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On the Control of an Aerial Manipulator Interacting with the Environment

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***Abstract*— This paper deals with the problem of modelling and controlling an innovative aerial manipulator, i.e. a vertical take-off and landing aircraft equipped with a fully-actuated robotic arm. This system is able to perform complex operations that require the physical interaction with the surrounding environment while remaining airborne. Once a detailed dynamical model in the planar case is provided, a control law able to govern all the degrees of freedom of the system is discussed. Beside the methodological contribution that is easily extendable to different combinations of UAVs and robotic arms, the effectiveness and main properties of the proposed control algorithm are illustrated with the help of an experiment that takes into account the case in which the manipulator is in contact with the surrounding environment.**

I. Introduction

In recent years, Unmanned Aerial Vehicles (UAVs) have become a popular research topic in the control and robotic community, mostly driven by the particular application in which such devices are employed, e.g. surveillance and data acquisition in areas that are dangerous for humans and not easily accessible to ground vehicles. Some of the most investigated topics include coordination with other ground and aerial vehicles [1], acrobatic flight [2], localization and reconstruction of unknown environments, [3]. On the other hand, this paper focuses on a different applicative scenario, namely the case in which the UAV is equipped with a manipulator to be able to interact, in a safe and constructive way, with the surrounding environment. The idea is to let the system accomplish real robotic manipulation tasks, while the UAV remains airborne. Among others, results on cooperative transportation [4], grasping [5], [6] and the European funded projects ARCAS [7] and AIRobots [8] are worth mentioning. This latter, in particular, aims at developing aerial platforms and control solutions specifically tailored towards applications in which physical interaction is required. A number of industrial applications, such as inspection of power plants, wind turbines, big public infrastructures, etc., motivates the interest towards this research direction.

In this work, the modeling and control of an innovative aerial robot that consists of a Vertical Take-Off and Landing (VTOL) airframe, more specifically a ducted-fan configuration [9], [10], [11], [12], and a fully-actuated 2 d.o.f robotic arm with prismatic joints, and an end-effector, are discussed. For its peculiar characteristics, the vehicle will be denoted as *aerial manipulator*. By focusing on the planar dynamics of the robot, the kinematical and dynamical model are investigated with reference to a particular applicative scenario that can be briefly described as follows. The human operator moves the UAV in a desired position in space, and then performs some inspection-by-contact tasks with the manipulator while the position of the UAV is kept constant. The environment is modeled as a vertical compliant surface. The robot is then moved with respect to the floating base: as a consequence, in this paper, the trajectory is directly given in joint space, which is the analogous of the (relative) working space due to the particular geometry of the manipulator. More complex mechanisms can be treated with minimum modifications. Since the UAV is under-actuated, the challenge is to compensate the coupling forces with the manipulator that appear either when the robot is moved in free-space, and when the end-effector is in contact situations. Inspired by classical vectored-thrust control for aerial robots [13], [14] the idea is to take directly into account for these coupling forces in the definition of the desired force control vector which is obtained by tilting the overall vehicle in the desired direction while applying a certain thrust. This approach naturally leads to a *cascade* control strategy [15], [16] in which the position controller, which takes into account all force components applied to the vehicle, plays the role of the outer-loop generating an attitude reference for the inner attitude loop.

A. Notation and Definitions

In this paper, let $\mathbb{R}$, $\mathbb{R}_{>0}$, and $\mathbb{R}_{\geq 0}$ denote the set of real, positive real and non-negative real numbers, respectively, and I_n, with $n > 1$, the n-dimensional identity matrix. Given $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm, while given a function $f : [0, \infty) \to \mathbb{R}^k$, with $k > 0$, we define $|f|_\infty := \sup_{t\in[0,\infty)} |f(t)|$, and $|f|_a := \limsup_{t\to\infty} |f(t)|$. Finally, given $v = [v_x, v_y]^{\mathrm{T}}$, with $v_y \in \mathbb{R}_{>0}$, we have that

$$\arg(v) := -\arctan\left(\frac{v_x}{v_y}\right).$$

Accordingly, v can be rewritten as

$$v = |v| \begin{bmatrix} -\sin\alpha \\ \cos\alpha \end{bmatrix}, \quad \alpha = \arg(v),$$

and the following equalities hold true

$$|R(\theta)v| = |v|, \qquad \arg(R(\theta)v) = \arg(v) + \theta,$$

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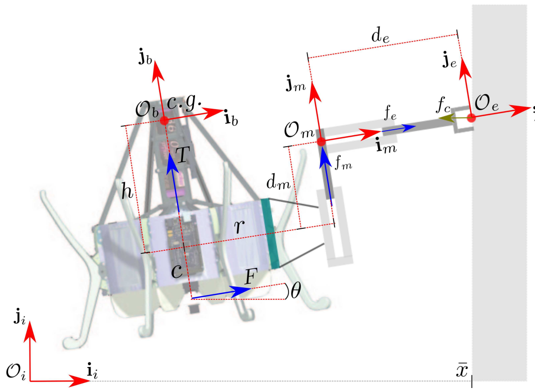

Fig. 1. The aerial manipulator: structure and mechanical parameters.

where $R(\theta) \in SO(2)$ is given by

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

being $\theta \in \mathbb{R}$. Moreover,

$$\frac{d}{dt}\arg(v) = \frac{1}{v^{\mathrm{T}}v} v^{\mathrm{T}} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \dot{v}. \quad (1)$$

At times, for sake of compactness, given $\theta \in \mathbb{R}$, S_θ, C_θ, and T_θ denote $\sin\theta$, $\cos\theta$ and $\tan\theta$, respectively.

In this paper, the notion of Input-to-State Stability (ISS) given in [15, Appendix C] and the *saturation functions* given in [15, Appendix D] are considered.

II. Dynamical Model of the Aerial Manipulator

A. Direct and differential kinematics of the aerial manipulator structure

The first step in modeling the robot consists in the definition of the forward kinematics. First of all, in order to make the paper readable, the following notation (related to the sketch in Fig. 1) is introduced:

- $\mathcal{F}_i := \{\mathcal{O}_i, i_i, j_i\}$, $\mathcal{F}_b := \{\mathcal{O}_b, i_b, j_b\}$, $\mathcal{F}_m := \{\mathcal{O}_m, i_m, j_m\}$, and $\mathcal{F}_e := \{\mathcal{O}_e, i_e, j_e\}$ represent the inertial reference frame, the reference frame attached to the center of gravity (*c.g.*) of the UAV, the reference frame attached to the first link of the manipulator and finally, the reference frame attached to the end-effector, respectively;
- $h_b := [0,\, -h]^{\mathrm{T}}$, $r_b := [r,\, 0]^{\mathrm{T}}$, and $c_b := [0,\, -c]^{\mathrm{T}}$ are three vectors of parameters expressed in the body frame $\mathcal{F}_b$ that represent the vertical and lateral displacements of the base plate of the prismatic manipulator with respect to the *c.g.*, the distance between the *c.g.* and the application point of the flap's force F, while $\bar{x}_i := [\bar{x},\, 0]^{\mathrm{T}}$ is the position of the contact surface with respect to the inertial frame;
- $p_b^i := [x,\, y]^{\mathrm{T}}$ represents the position of $\mathcal{O}_b$ with respect to $\mathcal{O}_i$, namely the position of the UAV expressed in the inertial reference frame;
- $p_m^b := [r,\, d_m - h]^{\mathrm{T}}$ is the position of $\mathcal{O}_m$ with respect to $\mathcal{O}_b$, while $p_e^m := [d_e,\, 0]^{\mathrm{T}}$ represents the position of $\mathcal{O}_e$ with respect to $\mathcal{O}_m$;
- θ is the attitude of the UAV;
- $q := [x,\, y,\, \theta,\, d_m,\, d_e]^{\mathrm{T}}$ denotes the vector containing the "joint" variables of the overall robot, i.e. UAV and manipulator;
- $u := [T,\, F,\, f_m,\, f_e]^{\mathrm{T}}$ is the vector containing all actuation forces of the overall robot, i. e. T is the propeller force, F is the flap's force, while f_m and f_e are the two forces acting on the manipulator.

The direct cinematic is easily computed by starting from the following three homogeneous transformations:

$$A_b^i := \begin{bmatrix} R(\theta) & p_b^i \\ 0_{1\times 2} & 1 \end{bmatrix}, \qquad A_m^b := \begin{bmatrix} I_2 & p_m^b \\ 0_{1\times 2} & 1 \end{bmatrix},$$
$$A_e^m := \begin{bmatrix} I_2 & p_e^m \\ 0_{1\times 2} & 1 \end{bmatrix}$$

having in mind that the generic matrix A_j^k describes the coordinate transformation of an applied vector from a reference frame $\mathcal{F}_j$ to another $\mathcal{F}_k$. As far as the construction of the dynamic model is concerned, it is important to compute the position of the origin $\mathcal{O}_m$ with respect to the inertial frame, i.e. p_m^i, and the position of the end-effector with respect to the same frame, i.e. p_e^i. It can be easily obtained that

$$T_m^i := A_b^i A_m^b = \begin{bmatrix} R(\theta) & p_m^i \\ 0_{1\times 2} & 1 \end{bmatrix},$$
$$T_e^i := A_b^i A_m^b A_e^m = \begin{bmatrix} R(\theta) & p_e^i \\ 0_{1\times 2} & 1 \end{bmatrix}$$

where

$$p_m^i := \begin{bmatrix} C_\theta r - S_\theta(d_m - h) + x \\ S_\theta r + C_\theta(d_m - h) + y \end{bmatrix}, \quad p_e^i := p_m^i + \begin{bmatrix} C_\theta d_e \\ S_\theta d_e \end{bmatrix}.$$

Finally, the Jacobian matrix $J(q)$ of the entire manipulator is determined by simply differentiating the direct kinematics equations:

$$J(q) := \\ := \begin{bmatrix} 1 & 0 & -S_\theta(d_e + r) - C_\theta(d_m - h) & -S_\theta & C_\theta \\ 0 & 1 & C_\theta(d_e + r) - S_\theta(d_m - h) & C_\theta & S_\theta \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (2)$$

The Jacobian matrix is fundamental for analyzing the physical interaction between aerial manipulator and environment. In fact, it is well-known that $J(q)$ is not only involved in the relation between joint and end-effector velocities, but also in the *static* relation, between joint and external forces/torques applied to the end-effector.

B. Model of the aerial manipulator in free flight & interaction scenario

The dynamical model of the system under investigation has been computed thanks to classical Lagrangian arguments. In particular, let us define the Lagrangian of the system as $\mathcal{L}(q,\dot{q}) := \mathcal{T}(q,\dot{q}) - \mathcal{U}(q)$, with $\mathcal{T}$ and $\mathcal{U}$ the kinetic and potential energy respectively. In compact form, the Lagrangian equations are given by

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = \xi \tag{3}$$

where $\xi := U(q)u - D\dot{q}$ contains all the generalized forces acting on each d.o.f., with

$$U(q) := \begin{bmatrix} -S_\theta & C_\theta & 0 & 0 \\ C_\theta & S_\theta & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

while D is a diagonal matrix with the viscous friction coefficients.

The total kinetic energy is the sum of two contribution (i.e., the UAV with mass M and moment of inertia J_{uav}, and the manipulator with mass and moment of inertia of the first and second joint, supposed to be equal and given by m_m, J_m, and mass and moment of inertial of the end-effector given by m_e, J_e), namely $\mathcal{T} := \mathcal{T}_{uav} + \mathcal{T}_{man}$, where

$$\begin{aligned} \mathcal{T}_{uav} &:= \frac{1}{2}\left[M(\dot{p}^i_{uav})^{\mathrm{T}}\dot{p}^i_{uav} + J_{uav}\dot{\theta}^2\right], \\ \mathcal{T}_{man} &:= \frac{1}{2}\left[m_m(\dot{p}^i_m)^{\mathrm{T}}\dot{p}^i_m + m_e(\dot{p}^i_e)^{\mathrm{T}}\dot{p}^i_e + (J_m + J_e)\dot{\theta}^2\right] \end{aligned}$$

while the overall potential energy of the system is given by $\mathcal{U} := \mathcal{U}_{uav} + \mathcal{U}_{man}$ with

$$\begin{aligned} \mathcal{U}_{uav} &:= -Mg_0^{\mathrm{T}}p^i_{uav}\,, \\ \mathcal{U}_{man} &:= -m_m g_0^{\mathrm{T}}p^i_m - m_e g_0^{\mathrm{T}}p^i_e\,. \end{aligned}$$

As depicted in Figure 1, the end-effector installed on the planar aerial manipulator may come into contact with the environment, modeled as a vertical compliant surface located at the position $\bar{x}$ with respect to the inertial reference frame $\mathcal{F}_i$. When the lateral position x^i_e of the end-effector is greater or equal than the position of the vertical surface, the environment applies to the end-effector a force f_c directed along the inertial x axis. Compliant contact models, such as Hunt-Crossley or Kelvin-Voight [17], can be employed to model the force f_c as a function of the mechanical characteristics of the contacts materia. In this case, to complete the description of the full order model, we define $h_e(q) := [f_c,\, 0,\, 0]^{\mathrm{T}}$.

C. Approximated dynamic model in the interaction scenario

As far as the manipulator is concerned, by assuming that the mass of the two links is negligible compared to the one of the end effector (i.e. $m_m \ll m_e := m$), the following approximated dynamical model can be obtained:

$$m\begin{bmatrix}\ddot{d}_e \\ \ddot{d}_m\end{bmatrix} = \begin{bmatrix} f_e \\ f_m \end{bmatrix} - R^{\mathrm{T}}(\theta)\begin{bmatrix} f_c \\ mg\end{bmatrix}. \tag{4}$$

Note that the two joint positions are driven by the control forces f_e and f_m, and are affected by the force f_c applied by the environment, and by the attitude θ of the vehicle.

The second subsystem is the dynamical model of the UAV that is driven by the thrust T and the torque $\tau := F\,c$. By assuming that the mass of the manipulator is negligible compared to the one of the vehicle, namely $m \ll M$ (in our set-up $M \approx 1.8\,\mathrm{kg}$, $m \approx 0.1\,\mathrm{kg}$), the lateral and longitudinal dynamics can be approximated as

$$M\begin{bmatrix}\ddot{x} \\ \ddot{y}\end{bmatrix} = -R(\theta)\begin{bmatrix} f_e \\ f_m\end{bmatrix} + R(\theta)\begin{bmatrix}0 \\ T\end{bmatrix} - \begin{bmatrix} 0 \\ Mg\end{bmatrix} \tag{5}$$

while the attitude dynamics are governed by

$$J_{uav}\ddot{\theta} = \tau + (d_m - h)f_e - (r + d_e)f_m\,. \tag{6}$$

Note that the lateral, the vertical and the attitude dynamics of the aerial vehicle are affected by the two forces f_e and f_m governing the manipulator.

Remark. In (5), the effect of the aerodynamic force F on the position dynamics of the vehicle has been omitted. This effect is often denoted as *small body force* in literature and it is a distinguishing feature of some VTOL configurations, including ducted-fan and helicopters [18]. In the analysis proposed in this work, their effect has been neglected. In fact, for a miniature system in which the inertia is characterized by small values [19], large angular accelerations are achieved with small values of F. $\triangleleft$

III. The Robust Control of the Aerial Manipulator

A. The Control Problem

The control problem tackled in this paper deals with the stabilization of the complete system when the manipulator tracks a desired trajectory defined, for simplicity, in the joint space. Furthermore, the control law has to be robust with respect to external non-modeled forces that are applied on the end-effector in case of interaction with the environment. Differently than in [20], the control algorithm has been developed without necessarily relying on the impedance control paradigm. In fact, as far as the control of the robotic arm is concerned, it is required that joint control actions f_m and f_e remain bounded in the presence of the external contact force f_c. This topic is discussed in Sect. III-B.

Either in case of free-flight and interaction with the environment condition, the coupling between manipulator and UAV takes place via the couple of (joint) forces f_m and f_e. The control of the UAV is developed in order to keep a constant position in space, and to compensate for the disturbance forces due to coupling. This is achieved thanks to a modified vector control strategy proposed in Sect. III-C. The idea is to define a control vector that considers the presence of the known forces f_m and f_e, so as to determine the thrust and the reference orientation required to maintain the desired configuration.

B. Robust Control of the Robotic Arm

Let $d_e^\star(t)$ and $d_m^\star(t)$ denote the reference position of the manipulator joint variables, and consider the following control law:

$$\begin{bmatrix} f_e \\ f_m \end{bmatrix} = R^{\mathrm{T}}(\theta) \begin{bmatrix} 0 \\ mg \end{bmatrix} + m \begin{bmatrix} \ddot{d}_e^\star \\ \ddot{d}_m^\star \end{bmatrix} + \\ + \kappa_m \left(\tilde{d}_e, \tilde{d}_m, \dot{\tilde{d}}_e, \dot{\tilde{d}}_m \right) \quad (7)$$

having defined

$$\tilde{d}_e := d_e - d_e^\star \,, \qquad \tilde{d}_m := d_m - d_m^\star$$

and where $\kappa_m : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is an error feedback controller that is designed by means of the following nested saturation control law

$$\kappa_m(\cdot) := \lambda_2^m \sigma \left(\frac{k_2^m}{\lambda_2^m} \left(\begin{bmatrix} \dot{\tilde{d}}_e \\ \dot{\tilde{d}}_m \end{bmatrix} + \lambda_1^m \sigma \left(\frac{k_1^m}{\lambda_1^m} \begin{bmatrix} \tilde{d}_e \\ \tilde{d}_m \end{bmatrix} \right) \right) \right) \quad (8)$$

in which, following [15, Appendix D], parameters k_1^m, k_2^m, λ_1^m, λ_2^m are selected as

$$\lambda_i^m = \epsilon_m^{(i-1)} \lambda_i^\star, \ k_i^m = \epsilon_m k_i^\star, \ i \in \{1, 2\} \,, \quad (9)$$

and where $k_i^\star$, $\lambda_i^\star$ are such that

$$\frac{\lambda_2^\star}{k_2^\star} < \frac{\lambda_1^\star}{4}, \quad 4 k_1^\star \lambda_1^\star < \frac{\lambda_2^\star}{4}, \quad 6 \frac{k_1^\star}{k_2^\star} < \frac{1}{24} \,, \quad (10)$$

with $\epsilon_m > 0$.

For the closed-loop manipulator error dynamics given by (4) driven by the control law (7)-(8) the following result holds true.

Proposition 1 *Consider system (4) driven by the control law (7)- (8) in which k_i^m and λ_i^m, $i = 1, 2$, have been selected according to (9) and (10) with $\epsilon_m > 0$. Then*

- $|[f_e, f_m]|_\infty \leq mg + m|[\ddot{d}_e^\star, \ddot{d}_m^\star]|_\infty + \sqrt{2}\lambda_2^\star \epsilon_m$;
- *there exist $\Gamma_1^m, \Gamma_2^m \in \mathbb{R}_{>0}$ such that*

$$\left| \frac{d}{dt} \kappa_m(\cdot) \right|_\infty \leq \Gamma_1^m \epsilon_m^2 + \Gamma_2^m \epsilon_m |f_c|_\infty \,. \quad (11)$$

- *there exist $\Delta(\epsilon_m) > 0$ and a class-$\mathcal{K}$ function γ_{ϵ_m} such that if $|f_c|_\infty \leq \Delta(\epsilon_m)$ then*

$$|[\tilde{d}_e, \tilde{d}_m]|_a \leq \gamma_{\epsilon_m} \left(|f_c|_a \right) \,. \quad (12)$$

The first item in the above proposition shows how the manipulator control forces f_e, f_m are bounded functions of time. The bound does not depend on the magnitude of the contact force f_c, thanks to the nested saturation control design introduced in (8). This property can be regarded as the *disturbance response* of the manipulator closed-loop system [21]. The second item is showing how the first order derivative of the feedback control law can be bounded by a value that does not depend on the current tracking error and its derivatives. This bound will be taken into account in the next section where the interconnection between the manipulator and the aerial platform is considered. Finally, the third item shows how, in the presence of the exogenous disturbance f_c, practical tracking of the desired trajectory $d_e^\star$, $d_m^\star$ is achieved. In the special case in which f_c is vanishing asymptotically, such as during free-flight maneuvers, the tracking becomes asymptotic.

C. Modified Thrust-Vectoring Control of the UAV

Let $x^\star$ and $y^\star$ denote the constant lateral and vertical desired positions of the center of gravity of the aerial vehicle. To stabilize the aerial platform in the desired configuration, we define the following *control vector*

$$v_c := \begin{bmatrix} 0 \\ Mg \end{bmatrix} - \kappa \left(\tilde{x}, \, \dot{\tilde{x}}, \, \tilde{y}, \, \dot{\tilde{y}} \right)$$

in which

$$\tilde{x} := x - x^\star \,, \qquad \tilde{y} := y - y^\star$$

are the position errors and $\kappa : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}^2$ is a feedback control law.

The control vector v_c can be applied to the vehicle position dynamics (5) by properly *vectorizing* the thrust produced by the propeller. More specifically, we compute a *control thrust* T_c and a *control attitude* θ_c in order to have

$$R(\theta_c) \begin{bmatrix} -f_e \\ T_c - f_m \end{bmatrix} = v_c \,.$$

The above relation can be separated into two parts, one considering the magnitude and the other considering the orientation of the two vectors, respectively. Accordingly, we obtain

$$\left| R(\theta_c) \begin{bmatrix} -f_e \\ T_c - f_m \end{bmatrix} \right| = |v_c| \quad (13)$$

and

$$\arg \left(R(\theta_c) \begin{bmatrix} -f_e \\ T_c - f_m \end{bmatrix} \right) = \arg(v_c) \,. \quad (14)$$

To compute a solution to (13)-(14) we consider the following assumptions

$$|v_c| > 0, \quad |T_c - f_m| > 0 \quad \forall t \geq 0 \quad (15)$$

and we obtain

$$\begin{aligned} T_c &= \sqrt{v_c^{\mathrm{T}} v_c - f_e^2} + f_m, \\ \theta_c &= \arg(v_c) + \arg([f_e, \, \sqrt{v_c^{\mathrm{T}} v_c - f_e^2}]^{\mathrm{T}}). \end{aligned} \quad (16)$$

While the control thrust T_c can be directly applied to the vehicle by choosing $T = T_c$, the control attitude θ_c is employed as a reference for the attitude stabilizing control law.

Remark. The control design proposed in this work draws inspiration from the *thrust-vectoring* control employed to

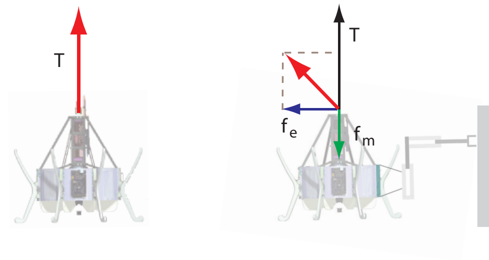

Fig. 2. The control force for a standard VTOL aerial vehicle (on the left) and for the aerial manipulator proposed in the paper (on the right).

stabilize the position of under-actuated aerial configurations (see, among others, [13]). The novelty of the design proposed here is the fact that the reaction forces applied by the manipulator back to the aerial platform are directly taken into account in the definition of the desired control force vector (see Figure 2). This fact allows the vehicle to stabilize, in the sense that will be specified in the following, a constant position while the manipulator is physical interacting with the environment. ◁

1) Position Controller: In order to stabilize the position dynamics of the aerial vehicle, we focus on the following nested saturation control law

$$\kappa\left(\cdot\right) := \lambda_2 \sigma\left(\frac{k_2}{\lambda_2}\left(\begin{bmatrix}\dot{\tilde{x}}\\ \dot{\tilde{y}}\end{bmatrix} + \lambda_1 \sigma\left(\frac{k_1}{\lambda_1}\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix}\right)\right)\right) \tag{17}$$

in which k_1, k_2, λ_1, λ_2 are selected as

$$\lambda_i = \epsilon^{(i-1)}\lambda_i^\star, \qquad k_i = \epsilon k_i^\star, \quad i \in \{1, 2\}\,, \tag{18}$$

where $k_i^\star$, $\lambda_i^\star$ are given by (10) and $\epsilon > 0$.

Along the lines of Proposition 1, the following preliminary result can be stated.

Proposition 2 *Consider the control law (17) in which k_i and λ_i, $i = 1, 2$, have been selected according to (18) and (10) with $\epsilon > 0$. Then*

- $|\kappa|_\infty \leq \sqrt{2}\lambda_2^\star\epsilon$;
- *let* $|[\ddot{d}_e^\star,\, \ddot{d}_m^\star]|_\infty \leq \bar{D}^\star > 0$. *Then there exist* $\Gamma_{\bar{D}^\star} \in \mathbb{R}_{>0}$ *such that*

$$\left|\frac{d}{dt}\kappa(\cdot)\right|_\infty \leq \Gamma_{\bar{D}^\star}\,\epsilon\,. \tag{19}$$

2) Design of the Attitude Controller: As a final step, the attitude control law for the aerial vehicle is designed. In particular, we define the control torque τ_c as

$$\tau_c := \tau_{FF}(d_e, d_m, f_e, f_m) + \tau_{FB}(\theta, \dot{\theta}, \theta_c) \tag{20}$$

in which

$$\tau_{FF}(d_e, d_m, f_e, f_m) = -(d_m - h)f_e + (r + d_e)f_m \tag{21}$$

is the feed-forward control action compensating for the reaction torque disturbance produced by the manipulator and

$$\tau_{FB}(\theta, \dot{\theta}, \theta_c) = -k_P\left((\theta - \theta_c) + k_D\dot{\theta}\right) \tag{22}$$

is the feedback stabilizing control law.

3) Stability Properties of the Aerial Platform: For the aerial vehicle, the following stability result holds true.

Proposition 3 *Let us consider the system (5)-(6) in which the control inputs T, τ are selected as $T = T_c$ and $\tau = \tau_c$. Let be $\epsilon > 0$, $\epsilon_m > 0$ and the manipulator references $d_e^\star$, $d_m^\star$ be chosen such that*

$$\begin{aligned} \sqrt{2}\lambda_2^\star\epsilon &\leq Mg - \underline{v}, \\ mg + m|[\ddot{d}_e^\star,\, \ddot{d}_m^\star]|_\infty + \sqrt{2}\lambda_2^\star\epsilon_m &\leq \underline{v} \\ |[\ddot{d}_e^\star,\, \ddot{d}_m^\star]|_\infty &\leq \bar{D}^\star \\ |[d_e^{\star,\,(3)}|_\infty &\leq \bar{D}_e^\star \end{aligned} \tag{23}$$

for some $mg < \underline{v} < Mg$, $\bar{D}^\star > 0$ and $\bar{D}_e^\star > 0$. Moreover let $|f_c|_\infty \leq \bar{F}$ for some $\bar{F} > 0$. Then, there exists $k_D^\star > 0$ and for all $k_D < k_D^\star$ there exists $k_P^\star(k_D) > 0$ and a class-$\mathcal{K}$ function γ_p such that

$$|[\tilde{x},\, \tilde{y}]|_a \leq \gamma_p\left(\frac{k_D}{k_P}\left(|d_e^{\star,\,(3)}|_a + |f_c|_a\right)\right)\,.$$

The result in the above proposition is showing how the aerial vehicle dynamics remain bounded in the presence of the reaction forces applied by the manipulator. The effect of these disturbances can be arbitrarily reduced by increasing the gain of the attitude control loop. Note that the exogenous disturbance include also the unknown contact force f_c, showing the effectiveness of the proposed design in tasks requiring physical interaction.

IV. Experimental Results

The purpose of this section is to validate all the previous theory, presented in Section III, showing some results obtained by carrying out a series of experiments. The overall Aerial Manipulator (see Fig. 3) consists in a Ducted Fan prototype rigidly connected to the base plate of a parallel Delta robot. In Table I are listed the main mechanical parameters of the structure according to the approximated dynamical model shown in Section II-C, while in Table II, are listed the control parameters employed in the control laws (8)-(9)-(17)-(18)-(22).

By referring to the following Figure 4, it is possible to notice the behavior of the UAV during two different phases composing the experiment: the first one, is the Free Flight (that happens approximately from 70 s to ≈ 105 s) in which the vehicle tracks in a good way the reference trajectories until the vertical surface; the second phase, that is the Docking/Contact phase, begins after ≈ 105 s, recognizable by observing the lateral position x that does not follow the right trajectory because of the presence of the obstacle. Furthermore, at the same time, the UAV tilts since the θ dynamics is directly influenced by the lateral error. The contact force also affects the vertical dynamics of the vehicle, as a small tracking error can be observed also for the vertical position y.

$M = 1.8\,\text{Kg}$	$J_{uav} = 0.019\,\text{Kg}\cdot\text{m}^2$
$m = 0.1\,\text{Kg}$	$c = 0.2\,\text{m}$
$h = 0\,\text{m}$	$r = 0.15\,\text{m}$
$T_{\text{sat}} = [0, 30\,]N$	$F_{\text{sat}} = \pm 3\,\text{N}$
$f_m^{\text{sat}} = \pm 7\,\text{N}$	$f_e^{\text{sat}} = \pm 7\,\text{N}$

TABLE I

STRUCTURAL PARAMETERS OF THE AERIAL MANIPULATOR PROTOTYPE

$(k_1^\star, k_2^\star) = (1, 150)$	$(\lambda_1^\star, \lambda_2^\star) = (5, 150)$
$(\epsilon, \epsilon_m) = (0.1, 0.05)$	$(k_P, k_D) = (30, 9)$

TABLE II

TABLE OF CONTROL PARAMETERS USED DURING EXPERIMENTS

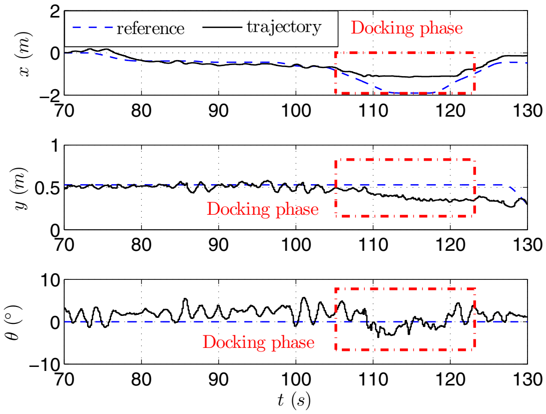

Fig. 4. Trajectories and references concerning the variables $x - y - \theta$ for the Ducted Fan UAV.

V. CONCLUSIONS AND FUTURE ACTIVITIES

In this work, the model and the control of an aerial manipulator, namely a ducted-fan aerial robot equipped with a fully-actuated robotic arm, able to accomplish operations requiring the physical interaction with the surrounding environment is presented. A control law, able to stabilize the system both in the free flight and in the presence of contacts, is discussed. Furthermore, the regulator is robust in the sense that it does not require the knowledge of the environment, neither the measurement of the interaction forces. Experiments show how docking to a surface can be achieved. Future activities deal with the extension of the proposed approach to the three-dimensional case.

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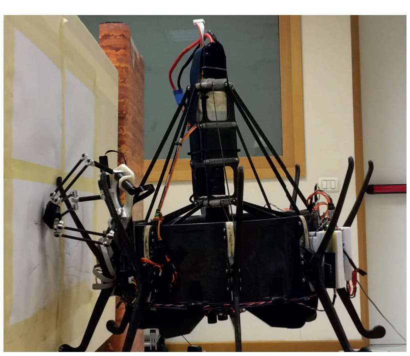

Fig. 3. Docking maneuver performed by the aerial manipulator (i.e. Ducted Fan prototype equipped by a Delta robot).

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