

Characterization of Grasp Quality Measures for Evaluating Robotic Hands Prehension

Beatriz León¹, Carlos Rubert², Joaquín Sancho-Bru³ and Antonio Morales²

Abstract—Many analytical metrics have been proposed to evaluate the quality of a grasp based on different criteria and principles. To use most of them in practical real applications, some operational parameters need to be determined: maximum and minimum values, normalization ratios, quality thresholds, robustness in front of position errors and, more importantly, relations between alternative metrics. This paper proposes a methodology to study and characterize the operational parameters that allow the use of several metrics in practical applications, and comparing them. The proposed approach uses exhaustive simulation testing to obtain statically significant results regarding the measurements of several quality metrics. This allows an informed setting of the practical operational values for each metric. Results are provided for a Barrett hand grasping a varied set of objects.

I. INTRODUCTION

Analytical approaches to robot grasp synthesis have focused on several problems through the years. Primarily, researchers have established a number of necessary conditions that a grasp must meet in order to be considered stable, being the force closure condition the most frequently used. In this context, a grasp is described as a set of contact points on the surface of an object, where the robot hand contacts and exerts forces and torques. There exists a vast literature focusing on the development of algorithms to find grasps under a variety of assumptions regarding contact models, object shapes and others. As a result of these algorithms many feasible grasps are often obtained, then the problem is to rank them in order to select one solution. This problem has been addressed by proposing many quality metrics based on different principles and heuristics. The purpose of these metrics is to give a unique quantitative assessment of the *quality* of a grasp. Suarez et al. [1] reviewed the available metrics classifying them according to underlying principles used for their definitions. In an alternative approach Chinelato et al. [2] adapted many of these metrics to characterize 2D vision-based grasps. Most recently León et al. [3] used a wide set of metrics to evaluate human hand prehension.

Among all these metrics, the most widely-used has become the ϵ -metric proposed by Ferrari and Canny [4], which measures the quality of grasp as the radius of the maximal origin-centred sphere fully contained within the

Grasp Wrench Space [5]. This metric has been implemented in several simulation tools like GraspIt![6] and OpenRAVE [7]. However, recent works [7], [8] show that grasps highly ranked by this metric perform poorly on real robots, and that the metric presents a high sensibility to positioning errors [9]. A reason for this may be that the metric is unable to capture the difficulty of grasp execution in real conditions or that the underlying principle in its design, which is to favour those grasps able to resist statically forces from any direction, is not enough to define a valid grasp.

In any case, it would be valuable to test whether other metrics can be better predictors of the stability of a grasp. However, for most of those metrics, there are few works that provide their formal definition, implementation and demonstration. In fact, there are many difficulties to use alternative metrics and to compare their results. First, despite many of them have clear theoretical maximum and minimum limits, in practice it can be observed that most values fall in narrower ranges. Related to this, there are no clear rules to set thresholds to separate *good* from *bad* grasps. Second, comparison between different metrics is difficult since each of them produces values in different units and scales. Third, there are no studies of the sensitivity of a metric to small variations in the pose of the target object. And last, since most metrics are based on different principles, it is unknown which of them are more convenient, or even whether they are correlated to each other. All these inconveniences limit the practical utility of most analytical metrics.

The objective of this work is to propose a methodology to study the different metrics and to characterize them. This would enable us to understand the ranges in which they move and use them to find appropriate values to normalize them, thus making them comparable. Additionally, the aim is to find out the robustness of the metrics by performing a sensitivity analysis and finally, find the minimum set needed to evaluate a robotic grasp. Our approach is based on the use of simulation to exhaustively analyse the values provided by a set of metrics and to obtain significant statistical results. In order to limit the variables on this preliminary study, the analyses will focus on grasps computed for a particular gripper, the Barrett hand, on 10 different objects.

II. GRASP QUALITY MEASURES

In previous work, we have reviewed, selected and adapted a set of 10 common grasp quality metrics to study different aspects of the grasp [3], [10]. Table I presents a brief summary of the metrics with the notation defined in Table II.

¹B. León is with the Adaptive Systems Research Group at the School of Computer Science, University of Hertfordshire, Hatfield Hertfordshire, AL10 9AB, UK b.leon@herts.ac.uk

²C. Rubert and A. Morales are with the Robotic Intelligence Laboratory at the Department of Computer Science and Engineering, Universitat Jaume I, 12006 Castellón, Spain {carlos.rubert,morales}@uji.es

³J. Sancho-Bru is with the Group of Biomechanics and Ergonomics at the Department of Mechanical Engineering and Construction, Universitat Jaume I, 12006 Castellón, Spain sancho@uji.es

TABLE I
SUMMARY OF SELECTED QUALITY METRICS

Name	Formula	Normalization	Units	Min	Max
Group A: Algebraic properties of G					
Q_{A1}	Smallest singular value of G[5]	$\sigma_{min}(G)$	-	0	-
Q_{A2}	Volume of G in the wrench space[5]	$\prod_{i=1}^r \sigma_i$	-	0	-
Q_{A3}	Grasp Isotropy Index[11]	$\sigma_{min}(G)/\sigma_{max}(G)$	Q_{A3}	0	1
Group B: Distribution of contact points					
Q_{B1}	Distance between the centroid of the contact polygon and the object's centre of mass[12], [13]	$distance(p, p_c)$	$1 - Q_{B1}/distance_{max}$	none	0 1
Q_{B2}	Area of the grasp polygon[14]	$Area(Polygon(p1, p2, p3, p4_P, p5_P))$	$Q_{B2}/Area_{max}$	none	0 1
Q_{B3}	Shape of the grasp polygon[11]	$\frac{1}{\theta_{max}} \sum_{i=1}^{n_f} \theta_i - \bar{\theta} $	$1 - Q_{B3}$	none	0 1
Group C: Algebraic properties of G					
Q_{C1}	Smallest maximum wrench to be resisted [4], [15]	$\min_{w \in CW} \ w\ $	$Q_{C1}/\sqrt{2}$	[force]	0 1
Q_{C2}	Volume of the convex hull[16]	$Volume(CW)$	$Q_{C2}/Volume_{max}$	none	0 1
Group D: Configuration of the manipulator					
Q_{D1}	Posture of hand finger joints[17]	$1/n_q \sum_{i=1}^{n_q} ((y_i - a_i)/(a_i - y_{iM}))^2$	$1 - Q_{D1}$	none	0 1
Q_{D2}	Inverse of the condition number of G_J [18], [19]	$\sigma_{min}(G_J)/\sigma_{max}(G_J)$	Q_{D2}	none	0 1

TABLE II
NOTATION

G	Grasp matrix
r	Rank of G
σ_{min}	Minimum singular value
σ_i	Nonzero singular values
σ_{max}	Maximum singular value
p	Centroid of contact polygon
p_c	Object centre of mass
p_i	Vertex of the grasp polygon
p_{ip}	Projected vertex of the grasp polygon on a plane
n_f	Number of fingers
θ_{max}	Sum of differences between the internal angles when the polygon has the most ill-conditioned shape and those of a regular polygon
θ_i	Inner angle at the vertex i of the grasp polygon
$\bar{\theta}$	Average angle of all inner angles of the grasp polygon
CW	Convex hull of the primitive wrenches
$w \in CW$	Generalized forces acting CW
$\ w\ $	Magnitude of a wrench
n_q	Number of joints of the hand
a_i	Middle range position of a joint
y_i	Angle of joint i
y_{iM}	Maximum angle limits of joint i
G_J	Grasp Jacobian matrix
$distance_{max}$	Maximum distance from the object's centre of mass to any point in the object's contour
$Area_{max}$	Maximum possible area of the hand calculated as the area of the polygon when the hand is fully opened
$Volume_{max}$	Maximum volume of the convex hull of the primitive wrenches

III. EXPERIMENTAL PLATFORM

A. Simulation Framework

The simulation platform chosen to perform the experiments is OpenHand [10], a simulation toolkit developed by the authors in which the selected quality metrics have been implemented, based on OpenRAVE [20].

B. The Barrett hand

In order to show how can we characterize the quality metrics, we chose a popular robot manipulator: the Barrett

hand [21] which has been widely used in many fields for applications ranging from industrial robotics for large assembly lines, to academics and research for simulating and studying robotic grasps. This hand has three fingers with two joints each, and two of them have an extra degree of freedom that is used to rotate the fingers around the palm (abduction). The model of this hand is available in OpenRAVE.

C. Objects

Different objects, commonly used for everyday tasks, have been selected for this study. Figure 1 presents the 10 selected objects, 5 with regular and 5 with irregular shapes. Most of the objects have been obtained from the KIT object database [22], except objects d and e that were modelled by us.

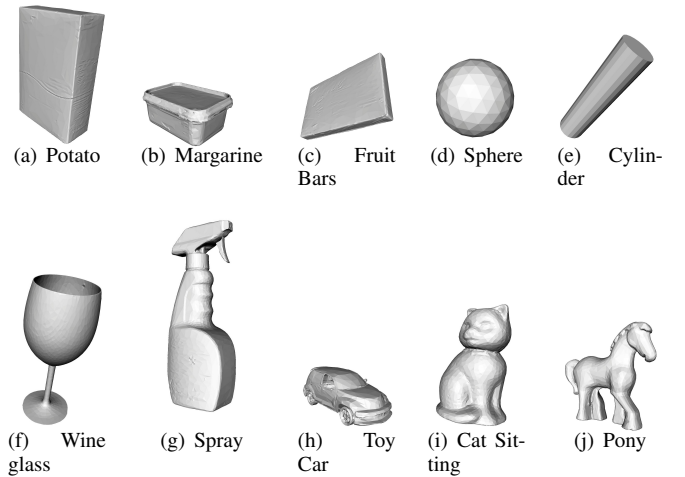


Fig. 1. Set of objects used to evaluate the grasp's quality with the different metrics

D. Grasp generation

In order to evaluate the variability of each quality metric, different grasps have been obtained for each one of the

selected objects. The grasps have been generated using the Grasping Module from the Database Generators in OpenRAVE. This module requires a set of parameters to define the conditions used in order to generate different grasp hypotheses. For each grasp hypothesis generated, it runs the grasp closure algorithm, calculates the contact information and makes an evaluation of force closure and finally saves the grasp in the database if it is successful.

The parameters used for generating the grasping database are: object to be used, vector containing values of DOF of each joint of the hand, vector of distances between object and manipulator, vector of rotations of the hand with respect of the target and a vector of approach directions from the hand to the object.

The approach rays parameter can be configured defining a delta space between points in the bounding box and also the angle of the vector to the surface, in this case we use a delta space of 0,2 and an angle of 0, which make the vector being normal to the object surface. As demonstrated in [23], [8] orthogonal approach angles are highly efficient for grasp success.

These vectors determine the amount of grasp positions to be generated and evaluated. The values of these parameters used in this work are detailed on Table III. These parameters have been used for the selected objects, creating ten different grasping databases with a total of 2716 stable grasps generated.

TABLE III
PARAMETERS USED TO GENERATE THE GRASPING DATABASES

Parameter	Value
Target	Each one of the 10 selected objects
Preshape (rad)	[0, 0, 0, 0]
Standoffs (m)	[0.01, 0.02]
Rolls (rad)	[0, 2 Π , Π /2]

E. Grasp closure algorithm

In order to evaluate each grasp with different quality metrics, a closure algorithm has been implemented in MATLAB to determine the contact information required. This algorithm loads the hand posture for each grasp and sets the preshape for each joint, then starts closing the hand at a constant speed and when it detects a collision between the target and the manipulator, stops the closure for this finger. Once the hand is closed around the object, it is possible to obtain all the quality metrics for each grasp with the geometry of the hand and the object, and the contacts vector obtained during the closure algorithm. The friction coefficient used for all objects was 0.4.

IV. MEASURES CHARACTERIZATION

In this section, a methodology is presented to study the different metrics and to characterize them. This is done by performing three analysis. First, a variability analysis will enable us to understand the ranges in which the metrics move and then use these ranges to find appropriate limits to normalize them, thus making the metrics comparable.

Second, a sensitivity analysis will enable us to find out the robustness of the metrics to errors in the input parameters. And finally, a correlation analysis will enable us to find the minimum set of metrics needed to achieve a global assessment of a robotic grasp.

A. Variability analysis

1) *Methods*: As mentioned in the previous section, we have selected 10 different objects with a variety of shapes and saved over 2000 grasps using the Barrett Hand. Each grasp has been simulated and with the contact information, we have obtained the value of the grasp quality for each of the 10 selected metrics. With these results we have analysed the range of variation for each metric.

2) *Results*: The maximum and minimum values obtained for each metric are presented in Table IV.

TABLE IV
MAX AND MIN VALUES OBTAINED FOR EACH METRIC

	A1	A2	A3	B1	B2	B3	C1	C2	D1	D2
Max	0.848	5.108	0.489	0.960	0.782	0.988	0.081	0.053	0.981	0.079
Min	0.016	0.003	0.011	0.215	0	0	0	0	0.573	0

Using these ranges, we have done a first normalization for each metric so that they have the best value of 1 and the worst value of 0. As example of the results obtained, Fig.3(a) shows the normalized quality metric obtained for all the grasps performed over the 10 objects using metric Q_{A1} .

It can be seen that an even distribution of the data over the range [0,1] has not been obtained, but instead they tend to concentrate in smaller ranges (from 0 to 0.5 in the case of Q_{A1}). As this was the case for several metrics, a different method was used to normalize them using statistical parameters. For each metric x_i , the mean and standard deviation were calculated to define the following minimum and maximum values to perform the normalization: $Min = mean(x_i) - std(x_i)$ and $Max = mean(x_i) + std(x_i)$. The resulting values for each metric are presented in Table V.

TABLE V
MAXIMUM AND MINIMUM VALUES OBTAINED USING STATISTICAL PARAMETERS

	A1	A2	A3	B1	B2	B3	C1	C2	D1	D2
max	0.371	1.847	0.214	0.848	0.365	0.839	0.033	0.016	0.913	0.023
min	0.072	0.207	0.042	0.576	0.110	0.482	0.010	0.001	0.780	0.004

With these parameters, we propose to classify the values of each metric grouping them as shown in Table VI.

These ranges enable us to differentiate those common values from the atypical ones and also we can determine for each metric, when a value is acceptable or not. Figure 2 shows the difference in ranges chosen using the two methods. It can be seen that using statistical parameters we can obtain smaller ranges discarding atypical values.

TABLE VI

RANGES TO CLASSIFY THE QUALITY OF A GRASP FOR EACH METRIC

Classification	Range
Bad quality	$[-\infty, 0]$
Fair quality	$[0, 0.5]$
Good quality	$[0.5, 1]$
Very good quality	$[1, \infty]$

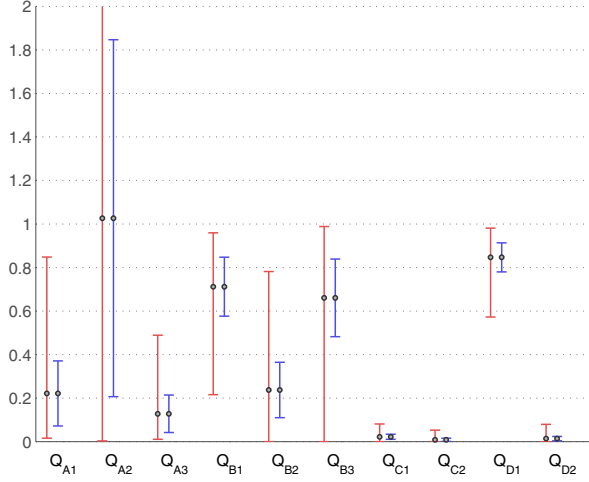


Fig. 2. Resulting ranges for each metric using two methods. Red: min and max using variability analysis and blue: min and max using statistical parameters.

As a result, histograms of the metrics using the second normalization method shows a more even distribution in the range from 0 to 1, as shown in Fig. 3(b) for metric Q_{A1} . Also note that there are a significant number of grasps that are left out of the limits and therefore there are higher number of grasp with *bad* and *very good* quality than those with *fair* quality.

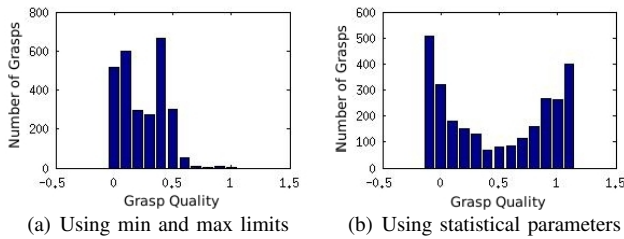


Fig. 3. Normalized histogram for Q_{A1}

B. Sensitivity analysis

In this analysis, we studied the robustness of each metric to small variations in the input parameters. The metrics selected for this study rely on the position of the hand with respect to the object and the information of the contact points. Errors in these parameters can occur due to small variations on the position of the hand that does commonly happen when transfer a grasp from the simulator to the real robot.

1) *Methods*: For this analysis, we have selected the first 10 grasp for each object that obtained the best quality

values according to each metric. Each of the initial postures was considered as the reference posture, and we introduced a variation for each of these grasp in the position and configuration of the hand with the aim of emulate the small unpredictable variations that may happen in real experiments.

The variations have been done in the position and rotation of the hand with respect to the object, as well as in the angle of the finger's abduction (see Fig. 4). The variation in the position has been done for each axis in increments of 1 mm in the range of ± 0.5 cm, the variation in the hand rotation has been done for each axis in increments of 0.01 rad in the range of ± 0.05 rad, and finally, the abduction DoF has been varied 0.5% each time to cover 5% of the total range. Therefore, we have defined 10 variations for each parameter which results in 70 new variations for each grasp, a total of 700 new grasps for each object and a total of 7000 grasps for the study.

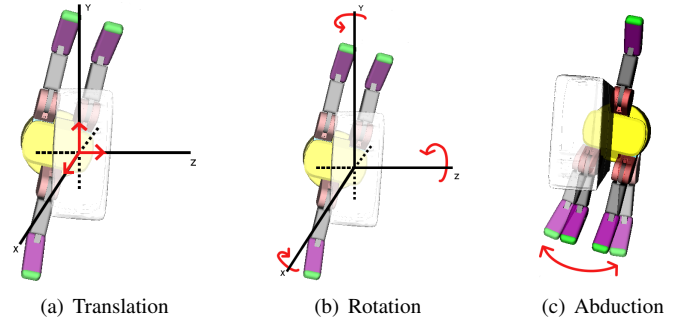


Fig. 4. Variations in the hand posture

A Sensitivity Index (SI) for each metric was obtained as the mean value of the standard deviations with respect to the metric calculated for the reference posture:

$$SI = \frac{1}{n} \sum_{x=1}^n \sigma_x$$

where n is the number of grasps for each object and σ_x the standard deviation calculated as:

$$\sigma_x = \frac{1}{n_v} \sqrt{\sum_{i=1}^{n_v} (x_i - x_0)^2}$$

where n_v is the number of variations of the initial posture, x_0 the value of the metric of the reference posture and x_i the value of the metric calculated for each variation. Finally, a Global Sensitivity Index (GSI) has been calculated for each metric, as the mean value of the SI per object previously calculated:

$$GSI = \frac{1}{n_o} \sum_{i=1}^n (SI_i)$$

where n_o is the number of objects and SI_i is the Sensitivity Index calculated for each object. In order to normalize GSI and give the result in percentage, the range of each metric obtained in the variability analysis has been used:

$$GSI_N = \frac{GSI}{\max - \min}$$

where \min y \max are the minimum and maximum value for each quality metric obtained in the variability analysis (Table V).

2) *Results*: The results of the Global Sensitivity Index calculated for the selected metrics are presented in Table VII. As we have used a very restrictive normalization, the sensitivity values reach values up to 40% of variation with respect of the reference posture. The results show that the metrics Q_{B3} and Q_{C2} are the most robust while Q_{A2} and Q_{D1} are the most sensitive.

TABLE VII
GSI (%) CALCULATED FOR EACH METRIC

Q_{A1}	Q_{A2}	Q_{A3}	Q_{B1}	Q_{B2}	Q_{B3}	Q_{C1}	Q_{C2}	Q_{D1}	Q_{D2}
29.58	38.62	30.62	23.23	37.60	12.20	30.60	16.30	37.01	29.66

Additionally, a summary of the results of the Sensitivity Index obtained for each object is presented in Fig. 5. The

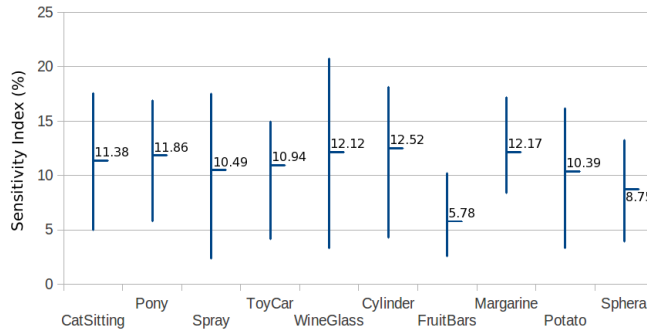


Fig. 5. Results of the SI per object over all metrics

FruitBars box shows the lowest values and smallest range while the *Wineglass* presented the highest variation, likely because of its division in three different shapes. However, there are no significant differences in the sensitivity obtained for the different objects even though the shape of the objects varied greatly. This can lead to the conclusion that, for future studies, there is no need to increase the number of objects as no significant differences are expected on the sensitivity of the metrics from object to object.

Finally, a comparison of the sensitivity was done according to the different variations originated due to errors in the position, rotation or joint's angles and the results are shown in Fig. 6.

The comparison shows that the sensitivity of the metrics obtained as a result of modifying the angles of the joints is significantly lower than modifying either the position or rotation of the hand. Therefore, we can conclude that to avoid variations in the quality of a grasp is important to focus on accurately adjust the arm movement so that the position and orientation of the manipulator to the object is as accurate as possible.

C. Correlation analysis

This analysis is needed as it is not efficient to calculate all reported quality metrics for evaluating a grasp when

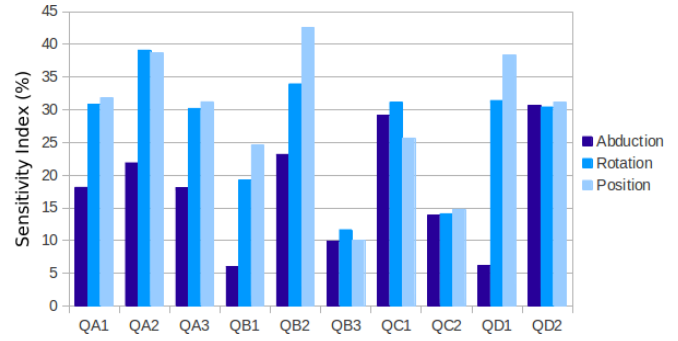


Fig. 6. Comparison of the SI when varying position, rotation and abduction of the fingers

performing several experiments and to analyse the high amount of data produced. To overcome this problem, a set of metrics may be chosen as it is expected that some of them provide similar information as they are formulated to evaluate the same aspect. The purpose of this analysis is to find the minimum set of indices that allows the evaluation of different aspects of the grasp. In previous work [3], we have found the minimum set of metrics needed to evaluate human grasp, therefore we can compare them with the ones we find for this robotic hand.

A Pearson correlation coefficient was calculated for each combination of metrics and the results are shown in Table VIII.

TABLE VIII
RESULTS OF THE STATISTICAL CORRELATION BETWEEN DIFFERENT QUALITY METRICS

	A1	A2	A3	B1	B2	B3	C1	C2	D1	D2
A1	1.00									
A2	0.93	1.00								
A3	1.00	0.93	1.00							
B1	0.25	0.15	0.25	1.00						
B2	0.29	0.35	0.29	-0.03	1.00					
B3	-0.56	-0.59	-0.57	-0.22	-0.52	1.00				
C1	0.00	-0.12	0.00	0.28	0.10	0.16	1.00			
C2	0.43	0.48	0.43	0.20	0.70	-0.53	0.27	1.00		
D1	0.63	0.61	0.63	0.16	0.73	-0.66	0.09	0.71	1.00	
D2	0.37	0.35	0.37	0.13	0.33	-0.33	0.05	0.30	0.41	1.00

Measures A1 and A3 show perfect correlation, and both metrics have an almost perfect correlation with index A2. Therefore, calculating only one of this metrics enable us to simplify the evaluation of grasps. Measure B2 has high correlations with metrics C2 and D1, whilst D1 also show high correlations with A1, A2, A3, B3 and C2. The other metrics showed to be independents so it is not possible to simplify more the evaluation of robotic grasping.

It is therefore recommended to group the metrics A1, A2 and A3, as their correlation is almost perfect, and simplify the calculation to only obtain A3 as this metric has already theoretical limits and it is more robust than A1 and A2. We also suggest to group metrics B2, C2 and D1, because their correlation is very high. Although D1 has the highest corre-

lation index within the other metrics, it is a sensitive metric with respect to errors in position and rotation, therefore we suggest to select C2 as it is the most robust of the three.

As a result of this analysis, the recommended set of six independent metrics needed for robotic grasping evaluation is: **A3, B1, B3, C1, C2, D2**. Each of these metrics can be associated, respectively, with a physical interpretation which describes the aspect being measured: *Restriction of the grip*, *Dynamic effects*, *Symmetry of the grasp*, *Ability to resist forces*, and *Manipulability*. These aspects are similar to the ones obtained in [10], except for the change of the metric D1 with metrics B3 and C2. This shows that the selection of metrics is very similar if what you want is to study human or robotic grasps, however there are some differences that might need to be taken into account.

V. CONCLUSIONS

In this paper, we proposed a methodology to characterize a set of metrics to evaluate the grasp quality of robotic hands. We have analysed the variability of each of the metrics, determining the ranges in which they vary, which allows to establish thresholds that classify the grasp quality with the selected objects and robot hand. The sensitivity analysis enabled us to determine the robustness of the quality metrics. This tells us how the results can vary when translating a grasp from simulation to real environment, and being exposed to position and rotation errors. An important result is that we did not observe any significant variation in the sensitivity of the metrics from object to object. Finally, a correlation analysis allowed us to select only six independent metrics, which we have related with a physical aspect of the grasp being measured. This enables to differentiate those metrics which are more relevant for the specific task to be performed.

It would be interesting for future work to extend the study to a variety of robotic hands and a increased number of objects that were used for the analyses of variability and sensitivity. This would enable us to more precisely define the ranges in which the results of the quality metrics move. Likewise, it would enable us to classify robot hands according to their behaviour with respect to different quality metrics, determining which obtained better scores for each metric and which are more robust to the different positioning errors. It would be also interesting to study the grasp strategy used showing results of the variation of the number of grasp points and normals used.

A final thought regards some recent studies that have raised concerns about the assessment utility of ϵ -metric (C1 in our study) which is the most widely-used metric. Diankov [20] showed that grasps with high quality values tend to fail when executed on real robots. Similarly Balasubramanian et al. [8] tested a number of grasp that were stable according to this metric and observed that they were less stable compared to those learned from human teaching. A fundamental work yet to be done is to replicate these studies with the whole set of metrics. This kind of tests can give us a strong evidence of the utility of the metrics as *quality* predictors.

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