as [20]

$$J(\hat{\boldsymbol{w}}) = \boldsymbol{\epsilon}^T\mathbf{R}^{-1}\boldsymbol{\epsilon}, \tag{16}$$

where $\mathbf{R} = diag(\lambda^{m-1}, ..., \lambda^0)$ and $\lambda \in (0, 1]$ is the forgetting factor.

To further enhance the estimation tracking performance of the RLS method, a directional forgetting RLS algorithm is introduced that forgets the past data solely in the direction from where new information comes [21]. The exponential and directional forgetting RLS algorithm, modified by Bittanti [22] for achieving a better estimation of time-variant parameters, can be described by incorporating a correction factor in the update of the covariance matrix as follows

$$\mathbf{P}(i) = \left[\mathbf{I} - \frac{\mathbf{P}(i-1)\boldsymbol{v}(i)\boldsymbol{v}^T(i)}{\beta^{-1}(i) + r(i)}\right]\mathbf{P}(i-1) + \delta\mathbf{I}, \tag{17}$$

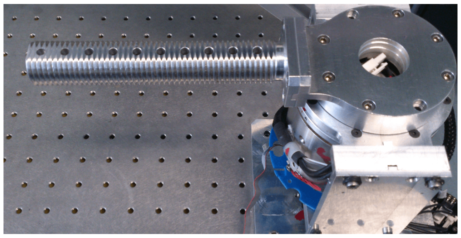

Fig. 2: An image of the VPDA system used for experiments

where $\delta \in [0, 0.01]$ is the correction factor and β is an auxiliary variable defined as

$$\beta(i) = \begin{cases} \lambda(i) - \dfrac{1-\lambda(i)}{r(i)} & \text{if } r(i) > 0 \\ 1 & \text{if } r(i) = 0 \end{cases} \quad (18)$$

When $\lambda = 1$, no forgetting factor is applied. However, any reduction in this parameter increases the effect of latest data on the estimation process as well as the sensitivity to noise, which is not desirable [23]. To address this, a variable forgetting factor that monitors prediction error was introduced. This factor decreases to a certain value λ_{min} when the prediction error grows rapidly, and increases to the maximum value, $\lambda = 1$ when the prediction error is small. The variable forgetting factor is expressed by [24]

$$\lambda(i) = \lambda_{min} + (1 - \lambda_{min}) \cdot 2^{L(i)}, \quad (19)$$

where λ_{min} is the minimum value chosen for the forgetting factor and

$$L(i) = -round(\rho\epsilon^2(i)/\sigma(i)), \quad (20)$$

in which ρ is a design parameter, λ_c is another constant forgetting factor, $round(.)$ rounds the element of $(.)$ to the nearest integer, and σ is the energy of a priori estimation error defined for moderating the sensitivity of the forgetting factor to the dynamic range of the error. It is updated by

$$\sigma(i) = \lambda_c \sigma(i-1) + \epsilon^2(i). \quad (21)$$

The proposed algorithm is based on the use of a variable weighting factor (19)-(21) in the exponential and directional forgetting RLS algorithms of (11)-(14) and (17)-(18).

III. Case Study

The performance of proposed damping estimator is experimentally validated using the VPDA unit introduced in [11] and named CompAct™, see Fig. 2.

A. Experimental System Description

The dissipative part of this system, is composed of a semi-active friction damper made of a flat steel plate acting against a friction ring made of Kevlar. This mechanism is actuated by four Noliac SCMAP04 piezoelectric actuators. The force on the friction plates is controlled using a dynamic compensation scheme, called a time-delay estimation [25]. This gives robust and accurate tracking performance. The transmission torque is measured using a customized strain-gauge-based sensor, and the relative rotational displacement of the friction plates is measured using a 12-bit encoder. Considering the mechanical model of this system, having measured the transmission torque τ_t and displacement ϕ, the dissipation torque is calculated from

$$\tau_d = \tau_t - Kz, \quad (22)$$

where K is the stiffness of compliant element.

The velocity is subsequently computed using a second-order differentiator filter, implemented on a digital signal processing (DSP) driver board, with a time constant of $0.01s$. Data acquisition of the system states, at the DSP, was 1 kHz. Communication between the DSP board and the high level controller, implemented on a PC104 used a real-time version of Linux (Xenomai), operating at 1 kHz.

B. Modeling and realization

In this part, the dissipation torque model used for the open-loop damping control is realized, and the accuracy of the identified model is evaluated. The dissipation torque τ_c generated by friction dampers can be modeled as

$$\begin{aligned} \tau_c(\dot{\phi}, F_n) &= C_p\dot{\phi} + C_a F_n \dot{\phi} \\ &+ \mu_p sign(\dot{\phi}) + \mu_a F_n sign(\dot{\phi}), \end{aligned} \quad (23)$$

where F_n is the normal force on the friction plates, C_p and μ_p are intrinsic viscous and coulomb friction coefficients and therefore independent of the force F_n, while C_a and μ_a are those generated by the application of this force.

Given a desired linear viscous damping D_d, the resulting dissipation torque is obtained from

$$\tau_d = D_d\dot{\phi}. \quad (24)$$

To achieve a dissipation act corresponding to the desired linear viscous damping D_d, the torque applied by the friction damper τ_c should be equal to that generated by viscous damper τ_d. From the equality $\tau_c = \tau_d$, (24) and (23), the desired normal force $F_{n,d}$ is obtained as

$$F_{n,d} = \frac{D_d - C_p - \mu_p\gamma(\dot{\phi})}{C_a + \mu_a\gamma(\dot{\phi})}, \quad (25)$$

where γ is an even, positive semi-definite function defined by[1]

$$\gamma(\dot{\phi}) = \frac{sign(\dot{\phi})}{\dot{\phi}}. \quad (26)$$

Due to the fact that $F_n \geq 0$, the aforementioned relation of the force (25) is valid as far as

$$\frac{D_d - C_p}{\mu_p} \geq \gamma(\dot{\phi}), \quad (27)$$

that expresses a condition for the minimum passive viscous damping $D_d \geq C_p$ and for the minimum velocity $\dot{\phi} \geq \mu_p/(D_d - C_p)$. In other words, given a desired viscous damping larger than the passive viscous damping of the system, the passive friction torque is higher than desired dissipation torque at very low velocities.

[1]It can be shown that $\lim_{\dot{\phi}\to\infty} \gamma(\dot{\phi}) = 0$ and $\lim_{\dot{\phi}\to 0^{\pm}} \gamma(\dot{\phi}) = \infty$.

TABLE I: Identified friction parameters.

$C_p[N.m.s/rad]$	$C_a[m.s/rad]$	$\mu_p[N.m]$	$\mu_a[m]$
0.0969	0.003253	0.4218	0.01416

TABLE II: Model Error for different tested force on the friction plates.

$F_n[N]$	0	55	135	230
$NRMSE$	22.83%	13.66%	17.87%	19.79%

To identify the model parameters, a set of simple experiments was carried out. Constant normal forces with magnitudes of 0 N, 55 N, 135 N and 230 N were applied on friction plates while harmonic relative motion of them were exerted. The batch LS solution (10) is used in which the vector of coefficients and input signals are defined by

$$\boldsymbol{w} = [C_p, C_a, \mu_p, \mu_a]^T, \tag{28}$$

$$\boldsymbol{v} = [\dot{\phi}, F_n\dot{\phi}, sign(\dot{\phi}), F_n sign(\dot{\phi})]^T. \tag{29}$$

The obtained parameters are recorded in Table I. The measured torque compared to the one estimated through the model is presented in Fig. 3. The normalized root-mean-square error (NRMSE) of the estimation is reported in Table II. This index is defined by

$$NRMSE = \frac{100}{max(\boldsymbol{\tau}_d) - min(\boldsymbol{\tau}_d)}\sqrt{\frac{1}{m}\boldsymbol{\epsilon}^T\boldsymbol{\epsilon}}. \tag{30}$$

To have an exploration into the accuracy of the friction model, change in error $\boldsymbol{\epsilon}$ normalized by the range of torque versus velocity is demonstrated in Fig. 4. It shows that the error in estimated model is mainly due to stiction occurring at low velocities. Excluding stiction zone, the normalized error is smaller than 10% provided that the force on the friction plates is not very small; however it grows up to 30% when forces of only few Newtons are exerted.

C. Damping Estimation Experiments

Experiments were carried out to evaluate the presented estimation algorithm. The excitation of the system is performed

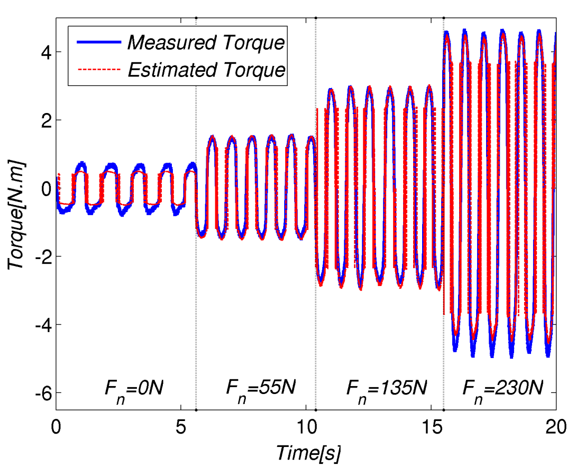

Fig. 3: Measured and Estimated dissipation torque versus time.

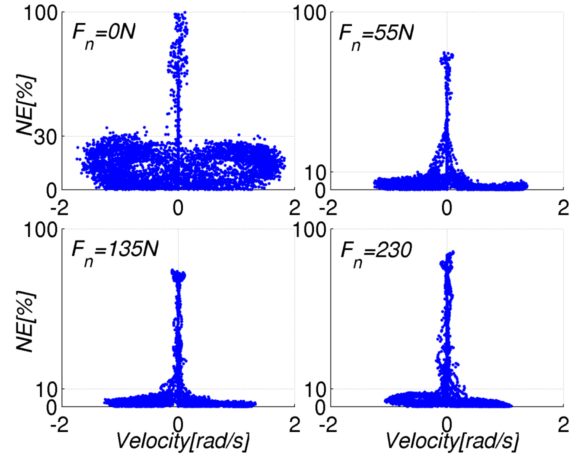

Fig. 4: Change in normalized error (NE) versus velocity for different forces on the friction plates.

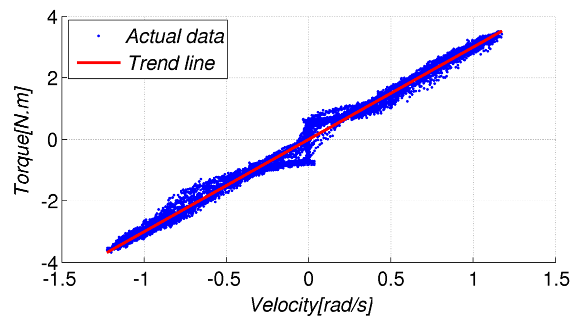

Fig. 5: Variation in dissipation torque versus the relative velocity of friction plates.

through manual motion of the link while the motor is position controlled to maintain a fixed position. To achieve a desired viscous damping D_d by this system, the normal force on the friction plates is modulated according to the force reference defined by (25). The parameters of the proposed estimator are set as follows: the initial value of the covariance matrix is chosen as $\mathbf{P}(0) = 10^3\mathbf{I}$ and that of elements of coefficients vector $\hat{\boldsymbol{w}}_\vartheta$ is simply set to one, the minimum forgetting factor and the constant forgetting factor were $\lambda_{min} = \lambda_c = 0.95$, the unity zone design parameter was set to $\rho = 10^3$, the correction factor was $\delta = 10^{-4}$, $\sigma(0) = 1$ and a third-order polynomial, $n = 2$, was chosen for the simplicity of implementation.

1) Constant Damping: The first experiment consists in estimating a constant viscous damping behavior. The value considered for the desired damping is $D_d = 3$ Nms/rad. Fig. 5 reports variation in dissipation torque versus change in velocity. Since a constant damping was regulated, based on (1), the slope of the trend-line of this graph represents a suitable approximation on real damping of the system. The overall actual damping of $D = 3.02$ Nms/rad is obtained with the NRMSE of 2.74%. Estimation results were also obtained by using the proposed approach with the NRMSE of 2.89% which is quite close to NRMSE of obtained overall damping (2.74%). The changes in the force on the friction plates and the relative velocity of them versus time are

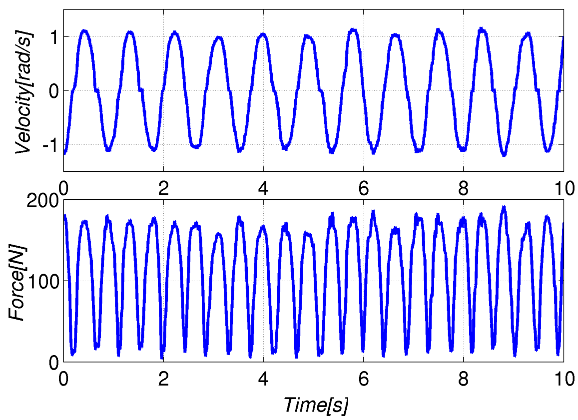

Fig. 6: Time history of the normal force and the relative velocity of clutch plates.

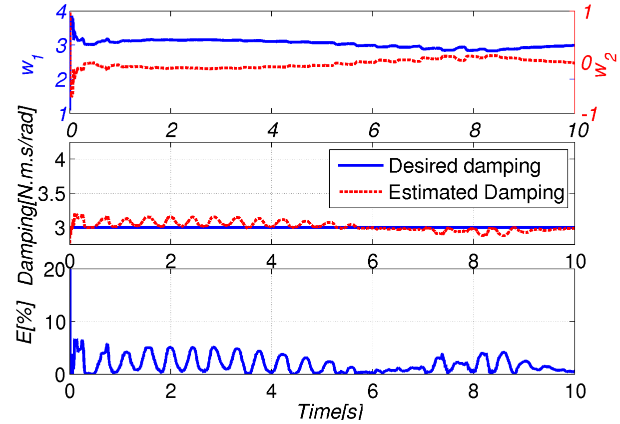

Fig. 8: Change in polynomial coefficients, i.e. w_1 and w_2 which are respectively denoted by (—) and (– –) in top graph, reference and estimated damping values and the error between them versus time.

demonstrated in Fig. 6. Estimated torque in compared to the measured one, and also the normalized error between these two, are presented in Fig. 7. It shows that the accuracy of estimation as the normalized error is below 1%.

Fig. 8 represents the change in the coefficients of estimated polynomial, the reference damping and the estimated one, and the error between these two. Results describes this error reaches a maximum of about 5% which is mostly due to unmodeled friction around zero-velocity area. The effect of stiction on generated dissipation torque can be also observed in Fig. 5.

2) Time-Variant Damping: In this experiment, the normal force on the friction plates is modulated based on producing a time-variant damping. The desired damping is defined by

$$D_d(t) = 1 + 1.4sin^2(0.05\pi t) + 1.7sin^2(0.1\pi t). \qquad (31)$$

Using the presented algorithm, damping estimation results are calculated with the NRMSE of 3.17%. Fig. 9 demonstrates the force on the friction plates, the measured and estimated torque, in addition to the corresponding normalized error between measured and estimated torques. The error in this experiment is also below 1% validating the accuracy of estimation when compared to the actual value.

The time history of desired damping in compared to measured value and the error between these two values, in addition to the polynomial coefficients, are presented in Fig. 10. It can be seen that error between the reference value and estimated one varies from lower than 10% in higher damping values to about 30% in lower magnitudes which is comparable to the accuracy of identified friction model in low and high forces, which was illustrated in Fig. 4.

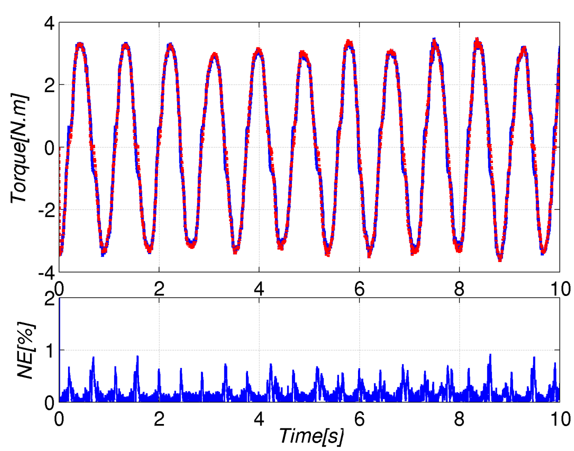

Fig. 7: Time history of measured (—) and estimated (–·–) damping torque and the normalized error between them.

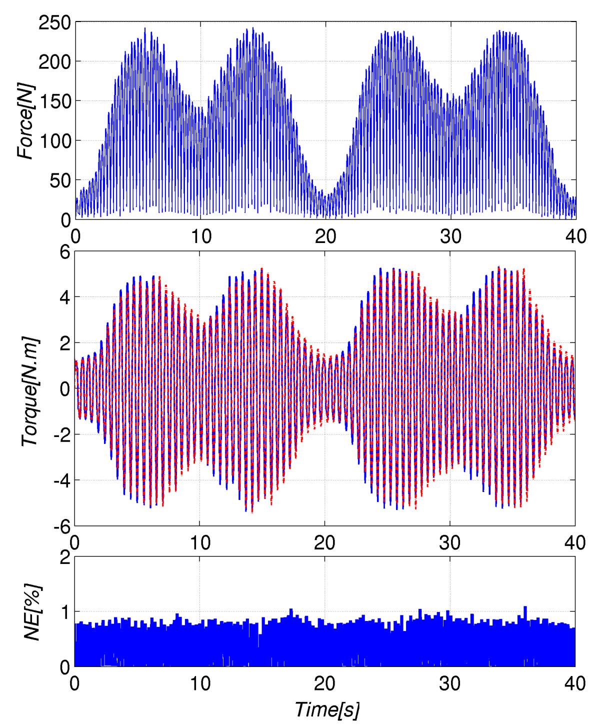

Fig. 9: Time history of normal force in top graph, measured (—) and estimated (– –) dissipation torque in middle graph, and the normalized error between measured and estimated values in bottom graph.

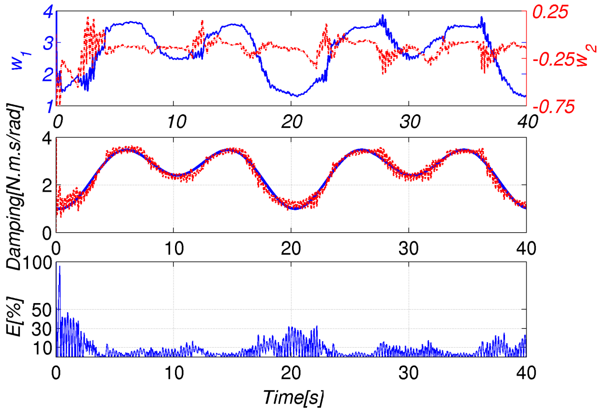

Fig. 10: Time history of polynomial coefficients, i.e. w_1 and w_2 which are respectively denoted by (−) and (−−) in top graph, reference (−) and estimated (−−) damping in middle graph, and the error between these two variables.

IV. Conclusion and Future Works

This article proposed a novel non-model based parametric estimator developed to measure the damping of a variable damping system both when it is time-variant or constant. This method constructs the dissipation torque polynomial as a function of the velocity, and the parameters of this estimator, i.e. polynomial coefficients, are updated based on the torque and velocity readings of the system. The performance of this approach in estimating the damping at various low and high frequencies were presented, and the effect of the main design parameter, i.e. forgetting factor, was analyzed. The method was experimentally implemented and validated on a VPDA unit controlled to produce both time-variant and constant damping. The results confirmed the efficacy of the method in estimating the damping level in real systems. Future work of the authors will focus on employing the proposed approach on designing an adaptive controller for accurately controlling the variable damping actuator.

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References

[1] G. A. Pratt and M. M. Williamson, “Series elastic actuators,” in *Intelligent Robots and Systems 95. ’Human Robot Interaction and Cooperative Robots’, Proceedings. 1995 IEEE/RSJ International Conference on*, vol. 1, 1995, pp. 399–406 vol.1.

[2] M. Garabini, A. Passaglia, F. Belo, P. Salaris, and A. Bicchi, “Optimality principles in variable stiffness control: The VSA hammer,” in *Intelligent Robots and Systems (IROS), 2011 IEEE/RSJ International Conference on*, 2011, pp. 3770–3775.

[3] L. Chen, M. Garabini, M. Laffranchi, N. Kashiri, N. G. Tsagarakis, A. Bicchi, and D. G. Caldwell, “Optimal Control for Maximizing Velocity of the CompAct Compliant Actuator,” in *Robotics and Automation (ICRA), 2013 IEEE International Conference on*, Karlsruhe (Germany), 2013, pp. 516–522.

[4] N. G. Tsagarakis, M. Laffranchi, B. Vanderborght, and D. G. Caldwell, “A compact soft actuator unit for small scale human friendly robots,” in *Robotics and Automation, 2009. ICRA’09. IEEE International Conference on*. IEEE, 2009, pp. 4356–4362.

[5] M. Laffranchi, N. G. Tsagarakis, F. Cannella, and D. G. Caldwell, “Antagonistic and series elastic actuators: a comparative analysis on the energy consumption,” in *Intelligent Robots and Systems, 2009. IROS 2009. IEEE/RSJ International Conference on*. IEEE, 2009, pp. 5678–5684.

[6] N. Kashiri, N. G. Tsagarakis, M. Laffranchi, and D. G. Caldwell, “On the stiffness design of intrinsic compliant manipulators,” in *Advanced Intelligent Mechatronics (AIM), 2013 IEEE/ASME International Conference on*. IEEE, 2013, pp. 1306–1311.

[7] M. Laffranchi, L. Chen, N. G. Tsagarakis, and D. G. Caldwell, “The role of physical damping in compliant actuation systems,” in *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, Oct. 2012, pp. 3079–3085.

[8] M. Laffranchi, N. G. Tsagarakis, and D. G. Caldwell, “Analysis and Development of a Semiactive Damper for Compliant Actuation Systems,” *Mechatronics, IEEE/ASME Transactions on*, vol. 18, no. 2, pp. 744–753, 2013.

[9] B. Vanderborght and *et al.*, “Variable impedance actuators: A review,” *Robotics and Autonomous Systems*, vol. 61, no. 12, pp. 1601–1614, 2013.

[10] F. Petit and A. Albu-Schaffer, “State feedback damping control for a multi DOF variable stiffness robot arm,” *Robotics and Automation (ICRA), 2011 IEEE International Conference on*, pp. 5561–5567, 2011.

[11] M. Laffranchi, N. Tsagarakis, and D. G. Caldwell, “A compact compliant actuator (CompAct) with variable physical damping,” in *Robotics and Automation (ICRA), 2011 IEEE International Conference on*. IEEE, 2011, pp. 4644–4650.

[12] A. Radulescu, M. Howard, D. J. Braun, and S. Vijayakumar, “Exploiting variable physical damping in rapid movement tasks,” in *Advanced Intelligent Mechatronics (AIM), 2012 IEEE/ASME International Conference on*. IEEE, 2012, pp. 141–148.

[13] M. Catalano, G. Grioli, M. Garabini, F. W. Belo, A. di Basco, N. Tsagarakis, and A. Bicchi, “A Variable Damping Module for Variable Impedance Actuation,” in *Robotics and Automation (ICRA), 2012 IEEE International Conference on*. IEEE, 2012, pp. 2666–2672.

[14] D. Erickson, M. Weber, and I. Sharf, “Contact stiffness and damping estimation for robotic systems,” *The International Journal of Robotics Research*, vol. 22, no. 1, pp. 41–57, 2003.

[15] N. Diolaiti, C. Melchiorri, and S. Stramigioli, “Contact impedance estimation for robotic systems,” *Robotics, IEEE Transactions on*, vol. 21, no. 5, pp. 925–935, 2005.

[16] G. Grioli and A. Bicchi, “A real-time parametric stiffness observer for VSA devices,” in *Robotics and Automation (ICRA), 2011 IEEE International Conference on*. IEEE, 2011, pp. 5535–5540.

[17] F. Flacco, A. De Luca, I. Sardellitti, and N. G. Tsagarakis, “On-line estimation of variable stiffness in flexible robot joints,” *Int. J. Rob. Res.*, vol. 31, no. 13, pp. 1556–1577, Nov. 2012.

[18] D. J. Sandoz and B. H. Swanick, “A recursive least-squares approach to the on-line adaptive control problem,” *International Journal of Control*, vol. 16, no. 2, pp. 243–260, 1972.

[19] R. M. Johnstone, C. Richard Johnson, R. R. Bitmead, and B. Anderson, “Exponential convergence of recursive least squares with exponential forgetting factor,” *Systems & Control Letters*, vol. 2, no. 2, pp. 77–82, 1982.

[20] L. Ljung, “System identication: theory for the user,” *Preniice Hall Inf and System Sciencess Series, New Jersey*, vol. 7632, 1987.

[21] M. Kárný, A. Halousková, J. Böhm, R. Kulhavý, and P. Nedoma, “Design of linear quadratic adaptive control: Theory and algorithms for practice,” *Kybernetika*, vol. 21, no. 7, pp. 1a–3, 1985.

[22] S. Bittanti, P. Bolzern, and M. Campi, “Exponential convergence of a modified directional forgetting identification algorithm,” *Systems & Control Letters*, vol. 14, no. 2, pp. 131–137, 1990.

[23] R. M. Canetti and M. D. España, “Convergence analysis of the least-squares identification algorithm with a variable forgetting factor for time-varying linear systems,” *Automatica*, vol. 25, no. 4, pp. 609–612, 1989.

[24] J. Vaèko and D. Kocur, “Fast Tracking RLS Adaptation Algorithms of the Second-Order Volterra Digital Filters,” *Radioengineering*, vol. 4, no. 1, 1995.

[25] J. Lee, M. Laffranchi, N. Kashiri, N. Tsagarakis, and D. Caldwell, “Model-Free Force Tracking Control of Piezoelectric Actuators: Application to Variable Damping Actuator,” in *Robotics and Automation, 2014. ICRA’14. IEEE International Conference on*, 2014.