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Control-Theoretic and Model-Based Scheduling of Crude Oil Transportation for Refinery Industry*

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***Abstract*—Some refineries need to process both low and high fusion point crude oil. Often storage tanks are located at two geographically different sites, one for low fusion point crude oil and the other for high one. With two storage sites, two pipelines are needed to transport different types of crude oil. It is difficult to enforce the constraints resulting from high fusion point oil transportation. Reducing such transportation cost is among the most important goals sought by refinery industry. This work for the first time studies this scheduling problem from a control-theoretic viewpoint. Due to the hybrid property of the process, the system is modeled by a hybrid Petri net. With this model, an efficient method is presented to schedule the transportation of high fusion point crude oil.**

I. INTRODUCTION

Due to the NP-hard nature for short-term scheduling of process industry [1], it is very challenging to schedule oil refining operations [9]. According to [9], the crude oil scheduling problem is one of the most challenging ones in an oil refinery plant. Scheduling crude oil operations needs to sequence not only the jobs, but also define them. Thus, heuristic and mate-heuristic methods that used in discrete production scheduling, such as genetic algorithm (GA), are very difficult to be applied if not impossible. One way to do so is to develop mathematical programming models and their solution. There are mainly two categories of such models: discrete-time and continuous-time ones. The former divides a scheduling horizon into a number of intervals with uniform time durations. An event, such as start and end of an operation, should happen at the boundary of a time interval. Then, an exact solution method is used to obtain an optimal short-term schedule [5, 7, 8, 13]. By such models, the uniform interval must be small enough so as to obtain acceptable accuracy. This often leads to an unmanageable number of binary variables for real-world applications, thereby making the problem or even impossible to solve [1, 10]. Continuous-time models are proposed as an alternative in [2-4] to reduce the number of discrete variables. With them, although the number of discrete variables is significantly reduced, one must handle nonlinear constraints [6]. Also, they need to know the number of discrete events that occur during the scheduling horizon, which is not the case in most industrial setting and thus iterative methods are necessary to determine such a number [1], thereby requiring prohibitive computation.

To make the problem solvable, all the mathematical models make special assumptions, which, unfortunately, make the solution inefficient or unrealistic for real world cases. Moreover, they often ignore critical constraints, i.e. oil residency time and high-fusion-point crude oil (H-oil for short) transportation constraints. This leads their solutions to be infeasible. Hence, one must search for other innovative methods.

From a control theory perspective, our prior work [11] can decompose the problem into two sub-problems in a hierarchical way. At the upper level, it finds a refining schedule to achieve such performance objectives as the minimum crude oil inventory and maximal productivity without generating a detailed schedule. At the lower level, it finds a detailed schedule to realize a given refining one. This eases the original short-term scheduling problem drastically.

In many refineries, all the types of crude oil are unloaded into their storage tanks first and then delivered to the charging tanks. There is only a pipeline used to transport crude oil from the former to the latter. Scheduling analysis is made for cases with and without H-oil [10, 11].

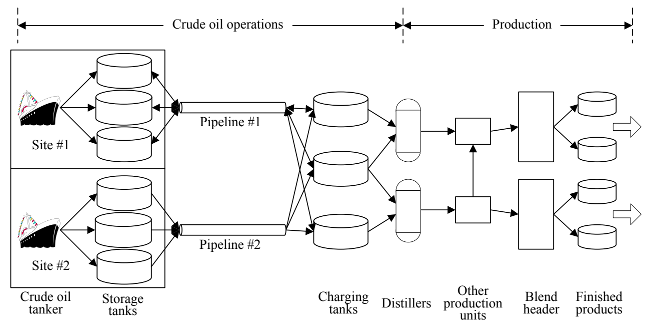

Figure 1. The illustration of 2-pipeline oil refinery process

For many other cases, due to the special requirement for H-oil unloading from a tanker, the storage tanks are located at two geographically different sites, as shown in Fig. 1. The tanks at Site #1 are used for the storage of crude oil with low fusion point oil, L-oil for short, and the ones at Site #2 are for H-oil. Pipeline #1 is used for the transportation of L-oil at Site #1 to charging tanks, while Pipeline #2 for the transportation of H-oil at Site #2. To transport H-oil, Pipeline #2 should be heated first by using hot L-oil that flows from charging tanks to storage tanks. After the transportation of a parcel of H-oil, Pipeline #2 goes to an idle state. However, H-oil cannot stay in the pipeline without flowing, since otherwise it would be frozen and block the pipeline. We call this requirement an H-oil transportation constraint. Thus, it is necessary to send a certain amount of L-oil from the storage tanks to the pipeline such that H-oil in the pipeline can go out of the pipeline.

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Clearly, the oil flow in this pipeline is bidirectional. Hence, some charging tanks should be reserved for oil transportation but not for charging which requires more charging tanks. To reduce the operational cost, it needs to transport a type of H-oil in storage tanks at a time to the refinery plant with a single setup. The question is whether this requirement can be met. Up to now, there is no rigorous research to answer it for such material flow in a refinery. This motivates us to conduct this study to answer this important question.

Next we introduce the short-term scheduling problem addressed in this paper. Then, Section III presents a hybrid PN to describe it. With the model, Section IV conducts scheduling analysis and proposes a control-theoretic scheduling method for the first time. Conclusions are given in Section V.

II. Problem Statement

A. Crude Oil Operations

At the crude oil operation stage, crude oil is carried to a dock near the refinery plant by tankers, where it is unloaded into storage tanks. It is then transported to charging tanks in the refinery plant via pipelines. The charging tanks are used to feed distillers for distillation.

Crude oil is normally unloaded into only an empty storage tank unless the same type of crude oil is in it. After a storage or charging tank is charged, before the crude oil in it can be discharged, it must stay in it for a certain amount of time. This time delay is called oil residency time (RT). Usually, a pipeline takes tens of kilometers long with capacity of tens of thousand cubic meters. It is full of oil all the time and cannot be emptied. Crude oil in it should be taken as inventory and cannot be simply neglected. At each site, there are a number of crude oil types to be processed. These types of oil are delivered from storage tanks to charging ones via a pipeline. Hence, it needs to switch from one type of oil to another from time to time. There may be a number of crude oil segments of different types that stay in a pipeline or are flowing through it. When crude oil is transported, the pipeline can feed one charging tank at a time. Besides, a tank cannot receive and send oil simultaneously.

Due to its transportation constraint, H-oil's transportation cost is high. Hence, when it is transported from storage tanks at Site #2 to charging tanks, it is desired that it can be transported as much as possible, or at least with an expected amount by a single setup.

In summary, scheduling such a system is subject to the following resource and process constraints. The former include: 1) the limited number and capacity of storage and charging tanks; 2) the limited flow rate of oil unloading and oil transportation through the pipelines; and 3) existence of various crude oil types in storage and charging tanks, and in coming tankers. The latter include: 1) a distiller kept in working all the time uninterruptedly; 2) at least one charging tank dedicated to feed a distiller at any time; 3) no simultaneous charging and discharging of a tank; 4) oil residency time constraints after charging; and 5) H-oil transportation constraint must.

B. Short-Term Scheduling Problem

To describe a short-term schedule from a control-theoretic perspective, we first introduce the concept of an operation decision (*OD*).

Definition 2.1: $OD = (COT, \zeta, S, D, INT = [a, b])$ is defined as an *operation decision*, where COT = crude oil type; ζ = volume of crude oil to be unloaded from a tanker to a storage tank, or transported from a source tank to a destination one, or fed from a charging tank to a distiller; S = the source; D = the destination; and INT is a time interval in which a and b are the start and end time points of the operation.

The flow rate in delivering crude oil during $[a, b]$ can be kept as a constant for a single operation. Thus, given volume ζ and time interval $[a, b]$ in an OD, $\zeta/(b - a)$ is determined and used as its flow rate.

Four types of *ODs* are identified, i.e., crude oil unloading, L and H-oil transportation, and feeding, denoted by ODU, ODL, ODH, and ODF, respectively, and their time interval is denoted as $[\alpha, \beta]$, $[\lambda, \mu]$, $[\gamma, \eta]$, and $[\omega, \pi]$, respectively. For ODU, S is a tanker and D is a storage tank. For ODL, S is a storage tank at Site #1, D is a charging tank. For ODH, S and D can be both storage at Site #2 and charging tanks. For ODF, S is a charging tank and D is a distiller. We use ODF_{ki} to denote the i-th OD for feeding distiller k for a schedule. To describe a short-term schedule, let $g = \zeta/(\beta - \alpha)$, $f = \zeta/(\mu - \lambda)$, $r = \zeta/(\eta - \gamma)$, and $h = \zeta/(\pi - \omega)$ denote flow rates for a tanker unloading, L- and H-oil transportation, and distiller feeding decided by *ODs*, respectively. Also let K be the set of distillers. Let $[\tau_0, \tau_1]$ be the schedule horizon that often lasts for a week or ten days. Given the system state at τ_0, i.e., the inventory of crude oil and state of all the devices, and information of tanker arrival, one needs to know if there exists a system state at a time point $\tau \in [\tau_0, \tau_1]$ and time interval $[\tau, \tau_d]$ such that during $[\tau, \tau_d]$ a required amount of H-oil at Site #2 can be transported to the charging tanks. Notice that $[\tau, \tau_d]$ may cross over two schedule horizons. Hence, we let $\Gamma = [\tau_0, \tau_2] = [\tau_0, \tau_1] \cup [\tau, \tau_d]$ be the schedule duration considered and Ψ be the volume of H-oil required to be transported. With Ψ known, it is useful to optimize a refining schedule. We use $ST1$, $ST2$, and CT to denote the storage tanks at Site #1 and #2, and charging tanks, respectively. Then, the short-term scheduling problem is to find a series of *ODs* as follows.

$$SCHD = \{ODU_1, \ldots, ODU_W, ODL_1, \ldots, ODL_Z, ODH_1, \ldots, ODH_U, ODF_1, \ldots, ODF_K\} \tag{2.1}$$

Subject to

$$\omega_{k1} = \tau_0,\ \pi_{k1} = \omega_{k2},\ \ldots,\ \pi_{k(i-1)} = \omega_{ki},\ \ldots,\ \text{and } \pi_{kn} = \tau_2, \text{ for } \forall k \in K \tag{2.2}$$

$\exists i$ and j with $1 \le i < j \le$ U such that

$$\begin{cases} COT_k \in ODH_k, \zeta_k \in ODH_k, S_k \in ODH_k, D_k \in ODH_k, \\ \qquad S_k \in ST2, D_k \in CT, for\, i \le k \le j \\ \zeta_i + \ldots + \zeta_j = \Psi \\ \tau = \gamma_i, \eta_i = \gamma_{i+1}, \ldots, \eta_{j-1} = \gamma_j, \eta_j = \tau_d \end{cases} \tag{2.3}$$

and the resource and process constraints given in Section II.A.

Constraint (2.2) requires that the schedule should cover the entire scheduling duration and a distiller cannot be stopped. Constraint (2.3) guarantees that the required amount of H-oil is transported by a single setup.

Because of a number of constraints and the combinatorial nature, Problem (2.1) is NP-hard. In fact, it is extremely difficult to find a feasible schedule. Notice that an operation decision *OD* in a schedule transfers the system from one state to another. If a state is reached such that one or more constraints are violated, this state is said to be an infeasible one. If not infeasible, it is feasible. If a state is feasible itself, but no matter what *ODs* are applied thereafter, the system will enter an infeasible state, such a state is called an unsafe state, and otherwise it is safe. A safe state guarantees the existence of a feasible schedule. Meanwhile, if a criterion to decide if a state is safe is known, it is easy to find a feasible schedule in a control theoretic perspective. With this idea, we find a model-based solution to the scheduling problem.

III. SYSTEM MODELING BY PETRI NET

To make scheduling analysis for crude oil operations in a control theory perspective, one has to build up a model. Because there are both discrete event and continuous variables, such a model is hybrid. To describe crude oil operations, we use the device models [11] for tanks and Pipeline #1, and develop a new hybrid PN for Pipeline #2 and the overall model for an entire system. A reader is referred as to [14] for the basic knowledge of PN.

We use both discrete and continuous places transitions. Colors are used to identify oil types. Also, the time aspect is described. Thus, the PN model is a kind of hybrid colored-timed PN (CTPN) defined as CTPN = ($P = P_D \cup P_C \cup P_E$, $T = T_D \cup T_T \cup T_C$, I, O, H, Φ, M_0), where P_D, P_C, and P_E are sets of discrete, continuous, and enforcing places; T_D, T_T, and T_C are sets of discrete, timed, and continuous transitions; I: $P \times T \rightarrow \{0, 1\}$ and O: $P \times T \rightarrow \{0, 1\}$ are input and output functions; H: $P \times T \rightarrow \{0, 1\}$ is an inhibitor function; $\Phi(p)$ and $\Phi(t)$ represent the color sets of the places in P and transitions in T; and M_0 is the initial marking. We use for discrete transition, for timed transition, for continuous transition, for continuous place, for discrete place, and for enforcing place. Then, an overall PN model is synthesized via the models for devices.

A. Models for Tanks and Pipelines

The PN models for a tank and Pipeline #1 are shown in Figs. 2-3, respectively. For detail, please see [11].

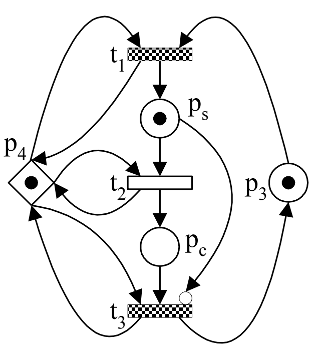

Figure 2. Petri net for a tank

The PN model shown in Fig. 4 is newly developed to describe the behavior of Pipeline #2 where places $p_{s1\text{-}sk}$ are for the tanks at Site #2 and $p_{c1\text{-}ck}$ for the charging tanks. As the model for Pipeline #1 in Fig. 3, continuous places p_1-p_3 are used to identify crude oil segments of different types in the pipeline. Because oil flow through Pipeline #2 bi-directionally, continuous transitions z_{O1} to z_{Ok} and x_{O1} to x_{Ok} are used for crude oil flowing out of and into the pipeline at the Site #2, while z_{I1} to z_{Ik} and x_{I1} to x_{Ik} for crude oil flowing into and out of the pipeline at the plant. Transitions z_2, z_1, x_2, and x_1 are used to describe the dynamic behavior of different oil segments in the pipeline. Let $X_I = \{x_{I1}, \ldots, x_{Ik}\}$, $X_O = \{x_{O1}, \ldots, x_{Ok}\}$, $Z_I = \{z_{I1}, \ldots, z_{Ik}\}$, and $Z_O = \{z_{O1}, \ldots, z_{Ok}\}$.

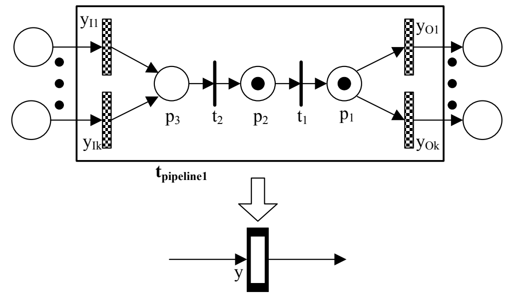

Figure 3. Petri net for Pipeline #1, called $t_{pipeline1}$

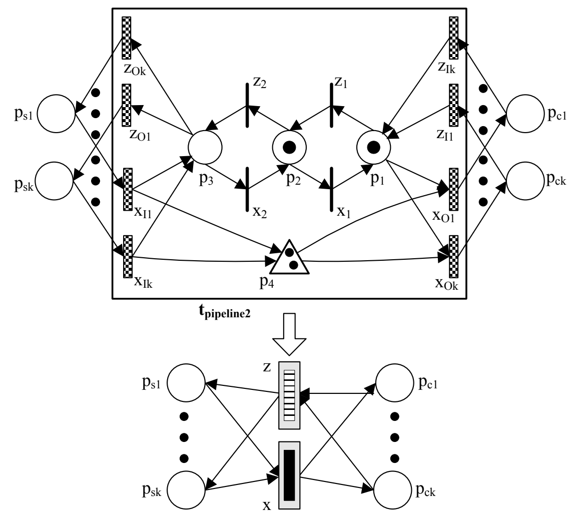

Figure 4. Petri net for Pipeline #2, called $t_{pipeline2}$

To describe an H-oil transportation constraint, an enforcing place p_4 is used to connect x_{Ii} and x_{Oi} in the model. Every time, a token enters p_3 by firing a transition in X_I, a token with the same color and volume goes into p_4 too. When there is a token representing an H-oil type in p_4, it enforces one of transition in X_O to fire, which requires firing one of transitions in X_I too. When the token representing an H-oil type in a continuous place in the model goes out of it by firing a transition in X_O, the token in p_4 is removed. In this way, the behavior of Pipeline #2 is well modeled. It can be simply described by two macro-transitions x and z for different oil flowing directions as shown in Fig. 4. Then, a place p in the model can be denoted as $p(x)$ or $p(z)$ when the crude oil is delivered from storage tanks to charging ones or in the opposite direction.

B. Overall PN Model

With the device models we develop the overall PN model for the system with two storage tanks for each site and two charging tanks shown in Fig. 5, where y represents Pipeline #1 and x and z together represent Pipeline #2. For simplicity, we omit the discrete place and its associated arcs, and the inhibitor arc in a tank PN model. A storage tank i at Site #1 must be discharged via Pipeline #1, it is described by $\{t_{i1}, t_{i2}, y, p_{is}, p_{ic}, p_{i3}\}$. A storage tank i at Site #2 must be discharged via Pipeline #2. However, it can be charged by a tanker or via Pipeline #2, as described by $\{t_{i1}, t_{i2}, x, p_{is}, p_{ic}, p_{i3}\}$ or $\{z, t_{i2}, x, p_{is}, p_{ic}, p_{i3}\}$. Similarly, a charging tank i is described by $\{\nu, t_{i1}, \psi, p_{is}, p_{ic}, p_{i3}\}$, where $\nu = y$ or x when it is charged via Pipelines #1 or #2,; and $\psi = t_{i2}$ or z when it feeds a distiller or is discharged via Pipeline #2. A type of oil from a tanker is modeled by a token entering p_1 (p_2) by firing t_1 (t_2).

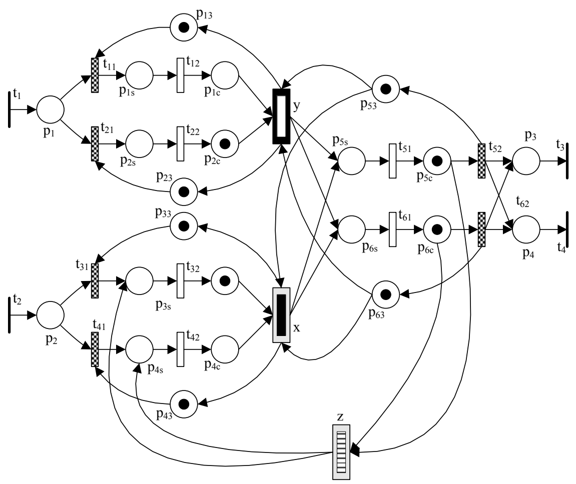

Figure 5. Petri net for the overall system

C. Transition Enabling and Firing Rules

Let ${}^\bullet t$ (${}^\bullet p$) denote the set of input places of transition t (input transitions of place p) and $t^\bullet$ ($p^\bullet$) the set of output places of t (output transitions of p). We use $M(p)$ to denote the number of tokens in p at M, regardless of the colors of the tokens, and $M(p, \varphi_i)$ the number of tokens with color φ_i in p. Further, let $V(M(p))$ be the token volume (the oil volume) in p at marking M. First we define the transition enabling and firing rules for discrete transitions. Notice that, in the model, for a discrete transition t, we have $|{}^\bullet t| = |t^\bullet| = 1$.

Definition 3.1: A discrete transition (including the discrete ones in y, x, and z) t with ${}^\bullet t = p_i$ and $t^\bullet = p_j$ is *enabled* at marking M if $M(p_i) \geq 1$ and $M(p_j) = 0$. When t fires, M is changed into M' with $M'(p_i) = M(p_i) - 1$ and $M'(p_j) = M(p_j)$.

This definition requires that t's output place be empty, which is similar to a finite capacity PN, but the tokens in the PN model here may be discrete or continuous. A timed transition is used only in the PN for a tank to meet the RT constraint before oil in a tank can be discharged.

Definition 3.2: A timed transition t with its delay Δ is *enabled* at marking M if $M(p_i) \geq 1$, $\forall p_i \in {}^\bullet t$. When t starts to fire at time τ, M is changed into M' such that: 1) at τ, $M'(p_i) = M(p_i) - 1$, if $p_i \in {}^\bullet t \cap P_D$, and $M'(p_i) = M(p_i)$ if $p_i \in {}^\bullet t \cap P_C$; 2) at time less than $\tau + \Delta$, $M'(p_i) = M(p_i)$, $p_i \in {}^\bullet t$, $M'(p_j) = M(p_j)$, $p_j \in t^\bullet$; 3) at $\tau + \Delta$, $M'(p_i) = M(p_i) - 1$ and $V(M'(p_i)) = 0$, $\forall p_i \in {}^\bullet t \cap P_C$; and 4) at $\tau + \Delta$, $M'(p_j) = 1$, $\forall p_j \in t^\bullet$ and $V(M'(p_j)) = V(M(p_j)) + V(M(p_i))$, $\forall p_i \in {}^\bullet t$, $p_j \in t^\bullet$, $p_i \in P_C$, $p_j \in P_C$, and $M'(p_j) = M(p_j) + 1$ if $p_j \in t^\bullet \cap P_D$.

When a timed transition fires, there may be a token in its output place p_j. Then, the token in its input place p_i enters p_j, these two tokens merge into one with the volume being their sum.

To distinguish multiple types of crude oil, colors are introduced into the model. Let color $\varphi \in \Phi$ denote a crude oil type. A token in place p representing crude oil type i has color φ_i and the volume for this token at M is denoted by $V(M(p, \varphi_i))$. A continuous transition must fire with a color. As discussed above, the volume of a token in p_3 in Fig. 2 models the capacity of a tank available. Let Θ denote the color for such a token. Further, let $\Phi_1 \subset \Phi$ be the set of colors for L-oil and $\Phi_2 \subset \Phi$ be the set of colors for H-oil with $\Phi_1 \cap \Phi_2 = \varnothing$ and $\Phi_1 \cup \Phi_2 \neq \varnothing$.

Definition 3.3: A continuous transition (including transitions in Y_I, Y_O, X_I, X_O, Z_I, and Z_O in y, x, and z) t is *enabled* with color φ_i at M if

1) $M(p, \varphi_i) \geq 1$ or $M(p, \Theta) \geq 1$, $\forall p \in {}^\bullet t$; and
2) If $K_j(p_s) \in t^\bullet$ for some j, $M(K_j(p_s), \varphi_i) \geq 1$ or $M(K_j(p_c), \varphi_i) \geq 1$, or $M(K_j(p_s)) = M(K_j(p_c)) = 0$.

By 1), we mean that, if $p \in P_C$, there must be crude oil with a right color or the tank to be charged must not be full. Place $p \in P_E$ implies that firing t consumes a token in p. By 2), we mean that the oil in $K_j(p_s)$ has color φ_i that is same as that in $p \in {}^\bullet t$. When a continuous transition t is enabled and triggered by an *OD*, it can then fire. This firing must be associated with a flow rate given by this *OD*, i.e., the flow of oil is governed by a flow rate. Assume that t's firing with color φ_i begins at time τ_s, ends at τ_e, $\tau \in [\tau_s, \tau_e]$, and the flow rate is f. Then, the marking changes are defined.

Definition 3.4: Firing an enabled continuous transition t at M transfers the system into M' according to the following rules.

At τ_s, if $p \in {}^\bullet t \cap P_D$, $M'(p) = M(p) - 1$ (3.1)

At τ_e, if $p \in t^\bullet \cap P_D$, $M'(p) = M(p) + 1$ (3.2)

At $\tau \in [\tau_s, \tau_e]$, if $p \in {}^\bullet t \cap P_C$ or $p \in P_E$, and $M(p, \varphi_i) \geq 1$

$$V(M'(p, \varphi_i)) = V(M(p, \varphi_i)) - (\tau - \tau_s)f \quad (3.3)$$

If $p \in {}^\bullet t \cap P_C$, and $M(p, \Theta) \geq 1$

$$V(M'(p, \Theta)) = V(M(p, \Theta)) - (\tau - \tau_s)f \quad (3.4)$$

If $p \in t^\bullet \cap P_C$ or $p \in P_E$, and $M(p, \varphi_i) \geq 1$

$$V(M'(p, \varphi_i)) = V(M(p, \varphi_i)) + (\tau - \tau_s)f \quad (3.5)$$

If $p \in t^\bullet \cap P_C$ or $p \in P_E$, and $M(p, \varphi_i) = 0$

$$M'(p, \varphi_i) = 1 \text{ and } V(M'(p, \varphi_i)) = (\tau - \tau_s)f \quad (3.6)$$

With the above PN model, some constraints are automatically guaranteed by the transition enabling and firing rules, but the first and fifth process constraints are not. Thus, liveness of the model is defined and used ensure their satisfaction. Let P_{dsl} denote the set of places representing h distillers and thus $|P_{dsl}| = h$.

Definition 3.5: The PN model for the system is said to be live if:

1) At any time τ for any $p_i \in P_{dsl}$, there exists $t_i \in {}^\bullet p_i$ such that t_i is firing or enabled and $\{t_1\} \cap \{t_2\} \cap \ldots \cap \{t_h\} = \varnothing$; and
2) If $M(p_4(x), \varphi_i) > 0$, $\varphi_i \in \Phi_2$, and $p_4(x) \in P_E$, at M, there exists at least a transition $t \in p_4(x)^\bullet$ such that it is enabled or firing.

By Definition 3.5, a non-live state is an infeasible state resulting from an infeasible schedule. Thus, it is necessary to schedule crude oil operations such that their PN model is live.

IV. Scheduling H-Oil Transportation

With the proposed PN model and system control theory, this work carries out scheduling analysis and discovers the conditions under which a short-term schedule can be found such that the net is live and the amount of H-oil at Site #2 can be transported to the charging tanks by a single setup. Let Λ denote the capacity of Pipeline #2, D_i denote distiller i, and υ, ρ, χ, f_{dsi}, and Ω be the maximal transportation rate of Pipeline #1, maximal one of Pipeline #2, minimal one of Pipeline #2, feeding rate to distiller DS_i, and oil residency time, respectively, and the amount of oil fed into D_i during the oil residence time $\alpha_i = \Omega \times f_{dsi}$. Further, let V_{hot} denote the necessary volume of hot oil that flows through Pipeline #2 satisfying $V_{hot} > \Lambda$. Because H-oil transportation begins at the time when hot oil from the charging tanks stops charging to Pipeline #2, we let this time point be τ_0 and discuss the scheduling problem with τ_0 being the starting time. Because the capacity of a pipeline is tens of thousand cubic meters, we have $\Lambda/\rho > \Omega$ and $(V_{hot} + \Lambda)/(2\rho) > \Omega$. Generally, $\forall i$, we have $\rho \geq f_{dsi}$, leading to $\Lambda > \rho \times \Omega \geq f_{dsi} \times \Omega = \alpha_i$. Among all the distillers, a refinery tends to use only one to process H-oil. Hence, this work addresses such a case and presents a method for the problem with the proofs omitted because of the limited space. We have the following result for L-oil transportation.

Proposition 4.1: For a (K+1)-distiller system with D_1 - D_K for processing L-oil and D_{K+1} being able to process H-oil, assume that: 1) the feeding rate of D_i, $i \in \{1, 2, \ldots, K\}$, for crude oil with color $\varphi_i \in \Phi_1$, and $f_{ds1} \neq \ldots \neq f_{dsK}$ and $f_{dsK} < f_{ds1} + \ldots + f_{ds(K\text{-}1)}$; 2) there are $3K$-1 charging tanks $\mathbb{T}_{1\text{-}(3K\text{-}1)}$ for L-oil feeding with $\mathbb{T}_{3(i\text{-}1)+1}$, $\mathbb{T}_{3(i\text{-}1)+2}$, and $\mathbb{T}_{3(i\text{-}1)+3}$ for D_i, $i \in \{1, 2, \ldots, (K\text{-}1)\}$, and $\mathbb{T}_{K\text{-}2}$ and $\mathbb{T}_{K\text{-}1}$ for D_K; 3) initially, the volume of oil with color $\varphi_1 \in \Phi_1$ in $\mathbb{T}_1$, $\mathbb{T}_2$, and $\mathbb{T}_3$ is $\zeta_1 = V_{hot}$ and $\zeta_2 = \zeta_3 = \Pi\alpha_1$ with the oil in $\mathbb{T}_1$ is heated, the volume of oil with color $\varphi_i \in \Phi_1$ in $\mathbb{T}_{3(i\text{-}1)+1}$, $\mathbb{T}_{3(i\text{-}1)+2}$, and $\mathbb{T}_{3(i\text{-}1)+3}$, $i \in \{2, \ldots, (K\text{-}1)\}$, is $\zeta_{3(i\text{-}1)+1} = \zeta_{3(i\text{-}1)+2} = H\alpha_i$, $\zeta_{3(i\text{-}1)+3} = 0$, $\zeta_{3K\text{-}2} = H\alpha_K$, and $\zeta_{3K\text{-}1} = 0$, and the oil in $\mathbb{T}_1$, $\mathbb{T}_2$, and $\mathbb{T}_{3(i\text{-}1)+1}$, $i \in \{2, \ldots, K\}$ is ready for discharging; and 4) $\upsilon \geq f_{ds1} + \ldots + f_{dsK}$. Then, there is a feasible schedule for feeding D_i, $i \in \{1, 2, \ldots, K\}$.

Based on Proposition 4.1, we have derived the following result.

Theorem 4.1: Assume that: 1) the conditions for feeding the L-oil processing distillers given in Proposition 4.1 are satisfied; 2) there is one distiller D_2 that is able to process H-oil with feeding rate f; 3) there are two charging tanks $\mathbb{T}_4$ and $\mathbb{T}_5$ for this distiller with capacities C_4 and C_5, respectively; 4) initially, the volume of oil with color $\varphi_2 \in \Phi_1$ in $\mathbb{T}_4$ and $\mathbb{T}_5$ are ζ_4 and ζ_5, respectively, with $\zeta_4/f \leq (V_{hot} + \Lambda)/\rho$ and $(\zeta_4 + \zeta_5)/f > (V_{hot} + \Lambda)/\rho + \Omega$; and 5) after finishing the oil in $\mathbb{T}_4$ and $\mathbb{T}_5$, D_2 starts processing H-oil with color $\varphi_3 \in \Phi_2$. Then, the amount of H-oil in Site #2 transported into the charging tanks by a single setup is $\psi = \max[\rho((\zeta_4 + \zeta_5)/f - (V_{hot} + \Lambda)/\rho - \Omega), C_4]$.

With two charging tanks $\mathbb{T}_4$ and $\mathbb{T}_5$, to make RT constraint satisfied, 1) $\mathbb{T}_4$ is emptied Ω time units earlier in feeding D_2 before $\mathbb{T}_5$ is filled, leading to an interruption of feeding D_2; or 2) $\mathbb{T}_4$ is emptied in feeding D_2 and $\mathbb{T}_5$ is filled at the same time, leading to the interruption of oil flowing in Pipeline #2. It is infeasible for both cases. Thus, at most one tank of H-oil can be transported to the charging tanks and it is not economically feasible. For the following discussion, we define a tank index function as

$$\xi(i) = \begin{cases} i \bmod 3 + 3, \text{if } i \bmod 3 \neq 0 \\ 6, \text{if } i \bmod 3 = 0 \end{cases} \tag{4.1}$$

Let $C_{max} = \max\{C_4, C_5, C_6\}$ and $C_{min} = \min\{C_4, C_5, C_6\}$. We have the following result.

Theorem 4.2: Assume that: 1) the conditions for feeding the L-oil processing distillers given in Proposition 4.1 are satisfied; 2) there is one distiller D_2 for processing H-oil with feeding rate f; 3) there are three charging tanks $\mathbb{T}_{4\text{-}6}$ for this distiller with capacities C_4, C_5, and C_6, respectively; 4) initially, the volume of oil with color $\varphi_2 \in \Phi_1$ in $\mathbb{T}_4$ and $\mathbb{T}_5$ are ζ_4 and ζ_5, respectively, with $\zeta_4/f \leq (V_{hot} + \Lambda + C_6)/\rho$ and $(\zeta_4 + \zeta_5)/f \geq (V_{hot} + \Lambda + C_6)/\rho + \Omega$, and $\mathbb{T}_6$ is empty; 5) $C_{max}/\rho \leq C_{min}/f$; and 6) after finishing the oil in $\mathbb{T}_4$ and $\mathbb{T}_5$, D_2 starts processing H-oil with color $\varphi_3 \in \Phi_2$. Then, there exists a feasible schedule such that the amount ψ of H-oil in Site #2 transported into the charging tanks by a single setup can be calculated by Algorithm 4.1.

Algorithm 4.1: Finding the amount of H-oil that can be transported into the charging tanks by a single setup with three charging tanks

1) Initialization: $\tau = (V_{hot} + \Lambda + C_6)/\rho$, $h = \zeta_4/f$, $g = (\zeta_4 + \zeta_5 + C_6)/f$, and $\psi = C_6$.
2) Filling $\mathbb{T}_4$: let $i = 4$
   a) If $\tau + C_4/\chi + \Omega \leq g$, $\tau = \tau + C_4/\chi$, $h = h + \zeta_5/f$, $g = g + C_4/f$, $\psi = \psi + C_4$, and $i = i + 1$. Then, go to 3);
   b) If $\tau + C_4/\rho + \Omega \leq g < \tau + C_4/\chi + \Omega$, $\beta = C_4/(g - \Omega - \tau)$, $\tau = \tau + C_4/\beta$, $h = h + \zeta_5/f$, $g = g + C_4/f$, $\psi = \psi + C_4$, and $i = i + 1$. Then, go to 3).
3) Let $j = \xi(i)$ and $k = \xi(i + 1)$
4) Filling $\mathbb{T}_j$
   a) If $h \leq \tau$ and $\tau + C_j/\chi + \Omega \leq g$, $\tau = \tau + C_j/\chi$, $h = h + C_k/f$, $g = g + C_j/f$, $\psi = \psi + C_j$, and $i = i + 1$. Then, go to 3);
   b) If $h \leq \tau$ and $\tau + C_j/\rho + \Omega \leq g < \tau + C_j/\chi + \Omega$, $\beta = C_j/(g - \Omega - \tau)$, $\tau = \tau + C_j/\beta$, $h = h + C_k/f$, $g = g + C_j/f$, $\psi = \psi + C_j$, and $i = i + 1$. Then, go to 3);
   c) If $h > \tau$ go to 5).
5) Return ψ.

To transport H-oil from Site #2 into the charging tanks as much as possible by a single setup, it is desirable to charge each tank to its full capacity. Assumption $(\zeta_4 + \zeta_5)/f \geq (V_{hot} + \Lambda + C_6)/\rho + \Omega$ guarantees that $\mathbb{T}_6$ can be charged to its capacity when it is charged for the first time. Because of $C_{max}/\rho \leq C_{min}/f$, if $\mathbb{T}_6$ is charged to its capacity for the first time, the following tanks can also be charged to their capacity only if they can be charged. Thus, assumption $(\zeta_4 + \zeta_5)/f \geq (V_{hot} + \Lambda + C_6)/\rho + \Omega$ is very important. This implies that we can select appropriate states to start the H-oil transportation such that the volume of oil transported is maximized.

Sometimes more H-oil should be transported from Site #2 to the charging tanks. Thus, more than three charging tanks for feeding DS_2 are necessary and we present the result for the four-charging-tank case. For the discussion, we first define tank index function as

$$\theta(i) = i \bmod 4 + 4 \tag{4.2}$$

Theorem 4.3: Assume that: 1) the conditions for feeding the L-oil processing distillers given in Proposition 4.1 are satisfied; 2) there is one distiller D_2 for processing H-oil with feeding rate f; 3) there are four charging tanks $\mathbb{T}_{4\text{-}7}$ for this distiller with capacities C_4 - C_7, respectively; 4) initially, the volume of oil with color $\varphi_2 \in \Phi_1$ in $\mathbb{T}_4$ and $\mathbb{T}_5$ are ζ_4 and ζ_5, respectively, with $\zeta_4/f \leq (V_{hot} + \Lambda + C_6 + C_7)/\rho$ and $(\zeta_4 + \zeta_5)/f \geq (V_{hot} + \Lambda + C_6)/\rho + \Omega$, and $\mathbb{T}_6$ and $\mathbb{T}_7$ are empty; 5) $C_{max}/\rho \leq C_{min}/f$; and 6) after finishing the oil in $\mathbb{T}_4$ and $\mathbb{T}_5$, D_2 starts processing H-oil with color $\varphi_3 \in \Phi_2$. Then, there exists a feasible schedule such that the amount ψ of H-oil in Site #2 transported into the charging tanks by a single setup can be calculated by Algorithm 4.2 below.

Algorithm 4.2: Finding the amount of H-oil that can be transported into the charging tanks by a single setup with four charging tanks

1) Initialization: $\tau = (V_{hot} + \Lambda + C_6 + C_7)/\rho$, $h = \zeta_4/f$, $g = (\zeta_4 + \zeta_5 + C_6 + C_7)/f$, and $\psi = C_6 + C_7$;
2) Filling $\mathbb{T}_4$: let $i = 4$
   a) If $\tau + C_4/\chi + \Omega \leq g$, $\tau = \tau + C_4/\chi$, $h = h + \zeta_5/f$, $g = g + C_4/f$, $\psi = \psi + C_4$, and $i = i + 1$. Then, go to 3);
   b) If $\tau + C_4/\rho + \Omega \leq g < \tau + C_4/\chi + \Omega$, $\beta = C_4/(g - \Omega - \tau)$, $\tau = \tau + C_4/\beta$, $h = h + \zeta_5/f$, $g = g + C_4/f$, $\psi = \psi + C_4$, and $i = i + 1$. Then, go to 3)
3) Let $j = \theta(i)$ and $k = \theta(i + 1)$
4) Filling $\mathbb{T}_j$
   a) If $h \leq \tau$ and $\tau + C_j/\chi + \Omega \leq g$, $\tau = \tau + C_j/\chi$, $h = h + C_k/f$, $g = g + C_j/f$, $\psi = \psi + C_j$, and $i = i + 1$. Then, go to 3);
   b) If $h \leq \tau$ and $\tau + C_j/\rho + \Omega \leq g < \tau + C_j/\chi + \Omega$, $\beta = C_j/(g - \Omega - \tau)$, $\tau = \tau + C_j/\beta$, $h = h + C_k/f$, $g = g + C_j/f$, $\psi = \psi + C_j$, and $i = i + 1$. Then, go to 3);
   c) If $h > \tau$ go to 5).
5) Return ψ.

With four charging tanks for DS_2, much more H-oil at Site #2 can be transported into the charging tanks. With ψ calculated, we can check if $\psi \geq \Psi$. If so the requirement can be met, and otherwise it is not.

This method has been applied to an industrial case study. A refinery has four distillers D_1-D_4 with D_2 being able to process H-oil. Initially, there are 11 charging tanks available for distiller feeding. The maximal flow rate of Pipeline #1 is υ = 1,250 tons/h, and the maximal and minimal flow rates of Pipeline #2 are ρ = 625 tons/h and χ = 420 tons/h, respectively. There is 134,000 tons of H-oil at Site #2 to be transported to the charging tans. By using the proposed method, it is verified that the conditions given in Proposition 4.1 are met and three tanks can be used to feed D_2. Then, by Theorem 4.2 and Algorithm 4.1, a short-term schedule is found and 134,000 tons of H-oil is able to be transported to the charging tanks by a single setup. Such a problem is extremely difficult, even is impossible, for an existing method to solve.

V. Conclusion

Short-term scheduling for crude oil operations in a refinery belongs to the so-called NP-hard problems. Besides, transportation of H-oil results in high cost because of the high-cost setup. Thus, it is desired that the volume to be transported by a single setup should be as large as possible. With two pipelines, one for L-oil and the other for H-oil transportation, the problem is further complicated. There is no existing model and method that deal with such issues to the best knowledge of the authors. In this work, by modeling the system with a hybrid Petri net, the conditions and efficient method are presented to obtain a feasible schedule and to maximize the amount of H-oil that can be transported. It shows that the amount of H-oil that can be transported is greatly dependent on the initial state. With a detailed schedule obtained, it is necessary to find an optimal refining schedule by minimizing crude oil inventory cost, middle and final product inventory cost, and maximizing production profit. This is our future work.

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