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Robust Stabilization Control of Unknown Small-scale Helicopters

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***Abstract*— In this paper, we address the attitude and vertical stabilization problem for small-scale helicopters. An emergency controller that would successfully stabilize the helicopter in a safe flight mode when a pilot/autopilot fails to control it, owing to unexpected reasons, is of outmost importance in flight control systems. In this direction, we propose a low complexity nonlinear control scheme that drives the angles and the vertical speed to zero with prescribed transient and steady state response, without incorporating any knowledge of the dynamic model parameters in the control design. The stereographic coordinates were employed to model the attitude state of the helicopter in an attempt to guarantee the safe stabilization for every possible initial orientation without introducing any representation singularities as in the Euler angles representation or increasing complexity as in conventional four element quaternions. Moreover, the transient and steady state performance of the proposed scheme is a priori determined even in the presence of external disturbances. Furthermore, the overall control scheme can be easily implemented on embedded flight systems equipped with low-cost sensors. Finally, simulation and experimental results on a realistic platform verify the efficacy of the proposed method.**

I. INTRODUCTION

During the last two decades, unmanned helicopters have gained a lot of ground in many outdoor and indoor applications (e.g., surveillance, search, rescue and remote inspection) owing to their intriguing capabilities of hovering, taking off/landing from/on any area and aggressive maneuvering in unknown and dynamic environments. However, the aforementioned attributes do not come at no cost. The helicopters require highly complicated control schemes owing to the complex system modeling that leads in multivariable, underactuated and highly nonlinear dynamic systems with couplings between all states. Moreover, their field of work (i.e., air) not only has unknown wind gusts but also increases the possibilities for a disastrous scenario. Thus, helicopters constitute a difficult but intriguing test-bed for designing and testing control algorithms that has gained a lot of attention by the research community.

Several works can be found in the related literature. Common approaches such as linearizing the model within a specific flight operation (i.e hover) and designing a linear robust controller have been proposed in [1] [2]. However, such controllers yield satisfactory response only around a local operating point. Towards achieving global or semi-global closed loop stability, nonlinear techniques have been used [3] [4]. Nevertheless, a priori knowledge of the accurate values of helicopter dynamic parameters is required, which is almost impossible for some of them (e.g., the aerodynamic coefficients). The aforementioned issue was addressed either by using neural network controllers in [5] [6], or robust nonlinear methods to handle large model uncertainties and external disturbances [7] [8].

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Fig. 1. CSL mini-Helicopter Platform (mini-Deadalus)

Since helicopters fly in unknown usually urban areas, it is important that they should be equipped with safety robust controllers in case of failure to prevent catastrophic consequences. Thus, our goal is to design a low-cost emergency controller that simultaneously stabilizes the attitude and the vertical speed of the helicopter in a normal flight mode. To achieve this, the designing method proposed here is based on some attributes suitable for the helicopters case. Firstly, the controller should be suitable for a wide range of different small-scale helicopters with unknown aerodynamic or other model parameters. For that reason any knowledge of the model parameters is not required for the proposed design. Also, using the stereographic coordinates to model the orientation of the helicopter, the emergency controller guaranties stability for every possible initial attitude state without increasing complexity or introducing representation singularities as in four elements quaternions or Euler angles, respectively. Moreover, the proposed approach described here achieves prescribed transient and steady state response even in the presence of external disturbances. Finally, the control algorithm is of low complexity and can be easily implemented to any embedded flight system.

The paper is organized as follows: The mathematical model of a helicopter and the main goal of this work are presented in Section II. In Section III the control framework along with the stability analysis is provided, while in Sections IV and V respectively, simulation and experimental results are given to verify the theoretical findings. Finally, we conclude in Section VI.

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II. PROBLEM FORMULATION

A. Helicopter Modeling

The nonlinear dynamic model of small-scale helicopters can be represented in a typical non-affine form as follows [3], [9]:

$$\dot{x} = f(x,u,w) \tag{1}$$

where $x = [\ V_b\ \omega_b\ \sigma\ a\ b\ ]^T \in \Re^{11}$ is the state vector that will be defined in the sequel, $u = [\ \delta_{col}, \delta_{lon}, \delta_{lat}, \delta_{ped}, \Omega_d\ ]^T \in \Re^5$ is the control input vector involving the normalized servo inputs of main (i.e., $\delta_{col}, \delta_{lon}, \delta_{lat}$) and tail rotor (i.e., δ_{ped}) as well as the desired main rotor speed (i.e., Ω_d) and $w \in \Re^3$ is the acting wind velocity vector expressed in the helicopter body frame. Employing the Newton-Euler equations for a 6-DOF rigid-body, the translational $V_b = [\ u_b\ v_b\ w_b\ ]^T$ and rotational $\omega_b = [\ p\ q\ r\ ]^T$ dynamics in body frame can be written as follows:

$$\begin{aligned} m\dot{V}_b &= -mS(\omega_b)V_b + f_b \\ J\dot{\omega}_b &= -S(\omega_b)J\omega_b + \tau_b \end{aligned} \tag{2}$$

where m is the helicopter mass, J is the diagonal matrix of moments of inertia in each axis, S denotes the well-known skew-symmetric matrix and f_b, τ_b represent the overall force and moment effects, acting on the body mass and expressed in the body frame, generated by the rotors and various aerodynamic phenomena.

Following the modeling presented in [9] regarding the terms f_b, τ_b, we obtain:

$$\begin{aligned} f_b &= [\ 0\quad 0\quad -T_M\ ]^T + f_{aero}(x,u,w) + f_{grav}(x) \\ \tau_b &= \begin{bmatrix} (K_\beta + h_M T_M)\, b \\ (K_\beta + h_M T_M)\, a \\ -l_T T_T \end{bmatrix} + \tau_{aero}(x,u,w) \end{aligned} \tag{3}$$

where T_M, T_T are the thrusts generated by the main rotor and tail rotor respectively, a, b are the flapping angles of main rotor, K_β is the stiffness of main rotor hub, h_M is the vertical distance from the center of main rotor disc to the center of gravity (CG), l_T is the horizontal distance from the center of tail rotor disc to CG, while f_{aero}, τ_{aero}, f_{grav} are nonlinear equations representing complex aerodynamic forces and moments as well as the gravity effect, respectively. A detailed presentation of the aforementioned terms can be found in [9].

In this work, we assume that the flapping derivatives are ultra-fast and equal to zero in steady-state. Hence, the flapping angles a, b are linearly dependent on the longitudinal and lateral servo movements. Moreover, the main and tail rotor thrusts are calculated using iterative nonlinear algebraic expressions related to the collective servos, states, wind velocities and rotors' aerodynamic coefficients [3]. Thus, we define the following expressions:

$$\begin{aligned} a = C_1\delta_{lon},\ & b = C_2\delta_{lat}, \\ T_M = g_1(x,\delta_{col},w),\ & T_T = g_2(x,\delta_{ped},w) \end{aligned} \tag{4}$$

where C_1, C_2 are unknown constants and g_1, g_2 are nonlinear functions with unknown coefficients. Furthermore, along a typical flight envelope owing to the mechanical properties of the rotor wings, when the collective command increases the corresponding yielded thrust also increases and vice versa, (i.e., the partial derivative of g_1, g_2 wrt to δ_{col} and δ_{ped} respectively remains strictly strictly positive) [2].

Finally, inspired by [10], we adopt the stereographic coordinates formulation to describe the rotational kinematics of the helicopter. Although the quaternion parameters $q = [\ n\ q_1\ q_2\ q_3\ ]^T$ have been used in the past to describe the rotational kinematics of a helicopter [11] [5] in an attempt to eliminate the singularity issues of Euler angles representation, an extra element (n) and a constraint equation ($n^2 + \|q\|_2^2 = 1$) are employed, thus increasing the number of coordinates necessary to describe the rotational kinematics. To heal the aforementioned deficiencies, the stereographic coordinates (i.e., $\sigma = [\sigma_1\ \sigma_2\ \sigma_3]^T \in \Re^3$) defined as: $\sigma_i = \frac{q_i}{n+1},\quad i = 1,2,3$ are employed.Finally, the rotational kinematics are written as:

$$\dot{\sigma} = \frac{1}{2}\mathrm{E}(\sigma)\,\omega_b \tag{5}$$

where $\mathrm{E}(\sigma) = \left(0.5\tilde{\sigma}I + \sigma\sigma^{\mathrm{T}} + S(\sigma)\right)$ and $\tilde{\sigma} = 1 - \sigma_1^2 - \sigma_2^2 - \sigma_3^2$.

B. Problem Statement

In this paper, the control objective is to design an emergency model free controller for small-scale helicopters that stabilizes simultaneously with prescribed performance the orientation (σ) and the vertical speed (w_b) as shown in Fig. 2. This approach refers to a scenario in which the ground pilot or autopilot loses the ability to control the helicopter, for example owing to potential communication malfunctions. In such case, a sophisticated control algorithm is required to be enabled in order to safely stabilize the helicopter in a normal flight mode. The main attributes of this algorithm should be: i) a model free design which significantly increases the portability of the control device, ii) a prescribed response which guarantees a priori a desirable control performance, iii) an implementation of significantly low complexity and iv) the robustness against external disturbances.

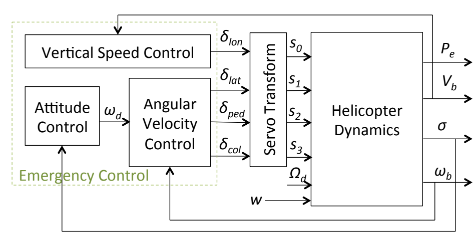

Fig. 2. The emergency controller proposed structure

III. Control Design

A. Prescribed Performance

It will be clearly demonstrated in the sequel, that the control design is connected to the prescribed performance notion that was originally employed to design neuro-adaptive controllers for various classes of nonlinear systems [12]–[14], capable of guaranteeing output tracking with prescribed performance. In this work, by prescribed performance, it is meant that the tracking error converges to a predefined arbitrarily small residual set with convergence rate no less than a certain predefined value. Thus, consider a generic scalar error $e(t)$. Prescribed performance is achieved if $e(t)$ evolves strictly within a predefined region that is bounded by decaying functions of time. The mathematical expression of prescribed performance is given, $\forall t \geq 0$, by the following inequalities:

$$-\rho(t) < e(t) < \rho(t) \tag{6}$$

where $\rho(t)$ is a smooth, bounded, strictly positive and decreasing function of time satisfying $\lim_{t\to\infty}\rho(t) > 0$, called performance function [12]. In this work, we choose an exponentially decreasing performance function $\rho(t) = (\rho_0 - \rho_\infty)e^{-lt} + \rho_\infty$ with ρ_0, ρ_∞, l appropriately chosen strictly positive constants. The constant $\rho_0 = \rho(0)$ is selected such that $\rho_0 > |e(0)|$. The constant $\rho_\infty = \lim_{t\to\infty}\rho(t)$ represents the maximum allowable size of the tracking error $e(t)$ at the steady state. Moreover, the decreasing rate of $\rho(t)$, which is affected by the constant l in this case, introduces a lower bound on the required speed of convergence of $e(t)$.

B. Control Scheme

Given any initial system state condition:

I. Attitude Controller

Select the exponentially decaying attitude performance functions $\rho_{\sigma_1}(t)$, $\rho_{\sigma_2}(t)$, $\rho_{\sigma_3}(t)$ that i) satisfy:

$\rho_{\sigma_j}(0) > \lvert\sigma_j(0)\rvert$	$\rho_{\sigma_j}(t) > 0$	$\lim_{t\to\infty}\rho_{\sigma_j}(t) > 0$

where $j \in \{1,2,3\}$, and ii) incorporate the desired attitude performance specifications regarding the steady state error and the speed of convergence; and design the desired angular velocities:

$$\omega_d = \begin{bmatrix} p_d \\ q_d \\ r_d \end{bmatrix} = -K_\sigma E^{-1}(\sigma) \begin{bmatrix} \ln\left(\frac{1+\frac{\sigma_1}{\rho_{\sigma_1}}}{1-\frac{\sigma_1}{\rho_{\sigma_1}}}\right) \\ \ln\left(\frac{1+\frac{\sigma_1}{\rho_{\sigma_1}}}{1-\frac{\sigma_1}{\rho_{\sigma_1}}}\right) \\ \ln\left(\frac{1+\frac{\sigma_1}{\rho_{\sigma_1}}}{1-\frac{\sigma_1}{\rho_{\sigma_1}}}\right) \end{bmatrix} \tag{7}$$

with a positive definite diagonal gain matrix K_σ.

II. Velocity Controller

Select exponentially decreasing velocity performance functions $\rho_p(t)$, $\rho_q(t)$, $\rho_r(t)$, $\rho_w(t)$ that satisfy:

a. $\rho_n(0) > \lvert n(0) - n_d(0)\rvert$	$\rho_n(t) > 0$	$\lim_{t\to\infty}\rho_n(t) > 0$

where $n \in \{p,q,r,w\}$, and design the servo commands as:

$$\begin{bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \end{bmatrix} = -K_\omega \begin{bmatrix} \ln\left(\frac{1+\frac{p-p_d}{\rho_p}}{1-\frac{p-p_d}{\rho_p}}\right) \\ \ln\left(\frac{1+\frac{q-q_d}{\rho_q}}{1-\frac{q-q_d}{\rho_q}}\right) \\ -\ln\left(\frac{1+\frac{r-r_d}{\rho_r}}{1-\frac{r-r_d}{\rho_r}}\right) \end{bmatrix} \tag{8}$$

$$\delta_{col} = k_w \ln\left(\frac{1+\frac{w_b}{\rho_w}}{1-\frac{w_b}{\rho_w}}\right)$$

with a positive definite control gain matrix K_ω and a positive control gain k_w.

Remark 1: The proposed control scheme does not incorporate the model parameters or knowledge of the external disturbances. Furthermore, no estimation (i.e., adaptive control) has been employed to acquire such knowledge. Moreover, compared with the conventional control approaches, the proposed methodology proves significantly less complex. Notice that no hard calculations are required to output the proposed control signals thus making its implementation straightforward.

C. Stability Analysis

Theorem 1: Consider the nonlinear model of a small scale helicopter (1)-(5) and any initial system condition. The proposed control scheme (7)-(8) guarantees that the attitude σ as well as the vertical speed w_b are stabilized with prescribed transient and steady state performance.

Proof: A brief sketch of the proof is presented here while the detailed mathematical derivations are given in the Appendix. First, let us first define the normalized errors:

$$\xi_{\sigma_1} = \frac{\sigma_1}{\rho_{\sigma_1}(t)},\ \xi_{\sigma_2} = \frac{\sigma_2}{\rho_{\sigma_2}(t)},\ \xi_{\sigma_3} = \frac{\sigma_3}{\rho_{\sigma_3}(t)} \tag{9}$$

$$\xi_p = \frac{p-p_d}{\rho_p(t)},\ \xi_q = \frac{q-q_d}{\rho_q(t)},\ \xi_r = \frac{r-r_d}{\rho_r(t)},\ \xi_w = \frac{w}{\rho_w(t)} \tag{10}$$

and the overall closed loop system state vector as:

$$\xi = \left[\xi_{\sigma_1}, \xi_{\sigma_2}, \xi_{\sigma_3}, \xi_p, \xi_q, \xi_r, \xi_w\right]^T.$$

Differentiating the normalized errors with respect to time and substituting (1)-(5) as well as (7)-(8), we obtain in a compact form, the dynamical system of the overall state vector:

$$\dot{\xi} = h(t,\xi) \tag{11}$$

where the function $h(t,\xi)$ includes all terms found at the right hand side after the differentiation of ξ. Let us also define the open set:

$$\Omega_\xi = \underbrace{(-1,1)\times\cdots\times(-1,1)}_{\text{7-times}}.$$

The proof proceeds in two phases. First, the existence of a maximal solution $\xi(t)$ of (11) over the set Ω_ξ for a time interval $[0,\tau_{\max})$ (i.e., $\xi(t) \in \Omega_\xi$, $\forall t \in [0,\tau_{\max})$) is ensured. Then, we prove that the proposed control scheme guarantees, for all $t \in [0,\tau_{\max})$: a) the boundedness of all closed loop signals as well as that b) $\xi(t)$ remains strictly within a compact subset of Ω_ξ, which subsequently leads by contradiction to $\tau_{\max} = \infty$. Hence, from (9) and (10), we conclude that:

$$-\rho_n(t) < n(t) < \rho_n(t), n \in \{\sigma_1, \sigma_2, \sigma_3, w\}$$

for all $t \geq 0$ and consequently arrive at the solution of the emergency control problem stated in Subsection II-B.

Remark 2: From the proof of the aforementioned theorem, it is worth noticing that the proposed control scheme achieves its goals without residing to the need of rendering $\frac{\bar{F}_i}{k_i}$, $i \in \{x,y,z,\psi,u,v,w,r\}$ arbitrarily small, through extreme values of the control gains k_i, $i \in \{x,y,z,\psi,u,v,w,r\}$. In this respect, the actual tracking performance, which is determined by the performance functions $\rho_{\sigma_1}(t)$, $\rho_{\sigma_2}(t)$, $\rho_{\sigma_3}(t)$, $\rho_w(t)$, becomes isolated against model uncertainties thus extending the robustness of the proposed control scheme.

IV. Simulation Results

In order to verify and clarify the proposed control scheme, realistic simulations were carried out using a complex nonlinear model of a small-scale helicopter, as shown in Fig. 3. The simulated dynamics have also includes the servo dynamics, wind gusts as well as the actual iterative nonlinear functions g_1, g_2 for the computation of thrust. (helicopter parameters can be found in [9]).

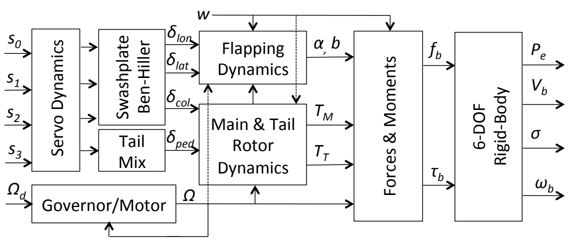

Fig. 3. Nonlinear helicopter full block diagram

The overall simulation duration is 5 sec and is divided into two phases. In the first phase (0-2 sec) the helicopter is uncontrolled and starts from an unknown point with unknown attitude. In the second phase (2-5sec) the emergency control is enabled in order to stabilize it in a safe state. The overall helicopter response is illustrated in Fig. 4.

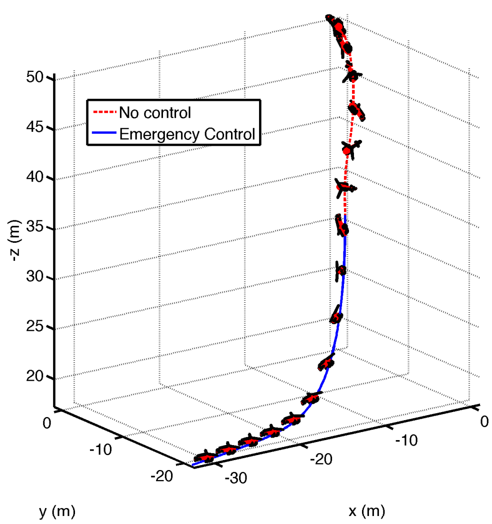

Fig. 4. Nonlinear helicopter model response before and after emergency control activation.

As it is verified in Fig. 4, although the emergency controller is enabled when the helicopter is in a highly critical attitude (i.e., all angles are greater than 90^o), it successfully manages to stabilize the orientation and drives the helicopter to a safe flight state. Moreover, notice that the stereographic coordinates remain inside the desired performance bounds after the controller activation (see Fig .5). Additionally, despite the external wind disturbances and the fact that no knowledge of the model parameters is utilized all velocities errors remain inside the performance bounds (see Fig. 6).

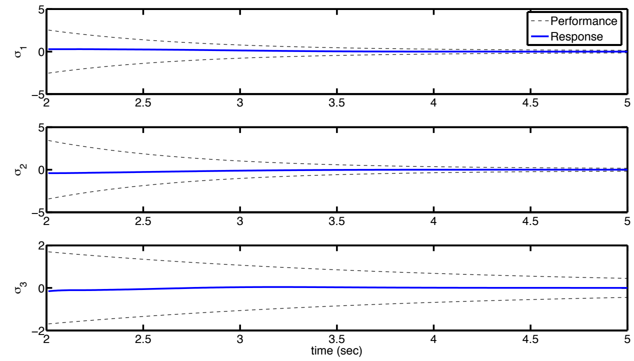

Fig. 5. Stereographic coordinates performance after emergency control activation.

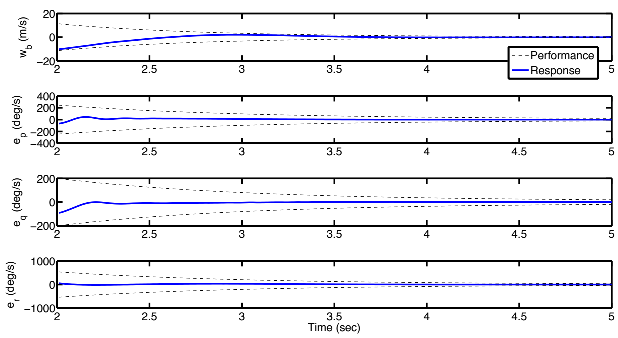

Fig. 6. 4-DOF error velocity performance after emergency control activation.

V. Experimental Results

A. CSL mini-Helicopter Platform

The CSL mini-helicopter platform named ”mini-Deadalus” is a flybarless version of the mini-Titan E325 helicopter with 325mm blades’ radius and a 3500kV electrical motor (Fig. 1). Moreover it is equipped with a speed controller which keeps the rotor speed to a desired value, and a 3-axis control system in order to control helicopter’s angular velocities. This helicopter was transformed into an autonomous aerial platform by attaching all the necessary sensors and devices. The main CPU (high-level board) is a mini-pc with four 1.7GHz ARM Cortex-A9 cores which run Linux with ROS [15]. In this unit the control algorithm is running every 20ms and the resulted commands are send to a low-level board. The low-level board is an 168MHz ARM Cortex-M4 μC, which is responsible to control the servos, send the appropriate commands to the helicopter’s embedded 3-axis controller, read all the available sensors and run low-level critical functions. The main sensors are a 3-axis accelerometer, a 3-axis gyroscope and 3-axis magnetometer that send the data to μC every 10ms where a sensor fusion finally calculates the orientation of the helicopter in quaternions form. Moreover a GPS sensor, a barometer and an altimeter complete the list of the available navigation sensors. Finally, the helicopter can send the requested data to a ground station - only for visualization reasons - or receive back high-level commands using a Wi-Fi module.

B. Preliminary Flight Tests

Owing to the fact that the platform has an already embedded controller for controlling the angular velocities, we started to test only the attitude controller as described in Section III-B in Eq. 7. Thus, an expert pilot was controlling the helicopter up to a desired rather conservative attitude and then the emergency controller is activated. For safety reasons, the pilot was controlling the collective command (δ_{col}) and the rotor desired speed (Ω_d) throughout the whole flight. For these initial tests three different scenarios were executed in which the pilot manually changed the roll, pitch and yaw angle of the helicopter and subsequently activated the controller. It should be noted that a wind gust ($6m/s$, W) was acting on helicopter throughout the whole experimental study. Applying the control parameters, $K_{\sigma_1} = K_{\sigma_2} = K_{\sigma_3} = 0.09$, $\rho_0 = 2|\sigma_0| + 0.1$, $\rho_\infty^{\sigma_1} = \rho_\infty^{\sigma_2} = 0.2$ and $\rho_\infty^{\sigma_3} = 0.16$, the helicopter responses are given in Fig. 7,8,9, respectively while a video is available in [16].

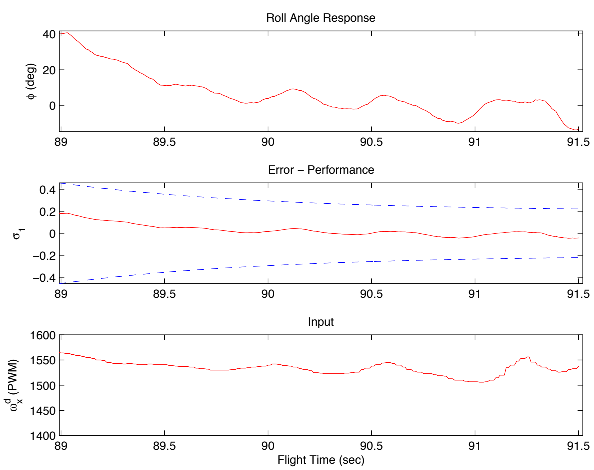

Fig. 7. Helicopter response, performance, input in 1^{st} flight test scenario.

As it was expected, despite the wind effect, which is clearly seen in Fig. 9 where the yaw angle is oscillating, the controller successfully stabilized the attitude of the helicopter with predefined performance.

VI. CONCLUSIONS

In this paper, we presented an emergency attitude and vertical stabilization scheme for small-scaled helicopters. The proposed controller drives, with prescribed transient and steady state response, the helicopter within a stable safe flight mode without requiring prior knowledge of the model parameters and despite the presence of exogenous disturbances. Future directions will be mainly focused on extending the emergency-case scenario towards stabilizing the orientation and the translational velocities in x-y, while controlling the altitude for a safe landing.

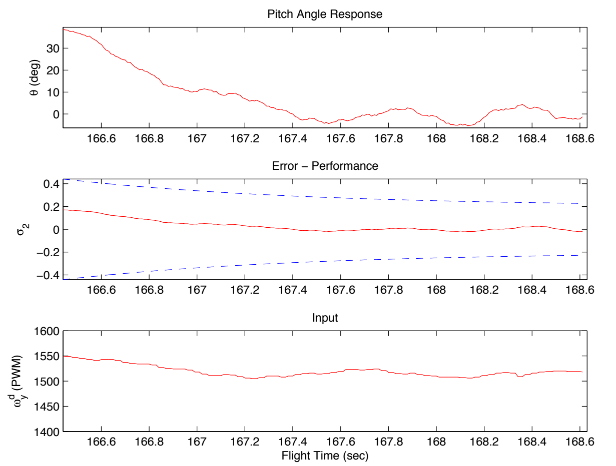

Fig. 8. Helicopter response, performance, input in 2^{nd} flight test scenario.

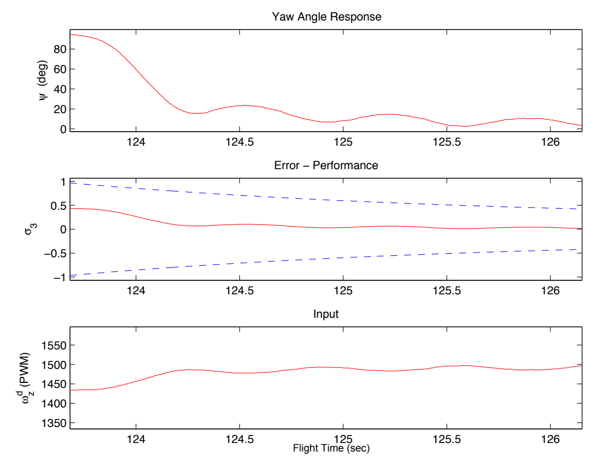

Fig. 9. Helicopter response, performance, input in 3^{rd} flight test scenario.

APPENDIX

Proof of Theorem 1

Phase A. The set Ω_ξ is nonempty and open. Moreover, owing to the selection of the performance functions $\rho_i(t)$ (i.e., $\rho_i(0) > |e_i(0)|$), $i \in \{\sigma_1, \sigma_2, \sigma_3, p, q, r, w\}$ we conclude that $\xi(0) \in \Omega_\xi$. Additionally, due to the smoothness of a) the system nonlinearities and b) the proposed control scheme, over Ω_ξ, it can be easily verified that $h(t,\xi)$ is continuous on t and continuous for all $\xi \in \Omega_\xi$. Therefore, the hypotheses of Theorem 45, stated in [17] (pp. 476), hold and the existence of a maximal solution $\xi(t)$ of (11) on a time interval $[0, \tau_{\max})$ such that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$ is ensured.

Phase B. We have proven in Phase A that $\xi(t) \in \Omega_\xi$, $\forall t \in [0, \tau_{\max})$ Therefore, the signals:

$$\varepsilon_i(t) = \ln\left(\frac{1+\xi_i(t)}{1-\xi_i(t)}\right), \ i \in \{\sigma_1, \sigma_2, \sigma_3, p, q, r, w\} \qquad (12)$$

are well defined for all $t \in [0, \tau_{\max})$. Consider now the positive definite and radially unbounded function $V_1 =$

$\frac{1}{2}\left(\varepsilon_{\sigma_1}^2+\varepsilon_{\sigma_2}^2+\varepsilon_{\sigma_3}^2\right)$. Differentiating with respect to time and substituting (5),(7),(10), after trivial algebraic manipulations, we obtain:

$$\dot{V}_1=\left[\frac{\varepsilon_{\sigma_1}}{\left(1-\xi_{\sigma_1}^2\right)\rho_{\sigma_1}(t)},\frac{\varepsilon_{\sigma_2}}{\left(1-\xi_{\sigma_2}^2\right)\rho_{\sigma_2}(t)},\frac{\varepsilon_{\sigma_3}}{\left(1-\xi_{\sigma_3}^2\right)\rho_{\sigma_3}(t)}\right]$$
$$\times\left(\frac{1}{2}K_\sigma\begin{bmatrix}\varepsilon_{\sigma_1}\\ \varepsilon_{\sigma_2}\\ \varepsilon_{\sigma_3}\end{bmatrix}+\frac{1}{2}E(\sigma)\begin{bmatrix}\xi_p\rho_p(t)\\ \xi_q\rho_q(t)\\ \xi_r\rho_r(t)\end{bmatrix}-\begin{bmatrix}\xi_{\sigma_1}\dot{\rho}_{\sigma_1}(t)\\ \xi_{\sigma_2}\dot{\rho}_{\sigma_2}(t)\\ \xi_{\sigma_3}\dot{\rho}_{\sigma_3}(t)\end{bmatrix}\right).$$

Furthermore, utilizing that $\xi(t)\in\Omega_\xi$, $\forall t\in[0,\tau_{\max})$ and the fact that $\dot{\rho}_{\sigma_1}(t)$, $\dot{\rho}_{\sigma_2}(t)$, $\dot{\rho}_{\sigma_3}(t)$, $\rho_p(t)$, $\rho_q(t)$, $\rho_r(t)$ and $E(\sigma)$ are bounded by construction, we conclude that $\dot{V}_1$ is negative when $|\varepsilon_i(t)|\leq\bar{\varepsilon}_i=\max\left\{\varepsilon_i(0),\frac{\bar{F}_\sigma}{\lambda_{\min}\left[\frac{1}{2}K_\sigma\right]}\right\} i\in\{\sigma_1,\sigma_2,\sigma_3\}$ for all $t\in[0,\tau_{\max})$, for a positive constant $\bar{F}_\sigma$. Thus, it can be easily concluded that:

$$-1<\frac{e^{-\bar{\varepsilon}_i}-1}{e^{-\bar{\varepsilon}_i}+1}\leq\xi_i(t)\leq\frac{e^{\bar{\varepsilon}_i}-1}{e^{\bar{\varepsilon}_i}+1}<1,\ i\in\{\sigma_1,\sigma_2,\sigma_3\}\quad(13)$$

for all $t\in[0,\tau_{\max})$. Additionally, the desired velocities p_d, q_d, r_d remain bounded for all $t\in[0,\tau_{\max})$. Thus, invoking (10), the boundedness of $p(t)$, $q(t)$, $r(t)$ for all $t\in[0,\tau_{\max})$ is also deduced. Finally, differentiating (7) with respect to time and after some algebraic manipulations it is straightforward to obtain the boundedness of $\dot{p}_d(t)$, $\dot{q}_d(t)$, $\dot{r}_d(t)$, $\forall t\in[0,\tau_{\max})$.

Similarly for the dynamic part of the vehicle (2), considering $V_2=\frac{1}{2}\left(\varepsilon_p^2+\varepsilon_q^2+\varepsilon_r^2+\varepsilon_w^2\right)$ and utilizing: i) owing to the mechanical properties of the wings the fact that $K_\beta+h_M g_1(\delta_{col})>\bar{K}$, $\frac{\partial g_1}{\partial_{\delta_{col}}}>\bar{g}_1$, $\frac{\partial g_2}{\partial_{\delta_{ped}}}>\bar{g}_2$ for some positive constants $\bar{K}$, $\bar{g}_1$, $\bar{g}_2$, ii) the fact that the velocities remain bounded for all $t\in[0,\tau_{\max})$ as well as iii) the smoothness of the model nonlinearities; substituting the control law (8) and employing the Mean Value Theorem, we obtain that $\dot{V}_2$ becomes negative whenever:

$$\left\|\begin{matrix}\varepsilon_p\\ \varepsilon_q\\ \varepsilon_r\\ \varepsilon_w\end{matrix}\right\|>\bar{E}=\frac{\bar{F}_d}{\lambda_{\min}\left[diag([J,m])^{-1}diag([\bar{K}C_2,\bar{K}C_1,l_T\bar{g}_2,\bar{g}_1])\right]}$$

for a positive constant $\bar{F}_d$. Hence, we conclude: i) that $|\varepsilon_i(t)|\leq\bar{\varepsilon}_i=\max\{\varepsilon_i(0),\bar{E}\}$, $i\in\{p,q,r,w\}$ for all $t\in[0,\tau_{\max})$, consequently ii) that:

$$-1<\frac{e^{-\bar{\varepsilon}_i}-1}{e^{-\bar{\varepsilon}_i}+1}\leq\xi_i(t)\leq\frac{e^{\bar{\varepsilon}_i}-1}{e^{\bar{\varepsilon}_i}+1}<1,\quad(14)$$

for some positive constants $\bar{\varepsilon}_i$, $i\in\{p,q,r,w\}$ as well as iii) the boundedness of the control law (8) for all $t\in[0,\tau_{\max})$.

Up to this point, what remains to be shown is that $\tau_{\max}=\infty$. Notice that (13) and (14) imply that $\xi(t)\in\Omega_\xi'$, $\forall t\in[0,\tau_{\max})$, where:

$$\Omega_\xi'=\prod_{i\in\{\sigma_1,\sigma_2,\sigma_3,p,q,r,w\}}\left[\frac{e^{-\bar{\varepsilon}_i}-1}{e^{-\bar{\varepsilon}_i}+1},\frac{e^{\bar{\varepsilon}_i}-1}{e^{\bar{\varepsilon}_i}+1}\right]$$

is a nonempty and compact set. Moreover, it can be easily verified that $\Omega_\xi'\subset\Omega_\xi$, $i\in\{\sigma_1,\sigma_2,\sigma_3,p,q,r,w\}$. Hence, assuming $\tau_{\max}<\infty$ and since $\Omega_\xi'\subset\Omega_\xi$, Theorem 45, stated in [17], dictates the existence of a time instant $t'\in[0,\tau_{\max})$ such that $\xi\left(t'\right)\notin\Omega_\xi'$, which is a clear contradiction. Therefore, $\tau_{\max}=\infty$. As a result, all closed loop signals remain bounded and moreover $\xi(t)\in\Omega_\xi'\subset\Omega_\xi$, $\forall t\geq 0$. Additionally, from (9), (13) and (14), we conclude that:

$$-\rho_n(t)<n(t)<\rho_n(t), n\in\{\sigma_1,\sigma_2,\sigma_3,w\}$$

for all $t\geq 0$ and consequently that prescribed performance is achieved, as presented in Subsection III-A, which completes the proof. Finally, it should be noticed that owing to the passivity of the aerodynamics in the longitudinal and latteral degrees of freedom, it can be easily deduced following a standard Input to State Stability (ISS) framework, that the corresponding states remain bounded for bounded external disturbances without altering the proposed control scheme.

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