

Explicit vs. Tacit Leadership in Influencing the Behavior of Swarms

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Abstract— Many researchers have employed some form of teleoperated leader to influence a robotic swarm; however, the way in which this influence is conveyed has not been well studied. Some researchers employ designated leaders that are known to be leaders by other members of the swarm and hence followed. Others do not impose a leader/follower distinction on the swarm's algorithms and instead choose to influence the swarm indirectly through controlling one or more of its members. Because the robustness of swarm behavior arises from its many distributed interactions, influence through designated leaders might render it susceptible to noise or disrupt its coherence by overriding these mechanisms. Conversely, limiting human influence to indirect control through the local effects of a leader might prove too sluggish to allow effective human control. This paper compares leader-based methods of each type, designated as Tacit leadership via *consensus* (no explicit leader/follower distinction) and Explicit leadership via *flooding* (influence propagating from leader takes precedence). These methods were compared in simulation and in human experiments finding that explicit leadership led to faster convergence in simulation and better performance in the experiments. Effects of noise were slightly more pronounced for Explicit leaders and cohesion slightly poorer.

I. INTRODUCTION

In the near future, swarms of robots may replace humans and single robots for a number of common tasks. Because swarms are made up of numerous robots that operate under scalable, distributed algorithms, they can cover more area more robustly than a single robot or teams of independent robots. This makes them suitable for jobs such as exploration and foraging [1], [2], [3], construction [4], [5], and fire fighting or HAZMAT situations [6], [7]. Indeed, in recent years, we have seen swarms move from a theoretical possibility to systems implemented on real robots in laboratory settings, such as those in [8], [9], [10].

Robots coordinated as swarms rely on simple control laws replicated across platforms which interact with each other to give rise to emergent organized behavior. Flocking behavior, for example, can be generated from three simple rules: 1) move away from any sensed robot closer than d_1 , 2) move toward any sensed robot further away than d_2 , 3) adjust heading to average heading of sensed robots. The balancing of attractive and repulsive forces and consensus on heading leads to a swarm that sticks together and moves in common, perhaps changing, directions.

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While there are some cases where swarms might act with full autonomy, many tasks require some sort of coordination between a human operator and the swarm. For instance, if the operator is using a swarm of unmanned vehicles (UVs) to survey a large outdoor area, the operator may want to alter several details of the UVs' operations, such as which areas should be explored first, what routes they should take, and how closely they should move with respect to each other. Because swarm algorithms are designed to work with large numbers of robots, individual teleoperation of each swarm member is not feasible.

Strategies for injecting human influence into a robotic swarm can be divided into global approaches which influence the swarm as a whole and bottom-up leader/follower approaches that influence the swarm through its members. Global approaches have been varied including broadcasts of parameters such as goal locations [11], "virtual leaders" that provide a reference signal for incorporation into local consensus [12], algorithm switching [11] [13] and beacons [11] or potential fields [14] to attract and channel the swarm.

This paper compares the effects of Explicit and Tacit leadership on robot swarms coordinating via local laws. Our contributions are as follows. In contrast to most leader/follower studies that have been limited to small (2-3) UV groups [15] [16] in which leaders and followers are in direct contact and where no propagation of leader influence is needed, our focus is on groups of moderate size of between 20-50 robots for which the leader may not be within sensor/communication range of all followers. Additionally, we performed controlled human experiments and compared performance on a variety of measures. This is a contribution over current research which has been conducted in simulation [1],[2],[3],[4],[5],[11],[12],[13], [14] and relied on global methods for exerting influence rather than control/teleoperation of a leader. Moreover, we examine a variant of Explicit leadership in which the leader's influence is propagated through the swarm via intermediaries rather than direct contact, much as orders might be propagated through a military chain of command.

II. RELATED WORK

Leader/follower strategies differ primarily between Explicit approaches in which the influence of leaders over followers is distinguished from their influence over one another and Tacit ones in which it is not. [15] provides an example of an Explicit leader. In their system the operator teleoperates a leading quadrotor while other quadrotors obeying their control laws follow behind. The leading quadrotor is equipped with an array of IR beacons that is used by the followers as a reference to maintain their relative

positions. Because the signal from the leader (IR beacon) is distinct and takes precedence over sensing mediating responses to other quadrotors (obstacle avoidance), the system employs Explicit leadership. Kira and Potter [13] provide an example of Tacit leadership in one of their human-in-the-loop experiments. In their simulation UAVs are controlled through physicomemetic control laws. To influence the swarm the operator creates and directs virtual agents that do not exist in the environment to interact with the real agents via the same force law mechanisms to influence swarm behavior. This is an example of Tacit leadership because the influence of the human directed leader is not distinguished from influences of other members of the swarm. Free selection of leader and Explicit leadership can also be combined as in [16] in which the operator selects a leader for a team of UAVs. A decision support algorithm proposes the leader based on optimal control calculations. The operator can designate a current follower as the next leader and the team will reconfigure itself and begin following the new leader. Finally agents can be assigned additional capabilities as in [17] who distinguishes stakeholders that respond to both human input and other agents from leaders (respond only to human input) and followers (respond only to other agents). In this case only the leader/follower relation remains fully Tacit while the stakeholder might be either a weakened form of Tacit leader (does not fully respond to operator input) or a variant of [12] global “virtual leader” strategy in a population containing only stakeholders.

III. APPROACH

We expect a rapid attenuation of a Tacit leader’s influence as it propagates while that of an Explicit leader should result in more rapid convergence. Two possible disadvantages of Explicit leadership, noise sensitivity and incoherence, have been raised. Goodrich [17] argues that a human operator may have poorer knowledge of the state of the swarm and environment than the agents themselves under conditions of sensor/communication noise, and therefore might make poorer judgments. The second objection is that by overriding local consensus, Explicit commands may disrupt the swarm’s coherence leading to loss of connections and expanding diameter. Goodrich argues instead that the operator should “work within the system” by injecting control through a small number of agents and allowing the system to adjust to these inputs over time.

In our experiments we will examine all three of these hypotheses:

H1: Explicit leadership will result in faster convergence to operator intent making swarms more responsive and easier to control

H2: Sensor noise will affect Explicit leadership to a greater extent than Tacit leadership (e.g.; there will be a noise x leadership type interaction)

H3: Swarms will exhibit less cohesion (more connected components and larger diameter) under Explicit leadership

IV. EXPERIMENT 1

Many swarm control problems ranging from formation following, to rendezvous to flocking can be reduced to

consensus problems. The ability of an operator input to influence the development of consensus within a swarm therefore provides a basic measure of capabilities for bringing the system into agreement with operator intent. For example if an operator sends an orientation command to a swarm, she would expect to see the robots’ orientations to begin changing and eventually settle at her intended value. Our first experiment compares the effects of propagating operator intent through either Explicit (tagged) or Tacit (indistinguishable) messages using *static* (robots are not moving) simulated networks.

We analyze the “consensus quality” based on:

Convergence time: How long does it take for the swarm to converge on the intended value/command of the operator?

Noise tolerance: How robust is the convergence algorithm in the presence of noisy communication?

and investigate the effects of:

Graph size and connectivity: What are the effects of graph size and graph connectivity on the convergence time?

The leader in the Tacit condition is a robot (node) that is not distinguished from its neighbors in any way but that retains its designated value (that intended by the operator) while other robots continue to adjust values to reflect those of their neighbors as dictated by the consensus algorithm.

In the Explicit condition, robots can distinguish between normal messages and tagged ones. Messages from the leader are tagged and update the internal information state of the recipient which remains constant thereafter. This tagged message is then propagated as the robot’s current value at the next step of the consensus algorithm

We define a swarm as a set of robots occupying spatial positions in a plane during time:

$$S(t) = \{s_i(t) | s_i(t) \in \mathbb{R}^2\}.$$

Robots also have an internal state which represents their knowledge at any moment in time. For example each robot’s internal state may reflect its set of internal rules (e.g. for switching between flocking and rendezvous behaviors). In our example this internal state is the robot’s orientation at each time step:

$$X(t) = \{x_i(t) | x_i(t) \in (-\pi, \pi)\}.$$

Our goal is to start from an arbitrary initial state $X(0)$, choose a random orientation value as our intention, x^* , send it to the swarm and wait until the swarm converges on our intention. Convergence is achieved if there exists a time τ , after which the internal values of all robots remain within an error tolerance range δ^* of x^* :

$$\forall t \geq \tau, \forall i \quad |x_i(t) - x^*| \leq \delta^*$$

Robots communicate with each other through a limited disk connectivity graph. In other words, robots that are in range κ can see each other and robots farther away would not be able to communicate directly. This results in a connectivity graph $G = (N, E)$ which has N nodes (the number of robots in our swarm) and there is an edge $e_{ij} \in E$ iff $\|s_i - s_j\| \leq \kappa$.

The operator chooses a random robot ϕ as her initial point of influence and updates the internal value of that robot with her desired random value: $x_\phi(0) = x^*$

We will refer to the Explicit Leader case as the flooding condition because propagation occurs in the manner of a flooding algorithm. In flooding, a swarm has an additional internal state set $P(t)$ which indicates which robots have received tagged messages. Thus after the user sends x^* to ϕ , we would have $p_\phi(0) = 1$ while $\forall i \neq \phi, p_i(0) = 0$. The internal states X and P are updated by this rule: $\forall i, j$ if $e_{ij} \in E$ and $p_j(t) = 1$ and $p_i(t) = 0$ then $p_i(t+1) = 1$ and $x_i(t+1) = x_j(t)$. In case a robot has more than one neighbors relaying tagged messages, it adopts the information from the first one.

In the Tacit control method, also known as consensus, the internal state of each robot is updated by averaging over internal values of all of its neighbors (including itself). The only robot that doesn't change its internal value is ϕ (i.e. the robot which receives x^* from operator). The consensus algorithm is based on the method presented in [18]. Here, we have a weight matrix W which defines the averaging coefficients. Thus at each time t we have:

$$X(t) = W^t X(0)$$

Instead of the optimal solution, we use a simpler approach which assigns a fixed weight to all edges. Xiao [18] demonstrates that if the swarm remains connected, any edge weight smaller than 1 will guarantee convergence. He also proves that the optimal constant edge weight is

$$\alpha^* = \frac{2}{\lambda_1(L) + \lambda_{n-1}(L)} \text{ where } \lambda_i(.) \text{ denotes the } i\text{th largest}$$

eigenvalue of a symmetric matrix and L is the Laplacian matrix of the connectivity graph G . In our experiments, we use this expression for the edge weight.

For the simulation experiments in both Explicit and Tacit conditions we assume that the swarm is operating in an obstacle free environment and always maintains its connectivity (even when robots move). We perform our analysis by assuming noiseless communication in a fixed connectivity graph. After comparing Explicit and Tacit methods and finding some lower bounds on the worst case convergence time, we expand our analysis by assuming noisy communication. As [19], [20] have demonstrated, even in the presence of noise, the consensus method will converge as long as a swarm's connectivity graph remains connected. Therefore our analysis focuses on its convergence time and its relation to the Explicit method's convergence time.

In the Flooding condition (Explicit leadership) the operator expresses intention to the leader and it propagates from there as a breadth first search (BFS) graph traversing algorithm. For a static graph the algorithm takes at most D steps, where D is the size of graph diameter, until the information reaches all the other. We assume that noise is uniformly distributed $E \in (-\delta, \delta)$ so that for static graphs the noise is at most $\pm D\delta$. For an error tolerance of δ^* , therefore, $\delta \leq \delta^*/D$.

In the Consensus condition information also propagates as a BFS and again requires D steps until the information from ϕ can affect the farthest robot, however, because the message's value has been affected by initial conditions of the intervening robots additional steps will be needed to reach convergence. Because error will be averaged rather than amplified an error tolerance of δ^* can be obtained provided $\delta \leq \delta^*$

In order to test Explicit and Tacit control approaches, we created random swarm configurations then analyzed their convergence time. The swarms were created by arbitrarily choosing from 1 to 50 robots and placing them randomly on a $200 \text{ m} \times 200 \text{ m}$ environment. Then starting with a connectivity range of $\kappa = [1, m]$, a connectivity graph was created by incrementing κ until all robots formed a single connected graph. Random internal state values were selected from $(-\pi, \pi)$ in order to simulate an orientation value. A random robot was selected as the leader to communicate the random intention x^* . Both Explicit and Tacit methods are performed by the swarm and their convergence time measured. An error acceptance value of $\delta^* = 0.1$ was used and results averaged over 10,000 experiments.

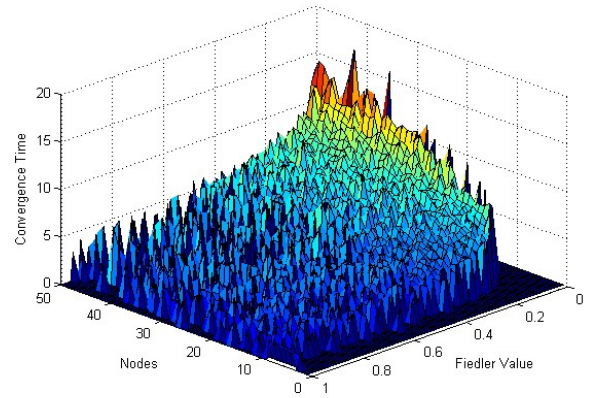
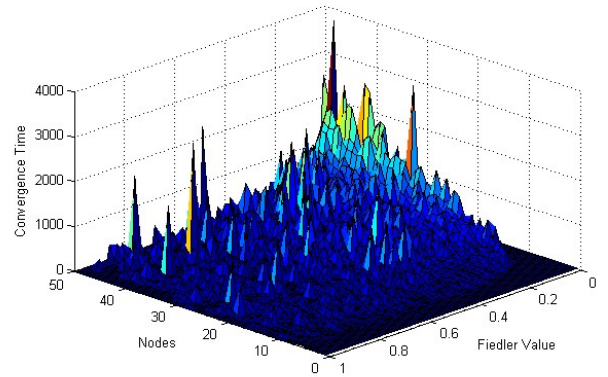


Figure 1. (a) Convergence times for Explicit leaders (flooding)



(b) Convergence times for Tacit leaders (consensus)

Figure 1. Convergence times- note 200:1 differences in scales and irregularity of times for Tacit leaders

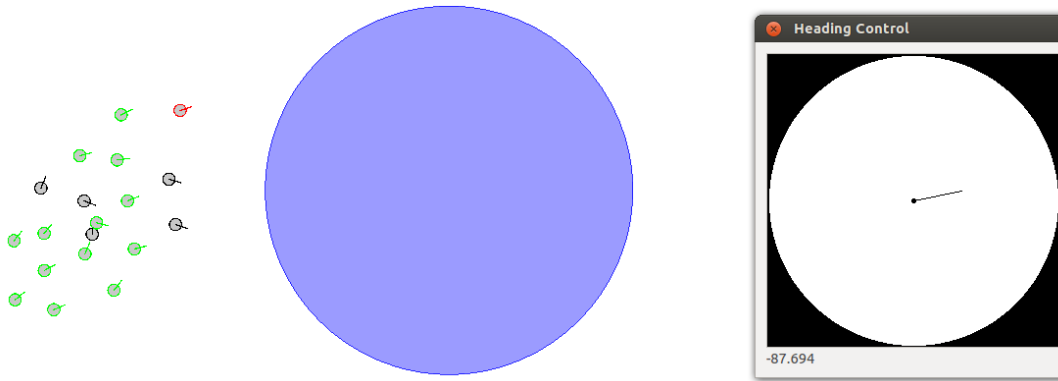


Figure 2. The swarm of robots (left) is steered to the goal region (center) by the user teleoperating the leader robot (shown in red) using the virtual joystick (right). Robots that have detected the goal heading and speed from the leader (or an intervening neighbor) and are moving in that direction are shown in green.

The convergence time for the Explicit and Tacit methods is presented in figures 1(a) and 1(b) respectively. The convergence times are compared based on the number of nodes in the graph and the Fiedler value. The Fiedler value is the second smallest eigenvalue of the Laplacian of the connectivity graph (i.e. $\lambda_1(L(G))$) [21]. A larger Fiedler value means that the graph is more connected and one has to remove more edges in order to cut the graph into independent components. While the Explicit method takes at most about 15-18 steps, the Tacit method takes much more time, sometimes even more than 2000 steps. The Explicit method is also much more robust. As the Fiedler value increases (the graph is more connected), the Explicit method converges faster regardless of the number of nodes. Also when the Fiedler value is small, the convergence time of the Explicit method has a linear relationship with the number of nodes in the graph. On the other hand, the Tacit method behaves differently. When the Fiedler value is small, the convergence time increases exponentially with the number of nodes. Moreover, even when the Fiedler value is high and the graph is well connected, sometimes the convergence time spikes. It seems that the Tacit method convergence time does not behave linearly in relation to the connectivity level of the graph.

The experiments had similar results for cases with moderate amounts of noise however when δ exceeded 10% of the error tolerance level δ^* the swarm frequently failed to converge in the Explicit leader condition. These results support hypotheses H1, namely faster convergence for Explicit leaders, and H2, namely better error tolerance for Tacit leaders. Hypothesis H3, greater coherence for Tacit leaders, was not tested in these experiments.

V. EXPERIMENT 2

In a follow up experiment we implemented flooding (Explicit leadership) and consensus (Tacit leadership) methods for propagating operator influence on a *human-in-the-loop testbed*. Our study investigated the ability of human operators to control a flocking swarm of robots in an open environment by teleoperating a single leader via a continuous velocity (i.e. heading and speed) command. The main task for the users was to survey a given area by guiding the swarm to *goal regions, which appeared dynamically* in

the environment. A goal region appeared at a random position only after another goal region has been visited by the swarm. Thus, the number of goal regions visited by the operator provided a natural measure of his or her performance. In addition, we investigated the effect of sensing error on the ability of the operator to control the swarm.

An open 100x100 meter environment in Stage v.3.2.2 [17] was used to simulate 20 P2AT robots. Each robot was equipped with a sensor providing speed and heading of neighboring robots within 4 meters as well as the presence of a tag in the flooding condition. In conditions with sensing error, simulated noise sampled from Gaussian distributions $N(0, 0.2)$ meters for location and $N(0, \pi)$ for orientation were added. The swarm of robots was initialized randomly in a 10x10 meter box, centered around the origin of the environment, with the leader at the origin with a random orientation.

The human operator's task was to steer the swarm to goal regions shown as blue circles in the environment (Figure 2). Once over half the swarm reached the goal region, a new goal appeared at a random position. The operator used a virtual joystick to control the heading and speed of the leader. Other robots moved according to local control laws.

Alignment: The alignment vector is determined by the propagation method (flooding or consensus). For the flooding propagation condition, each robot determines if any of its neighbors have tagged values. When a robot senses tagged values it sets its velocity and alignment vectors, (x_a, y_a) , to match the velocity and heading of that neighbor. Once that velocity and heading is matched (to within 15 degrees), the robot's values are returned as tagged. In the event that neither the leader nor any privileged neighbors are detected, no alignment vector is used and the robot continues on its previous path. For the consensus propagation method, each robot averages the speed and heading of each neighbor it can sense, and then sets its speed and alignment vector (x_a, y_a) to that average speed.

Cohesion and Repulsion: In addition to sensing neighbors' velocities, the robots also sense neighbors' positions to maintain swarm cohesion and avoid inter-robot collisions by using cohesion and repulsion laws that allow the swarm to flock together.

Eighteen human operators participated in the study. Each participant received 3 minutes of training on each of the four conditions (flooding with and without error, and consensus with and without error). Following training, participants completed the four 10 minute experimental trials in a random order completing a NASA-TLX workload questionnaire [22] to assess the workload of each trial.

A. Results

The main measure of success for participants was the number of goal regions reached. There was a significant difference between each of the four conditions ($F = 72.45$, $p < .001$, see Figure 2). Participants were most successful in

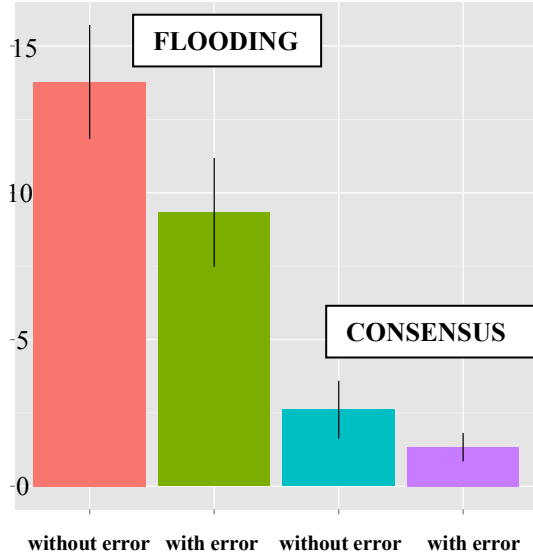


Figure 3. Number of Goals Reached

the flooding without error condition, where they reached an average of 13.78 goal regions. This was significantly more than the flooding with error condition ($M = 9.33$, $t = 3.49$, $p = .001$). Furthermore, participants were significantly more successful in reaching the goal regions in the flooding ($M = 11.56$) than the consensus ($M = 1.97$) conditions overall ($t = 12.26$, $p < .001$). Within the consensus conditions, goals reached in the consensus without error condition ($M = 2.61$) were significantly higher than in consensus with error ($M = 1.33$, $t = 2.46$, $p = .021$). These findings are also confirmed by the results of the NASA-TLX workload questionnaire ($F = 26.18$, $p < .001$) which found workload higher in the consensus ($M = 69.78$) than the flooding conditions ($M = 39.24$, $t = 7.84$, $p < .001$), and higher in the error ($M = 60.05$) than the non-error conditions ($M = 48.97$, $t = 2.14$, $p = .039$).

To examine cohesion we looked at connectivity and swarm diameter across the conditions. Connectivity was measured by Fiedler value and swarm diameter as the furthest distance between any two robots in the largest connected component of the swarm. Conditions without error showed significantly more connectivity ($M = 0.27$) than conditions with error ($M = 0.17$, $t = 2.55$, $p = .013$). The only two conditions that showed significant differences in connectivity were the consensus without error condition ($M = 0.30$) and the flooding with error condition ($M = 0.15$, $t = 2.34$, $p = .027$) although connectivity between consensus

($M = 0.24$) and flooding ($M = 0.20$) were not significant ($t = 0.92$, $p = .359$). There were significant differences between the conditions for swarm diameter, however ($F = 20.48$, $p < .001$, see Figure 3). The diameter of the swarm was larger in the flooding conditions ($M = 10.79m$) than in the consensus conditions ($M = 9.60m$, $t = 4.26$, $p < .001$). Similarly, the error conditions had a higher diameter ($M = 10.84m$) than the non-error conditions ($M = 9.54m$, $t = 4.81$, $p < .001$).

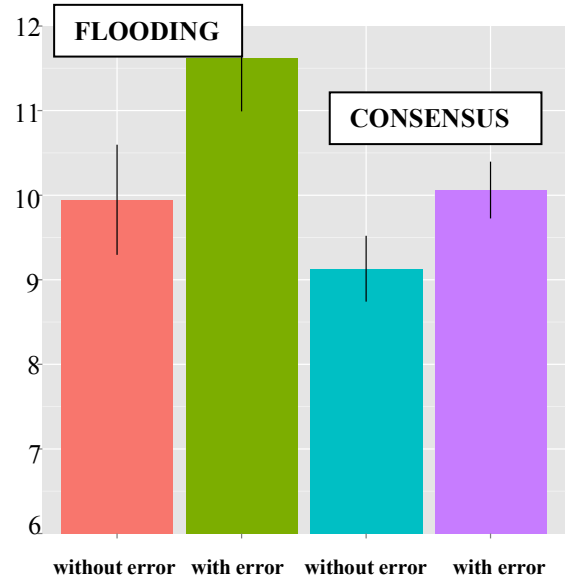


Figure 4. Diameter of Swarm

VI. DISCUSSION

Overall, the results show that an Explicit leader with a flooding method of propagating information is more effective than a Tacit leader relying on existing control laws in influencing a swarm to match an operator's intent. The first experiment comparing flooding and consensus found a ratio of approximately 200:1 in convergence times favoring flooding. This advantage was less pronounced in the human subject experiment which found only a 7:1 advantage for flooding in moving the swarm between goal regions. The sluggishness of the swarm's response to control through a Tacit leader is borne out both by the better performance in the Explicit leader conditions in moving swarms between target regions within a time limit, and workload ratings which were twice as high for Tacit leaders. Such high reported workload ratings are common for extremely low gain systems. These findings provide strong support for our first hypothesis, H1: Explicit leadership will result in faster convergence to operator intent making swarm more responsive and easier to control.

Evidence for our second hypothesis, H2: Sensor noise will affect Explicit leadership to a greater extent than Tacit leadership, is weak and equivocal. Our simulation data show that sensor noise has little differential effect on performance up until very high levels where the flooding algorithm may fail to converge. The human subject data provided similar results. Despite operators reaching fewer goals in the

flooding with error condition, a corresponding drop in performance was found using the consensus method. The absence of an interaction between leadership type and the presence of noise indicates no differential advantage in error tolerance for Tacit leadership. In fact, flooding retained its 7:1 advantage over consensus in the noise conditions.

The third hypothesis, H3: Swarms will exhibit less cohesion under Explicit leadership, was better supported by our study. Because connectivity was maintained in both the static and subsequently run dynamic graph conditions in the simulation this hypothesis could only be tested in the second experiment. In the consensus conditions, the swarms were both more compact (smaller diameter) and had a more connected sensing graph (higher Fiedler value). This could provide significant benefit if bandwidth between swarm members is at a premium, as a more highly connected network would allow for more messages to be passed through the swarm. Denser swarms could also be beneficial in cases of operation in obstacle-filled spaces, such as surveying the ocean floor or with small unmanned ground vehicles exploring urban and indoor environments.

While sensor noise had only minor effects in our experiments there may be other conditions under which Goodrich's [17] contention that "a human operator may have poorer knowledge of the state of the swarm and environment than the agents themselves" could lead to different results. In a cluttered environment, for example, averaging with neighbors might aid in moving around obstacles while persisting at an operator dictated heading might not. We would like to conduct further simulations imposing local constraints as well human control in cluttered environments to investigate these possibilities.

VII. CONCLUSIONS

One conclusion that may be drawn from this study is that it is important to consider mechanisms for exerting human influence when designing algorithms for coordinating swarms. While consensus algorithms have valuable guarantees when operating independently they may be less subject to human influence than less robust forms of coordination. Our finding of the relative robustness to noise of non-consensus influence propagation is encouraging. However, the loss of coherence resulting from imposing an external value on the consensus process may be a necessary price for improving the responsiveness of swarms whether through propagation or a global method such as broadcast.

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