2014 IEEE International Conference on Robotics & Automation (ICRA)
Hong Kong Convention and Exhibition Center
May 31 - June 7, 2014. Hong Kong, China

Coordinated Patterns of Underactuated Ships Along Closed Orbits*

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Abstract

This paper considers the problem of directing a family of underactuated ships to formation tracking a set of closed orbits and achieve attitude synchronization. It shows that our previous concentric compression design is useful to the coordinated motion of underactuated ships. The condition that the total linear speed of each ship is greater than zero is ensured by introducing a potential function. Asymptotical stabilization is proved when the inter-ship communication topology is bidirectional. The theoretical result is proved by a numerical example.

1. INTRODUCTION

Current applications, especially ocean sampling, have aroused the great attention of researchers to study the cooperative control of a fleet of sensor-equipped autonomous ships for optimizing information collection [1]. For the purpose of improving data quality and minimizing operator intervention, the trajectory of each coordinated ship is well-planned based on the spatial and temporal variability in the field and multiple ships should cooperative move along given orbits in the desired pattern. Therefore, the control of coordinated motion along a set of given orbits is widely discussed in last decade.

Most results and design methodologies in earlier works focus on the coordinated motion control of mobile robots and simple full-actuated vehicles [2, 3]. As we all know, even for one vehicle, the path following control for the mobile robot or the simple fully-actuated vehicle can not directly extend to deal with the underactuated ship, thus the discussion of coordinated control for multiple underactuated ships along orbits is still relatively small. In [4], a leader-following strategy is proposed to give a solution to the cooperation of multiple underactuated ships. The virtual structure approach is introduced in [5]. Such a structure leads to the rigid geometric relationship among ships. In order to adapt to different formations, Ghabcheloo *et.al* [6] decouple the path following and formation motion by constructing an extra upgrade scheme for synchronizing the parametrization states (e.g., curve parameters) of given paths. Then small-gain theorem is used to analyze the stability of the whole system.

With the technological development of ocean sampling and the launching of the scientific experiments [1], a novel geometric extension design has emerged on the coordinated motion control around orbits recently [7, 8, 9]. Zhang and his colleague [7] firstly propose the geometric extension design to extend given curve to be a set of level curves of the orbit function. Orbit tracking and formation motion are achieved by forcing the value of the orbit function and the relative arc-lengths to the desired values, respectively. To maintain the same geometric topology among the extended curves and the given curve, a new method named concentric compression design [8] is proposed to solve the coordinated control of multiple unicycles along convex and closed curves. Similar ideas are used to control fully-actuated surface vessels [9]. However, the result of the geometric extension design for dealing with underactuated ships motion along given orbits is not established yet.

In the event that the (near) optimal sampling trajectory for each ship in ocean is often designed as a closed planar curve [1], we consider the navigation of ships formation moving along a set of closed orbits with attitude synchronization. Attitude synchronization is discussed in this paper due to the requirements of some practical requirements such as underway replenishment and image acquisition [10], which is one of the differences between this paper and the recent works [2, 4, 5, 7, 8]. The ship under consideration is a more realistic underactuated model [11], which is suitable to describe the autonomous underwater vehicle or

**This work was supported by the National Natural Science Foundation of China under grants 61203356, 61273110, 61105113, 60934006, and Doctoral Fund of Ministry of Education of China under grant 20110092120025.

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the surface vessel. Compared with the simple particle [7, 9] and the nonholonomic vehicle [8], the underactuated ship is more difficult to control. Therefore, the dynamics of the underactuated ship is rewritten in the flow frame by using the ship's total linear speed and the total orientation of ship motion with reference to [12] at first. Unlike the behavior of assuming the nonzero ship's total velocity [12], a potential function used in collision avoidance [13] is brought into the control design which makes the total velocity greater than zero. Next, the surge force is decomposed into two components for formation motion and attitude synchronization, respectively. Finally, the cooperative control law is designed based on the backstepping technique and the asymptotical stability result is obtained according to the extension of Barbalat's lemma [14].

This paper is organized as follows. Section 2 formulates the control problem. In Section 3, a controller is designed based on the backstepping technology. Simulation results are provided in Section 4. Section 5 gives the conclusion.

2. PROBLEM FORMULATION

2.1. Underactuated Ship's Model

Consider a coordinated control system comprised by n underactuated ships. Each underactuated ship has the surge force τ_i^u and the yaw moment τ_i^r in the inertial reference frame $\mathscr{W}$. Let $\vec{p}_i = [p_i^x, p_i^y]^T$ be the position of the ith ship. Also let ψ_i denote its yaw angle. The variables u_i, υ_i and r_i denote the surge, sway and angular velocities, respectively. The dynamics of the underactuated ship (INFANTE AUV [11]) can be written as

$$\begin{cases} \dot{p}_i^x = u_i \cos\psi_i - \upsilon_i \sin\psi_i \\ \dot{p}_i^y = u_i \sin\psi_i + \upsilon_i \cos\psi_i \\ \dot{\psi}_i = r_i \\ \dot{u}_i = \frac{1}{m_i^u}\left(\tau_i^u - d_i^u\right) \\ \dot{\upsilon}_i = -\frac{1}{m_i^\upsilon}\left(m_i^{ur} u_i r_i + d_i^\upsilon\right) \\ \dot{r}_i = \frac{1}{m_i^r}\left(\tau_i^r - d_i^r\right) \end{cases} \tag{1}$$

with $m_i^u = m_i - \dot{X}_i^{\dot{u}}$, $d_i^u = -X_i^{u\upsilon} u_i^2 - X_i^{\upsilon\upsilon}\upsilon_i^2$, $m_i^\upsilon = m_i - \dot{X}_i^{\dot{\upsilon}}$, $d_i^\upsilon = -Y_i^\upsilon u_i \upsilon_i - Y_i^{\upsilon|\upsilon|}\upsilon_i|\upsilon_i|$, $m_i^r = I_i^z - Z_i^{\dot{r}}$, $d_i^r = -Z_i^\upsilon u_i \upsilon_i - Z_i^{\upsilon|\upsilon|}\upsilon_i|\upsilon_i|$, $m_i^{ur} = m_i - Y_i^r$ where m_i denotes the system mass, I_i^z is the moment of inertia w.r.t. the z-axis, $X_i^{\cdot\cdot}$, $Y_i^{\cdot\cdot}$ and $Z_i^{\cdot\cdot}$ are hydrodynamic derivatives, respectively.

By referring [12], we define $\upsilon_i^F = \sqrt{u_i^2 + \upsilon_i^2}$ and $\psi_i^F = \psi_i + \beta_i$ as the total line speed of ship and the total orientation of motion with respect to the horizontal axis of $\mathscr{W}$, respective, where $\beta_i = \arctan\left(\frac{\upsilon_i}{u_i}\right) \in (-\pi/2, \pi/2)$ is the sideslip angle. Then the dynamics (1) of the underactuated ship can be rewritten in the flow frame $\mathscr{F}_i$ as

$$\begin{cases} \dot{\vec{p}}_i = \upsilon_i^F \vec{x}_i \\ \dot{\vec{x}}_i = (r_i + \dot{\beta}_i)\vec{y}_i \\ \dot{\vec{y}}_i = -(r_i + \dot{\beta}_i)\vec{x}_i \\ \dot{\upsilon}_i^F = \tau_i^F + d_i^F \\ \dot{\beta}_i = \tau_i^\beta + d_i^\beta \\ \dot{r}_i = \frac{1}{m_i^r}\tau_i^r - \frac{d_i^r}{m_i^r} \end{cases} \tag{2}$$

where $\vec{x}_i = \left[\cos\psi_i^F, \sin\psi_i^F\right]^T$ denotes the total motion direction of the ith ship and the normal vector $\vec{y}_i$ is perpendicular to $\vec{x}_i$ so that it points to the reader, $d_i^F = \frac{\cos\beta_i}{m_i^u} d_i^u - \frac{\sin\beta_i}{m_i^\upsilon}\left(m_i^{ur} u_i r_i + d_i^\upsilon\right)$, $d_i^\beta = -\frac{\sin\beta_i}{m_i^u \upsilon_i^F} d_i^u - \frac{\cos\beta_i}{m_i^\upsilon \upsilon_i^F}\left(m_i^{ur} u_i r_i + d_i^\upsilon\right)$, and the surge force τ_i^u is divided into

$$\tau_i^F = \frac{\cos\beta_i}{m_i^u}\tau_i^u,\ \tau_i^\beta = -\frac{\sin\beta_i}{m_i^u \upsilon_i^F}\tau_i^u. \tag{3}$$

τ_i^F is the component where τ_i^u is projected onto $\vec{x}_i$ and thus we call it the total acceleration. τ_i^β is the component where τ_i^u is projected onto $\vec{y}_i$, and we call it the total attitude input because it forces β_i diversification. In this paper, β_i is regarded as the attitude of the ith ship because the variation of β_i with a constant linear speed leads to various sway-surge velocity combinations. The translation between τ_i^u and $\left\{\tau_i^F, \tau_i^\beta\right\}$ is suitable when $\upsilon_i^F(t) > 0$.

Hereinafter we directly focus on the design of the total acceleration τ_i^F and the total attitude input τ_i^β, the the surge force τ_i^u can be computed according to

$$\tau_i^u = sgn(\tau_i^F)\sqrt{\left(m_i^u \upsilon_i^F \tau_i^\beta\right)^2 + \left(m_i^u \tau_i^F\right)^2}. \tag{4}$$

2.2. Concentric Compression Design

Consider that the given orbit $\mathscr{C}_{i0}$ associated with the ith ship is a simple, closed, regular curve with nonzero curvature. According to Lemma 1 in [9], the given orbit $\mathscr{C}_{i0}$ can be extended to be a set of level curves of an orbit function $f_i : \Omega_i \to (-\varepsilon_i, \varepsilon_i)$ by concentric compression where $\nabla f_i \neq 0$ for all points in Ω_i and $\varepsilon_i > 0$. It means that each curve $\mathscr{C}_{ic}(\phi_i, c)$ in Ω_i is a level orbit of $f_i(\cdot)$ with its value belonging to a constant c, that is, $f_i(z) = c$ if $z \in \mathscr{C}_{ic}$, and the orbit value (the value of $f_i(\cdot)$) associated to the given orbit $\mathscr{C}_{i0}$ is zero. Also the open set can be expressed as $\Omega_i = \left\{\vec{p}_i \in \mathbb{R}^2 \,||f_i(\vec{p}_i)| < \varepsilon_i\right\}$.

2.3. Design Method

According to concentric compression design, it is obvious that the path following control should drive the orbit value $f_i(\vec{p}_i)$ and the error direction $\alpha_i \in (-\pi, \pi]$ between the ship's total motion direction $\vec{x}_i$ and the tangent vector $\vec{T}_i$ to the orbit to 0 asymptotically, and at the same time, ensure the trajectory of each ship limited in the set Ω_i, that is

$$\lim_{t\to\infty} f_i(\vec{p}_i(t)) = 0,\ \lim_{t\to\infty} \alpha_i(t) = 0,\ |f_i(\vec{p}_i(t))| < \varepsilon_i. \quad (5)$$

To coordinated motion along the given orbits. Each ship should cooperate with its neighbors. To this end, we let $\mathscr{G} = \{\mathscr{V}, \mathscr{E}\}$ be the bidirectional graph induced by the inter-ship communication topology, where $\mathscr{V}$ denotes the set of n ships and $\mathscr{E}$ is a set of data links. Let $\mathscr{N}_i$ denote the neighbor set of the ith ship. Through the paper, we assume $\mathscr{N}_i$ are time-invariant. Two matrices such as the adjacency matrix $A = [a_{ij}]$ and the Laplacian matrix $L = [l_{ij}]$ are used to represent the graph.

The description of the formation on given orbits is based on the consensus design, which is widely applied in recent works [2, 8]. For example, consider a formation tracking on a set of concentric superellipses

$$\frac{1}{a_i}\left[\left(2p_i^x p_i^y\right)^4 + \left((p_i^x)^2 - (p_i^y)^2\right)^4\right]^{\frac{1}{8}} = 1, \quad (6)$$

with different a_i, in-line formation can be maintained if $\xi_i = s_i/a_i$ reach consensus where the starting point for each orbit is selected as the intersection of the orbit with the horizontal axis. From above discussion, we choose a generalized arc-length $\xi_i(t)$ defined in Assumption 1 for the more general formation patterns and it is said that the formation is maintained when

$$\lim_{t\to\infty} (\xi_i(t) - \xi_j(t)) = 0. \quad (7)$$

In reality, it is required that the formation moves along the orbits in some specified speed. Note that the deviation of the generalized arc-length $\eta_i(t) = \mathbf{d}\xi_i/\mathbf{d}t$ on the given orbit reflects the orbital speed of ship. This is due to the fact that $\eta_i(t)$ defined in (15) is a product obtained by multiplying the total linear speed of a ship and a parameter with respect to the desired formation pattern. Therefore $\eta_i(t)$ is regarded as the generalized orbital speed. To accomplish formation motion along given orbits in accordance with the desired speed, it is required that

$$\lim_{t\to\infty} \eta_i(t) = \eta^*(t) \quad (8)$$

where $\eta^*(t) > 0$. Some practical missions, underway replenishment and image acquisition [10], require attitude synchronization. It is said the attitude synchronization is achieved when

$$\lim_{t\to\infty} \left(\bar{\beta}_i(t) - \bar{\beta}_j(t)\right) = 0, \quad (9)$$

where $\bar{\beta}_i = \beta_i - \beta_i^*$ and $\beta_i^* \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is the desired attitude scalar. In addition, owning to the condition of ship's model transformation, we need

$$\eta_i(t) > 0. \quad (10)$$

Assumption 1: ξ_i is a C^2 smooth function of s_i such that all $\partial\xi_i/\partial s_i$ is bounded and greater than 0 and $\partial^2\xi_i/\partial s_i^2$ is uniformly bounded.

Coordinated control problem around orbits: Design a path following controller τ_i^r, the total acceleration τ_i^F for formation motion and the total attitude input τ_i^β for attitude synchronization based on local neighbor-to-neighbor information such that (5),(7)-(10) are satisfied, where $i \in \mathscr{V}$, $j \in \mathscr{N}_i$.

3. MAIN RESULTS

3.1. Coordinated Control Model

Define $\vec{N}_i = -\frac{\nabla f_i}{\|\nabla f_i\|}$ as the normal vector to each level orbit where ∇ denotes the vector differential operator. Also let $\vec{T}_i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{N}_i$ be the tangent vector to each level orbit. The direction error α_i between $\vec{x}_i$ and $\vec{T}_i$ can be defined as

$$\sin\alpha_i = \vec{y}_i \cdot \vec{T}_i = -\vec{x}_i \cdot \vec{N}_i. \quad (11)$$

The time-derivative of (11) yields

$$\dot{\alpha}_i = \upsilon_i^F\left(\kappa_i^a \cos\alpha_i + \kappa_i^b \sin\alpha_i\right) - \left(r_i + \dot{\bar{\beta}}_i\right) \quad (12)$$

where $\kappa_i^a = \frac{1}{\|\nabla f_i\|}\vec{T}_i \cdot \nabla^2 f_i \vec{T}_i$, $\kappa_i^b = -\frac{1}{\|\nabla f_i\|}\vec{T}_i \cdot \nabla^2 f_i \vec{N}_i$ and $\nabla^2 f_i$ is the Hessian matrix of $f_i(\vec{p}_i)$. When the ith ship moves in the set Ω_i, the dynamics of f_i is given by

$$\dot{f}_i = \nabla f_i \cdot \dot{\vec{p}}_i = \nabla f_i \cdot \upsilon_i^F \vec{x}_i = \upsilon_i^F \|\nabla f_i\| \sin\alpha_i. \quad (13)$$

Since the motion of the ith ship projected to $\vec{T}_i$ causes the change of arc-length s_i while the motion along the direction of concentric compression causes the orbit change which also induces variation in the arc-length, the arc-length s_i measured from the starting point can be written as $s_i(f_i, \phi_i)$. Furthermore, we can write $s_i(f_i, \phi_i) \triangleq \int_{\phi_i^*}^{\phi_i} \frac{\partial s_i(f_i,\tau)}{\partial\tau} d\tau$ where the starting point of each level orbit in Ω_i is selected according to the same value of arc-length parameter ϕ_i^* corresponding to the starting point $\vec{p}_i^*$ of the given orbit $\mathscr{C}_{i0}$. When the ship moves, the dynamics of $\xi_i(t)$ is

$$\dot{\xi}_i = \frac{\partial \xi_i}{\partial s_i} \upsilon_i^F \left(\cos\alpha_i + \frac{\partial s_i}{\partial f_i} \|\nabla f_i\| \sin\alpha_i \right). \tag{14}$$

Notice that $\alpha_i = 0$ implies that $\dot{\xi}_i = \frac{\partial \xi_i}{\partial s_i} \upsilon_i^F$. We define η_i as

$$\eta_i = \frac{\partial \xi_i}{\partial s_i} \upsilon_i^F. \tag{15}$$

Then $\dot{\xi}_i$ can be written as

$$\dot{\xi}_i = \eta_i + d_i^\eta \tag{16}$$

where $d_i^\eta = \eta_i \left(-2\sin^2 \frac{\alpha_i}{2} + \frac{\partial s_i}{\partial f_i} \|\nabla f_i\| \sin\alpha_i \right)$ and the derivative of η_i is

$$\dot{\eta}_i = \frac{\partial \xi_i}{\partial s_i} \left(\tau_i^F + d_i^F \right) + \frac{\partial^2 \xi_i}{\partial s_i^2} \upsilon_{F_i} \left(\eta_i + d_i^\eta \right). \tag{17}$$

From (2), the attitude model $\bar{\beta}_i$ is

$$\dot{\bar{\beta}}_i = d_i^\beta + \tau_i^\beta. \tag{18}$$

3.2. Backstepping Design

Step1. Convergence of $f_i, \alpha_i, \xi_i - \xi_j, \eta_i - \eta^*, \bar{\beta}_i - \bar{\beta}_j$**:** Consider the following control Lyapunov function

$$V_I = \sum_{i=1}^n h_i(f_i) - \sum_{i=1}^n \ln\left(\cos^2 \frac{\alpha_i}{2}\right) + \frac{k_0}{4} \sum_{i=1}^n \sum_{j=1}^n a_{ij} (\xi_i - \xi_j)^2 + \sum_{i=1}^n \left(\ln\left(\frac{\eta_i}{\eta^*}\right) + \frac{\eta^*}{\eta_i} - 1 \right) + \frac{k_1}{4} \sum_{i=1}^n \sum_{j=1}^n a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right)^2 \tag{19}$$

where $k_0, k_1 > 0$ and $h_i(f_i)$ is a C^2 smooth, nonnegative function on $(-\varepsilon_i, \varepsilon_i)$. $h_i(f_i)$ and $\nabla h_i = \mathrm{d}h_i / \mathrm{d}f_i$ also satisfy the following conditions: (C1)$h_i(f_i) \to +\infty$ and $\nabla h_i \to -\infty$ as $f_i \to -\varepsilon_i$. (C2) $h_i(f_i) \to +\infty$ and $\nabla h_i \to +\infty$ as $f_i \to \varepsilon_i$. (C3) $h_i(f_i) = 0$ if and only if $f_i = 0$.

From (19), the time derivation of V_I is

$$\dot{V}_I = \sum_{i=1}^n \tan\frac{\alpha_i}{2} \left(\Delta_i^r - r_i\right) + \sum_{i=1}^n (\eta_i - \eta^*) \left(\frac{1}{\eta_i^2} \frac{\partial \xi_i}{\partial s_i} \tau_i^F + \Delta_i^F \right) + \sum_{i=1}^n \left(\tau_i^\beta + d_i^\beta \right) \left(k_1 \sum_{j=1}^n a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan\frac{\alpha_i}{2} \right) \tag{20}$$

where

$$\Delta_i^r = \upsilon_i^F \left(\kappa_i^a \cos\alpha_i + \kappa_i^b \sin\alpha_i \right) + 2\upsilon_i^F \nabla h_i \|\nabla f_i\| \cos^2 \frac{\alpha_i}{2} + k_0 \eta_i \left(-\sin\alpha_i + 2\frac{\partial s_i}{\partial f_i} \|\nabla f_i\| \cos^2 \frac{\alpha_i}{2} \right) \sum_{j=1}^n a_{ij} (\xi_i - \xi_j),$$

$$\Delta_i^F = \frac{1}{\eta_i^2} \left(\frac{\partial \xi_i}{\partial s_i} d_i^F + \frac{\partial^2 \xi_i}{\partial s_i^2} \upsilon_i^F \left(\eta_i + d_i^\eta \right) \right) - \frac{1}{\eta_i \eta^*} \dot{\eta}^* + k_0 \sum_{j=1}^n a_{ij} (\xi_i - \xi_j).$$

Here we first use the ship angular velocity r_i as the virtual control $\bar{r}_i$, the total acceleration τ_i^F and the total attitude input τ_i^β to fulfill the coordinated control problem. We choose $\bar{r}_i$, τ_i^F and τ_i^β as follows:

$$\bar{r}_i = \Delta_i^r + k_2 \sin\frac{\alpha_i}{2}, \tag{21}$$

$$\tau_i^F = -\eta_i^2 \left(\frac{\partial \xi_i}{\partial s_i} \right)^{-1} \left(\Delta_i^F + k_3 (\eta_i - \eta^*) + k_4 \sum_{j=1}^n a_{ij} (\eta_i - \eta_j) \right), \tag{22}$$

$$\tau_i^\beta = -k_5 \left(k_1 \sum_{j=1}^n a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan\frac{\alpha_i}{2} \right) - d_i^\beta, \tag{23}$$

where $k_m > 0, m = 2, \ldots, 5$.

To accomplish the controller τ_i^r , the error variable $r_i^e = r_i - \bar{r}_i$ is defined and should be driven to zero. We re-write $\dot{V}_I$ as

$$\dot{V}_I = -k_2 \sum_{i=1}^n \frac{\sin^2 \frac{\alpha_i}{2}}{\cos \frac{\alpha_i}{2}} - k_4 \left(\vec{\eta} - \eta^* \vec{1}_n \right)^T L \left(\vec{\eta} - \eta^* \vec{1}_n \right) - k_3 \sum_{i=1}^n (\eta_i - \eta^*)^2 - k_5 \sum_{i=1}^n \left(k_1 \sum_{j=1}^n a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan\frac{\alpha_i}{2} \right)^2 + \sum_{i=1}^n r_i^e \Delta_i^e \tag{24}$$

where $\vec{\eta} = [\eta_1, \ldots, \eta_n]^T$, $\vec{1}_n = [1, \ldots, 1]^T$ and

$$\Delta_i^e = -\tan\frac{\alpha_i}{2} + \frac{1}{\eta_i^2} \frac{\partial \xi_i}{\partial s_i} \frac{\sin\beta_i}{m_i^\upsilon} m_i^{ur} u_i (\eta_i - \eta^*) + \frac{\cos\beta_i}{m_i^\upsilon \upsilon_i^F} m_i^{ur} u_i \left(k_1 \sum_{j=1}^n a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan\frac{\alpha_i}{2} \right). \tag{25}$$

Step2. Backstepping for r_i^e**:** We define the second control Lyapunov function such as

$$V_{II} = V_I + \sum_{i=1}^n (r_i^e)^2. \tag{26}$$

Differentiating (26) along the solution of (22) and (23), we have

$$\dot{V}_{II} = -k_2 \sum_{i=1}^{n} \frac{\sin^2 \frac{\alpha_i}{2}}{\cos \frac{\alpha_i}{2}} - k_4 \left(\vec{\eta} - \eta^* \vec{1}_n\right)^T L \left(\vec{\eta} - \eta^* \vec{1}_n\right)$$
$$- k_3 \sum_{i=1}^{n} (\eta_i - \eta^*)^2 - k_5 \sum_{i=1}^{n} \left(k_1 \sum_{j=1}^{n} a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan \frac{\alpha_i}{2}\right)^2$$
$$+ \sum_{i=1}^{n} r_i^e \left(\frac{1}{m_i^r} \tau_i^r - \frac{d_i^r}{m_i^r} - \dot{\bar{r}}_i - \Delta_i^e\right). \tag{27}$$

We design the yaw force τ_i^r as follows:

$$\tau_i^r = m_i^r \left(\frac{d_i^r}{m_i^r} + \dot{\bar{r}}_i + \Delta_i^e - k_6 r_i^e\right) \tag{28}$$

where $k_6 > 0$, which yields

$$\dot{V}_{II} = -k_2 \sum_{i=1}^{n} \frac{\sin^2 \frac{\alpha_i}{2}}{\cos \frac{\alpha_i}{2}} - k_4 \left(\vec{\eta} - \eta^* \vec{1}_n\right)^T L \left(\vec{\eta} - \eta^* \vec{1}_n\right)$$
$$- k_3 \sum_{i=1}^{n} (\eta_i - \eta^*)^2 - k_5 \sum_{i=1}^{n} \left(k_1 \sum_{j=1}^{n} a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right)\right.$$
$$\left. - \tan \frac{\alpha_i}{2}\right)^2 - k_6 \sum_{i=1}^{n} (r_i^e)^2 \leq 0. \tag{29}$$

3.3. Stability Analysis

Theorem 1: Consider level orbits of the orbit function constructed by concentric compression and the generalized arc-lengths satisfy Assumption 1. Suppose the initial conditions of underactuated ships make the initial value of V_{II} given in (26) finite. Assume $\eta^*(t)$ is bounded and greater than 0 for all t and $\dot{\eta}^*(t)$ are uniformly bounded. The coordinated control problem around orbits is solved by the yaw force τ_i^r as (28), the total acceleration τ_i^F as (22) and the total attitude input τ_i^β as (23) if the communication topology is connected.

Proof: The set $\Phi = \{(f_i, \alpha_i, \xi_i - \xi_j, \eta_i - \eta^*, \bar{\beta}_i - \bar{\beta}_j, r_i^e) | V_{II} \leqslant c\}$ such that $V_{II} \leq c$, for $c > 0$, is closed by continuity. It is easily to check that the closed-loop system is Lipschitz continuous on the set Φ and a solution exists and is unique.

Since the value of V_{II} is time-independent and non-increasing, we conclude that if the initial value of V_{II} is finite, the entire solution stays in Φ and then $\upsilon_i^F = \left(\frac{\partial \xi_i}{\partial s_i}\right)^{-1} \eta_i$ is bounded away from zero due to $\eta_i(t) > 0$ with Assumption 1. At the same time, $|f_i(\vec{p}_i(t))| < \varepsilon_i$ is satisfied by (C1) and (C2). With help of the invariance-like theorem, it follows that the trajectories of closed-loop system will converge to the set inside the region $E = \left\{\left(f_i, \alpha_i, \xi_i - \xi_j, \eta_i - \eta^*, \bar{\beta}_i - \bar{\beta}_j, r_{e_i}\right) \middle| \dot{V}_{II} = 0\right\}$ that is

$$\alpha_i = 0,\ \eta_i = \eta^*,\ r_i^e = 0, \tag{30a}$$

$$k_1 \sum_{j=1}^{n} a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) - \tan \frac{\alpha_i}{2} = 0, \tag{30b}$$

$$\left(\vec{\eta} - \eta^* \vec{1}_n\right)^T L \left(\vec{\eta} - \eta^* \vec{1}_n\right) = 0 \Rightarrow \eta_i = \eta_j. \tag{30c}$$

Firstly, we will prove $\bar{\beta}_i - \bar{\beta}_j \to 0$ as $t \to \infty$. Because $\alpha_i \to 0$, from (30b) we have

$$k_1 \sum_{j=1}^{n} a_{ij} \left(\bar{\beta}_i - \bar{\beta}_j\right) \to 0 \tag{31}$$

as $t \to \infty$. Because (31) is tenable for all the ship, we conclude $L\vec{\bar{\beta}} \to 0$ where $\vec{\bar{\beta}} = \left[\bar{\beta}_1, \ldots, \bar{\beta}_n\right]^T$. Then $\bar{\beta}_i - \bar{\beta}_j \to 0$ is stated when the communication topology is connected. Then on the set E, we have

$$\dot{f}_i = 0, \tag{32a}$$

$$\dot{\alpha}_i = -2\nabla h_i \upsilon_i^F \left\|\nabla f_i\right\| - 2k_0 \eta^* \frac{\partial s_i}{\partial f_i} \left\|\nabla f_i\right\| \sum_{j=1}^{n} a_{ij} \left(\xi_i - \xi_j\right), \tag{32b}$$

$$\dot{\xi}_i - \dot{\xi}_j = 0, \tag{32c}$$

$$\dot{\eta}_i - \dot{\eta}^* = -(\eta^*)^2 k_0 \sum_{j=1}^{n} a_{ij} \left(\xi_i - \xi_j\right), \tag{32d}$$

Next we will show $\xi_i - \xi_j \to 0$ as $t \to \infty$. On the set E, from (32c) one gets that $\xi_i - \xi_j$ is constant. Applying the extension of the Barbalat lemma in [14], from (32d) one gets $\dot{\eta}_i - \dot{\eta}^* = -k_0 \left(\eta^*\right)^2 \sum_{j=1}^{n} a_{ij} \left(\xi_i - \xi_j\right) \to 0$ because η^* is uniformly continuous and bounded. Due to $\eta^* \neq 0$, $L\vec{\xi} = 0$ where $\vec{\xi} = \left[\xi_1, \ldots, \xi_n\right]^T$, which implies that $\xi_i - \xi_j \to 0$ as $t \to \infty$ due to the fact that the communication topology is connected.

Because $\xi_i - \xi_j \to 0$ as $t \to \infty$, the equation of the closed-loop system for α_i on the set E is changed as

$$\dot{\alpha}_i = -2\nabla h_i \upsilon_i^F \left\|\nabla f_i\right\|. \tag{33}$$

It is easy to check that $\lim_{t\to\infty} \upsilon_i^F = (\partial \xi / \partial s_i) \eta^* > 0$ is uniformly continuous and bounded from Assumption 1. The details can be found in [8]. From (32a), f_i tends to a constant and thus ∇h_i tends to a constant. Therefore we conclude that $-2\nabla h_i \upsilon_i^F \left\|\nabla f_i\right\|$ is uniformly continuous. From (33) we have $\dot{\alpha}_i \to 0$ as $t \to \infty$ based on the extension of the Barbalat lemma [14]. Due to $\lim_{t\to\infty} \upsilon_i^F \left\|\nabla f_i\right\| \neq 0$, $\nabla h_i \to 0$. By (C3), f_i tends to 0.

4. Simulation Results

In this section, we apply the proposed control law to coordinate four underactuated ships moving along the closed orbits. The neighbors of each ship is that $\mathcal{N}_1 = \{\mathcal{V}_2, \mathcal{V}_3\}, \mathcal{N}_2 = \{\mathcal{V}_1, \mathcal{V}_3, \mathcal{V}_4\}, \mathcal{N}_3 = \{\mathcal{V}_1, \mathcal{V}_2\}, \mathcal{N}_4 = \{\mathcal{V}_2, \}$, and the parameters of ship model and the control parameters are selected as $m_i^u = 24, m_i^\upsilon = 34, m_i^r = 3, m_i^{u\upsilon} = 1, d_i^u = 2, d_i^\upsilon = 7, d_i^r = 0.5$ and $k_0 = 20, k_j = 10, j = 1, \ldots, 5$, reflectively.

The given orbits are a set of concentric superellipses (non-convex orbits) in the form (6) where $a_i = 3 + 0.5\,(i-1)$, $i = 1, \ldots, 4$. In this case, the nominated pattern is that forming an in-line formation with attitude synchronization and the reference velocity $\eta^*(t) = 1 + 0.1\sin(t)$. The starting points are defined as the intersection of the orbits with the positive horizontal axis of $\mathscr{W}$. According to the inline formation, we choose $\xi_i = s_i/a_i$. Also, the relative desired attitude scalar $\beta_i^* = 0$ The movement of underactuated ships is shown in of Fig. 1. Fig. 2 shows each general orbital velocity η_i converges to the reference.

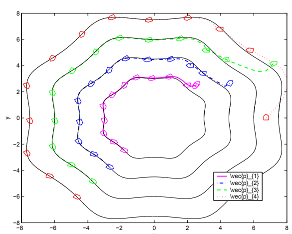

Figure 1. Plot of movements.

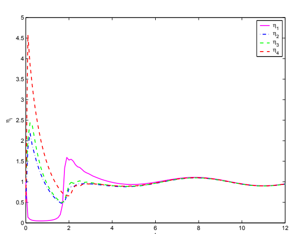

Figure 2. Plot of η_i

5. CONCLUSIONS

In this paper, the previous geometric extension design [8] is developed to deal with coordinated control of multiple underactuated ships around the closed curves. It removes the assumption of nonzero total line velocity of each ship through using the potential function.

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