Coordination of a Nonholonomic Mobile Platform and an On-board Manipulator

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Abstract—Mobile manipulators provide more advantages and flexibility in a wide range of applications than standard manipulators by introducing mobility. However, adding mobile platforms to standard manipulators, especially nonholonomic mobile platforms, introduces new challenges to the system modeling and control. Most existing methods for mobile manipulators do not consider the performance difference between the mobile platform and the manipulator and therefore cannot handle the uncertain and unexpected events happened in both the mobile platform and the manipulator. This paper introduces a planning and control method in a perceptive reference frame for a nonholonomic mobile manipulator to efficiently handle uncertain and unexpected events. The experimental results on a nonholonomic mobile manipulator demonstrate the effectiveness and advantages of the designed method.

I. INTRODUCTION

Mobile manipulators integrate the advantages of the mobile platform and the manipulator and have been widely used in many areas including industrial manufacturing, hazardous material operations, domestic service, etc. [1-3]. The working space of a standard manipulator can be enlarged by introducing a mobile platform, commonly a nonholonomic mobile platform. However, this integration of two different systems also introduces new challenges. First, the models for the mobile platform and manipulator are different. The manipulator is usually a holonomic system but the mobile platform may be subject to nonholonomic constraints. A method is required to incorporate the nonholonomic constraints during the system modeling. Second, the integrated system is highly redundant and the redundancy resolution scheme is required. Third, from the view of practical implementation, even if the above two theoretical problems are perfectly solved, the control performance may still be affected by uncertain and unexpected events in both the mobile platform and the manipulator.

Many studies have been conducted on the modeling and control of nonholonomic mobile manipulators. There are commonly two ways to model the kinematic system with nonholonomic constraints. One way is to directly add the constraints to the velocity kinematic model [4][5]. Another more efficient way is to model the system to explicitly entail the admissible motions with respect to the nonholonomic

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constraints [6][7], which is also used in this paper. After the model is obtained, the nonholonomic mobile manipulator system is usually redundant for a given end-effector task which usually has no more than six dimensions. Then, the redundancy resolution methods for standard manipulators can be extended to the nonholonomic mobile manipulator including Extended Jacobian method [8], task priority method [9], Reduced Gradient based method [10], Singularity-Robust method [11], Damped Least-Squares Inverse Jacobian method [12], etc. A task-space or end-effector space closed-loop controller can then be incorporated into the redundancy resolution schemes to make the robot track the desired end-effector motions.

However, in practical implementation, the task-space closed-loop controllers with redundancy resolution cannot always guarantee the best performance of the end-effector tracking because of the uncertain and unexpected events in both the mobile platform and the manipulator. First, the mobile platform and the manipulator have different structure characteristics and also work in completely different environments. This results in different motion dynamics and errors for the mobile platform and the manipulator. However, the task-space controller is not aware of such differences. This can affect the tracking performance of the end-effector. In addition, either or both of the mobile platform and the manipulator may be stopped by unexpected events such as an obstacle. Since the task-space controller is not aware of these issues as well, it will generate larger end-effector errors and even lead to stability problems if no safety mechanisms are designed ahead.

To address these problems, coordination between the mobile platform and on-board manipulator is required. The major contribution of this paper is to present a planning and control method in a perceptive reference frame to online coordinate a nonholonomic mobile platform and a manipulator for the practical application. The method can handle uncertain and unexpected events in both the mobile platform and the manipulator. First of all, the kinematic model of the nonholonomic mobile manipulator is derived. Based on the model, traditional task-space kinematic control methods with redundancy resolution are then introduced. By using a perceptive reference, the traditional methods are converted to online and closed-loop planning and control processes driven by the system outputs. This makes the system be able to be aware of uncertain and unexpected events and then efficiently handle them. Therefore, the performance of the nonholonomic mobile manipulator can be significantly improved including the end-effector tracking errors and system

safety.

II. KINEMATICS OF THE NONHOLONOMIC MOBILE MANIPULATOR

A. Kinematic Modeling

The nonholonomic mobile manipulator studied in this paper contains a 4-wheel drive mobile platform and a 7-DOF on-board manipulator as shown in Fig. 1. The coordinate frames and variables are defined as below:

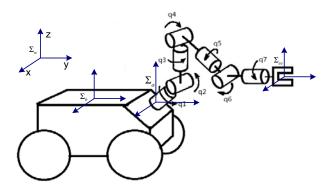


Fig. 1. Nonholonomic Mobile Manipulator

- Σ_w : World reference frame;
- Σ_a : Manipulator base frame;
- Σ_b: Body attached frame at the horizontal center of the mobile platform, which origin represents of the abstraction of the mobile platform;
- Σ_{ee}: Frame fixed at the end-effector center, which origin represents the abstraction of the end-effector;
- $r = [x_w, y_w, z_w, O_w, A_w, T_w]^T$: Generalized endeffector position with respect to Σ_W ;
- $r_a = [x_a, y_a, z_a, O_a, A_a, T_a]^T$: Generalized endeffector position with respect to Σ_a ;
- $\bar{r}_b = [x_b, y_b, \theta_b]^T$: Position and orientation of the mobile platform with respect to Σ_W ;
- $r_b = [d_b, \theta_b]^T$: Traveled distance and orientation of the mobile platform with respect to Σ_W ;
- $\dot{r}_b = [v_b, \omega_b]^T = [\dot{d}_b, \dot{\theta}_b]^T$: Longitudinal and turning velocities of the mobile platform;
- $q_a = [q_1, q_2, q_3, q_4, q_5, q_6, q_7]^T$: Joint variables of the manipulator;
- $u_b = [u_l, u_r]^T$: Joint velocity of the mobile platform;
- $u_a = [u_1, u_2, u_3, u_4, u_5, u_6, u_7]^T$: 7 Joint motor velocities of the manipulator;
- $u = \begin{bmatrix} u_b & u_a \end{bmatrix}^T$: Complete motor velocities of the mobile manipulator;

The forward kinematics of the 7-DOF manipulator with respect to Σ_a can be obtained by

$$y_a = h_a(q_a) \tag{1}$$

Using the definition $\bar{r}_b = [x_b, y_b, \theta_b]^T$ and (1), the forward kinematics of the mobile manipulator with respect to Σ_w can be obtained by

$$r = h(\bar{r}_b, q_a)$$

$$= \bar{r}_b + T_a^w(\theta_b) h_a(q_a)$$
(2)

where $T_a^w(\theta_b)$ is a transformation matrix which transfers the manipulator base frame Σ_a to the frame whose origin coincides with the origin of Σ_b and whose axes are parallel to the axes of Σ_w . The only variable in the $T_a^w(\theta_b)$ is θ_b .

Through differentiation on both sides of (2), the velocity kinematics can be obtained by

$$\dot{r} = \frac{\partial h}{\partial \bar{r}_b} \dot{\bar{r}}_b + \frac{\partial h}{\partial q_a} \dot{q}_a
= J_b \dot{\bar{r}}_b + J_a \dot{q}_a$$
(3)

The mobile platform is constrained by a nonholonomic constraint described by

$$\dot{x}_b \sin(\theta_b) - \dot{y}_b \cos(\theta_b) = 0 \tag{4}$$

which implies that the mobile platform cannot have a velocity along the lateral direction with respect to its body. One way to handle this constraint is to attach the constraint description to the velocity kinematic model by adding a new row described by

$$0 = \begin{bmatrix} \sin(\theta_b) & -\cos(\theta_b) & 0 & 0_{7\times 1} \end{bmatrix} \begin{bmatrix} \dot{\bar{r}}_b \\ \dot{q}_a \end{bmatrix}$$
 (5)

Another more efficient way is to modify the kinematic model to explicitly entail the admissible velocities of the mobile platform with respect to its nonholonomic constraint. The admissible velocities actually contain a longitudinal velocity parallel to the mobile platform and an angular velocity for horizontally turning the mobile platform. Therefore, define these two admissible velocities by $\dot{r}_b = [v_b, \omega_b]^T$ and we have

$$\dot{r} = J_b J_b^{adm} \dot{r}_b + J_a \dot{q}_a \tag{6}$$

where J_b^{adm} is the transformation matrix for the transformation from the admissible velocity space to the operational velocity space, which is expressed by

$$J_b^{adm} = \begin{bmatrix} \sin(\theta_b) & 0\\ \cos(\theta_b) & 0\\ 0 & 1 \end{bmatrix}$$
 (7)

For both the mobile platform and the manipulator, the ultimate outputs for kinematic control should be joint motor velocities. For the manipulator, there is a one-to-one correspondence between the joint motor velocities u_a and the joint variables q_a . So we have

$$u_a = \dot{q}_a \tag{8}$$

For the mobile platform, there are four motors with two mounted on both left and right sides, so it has four joint motor velocities u_{l1} , u_{l2} , u_{r1} and u_{r2} . Because of the special installation of these motors, considering no slippage in longitudinal direction, it is required that the two motor velocities on the same side must be identical, which is described by

$$u_{l1} = u_{l2} u_{r1} = u_{r2}$$
 (9)

For simplicity, use $u_b = [u_l, u_r]^T$ to represent the motor velocities on the left and right sides and the following velocity kinematics can be obtained

$$\dot{r} = J_b J_b^{adm} J_b^u u_b + J_a u_a
= \begin{bmatrix} J_b J_b^{adm} J_b^u & J_a \end{bmatrix} \begin{bmatrix} u_b \\ u_a \end{bmatrix}$$

$$= J_m u$$
(10)

where J_b^u is a constant transformation matrix for transforming u_b to \dot{r}_b , which can be easily obtained based on the dimension parameters of the mobile platform, and J_m is the final Jacobian of the nonholonomic mobile manipulator.

B. Kinematic Control

In (10), u contains nine joint motor velocities of the mobile platform and the manipulator. The end-effector velocity \dot{r} is usually a variable of no more than six dimensions. Therefore, the mobile manipulator system is highly redundant. For a given desired end-effector velocity \dot{r}^d , there exist many feasible solutions of u to achieve it. The redundancy resolution methods for standard manipulators can be extended to the kinematic control of the nonholonomic mobile manipulator.

For a given end-effector velocity \dot{r} , the formula of the most commonly used kinematic control methods with redundancy resolution can be expressed by

$$u = J_m^{\dagger} \dot{r} + (I - J_m^{\dagger} J_m) u_0 \tag{11}$$

where J_m^{\dagger} is the pseudoinverse of J_m , $I-J_m^{\dagger}J_m$ is the orthogonal projection operator which projects the joint motor velocity vector into the null space of J_m , and u_0 is the designed joint motor velocity for the secondary task. The first item is in the operational space and the goal is to achieve the end-effector velocity. The second item is in the null space and the goal is to satisfy some secondary tasks. It could be to achieve a designed joint motor velocity or, more commonly, to optimize a criterion that is related to the robot states.

Furthermore, when a desired trajectory is given by $\dot{r}^d(t)$ and $r^d(t)$, a task-space closed-loop controller [11][13] can then be obtained by replacing \dot{r} in controller using

$$\dot{r} = \dot{r}^d(t) + K_r(r^d(t) - r_c) \tag{12}$$

where K_r is a constant gain matrix and r_c is the measurement of current end-effector position. This closed-loop design is adopted in most kinematic controllers of non-holonomic mobile manipulators. However, the closed-loop control in task space cannot always guarantee the best performance of the system since there may exist some uncertain and unexpected events in both the mobile platform and the manipulator. When the joint motor velocities are generated from the kinematic controller, they are sent to the dynamic motor controllers of the mobile platform and the manipulator separately. There usually exist two types of uncertain and unexpected events in both the mobile platform and the manipulator.

The first type relates to uncertain events in the dynamics. The manipulator is relatively light-weight and usually works in a contact-free environment that has relatively small motor disturbance. This leads to fast dynamics and small errors. In contrast, the mobile platform is much heavier and works in an unstructured environment where many unknown factors can introduce large disturbances, such as the contact ground condition or on-board manipulator motions. This leads the mobile platform to slow dynamics and larger errors. These differences, however, are never considered in the task-space closed-loop controller. These problems will result in large end-effector errors when the differences are obvious.

The second type of relates to unexpected events in both robotic systems. When the mobile platform or manipulator is stopped by unexpected events, such as an obstacle, only one of them is movable in this situation. However, the task-space kinematic controller is not aware of this and will continue generating velocities for both the mobile platform and the manipulator, one of which, however, can never be achieved. This will lead to large errors of end-effector. Moreover, when both the mobile platform and the manipulator are stopped, The desired end-effector velocities and position trajectories will still keep evolving because they are both functions of time and the time never stops. This will also make the timebased task-space kinematic controller continue to generate larger and larger joint motor velocities as time increasing, which will make the system unstable if no safety mechanisms are designed ahead. This is because the desired end-effector velocities and position trajectories are both functions of time and will always keep evolving because the time never stops.

Therefore, in order to achieve the best possible performance under these uncertain and unexpected events, coordinated control between the mobile platform and the manipulator and an automatic safety mechanism is required. These two requirements can actually be achieved at the same time through the planning and control in a perceptive reference frame which is introduced in the next section.

III. COORDINATION OF THE NONHOLONOMIC MOBILE MANIPULATOR

A. Planning and Control Theory in the Perceptive Reference Frame

In traditional planning and control, the planning is usually an open-loop process. The time variable is used as the motion reference and the motion plan is parameterized by time based on the defined tasks. Then, the planner generates the desired instant output to the control system by plugging the current referenced time into the time-based motion plan. All uncertain and unexpected events are left to the control system to handle. If these events are not considered in the action plan, then the controller alone is not able to handle them. This is especially true when it is working in an unstructured environment. Therefore, it is very important to incorporate the planning and control processes based on system output measurement to handle these events and achieve the best possible performance.

The planning and control in the perceptive reference frame aims to handle uncertain and unexpected events in both planning and control levels. The basic idea is to model the system planning and control based on a non-time reference related to the physical system outputs instead of time t. The new reference is named perceptive reference and usually represented by s. Based on this idea, we have designed the multi-robot coordination [14][15] and teleoperation [16] in our previous work. In this paper, it is modified and extended to the coordinated control of the nonholonomic mobile manipulator. The coordinated control mainly contains three steps described as below.

B. Generation of Motion Plans

The desired motion of the end-effector is given by a human operator using a spaceball. The spaceball can generate 6-dimensional operational commands including 3-dimensional translation and 3-dimensional rotation. Scaled sampled data of these commands are used as the desired velocity of the end-effector. Given a time-based desired trajectory $\dot{r}^d(t)$, define the perceptive reference s as the distance that the end-effector travels along the desired trajectory, the time-based trajectory can be converted to a non-time based trajectory parameterized by s: $\dot{r}^d(s)$. Using the task-space closed-loop kinematic control, the joint motor velocities of the mobile manipulator, u, can be obtained

$$u = U(\dot{r}^d(s)) \tag{13}$$

where u contain velocities of both the mobile platform and the manipulator, u_b and u_a . Furthermore, the desired velocities for the mobile platform and the manipulator, $\dot{r}_b^d(s)$ and $\dot{r}_a^d(s)$, can be obtained by

$$\dot{r}_b^d(s) = J_b^u U_b(\dot{r}^d(s))
\dot{r}_a^d(s) = J_a U_a(\dot{r}^d(s))$$
(14)

where the functions U_b and U_a are decomposed from the function U. From this relationship, given a desired trajectory of the end-effector $\dot{r}^d(s)$, we can find its corresponding desired trajectories of the mobile platform and the manipulator using a chosen kinematic control method. They are parameterized by s and represented by $\dot{r}^d_b(s)$, $r^d_b(s)$, $\dot{r}^d_a(s)$ and $r^d_a(s)$.

C. Generation of Motion Reference for Coordinated Control

The goal is to minimize the end-effector errors and guarantee the system safety under uncertain and unexpected events. This can be achieved by the coordination which requires that both the mobile platform and the manipulator work in a planned manner to achieve the desired motion of the end-effector. To be specific, for a given point along the path of the end-effector, both the mobile platform and the manipulator should be in their desired positions corresponding to the given point. To achieve this requirement, the perceptive reference can be generated as following. When the tracking performances of the mobile platform and the manipulator differ because of uncertain events in the dynamics, e.g., one tracks its desired motion slower than the other, the perceptive reference could be designed to slow down the motion of the faster one until the slower once catches up with it, such that

the end-effector errors can always be minimized. If one of or both the mobile platform and the manipulator are stopped by unexpected events, the perceptive reference could be designed to stop both motions of the mobile platform and the manipulator until the unexpected events are removed, such that the safety of the system can be guaranteed. Therefore, given the outputs of the mobile platform and the manipulator, r_a and r_b , to achieve this coordination, the calculation of the best perceptive reference can be designed as

$$s^* = \min\{s_b, s_a\}$$

$$s_b = \arg\min_{s} \left\{ (r_b^d(s) - r_b)^T W_b(r_b^d(s) - r_b) \right\}$$

$$s_a = \arg\min_{s} \left\{ (r_a^d(s) - r_a)^T W_a(r_a^d(s) - r_a) \right\}$$
(15)

where s_b represents the motion tracking status of the mobile platform and is calculated based on the mobile platform outputs; s_a represents the motion tracking status of the manipulator and is calculated based on the manipulator outputs; W_a and W_b are the weighting matrices to weigh the coordination errors based on the specific coordination requirements.

D. Planning and Control for Coordination

Once the best perceptive reference is generated based on the system output measurement, the planning for generating the desired instantaneous inputs for the mobile platform and the manipulator is achieved by simply plugging the perceptive reference into their desired motion plans. The joint motor velocities for the mobile manipulator can then be designed by

$$u = \begin{bmatrix} u_b & u_a \end{bmatrix}^T u_b = (J_b^u)^{\dagger} (\dot{r}_b^d(s^*) + k_b (r_b^d(s^*) - r_b)) u_a = (J_a)^{\dagger} (\dot{r}_a^d(s^*) + k_a (r_a^d(s^*) - r_a))$$
 (16)

where k_b and k_a are positive constants. The new velocity inputs are then passed to the dynamic motor controllers of both the mobile platform and the manipulator for executions.

The planning and control for coordination in the perceptive reference frame is schematically described as shown in Fig. 2. The block of Motion Plan Generation generates the motion plans described by s for both the mobile platform and the manipulator based on the desired trajectory of the end-effector. The system outputs are mapped to a best perceptive reference s^* which carries the current states of both the mobile platform and the manipulator. The block of Coordination Planner generates the desired instantaneous inputs for the system based on the original motion plans and the best perceptive reference s^* . Then, the block of Coordination Controller computes new joint motor velocities to the dynamic motor controllers of both the mobile platform and the manipulator to achieve the desired instantaneous output. It is easily seen that, at a given instant of time, the desired system inputs are functions of current system outputs, which makes the planning a closed-loop process to be able to handle some uncertain and unexpected events according to the original motion plans.

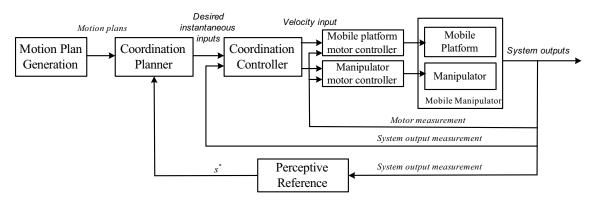


Fig. 2. Planning and control for coordination in the perceptive reference frame

Notice that if there are no uncertain or unexpected events in both the mobile platform and the manipulator, the planning and control in the perceptive reference frame is equivalent to the traditional task-level control methods. This is because the motion plans for coordination are actually generated based on the traditional task-level control method with redundancy resolution. However, when uncertain or unexpected events occur, the proposed planning and control method will take effect and make the system have a better performance.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

A. Experimental setup

In order to verify the effectiveness and advantages of the proposed coordinated control method in the perceptive reference frame, both traditional control method and proposed method were implemented on a nonholonomic mobile manipulator which was built at Michigan State University (MSU). As shown in Fig. 3, the robot consists of a 4-wheel Segway mobile platform and a 7-DOF Schunk manipulator. The mobile platform, a heavy and rugged system with slow dynamics and less accurate performance, moves on the unstructured ground. In contrast, the manipulator, a lightweight and precise system with fast dynamics and more accurate performance, moves in a contact-free environment. Notice that small errors of the mobile platform, in particular the turning errors, will result in larger position errors of endeffector.

B. Experimental Results of Coordinated Control

The first experiment was to make the end-effector track a straight-line trajectory along the y axis. The trajectory could not be achieved by either the sole manipulator or the sole mobile platform because the manipulator might be out of reach and the mobile platform could not move in the lateral direction due to the nonholonomic constraint. However, it could be achieved by using the mobile platform and the manipulator together. Both the traditional kinematic control method [17] and the proposed method were implemented on the robot system. The traditional method took maximizing the manipulability index as the secondary task. The proposed method was based on the same kinematic control method

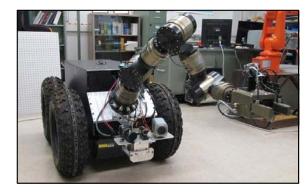


Fig. 3. MSU nonholonomic mobile manipulator

but introduced the coordinated control in the perceptive reference frame. Because the yaw angle tracking errors of the mobile platform and the manipulator were most different, to better demonstrate the effectiveness of the proposed method without loss of generality, the yaw angle tracking error of the end-effector were selected for the comparison.

Fig. 4 shows the errors of the traditional kinematic control method and the proposed coordinated control method in the upper plot and lower plot respectively. It can be seen that both position and orientation errors of the proposed method were always smaller than the traditional method. When using the traditional method, in the beginning, the mobile platform started much more slowly than the manipulator, which caused a large error in the end-effector. During the motion, the errors were also larger due to the different errors of the mobile platform and the manipulator. In contrast, using the proposed method, the errors were kept in a small range for the entire time. When the obstacle stopped the mobile platform at around 6.5 s, it made the perceptive reference s^* stop evolving and consequently caused the stop of both the mobile platform and the manipulator. Therefore, the error remained constant. At around 8 s when the obstacle was removed, s^* automatically started evolving and the motions were automatically recovered. The entire process was automatic and no replanning or resetting of the system was required.

Besides the straight-line motion, both methods were also conducted for an arbitrary trajectory which was generated through arbitrarily moving a space-ball joystick. Fig. 5 shows the results of the traditional method and the proposed method in the upper plot and lower plot respectively. It can be seen that the errors of the former were much larger than the latter. There were some large error segments in the traditional control. This usually happened when large changes occurred in the motion, e.g., large changes of velocity amplitude or direction. This was mainly caused by different dynamics and errors of the mobile platform and the manipulator. However, using the proposed method, the errors were kept in a smaller range for the entire process due to the coordination between the mobile platform and the manipulator in the perceptive reference frame.

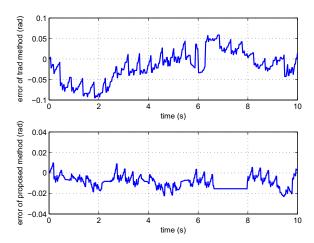


Fig. 4. Tracking errors for straight-line motion

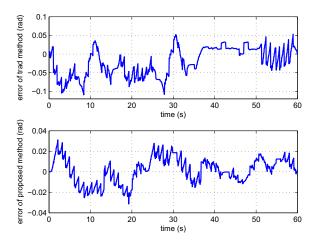


Fig. 5. Tracking errors for arbitrary motion

V. CONCLUSIONS

A new planning and control method for coordinated control of a nonholonomic mobile manipulator has been proposed in this paper. The method models the planning and control based on the perceptive reference, which is related to the system outputs. It makes the planning process become closed-loop and online adaptively to the real-time system outputs. The significance of the method is that it can handle uncertain and unexpected events encountered by both the mobile platform and the manipulator. The method provides a mechanism to plan and control the robot system according to real-time system outputs. For this reason, it can be extended to many other robotic systems to achieve better performance and handle more uncertain and unexpected events through incorporating more information into the perceptive reference.

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