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Control of Hopping Through Active Virtual Tuning of Leg Damping for Serially Actuated Legged Robots

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Abstract— **Spring-mass models have been very successful both in describing and generating running behaviors. Control of system energy within these models takes different forms, among which the use of a linear actuator in series with the leg spring has been preferred by many recent monopedal and bipedal platform designs due to its relative robustness and simplicity. However, the validity of the well-known Spring-Loaded Inverted Pendulum (SLIP) model of running for such platforms can only preserved under a specific family of control strategies to drive the actuator. In this paper, we propose a novel controller in this family, based on the idea of embedding a lossy SLIP template with a tunable damping coefficient into the linearly actuated compliant leg system. We show that this maximizes energy input within a single stride, allows decreased power requirements on the actuator compared to alternative methods, while preserving the validity and accuracy of analytic solutions and model-based controllers. We present systematic simulations to establish performance gains with respect to the commonly investigated method of modulating leg stiffness. Enabling the most efficient use of actuator power in this manner while preserving analytic tractability will allow efficient and accurate control of running for linearly actuated compliant leg designs.**

I. INTRODUCTION

Design, analysis and accurate control of legged robot behaviors have been among difficult challenges faced by robotics researchers. This line of inquiry brought together researchers from different areas, including biomechanics [1], dynamical systems [2], control theory [3], mechanical engineering [4] and materials science [5], towards the goal of building high performance autonomous legged machines. Surprisingly, simple hybrid spring-mass models have been able to capture the essence of basic running behaviors, embodied by the now widely accepted Spring-Loaded Inverted Pendulum (SLIP) model [6]. Since its early discovery [7], to its first instantiation on physical robot platforms [8], with more recent detailed analysis [9–12], as well as extensions and adoption by morphologically different platform designs [8, 13–16], this model continues to provide a rich context in which running behaviors can be studied and implemented.

One of the challenges in the embodiment of the fully passive SLIP model in physical robots [17, 18] is the manner in which system energy can be managed through active components. In some cases, such as the flight precompression of the leg in the Bow-Leg platform [13], this is accomplished without compromising the passivity of the stance phase. Another approach which remains within the confines of the ideal model has been active modulation of spring stiffness during stance, implemented by some recent platform designs such as the BiMASC [19] and MABEL [20] platforms. Alternatively, for platforms that use a hip actuator as the only energy input, extensions to the model were necessary to support controller design and stability analysis [21].

Among the best combinations of mechanical simplicity and minimal deviations from the ideal spring-mass model are designs that incorporate a linear positional actuator in series with the leg spring. Raibert's hoppers were similar to this design in their use of a pneumatic actuator in series with the spring [8, 22]. The difficulty of accurate position control for these actuators was addressed by the ARL Monopod [14], which uses a DC motor coupled with the spring through a ball screw. More recent instantiations of this idea can be observed in the ATRIAS biped [16] as well as other small experimental platforms [23]. Recent efforts include using principles behind series-elastic actuators, considering the combination of the spring and the actuator as a force transducer [24, 25]. Improvements in rough terrain mobility, as well as new analytically tractable solutions to resulting dynamics have been reported in the literature [26, 27].

In this paper, we propose a novel method that bridges the analytic gap between such serially actuated compliant leg designs and the passive SLIP model. In particular, we consider the lossy SLIP model, a generalization of the ideal, conservative model, as a basis for our locomotion controllers. Our recent work on approximate but accurate analytic solutions to the dynamics of this system have been instrumental in the construction of adaptive, model-based controllers and state estimators for running behaviors [11]. We now propose to actively modulate the leg force through the series elastic leg actuation to obtain a virtual SLIP model whose viscous damping coefficient can be selected as desired by a high level gait controller.

Active control of damping is traditionally considered for vibration control in various application areas [28–30], wherein dissipative forces are actively introduced into the system to attenuate unwanted vibrations. In contrast, systems with different, possibly negative, damping coefficients at different parts of their state space have been used to induce stable oscillatory behavior that are useful for many application domains [31], including locomotion [32]. The novelty of our approach lies in our goal of embedding a simple, analytically tractable *template* system within a more complex *anchor* structure [17]. As a result, we are able to ensure the applicability of analytic models and gait controllers designed

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for the lossy SLIP model, providing a useful abstraction for the control of running behaviors. We also show, through systematic simulation studies, that energy efficiency, control accuracy as well as power requirements on the leg actuation resulting from this strategy are better than its most common alternative, variable stiffness [33], for regulating system energy when actuator limitations are considered. For simplicity of the presentation, we limit the scope of this paper to a vertical model that captures all relevant issues in controlling the energy for a hopping system, noting that our method can readily be generalized to planar running behaviors thanks to the weak coupling between radial and angular dynamics as described in our earlier work [11].

II. Control of Hopping Through Active Damping

A. The Anchor: Vertical SLIP with Linear Actuation

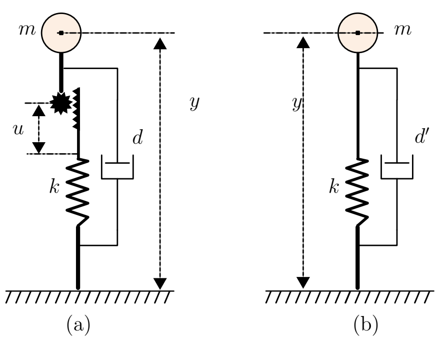

Fig. 1. (a) VA-SLIP: Vertical SLIP model with linearly actuated compliant leg and physical damping coefficient d, (b) VD-SLIP: Vertical SLIP template with a tunable, possibly negative, damping coefficient d'.

The mathematical model for the vertical hopper with series actuation in the leg, which we call VA-SLIP, is illustrated in Fig. 1.a. It incorporates a linear actuator in series with the leg spring in order to inject or remove energy from the system. This actuator is assumed to be position controlled, with its length u considered as a control input. Similar to planar SLIP models, this model alternates between stance and flight phases, that can further be decomposed into compression and decompression, and ascent and descent, respectively. Touchdown, liftoff, bottom, and apex events, respectively identified with subscripts td, lo, b, and a, mark transitions among these sub-phases and are defined in the same way as the ideal SLIP model [11].

The VA-SLIP model follows ballistic trajectories during flight, captured by the dynamics $\ddot{y} = -g$. Stance dynamics, however, are those of a second-order spring-mass system with a constant forcing term due to gravity, and the spring length modulated through the actuator input, taking the form

$$\ddot{y} = -g - \frac{k}{m}(y - y_0 - u) - \frac{d}{m}\dot{y} \,. \tag{1}$$

Since we focus on a purely vertical system, a Poincaré section at the apex point yields the one dimensional apex state $X := y_a$. As usual, we define the apex return map as a combination of four components as

$$X_{k+1} = \mathbf{f}(X_k, u) := {}^{a}_{l}\mathbf{f} \circ {}^{lo}_{b}\mathbf{f}_u \circ {}^{b}_{td}\mathbf{f}_u \circ {}^{td}_{a}\mathbf{f}(X_k), \tag{2}$$

where ${}^{td}_{a}\mathbf{f}$, ${}^{b}_{td}\mathbf{f}_u$, ${}^{lo}_{b}\mathbf{f}_u$, and ${}^{a}_{l}\mathbf{f}$ denote descent, compression, decompression, and ascent phases, respectively, and X_k denotes the k^{th} apex state. Maps for compression and decompression are subscripted with u since they depend on the particular choice of control strategy during stance.

B. The Template: Vertical SLIP with Tunable Damping

In contrast to the actuated VA-SLIP model, which closely corresponds to the physical robot implementation, Fig. 1.b shows the passive SLIP model with a tunable damping coefficient d', which we call VD-SLIP. We will use this model as a dynamical template to interface our high level gait controller for the actuated physical system. Flight dynamics for this model are identical to those of the actuated model, but stance dynamics take the form

$$\ddot{y} = -g - \frac{k}{m}(y - y_0) - \frac{d'}{m}\dot{y} \,, \tag{3}$$

excluding the actuation and incorporating a different damping coefficient, d', considered to be a control input.

For the planar lossy SLIP model, our previous work revealed an accurate approximation for the return map [11]. Unlike the planar SLIP model, however, vertical hopper dynamics admit an exact solution that we outline in the rest of this section. Energy is conserved during flight, yielding descent and ascent maps as

$$[y_{td}, \dot{y}_{td}]_k^T = {}^{td}_{a}\mathbf{f}(y_{a,k}) = [y_0, 2g\sqrt{y_{a,k} - y_0}]^T \tag{4}$$

$$y_{a,k+1} = {}^{a}_{l}\mathbf{f}([y_{lo}, \dot{y}_{lo}]_k^T) = \dot{y}_{lo,k+1})^2/(2g) \,. \tag{5}$$

Here, we assume that the actuator position is reset with $u = 0$ prior to touchdown. Return maps for compression and decompression require solving (1). Assuming underdamped dynamics with $d'^2 - 4km < 0$, which is actually necessary to ensure liftoff and enable locomotion, we have

$$y(t) = M\, e^{-\xi\omega_n t} \cos\left(\omega_n \sqrt{1-\xi^2}\, t + \phi\right) + \frac{F}{\omega_n^2} \tag{6}$$

$$\dot{y}(t) = -M\omega_n\, e^{-\xi\omega_n t} \cos\left(\omega_n \sqrt{1-\xi^2}\, t + \phi + \phi_2\right), \tag{7}$$

where $M := (c_1^2 + c_2^2)^{1/2}$, $\phi := -\mathrm{atan}(c_2/c_1)$, $\phi_2 := -\mathrm{atan}(1-\xi^2)^{1/2}/\xi$, $\omega_n := \sqrt{k/m}$, $\xi := d'/(2\sqrt{km})$, $F = -g + \omega_n^2 y_0$, and $c_{1,2}$ obtained from initial conditions as

$$c_1 = g/\omega_n^2, \tag{8}$$

$$c_2 = (\dot{y}_{td} + c_1\xi\omega_n)/(\omega_n\sqrt{1-\xi^2}) \,. \tag{9}$$

The bottom transition occurs at the end of compression with

$$t_b = (\pi/2 - \phi - \phi_2)/(\omega_n\sqrt{1-\xi^2}) \,. \tag{10}$$

During decompression, liftoff occurs when the ground reaction force becomes positive, with

$$k(y - y_0) + d'\dot{y} = 0 \,. \tag{11}$$

Adopting the approximation proposed in [11] for the exponential term in (7) to its value at the bottom time t_b as $e^{-\xi\omega_n t} \approx e^{-2\xi\omega_n t_b}$, the liftoff time takes the form

$$t_{lo} = \frac{2\pi - \mathrm{acos}(k(y_0 - F/\omega_n^2)e^{2\xi\omega_n t_b}/\left(\overline{M}M\right)) - \phi - \phi_3}{\omega_n\sqrt{1-\xi^2}} \tag{12}$$

where $\overline{M} := \sqrt{k^2 - 2kd'\omega_n \cos\phi_2 + d'^2\omega_n^2}$ and $\phi_3 := -\mathrm{atan}(d'\omega_n \sin\phi_2/(k - d'\omega_n \cos\phi_2))$.

These derivations complete the Poincaré map, allowing us to formulate gait controllers focusing on the virtual damping coefficient d' as a control input in Sec. II-D.

C. Embedding VD-SLIP into VA-SLIP

As noted in Sec. II-A, the linear actuator in the VA-SLIP model is assumed to be position controlled, meaning that the control input u can be chosen as desired. Using this model to realize VD-SLIP dynamics with a desired damping coefficient d^* requires that the acceleration felt by the robot body is identical for both, leading to the constraint

$$-g - \frac{k}{m}(y - y_0 - u) - \frac{d}{m}\dot{y} = -g - \frac{k}{m}(y - y_0) - \frac{d^*}{m}\dot{y} \,. \quad (13)$$

Solving this equation for the control input u hence yields the continuous stance controller for the VA-SLIP anchor model to properly realize VD-SLIP dynamics as

$$u_e(t) = \frac{d - d^*}{k}\dot{y}(t), \quad (14)$$

where d^* denotes the desired virtual damping coefficient. Note that since this embedding exactly realizes VD-SLIP dynamics, the analytic return map derivations of Sec. II-B remain valid with d' replaced with d^*.

Close inspection of (14) shows that energy input from the actuator resulting from this embedding controller will be spread throughout the stance phase in contrast to methods that require abrupt step commands to the actuator such as those used by Raibert's hoppers or the ARL Monopod. Consequently, this method will present substantial benefits in power requirements on the actuator compared to the alternative method of adjusting spring stiffness at bottom that can also preserve the validity of analytic solutions [6, 34].

D. High-Level Control of Energy

Now that we have active control over the damping coefficient of the VD-SLIP template, we can use it for gait control. For vertical hoppers, this is equivalent to controlling system energy, with positive and negative values of d' used for decreasing and increasing the energy level of the system, respectively.

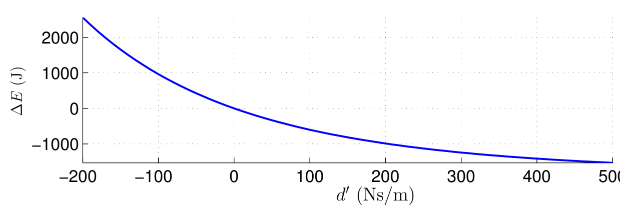

Fig. 2. Monotonic dependence of the change in VD-SLIP energy, ΔE on the virtual damping coefficient d'.

We begin by noting that the return map for the VD-SLIP model now accepts d' as a control parameter, rewritten as

$$X_{k+1} = \bar{\mathbf{f}}(X_k, d') \,. \quad (15)$$

With this in mind, we formulate our gait controller in a deadbeat structure, relying on the inverse of the return map to yield

$$d^* = \mathbf{f}^{-1}(X_k, X^*) \,, \quad (16)$$

where X^* denotes the desired apex height to be achieved in a single stride. Even though the return map derived in Sec. II-B does not readily admit an analytic inverse with respect to the damping coefficient, the one-dimensional and monotonic relation between d' and the change in energy within the stride $\Delta E := mg(y_{a,k+1} - y_{a,k})$, illustrated in Fig. 2, admits an efficient numerical solution. This takes the form of a simple root finding problem expressed in liftoff coordinates as

$$d^* = \mathrm{solve}_{d'}\left(g\, y_{lo,k}(d') + \frac{1}{2}\dot{y}^2_{lo,k}(d') - g\, X^* = 0\right) \,, \quad (17)$$

where the dependence of liftoff states on the control input d' are explicitly shown. Once d^* is computed, it can be virtually realized by (14). Fig. 3 illustrates an example run with this strategy marked as the solid blue line with $X_k = 1.2m$ and $X^* = 1.5m$ for a human sized model with $y_0 = 1m$, $m = 70kg$, $k = 10kN/m$, and $d = 100Ns/m$.

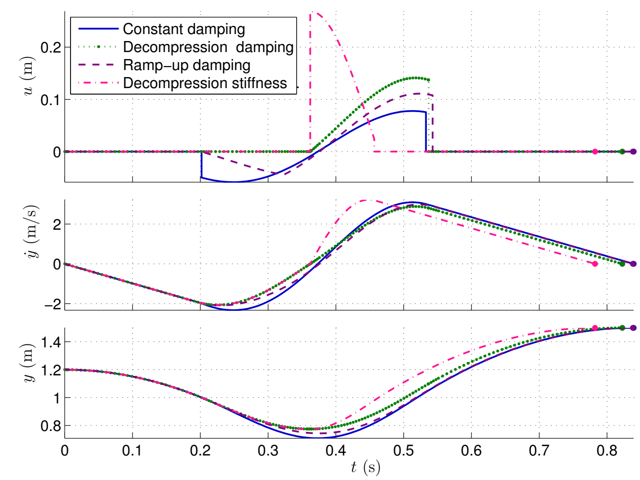

Fig. 3. Example simulation plots of actuator input (top), vertical velocity (middle) and vertical position (bottom) with constant d' throughout stance (solid blue), constant d' during decompression only (dotted green), initial ramp-up for d' (dashed magenta) and step stiffness change at bottom (dot-dashed red). End of the stride in each case is marked with a colored dot. Simulations were run for a hopper with $y_0 = 1m$, $m = 70kg$, $k = 10kN/m$, and $d = 100Ns/m$.

E. Variable Stiffness Control

One of the more commonly used alternatives for analytically tractable control of the SLIP model relies on changing the effective spring constant of the leg, either through mechanical linkages [33] or using active force control on a structure such as the VA-SLIP model. In the latter case, the actuator position required to realize a desired stiffness k' takes the form

$$u_s(t) = \frac{k - k'}{k}(y(t) - y_0) \,. \quad (18)$$

Usually, analytic tractability is obtained by assuming a step change in stiffness at the bottom instant, enabling the stance map to be studied in two parts. An example run with the VA-SLIP model under this controller is shown in Fig. 3 as the dot-dashed red plot. In subsequent sections, we will provide a comparative study of this method in relation to our method.

F. Actuator Limitations and Practical Considerations

As evident from the example run in Fig. 3 with constant virtual damping d', there are discontinuities in $u(t)$ at touchdown and liftoff. Even though the discontinuity at liftoff is not a problem since the actuator can be quickly rewound during flight, the necessity of having $y = y_0$ at touchdown requires $u(t_{td}) = 0$ at touchdown. The stiffness control method of (18) exhibits a similar discontinuity in actuator position at the bottom instant.

Such discontinuities clearly cannot be realized by any physical position-controlled actuator. Consequently, we must consider controller performance under actuator limitations in order to ensure their practical applicability. In particular, we will consider a realistic DC motor model wherein the actuator speed is limited by its force as

$$|\dot{u}(t)| < \dot{u}_{max}(1 - |F|/F_{max}) , \qquad (19)$$

where F denotes the current force exerted on the actuator by the leg spring, and F_{max} is the maximum actuator force. In the following sections, we will propose two modifications to our method to address this practical limitation without invalidating the analytic solutions for the VD-SLIP template dynamics.

1) Energy Pumping Limited to Decompression: Inspection of (14) shows that $u(t_b) = 0$ at bottom, independent of the value of d^*. Consequently, the discontinuity at touchdown imposed by the gait controller in Sec. II-D can be eliminated if the active tuning of damping is limited to decompression alone. Fortunately, the solutions of Sec. II-D can be adapted to this scenario by decomposing the stance phase into its sub-maps. Fig. 3 shows the resulting simulation as the dotted green line in comparison with the other methods.

The clear drawback of this modification is that no energy is injected into the system during compression, effectively increasing power requirements on the actuator as is evident from the larger actuator displacements in Fig. 3. Nevertheless, analytic solutions to stance dynamics are preserved, enabling accurate deadbeat control.

2) Initialization of Actuator with Ramp Input: In order to use the entirety of the stance phase for energy regulation, we propose to incorporate an "initialization" phase for the actuator immediately after touchdown, until it reaches the position required by (14). In particular, we use

$$u_r(t) = \gamma \,\mathrm{sign}(d - d^*)(y - y_0) , \qquad (20)$$

which is zero at touchdown, and thanks to its monotony, catches up with the ideal actuator position command within the compression phase. The parameter γ can easily chosen be chosen such that $|\dot{u}(t)| < \dot{u}_{max}$ throughout compression since $\dot{y}(t)$ will be monotonically increasing. Interestingly, this initialization phase corresponds to a lossy SLIP model with a different stiffness whose dynamics take the form

$$\ddot{y} = -g - \frac{k}{m}(1 - \gamma\,\mathrm{sign}(d - d^*))(y - y_0) - \frac{d}{m}\dot{y} , \qquad (21)$$

which are the same as those of the VD-SLIP model, making the analytic solutions of Sec. II-B immediately usable. The only challenge is to find the transition time t_{sw} such that $u_r(t_{sw}) = u_e(t_{sw})$, which is also relatively straightforward thanks to the analytic nature of solutions in both cases. The composition of solutions for this initialization phase with the rest of the stance yields an analytic return map, on which the gait controller of Sec. II-D can be applied. Fig. 3 shows an example simulation as the dashed magenta line showing the ramp-up initialization method in comparison with others.

III. Controller Performance and Efficiency

In this section, we present a comparative study of the energetic performance and controller accuracy for gait control methods we described in Sec. II. We first evaluate the energetic effectiveness of all four methods in Sec. III-A based on the maximum amount of energy they can inject into the system within a single stride. We then investigate the accuracy of single-stride deadbeat control in Sec. III-B. Finally, we compare power requirements of these methods on the actuator, establishing that the tunable damping methods allow platform designs with smaller actuators and better efficiency. All our simulation results in this section consider a human-sized platform with $y_0 = 1m$ and $m = 70kg$.

A. Effectiveness of Energy Input

One of the physical constraints associated with robot designs using a linear actuator in series with the leg spring is the limited range of displacements for the actuator. In this section, we establish upper bounds on the amount of energy that can be injected into the VA-SLIP system using all four control methods described in Sec. II by considering only the range constraint $|u(t)| < u_{max}$, assuming that the actuator can otherwise supply required forces and velocities.

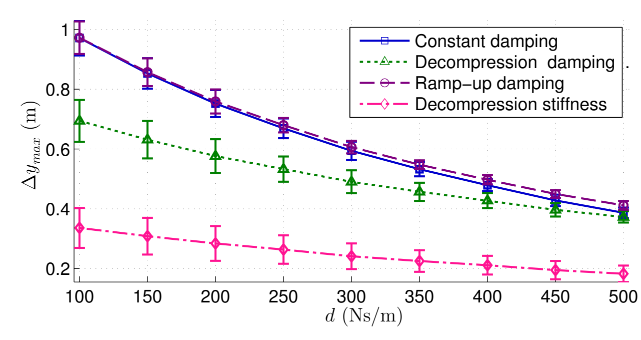

Fig. 4. Maximum height gain $\Delta y = y^* - y(0)$ that can be achieved using different control strategies as a function of the physical leg damping d with $u_{max} = 0.2m$, averaged over $y(0) \in [1.1, 2.5]m$, $k \in [10, 40]kN/m$

Fig. 4 shows our results from simulation runs with $u_{max} = 0.2m$, starting from initial conditions in the range $y(0) \in [1.1, 2.5]m$ and different leg stiffnesses in the range $k \in [10, 40]kN/m$. Maximum achievable increase in apex height was computed for each controller enforcing $|u(t)| < u_{max}$, and the results were averaged for different physical damping coefficients in the range $d \in [100, 500] Ns/m$. As expected, the stiffness controller has the lowest energy injection performance since it is limited to decompression and requires a large initial displacement. For similar reasons, the decompression-only damping controller is limited in its energy injection capability. However, both the ideal damping controller and the ramp-up damping have similar performance, with up to $1m$ height gain in a single stride, showing that they can effectively use the entire stride to inject energy into the system for the best effectiveness.

B. Accuracy of Deadbeat Control

Having established that the ramp-up damping controller outperforms its alternatives in being able to supply the most energy within a single stride, we now investigate its accuracy when additional actuator limitations described in Sec. II-F are introduced. To this end, we use a maximum actuator velocity of $\dot{u}_{max} = 2m/s$ and a maximum force of $F_{max} = 10kN$, motivated by motor choices in the ATRIAS 2.1 robot scaled to the size and weight of our platform. We run simulations covering initial conditions and spring stiffnesses in the same ranges as Sec. III-A, while considering height difference commands in the range $\Delta y := y^* - y(0) \in [-0.5, 0.5]m$. We evaluate controller performance with a *percentage error*, defined as

$$PE_{y^*} := \frac{|y^* - y_1|}{y^*}, \tag{22}$$

where y_1 denotes the next apex height reached at the end of the stride. Fig. 5 shows average percentage error across different simulations with respect to desired height differences.

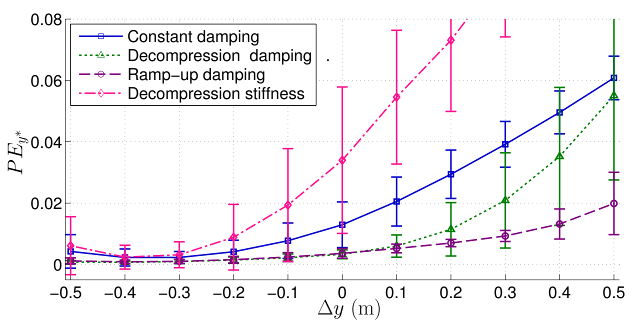

Fig. 5. Percentage height tracking error using four different control strategies with $u_{max} = 0.2m$, $\dot{u}_{max} = 2m/s$ and $F_{max} = 10kN/m$, averaged across $d \in [100, 500]Ns/m$, $y(0) \in [1.1, 2.5]m$, $k \in [10, 40]kN/m$.

Our results show that the accuracy of the stiffness controller is the worst since it requires a step change at bottom which is impossible to realize with realistic actuators. The bottom instant is also when the spring force is largest, challenging the actuator's power constraints. Similarly, since the ideal damping controller also assumes a discontinuous actuator position at touchdown, its accuracy worsens as the desired height difference increases. Damping control limited to decompression does well in a certain range, until its demand on the actuator speed exceeds practical limits beyond which its performance degrades substantially. Since the ramp-up damping controller eliminates discontinuities in the desired actuator position and spreads out energy pumping throughout the entire stance, its accuracy remains consistently low for a large range of desired height differences. Finally, it is informative to note that when the desired height difference is close to $-0.35m$, the physical damping in the system does most of the work, eliminating the need for explicit control and increasing the accuracy of all controllers.

C. Actuator Power Requirements

Finally, this section presents improvements in actuator power requirements under each of the four different gait control methods. To this end, we focus on a single choice of leg parameters with $k = 10000N/m$ and $d = 100Ns/m$, using an initial robot height of $y(0) = 1.6m$ with different commanded height differences in the range $\Delta y \in [-0.5, 0.5]m$. Actuator limits were chosen to be the same as Sec. III-B, with maximum and average power requirements through the stride, P_{max} and P_{avg} respectively, defined as

$$P_{max} := \max_{t \in [t_t, t_l]} (|\dot{u}(t) F_{spring}(t)|) \tag{23}$$

$$P_{avg} := \frac{1}{t_f} \int_{t_t}^{t_l} |\dot{u}(\tau) F_{spring}(\tau)| d\tau, \tag{24}$$

where t_t and t_l denote the touchdown and liftoff times, and $F_{spring}(t)$ denotes the spring force at time t. Fig. 6 shows our simulation results.

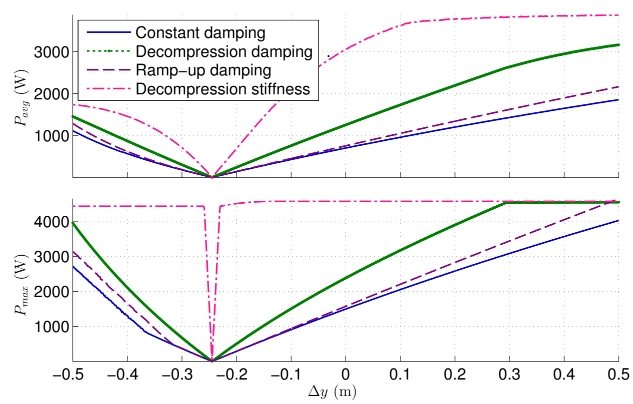

Fig. 6. Dependence of average (top) and maximum (bottom) actuator power on the commanded height difference for four different controllers. Simulations were run for a particular leg with $k = 10000N/m$ and $d = 100Ns/m$ and an initial height $y(0) = 1.6m$.

Our results show that the stiffness control has the highest power requirements as expected, attempting to inject most of the energy at bottom where the spring force is largest. The damping adjustment limited to decompression performs better but still has substantial power requirements. The ideal damping control and the ramp-up damping perform best, with a dramatic reduction in power requirements for small changes in apex height, until the actuator saturates altogether beyond $\Delta y > 0.5m$. Note, also, that the natural damping in the system around $\Delta y \approx -0.25m$ eliminates the need for any actuation, which is why power requirements on the actuator all vanish at that point.

These comparative results show that the novel gait control method we proposed through active tuning of the system's damping coefficient has the best performance in terms of effective energy injection, controller accuracy and power requirements. As such, it provides a robust, effective and accurate method for controlling running behaviors.

IV. Conclusion

In this paper, we introduced a new control strategy for running robots that incorporate a linear actuator in series with the leg compliance. Our strategy is based on tuning the damping of a virtual unactuated leg attached to the robot body, preserving the spring-mass-damper structure and solutions of radial SLIP dynamics [11]. We present our derivations and controller design on a simpler, vertical model, noting that they can readily be extended to planar running models. Our simulation studies show that the proposed strategy and its extensions to address physical considerations yield more efficient energy injection within a stride, smaller power requirements on the actuator and better control accuracy under physical actuator limitations in comparison to an alternative control strategy that relies on active regulation of leg stiffness.

We expect to verify our results on an experimental platform in the near future. In the long term, we believe that this method will provide a sufficiently efficient and accurate control strategy for locomotion to support physical implementations of model-based adaptive control, state estimation and reactive footstep planning algorithms.

Acknowledgments

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