2014 IEEE International Conference on Robotics & Automation (ICRA)
Hong Kong Convention and Exhibition Center
May 31 - June 7, 2014. Hong Kong, China

Resonance-driven dynamic manipulation: dribbling and juggling with elastic beam.

Alexander Pekarovskiy, Kunal Saluja, Rohan Sarkar and Martin Buss

Abstract— **This paper presents a new device and a method for dynamic manipulation. The device consists of a planar robotic arm and an elastic beam as an end-effector. Using it the elastic end-effector will tend to increase performance and energy efficiency while executing dynamic and repetitive tasks. Through the control of the beam vibration and resonant modes, we modify the state of manipulated objects. For lightweight objects the control is provided through the intermittent contacts without changing dynamics of the beam. However, we show that by using proper synchronization technique continuous-phase contacts are also possible. Juggling and dribbling of a ball are considered to be an alternating non-prehensile catching and throwing task. Such alternating decelerating and accelerating impacts on the ball and the curvature of the beam at the time of impact will stabilize the cyclic orbit of the ball. By proper analysis of continuous-time contact and dynamics of the beam we establish a rhythmic movement of the system. With the variation of frequency and amplitude of the beam it is possible to switch between different dynamic actions such as juggling, dribbling, throwing, catching and balancing.**

I. INTRODUCTION

Most of the robotic tasks are related to maximizing the rigidity of an end-effector to increase the performance of the robot. To increase the throughput of the manipulation tasks it is common to damp the vibration of the flexible structures to a reasonable level.

We are chasing the same goal, but by introduction of elasticity in the end-effector. At the same time we aim to get specified repeatability and accuracy in handling an object, especially for periodic manipulation tasks like dribbling and juggling. With such an end-effector, the vibration at high speeds is not being damped but is rather being controlled to obtain the prescribed properties for steady state. It is desirable to handle the widest possible range of objects with the robot end-effector. This minimizes changeover requirements, reduces costs for multiple tools and addresses weight and size considerations. Besides, the intrinsic elasticity increases the performance and energy efficiency while performing such dynamic and repetitive tasks.

In manufacturing processes there are various manipulative tasks with deformable linear objects (DLOs). Multiple works reported the results on manipulation of DLOs and the subsequent usage of sensory systems to reach a particular goal.

A. Pekarovskiy and M. Buss are with the Institute of Automatic Control Engineering, Technische Universität München, Munich, Germany, 80290 {a.pekarovskiy, mb}@tum.de

K. Saluja is a Mechanical engineering undergraduate in Birla Institute of Technology and Science, Pilani, India. {kunal15saluja}@gmail.com

R. Sarkar is an Instrumentation and Electronics Engineering undergraduate in Jadavpur University, Kolkata, India. {rohansarkar028}@gmail.com

Khalil [1] presented the usage of intelligent multi-sensory systems that combine vision data with force and tactile measurements to achieve proper interaction between the robotic manipulator and the deformable object. Yue and Henrich [2] carried out a research on the vibration of deformable objects and the task of inserting them in a hole by fast manipulation with a 6 DOF force/torque sensor mounted at the wrist of the robot arm.

Walking, running and jumping are some of the various rhythmic tasks performed by humans in their daily lives. These are of great importance in robotics, which aim to emulate such a periodic behavior. The knowledge of various dribbling or juggling models provide a theoretical basis for many complex tasks. Juggling is a rhythmic yet confined operation which involves sensomotoric coordination and several phase-locking relationships between the juggled object and the end-effector.

In robotics juggling has been attributed to be a challenging and dexterous task and was a benchmark tool for the field of dynamic manipulation for many years. The first juggling robot in print was presented in the work of Aboaf *et al.* [3], where they discussed learning algorithm for juggling. Rizzi and Koditchek [4] implemented a stable system for ping-pong ball juggling using stereo vision and 3 DOF robot. Schaal and Atkeson [5] investigated an open-loop control applied to different dynamical manipulation tasks. Buehler *et al.* [6], [7] analyzed the class of repeating robot-environment interactions, including discontinuities caused by the impact. They introduced a feedback law called the mirror algorithm that defines the trajectory of the paddle as it is mirrored about the desired impact height and also a scaled version of the ball trajectory. Consequently, this requires constant tracking of the ball position. They showed that the apex height of the ball is stabilized. Interestingly, the resulting paddle trajectory is controlled by accelerating at impact, which contrasts the result of Schaal and Atkeson [5] who analyzed the stabilizing property of a decelerating paddle for an open-loop bouncing ball system.

A number of papers dealt with the spatial elastic dribbling. For example, Baetz and Mettin *et al.* [8], [9] discussed continuous-time control of the task. Haddadin *et al.* [10] investigated potential energy storage of the elastic plate using passive compliance of the hand to facilitate blind dribbling task. Reist and D'Andrea [11] designed a juggling robot that is able to vertically bounce a completely unconstrained ball without any sensing in all the 3 dimensions. They took the impact time measurements as feedback and proved that the closed-loop performance is only marginally improved as compared to open-loop control. An aluminum paddle of a particular fixed curvature was decelerated at the time of impact to stabilize the impact. Haddadin *et al.* [10] also considered only the bending of the elastic material. Modeling of elastic structures was done by Barnoski [12] who discussed the response of elastic beam to the base excitation.

However, most of the work done on juggling and dribbling is using a rigid paddle but no work has been done as to how a continuously changing curvature of the elastic material can affect the stability. The use of resonant modes of an elastic material would be an entirely new approach in this direction. Various dynamic tasks use different end-effector design in each case suited to the particular task to be achieved. By exciting the elastic beam, see Fig. 1, with proper amplitudes and frequency, we can use the same end-effector to perform a variety of dynamic tasks like dribbling, balancing, batting, juggling and catching.

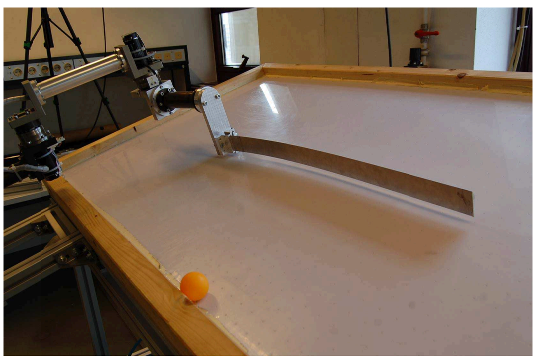


Fig. 1: End-effector as an elastic beam.

The paper is organized as follows: In Section II the modeling of the elastic beam as an end-effector, the beam and part dynamics, the resonance conditions and the collision model are described. In Section III motion planning and control of the beam are investigated. In Section IV the simulation verification of hybrid dynamics task is provided. Finally, in Section V our results are concluded and future work is discussed.

II. Modeling

Several assumptions are assumed for proper modeling:

A1 The object is modeled as a rigid ball.
A2 There is no air-friction.
A3 The end-effector is modeled as a uniform elastic beam.
A4 The collision is elastic with coefficient of restitution c_r.
A5 The damping in the beam is omitted.

The presented end-effector broadens the dynamic capability of the periodic manipulation tasks. The main advantages are energy storage due to elastic nature of the beam and large contact area for manipulation of multiple objects. Intermittent and continuous-time juggling is possible by proper control of relative velocities, curvature and contact properties between the object and the beam. Since the beam is represented as a long thin stick, it can be modeled as a cantilever beam. To excite different modes of vibration the beam is clamped from one side and free from another, hence the beam undergoes free vibrations.

A. *Model of the elastic beam as an end-effector*

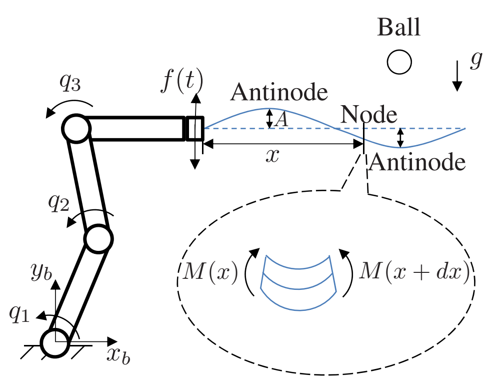
The slender beam has a high length to breadth ratio. The shear deformation and rotary inertia effects are neglected based on the Euler-Bernoulli beam assumptions. The Euler-Bernoulli beam is subjected to the translation $y(t)$ of its base in the transverse direction. Deformations of the geometrically uniform thin beam are assumed to be small.

B. *Beam dynamics*

The general equation [12] for transverse vibration is defined as

$$EI\frac{\partial^4 y(x,t)}{\partial x^4} + m\frac{\partial^2 y(x,t)}{\partial t^2} = m\omega^2 A\sin\omega t, \qquad (1)$$

where E is the Young's modulus, I is the cross section area moment of inertia about the bending axis, m is a mass per unit length of the beam, A is an amplitude of sinusoidal excitation, ω is a frequency of excitation and y is a vertical displacement of the beam relative to the base from its equilibrium position, see Fig. 2.



Parameter	Value
Beam dimensions, w*h*l	$0.5mm$*$40mm$*$600mm$
E	$1600N/mm^2$
I	$41.67mm^4$
m	$0.077kg/m$
c_r	0.85
Ball mass, m_b	$2.7g$

Fig. 2: Experimental setup model and table with parameters for elastic beam and object. Beam undergoes vertical base excitation $f(t) = A\sin\omega t$.

According to the modal theory, the solution to (1) is

$$y(x,t) = \sum_i \phi_i(x) r_i(t), \qquad (2)$$

where $\phi_i(x)$ denotes the ith normal mode of the physical system and $r_i(t)$ denotes the ith generalized coordinate.

Using the generalized coordinate approach to solve the given problem we get

$$\ddot{r}_i(t) + \omega_j^2 r_i(t) = \frac{\bar{F}_j}{\bar{M}_j}, \tag{3}$$

where $\bar{F}_j$ is the generalized force and $\bar{M}_j$ is the generalized mass

$$\bar{F}_j = \int_o^l \phi_i(x) f(x,t) \mathrm{d}x$$
$$\bar{M}_j = \int_o^l m\phi_i(x)\phi_j(x)\mathrm{d}x, \tag{4}$$

and

$$\omega_i(x) = a\cos\lambda_i x + b\sin\lambda_i x + c\cosh\lambda_i x + d\sinh\lambda_i x, \tag{5}$$

where $(\lambda_i l)^4 = \frac{ml^4}{EI}\omega_i^2$. For a given mode i, equation (4) becomes

$$\ddot{r}_i(t) + \omega_i^2 r_i(t) = \frac{2A}{l}\omega^2 \frac{\alpha_i}{\lambda_i}\sin\omega t, \tag{6}$$

whose steady-state solution is given as

$$r_i(t) = 2A\frac{\alpha_i}{\lambda_i}\left(\frac{\omega}{\omega_i}\right)^2 \frac{1}{1-(\frac{\omega}{\omega_i})^2}\sin\omega t \tag{7}$$

C. Boundary conditions

Since our system resembles a cantilever beam structure the following boundary conditions are applicable

$$y(0,t) = 0$$
$$\frac{\partial y}{\partial x}(0,t) = 0$$
$$EI\frac{\partial^2 y(l,t)}{\partial x^2} = 0$$
$$EI\frac{\partial^3 y(l,t)}{\partial x^3} = 0, \tag{8}$$

where the first condition signifies zero displacement at the fixed end, the second condition signifies zero slope at the fixed end, the third condition signifies zero bending moment at the free end and the fourth condition signifies zero shear force at the free end. On substituting conditions from (8) on (4) we get a square homogeneous matrix of the form

$$\begin{bmatrix} \cos\lambda_i l + \cosh\lambda_i l & \sin\lambda_i l + \sinh\lambda_i l \\ -\sin\lambda_i l + \sinh\lambda_i l & \cos\lambda_i l + \cosh\lambda_i l \end{bmatrix} \begin{bmatrix} d \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{9}$$

where d and c are constants that are taken from (5). Setting the determinant equal to zero, we yield $(\cos\lambda_i l)(\cosh\lambda_i l) + 1 = 0$ whose solution yield the model frequencies of the beam. The parameter $\phi(x)$ is obtained as

$$\phi_i(x) = d\left[\cosh\lambda_i x - \cos\lambda_i x + \tfrac{c}{d}(\sinh\lambda_i x - \sin\lambda_i x)\right], \tag{10}$$

where $\frac{c}{d} = \alpha_i$.

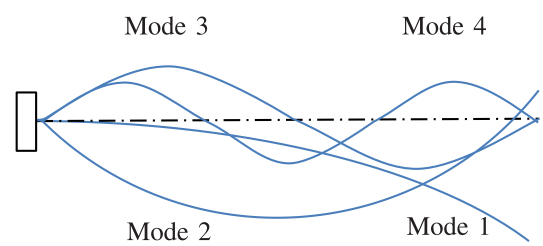
Using equations (9) and (10) and substituting them into (2), we get

$$y(x,t) = 2A\sum_{i=1,2,3\ldots}^{\infty}\left(\frac{\omega}{\omega_i}\right)^2 \frac{1}{1-(\frac{\omega}{\omega_i})^2}\frac{\alpha_i}{\lambda_i l}\phi_i(x)\sin\omega t. \tag{11}$$

Equation (11) describes the complete behavior of the beam at every point in time. It is a convergent sequence that tends to a particular solution.

D. Vibration modes conditions

Different excitation frequencies and amplitudes of the robot tip point result in various standing waves. Four different modes of vibration are presented in Fig. 3. The described modes are achieved only with the excitation of resonant frequencies.



Mode	Frequency, Hz
Mode 1	4.5818
Mode 2	28.7158
Mode 3	80.433
Mode 4	157.5811

Fig. 3: Vibration modes and standing waves.

E. Part dynamics

Collisions between the objects are assumed to be partially elastic such that the time period between successive collisions is sufficient for the beam to regain its vibration mode. The mass of the object is considered to be negligible to cause any disturbance to the current mode of beam vibration.

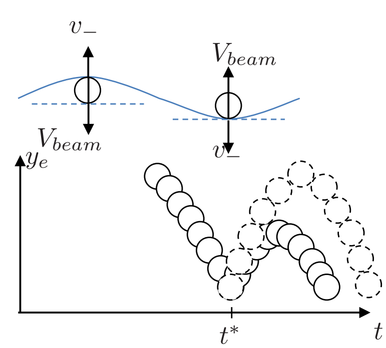


Fig. 4: Physical model of the collision. $V_{beam} = \dot{y}(x, T_{ball})$ is a velocity of the beam in the antinode.

Part dynamics depends on several parameters such as coefficient of restitution and contact properties. Various contact cases result in restitution that is served with $v_+ = c_r v_-$,where v_- and v_+ are velocities before and after collision. Next state of the ball after i-th impact is described by

$$v_{i+1-} = -gT - c_r v_{i-}, \tag{12}$$

when the velocity of the beam is 0, meaning that there is no energy injection.

III. Control design and motion planning

Through the continuous sinusoidal excitation of the beam it becomes possible to generate modes of vibration. The idea for this object manipulation is to fulfill some periodic goal. The classical goal for the vertical juggling is to support constant maximum height of the ball. Or to keep stable dribling for some predefined end-effector height above the floor. In control terms it means that the limit cycle of the task has to converge to some equilibrium point. Since the object gets in contact with the beam, the system transforms to a hybrid one with transient conditions. In this paper we limit ourselves only to discussion about juggling and dribbling in the vicinity of the beam antinodes.

A. Ball trajectory determination for juggling

Juggling task requires initialization procedure to obtain the ball trajectory. Position and velocity data can be obtained directly using tracking system.

The data needed can be also indirectly achieved with a force/torque measurements. The ball is released from some height h and longitudinal distance from the base r. The robot holds the beam in a horizontal plane without any vibration at any instant of time. For this measurement we make an assumption that the beam is rigid. Afterwards we read measurements from the two consecutive collisions

$$\begin{aligned} -v_{1-} &= \frac{1}{m_b} \int_{t_{st1}}^{t_{f1}} \frac{M_{y1}}{r} dt \\ -v_{2-} &= \frac{1}{m_b} \int_{t_{st2}}^{t_{f2}} \frac{M_{y2}}{r} dt, \end{aligned} \tag{13}$$

where M_{y1} and M_{y2} are torque readings from force/torque sensor in the wrist of the robot, t_{st} and t_f are initial and final time of a single impact. The time between two collisions $T_{21} = t_{f2} - t_{st1}$ is obtained by measuring the distance between two splashes in the sensor readings.

The distance r is obtained by substituting T_{21} in equation of ball dynamics (12)

$$-r(-gT_{21} - c_r v_{1-}) = -r v_{2-}. \tag{14}$$

Finally we can obtain initial position of the vertically released object

$$\begin{aligned} h^* &= \frac{v_{1-}^2}{2g} \\ r^* &= \frac{\frac{1}{m_b} \int_{t_{st2}}^{t_{f2}} M_{y2} dt + (\frac{1}{m_b} \int_{t_{st1}}^{t_{f1}} M_{y1} dt) c_r}{gT}. \end{aligned} \tag{15}$$

Right after the second collision, the ball release coordinates r^* and h^* are determined.

B. Motion planning for periodic reference trajectory

To be able to control the periodic contact we have to synchronize vibration of the beam with the period of the ball. The conditions for synchronization are as follows

$$\begin{aligned} T_{ball} &= kT_{beam}, k \in \mathbb{Z} \\ y(x,t) &< h(t), 0 < t < T_{ball} \\ y(x,t) &= h(t), t = T_{ball}. \end{aligned} \tag{16}$$

For simplification one can choose $T_{ball} = T_{beam}$, then the two other conditions are automatically fulfilled.

$$\begin{aligned} T_{ball} &= -\frac{2v_{1-}}{g} \\ \omega_b &= \frac{1}{T_{ball}}. \end{aligned} \tag{17}$$

Velocity of the beam v_{beam} at the point x is calculated as

$$\begin{aligned} v_+ &= \frac{2h_{max} + g\frac{T_{ball}^2}{4}}{T_{ball}} \\ v_- &= v_+ - gT_{ball} \\ v_{beam} &= -\frac{v_+ + v_- c_r}{c_r}. \end{aligned} \tag{18}$$

By comparing variables from (2), (17) and (18) we get the required parameters for desired beam excitation

$$\begin{aligned} \dot{y}(x, T_{ball}) &= \frac{\partial y}{\partial x}(x, T_{ball}) = v_{beam} \\ \omega_b &= \frac{1}{T_{ball}} = \frac{k}{T_{beam}}, \end{aligned} \tag{19}$$

In general case ω_b should be a multiple of the resonant frequency of chosen mode i. Using this data we get A and ω_b and thus we derive $f(t)$. Afterwards $f(t)$ is tracked with the robot controller, see Fig. 5.

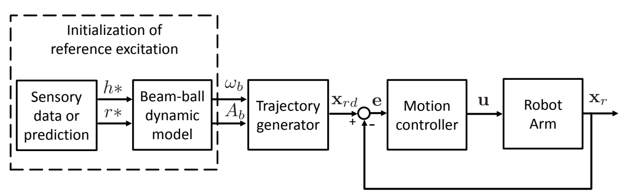


Fig. 5: Control structure

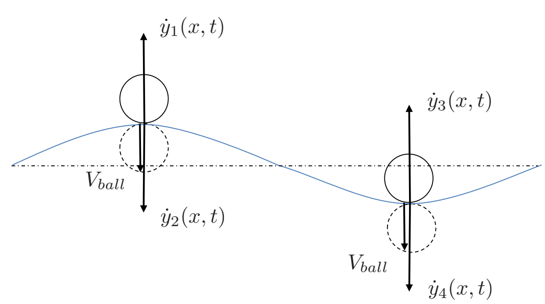


Fig. 6: Five possible cases of ball-beam contact in the antinodes, solid for juggling and dashed for dribbling. First, antinode is higher than the base line and decelerating, Second, higher than the base line and accelerating, Third, lower than the base line and decelerating, Fourth, higher than the base line and accelerating, and finally that it is exactly at the base line and acceleration is zero. All contacts for dribbling are symmetrical to the juggling.

For our elastic beam manipulation we would like to discuss two intrinsically different juggling scenarios one with intermittent and another with continuous-time contact, see Fig 6.

C. Intermittent contact

In this scenario the ball is periodically hitting the beam in the antinode. There are five possible cases. Case one is marginally stable but practically becomes unstable because of any disturbance. From [13] it is known that one of the required conditions for the stable limit cycle is the non-positive acceleration of the batting point, therefore second and third cases are leading to instability. The fifth case is the vertical collision that happens only when the beam deflection is zero, i.e. the beam crosses the base line. Collision with the horizontal line where $\ddot{y}(x,t) = 0$ works for all modes but even in the ideal case we can guarantee only marginal periodic stability. Finally, case four will be discussed in the following section.

D. Continuous-time contact

We can show more complex behavior with continuous-time phase for $\ddot{y}(x,t) < 0$, $\dot{y}(x,t) < 0$. By matching the velocities at some predefined height h_c, the beam catches the ball and decelerates till $\dot{y}(x,t) = 0$. At the minimum point for juggling and maximum for dribbling the switching between catching and throwing occurs. The beam carries the object and releases it exactly at the base line, since acceleration changes its sign there.

An important feature of such manipulation is that in case of small perturbations in position or/and velocity the catching and throwing phases will tend to passively balance the ball to the antinode due to the beam convexity before the release. Of course, it is also possible to use these continuous-time phases for updating and improving feedback control policies, however it is not within the scope of this paper.

E. Hybrid dynamics

Hybrid dynamics transition presented here is similar to [8] for underactuated continuous-time system, see Fig. 7. State of the system is described as

$$\mathbf{X} = \begin{bmatrix} y(x,T_{ball}) & y_{Ball} & \dot{y}(x,T_{ball}) & \dot{y}_{Ball} \end{bmatrix}$$
$$\mathbf{X}(i+1) = A\mathbf{X}(i). \tag{20}$$

Every transition phase is characterized by a switching surface S_i^j and by a reset map $\mathbf{X}_j^+ = \Delta_i^j(\mathbf{X}_i^-)$, where $\mathbf{X}_j^-$ and $\mathbf{X}_i^-$ are the states after and before the transition.

F. Local stability

Sketch of proof. To investigate stability properties of our hybrid system, state space dynamic equations (20) have to be linearized around equilibrium point, see e.g. [13]. If for all eigenvalues holds $\max_i(|\lambda_i|) \leq 1$ then the hybrid system is locally stable.

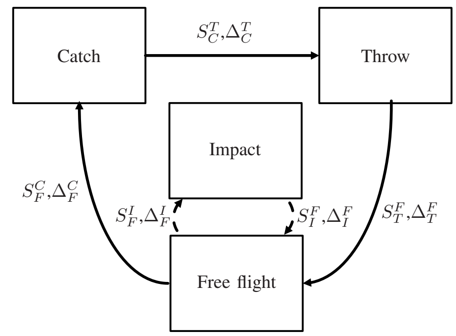


Fig. 7: Evolution chart for the hybrid system. Floor impact occurs only for dribbling

Switching surfaces	Reset maps
$S_C^T = \dot{y}(x,T_{ball})$	$\mathbf{X}_T^+ = \Delta_C^T = \mathbf{X}_C^-$
$S_T^F = y(x,T_{ball})$	$\mathbf{X}_F^+ = \Delta_T^F = \mathbf{X}_T^-$
$S_F^C = y(x,T_{ball})$	$\mathbf{X}_C^+ = \Delta_F^C = \begin{bmatrix} \multicolumn{3}{c}{\mathbf{I_3}} & \mathbf{0}_{3x1} \\ 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{X}_F^-$
$S_F^I = S_I^F = y_{Ball} - r_{Ball}$	$\mathbf{X}_I^+ = \Delta_F^I = \mathrm{diag}(1\ 1\ 1\ -c_r)\mathbf{X}_F^-$ $\mathbf{X}_F^+ = \Delta_I^F = \mathbf{X}_I^-$

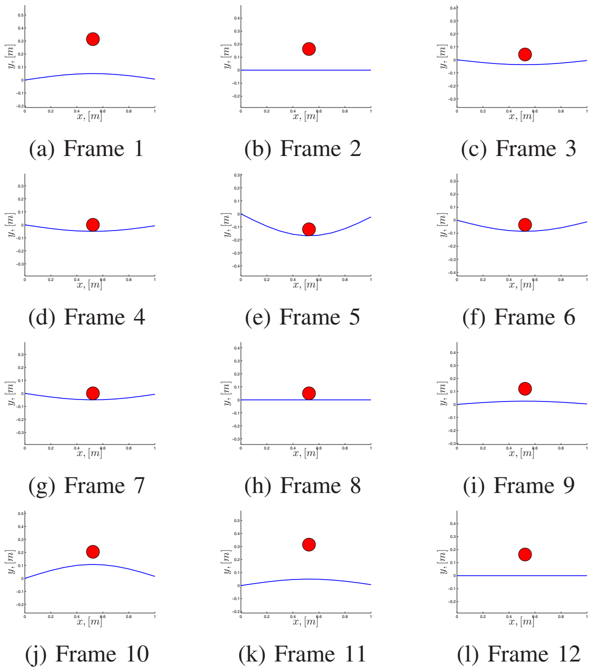
Fig. 8: Table with switching surfaces and reset maps for hybrid dynamics.

IV. Simulation

The derived model for the system with continuous contact is tested in a simulation. This results in a particular scenario of alternating ball catching and throwing to perform juggling and dribbling as discussed in Sec. III.D. In case of juggling the ball is released from a particular height and then caught at the instance when the velocity of the beam and the ball are equal, ensuring smooth catching of the last. The beam then moves back to its initial position, imparting velocity to the ball which then rises back to its maximum height. The same holds for dribbling, but only with additional impact with the ground instead of going to prescribed height. The simulation results are presented in Fig. 9 and 10.

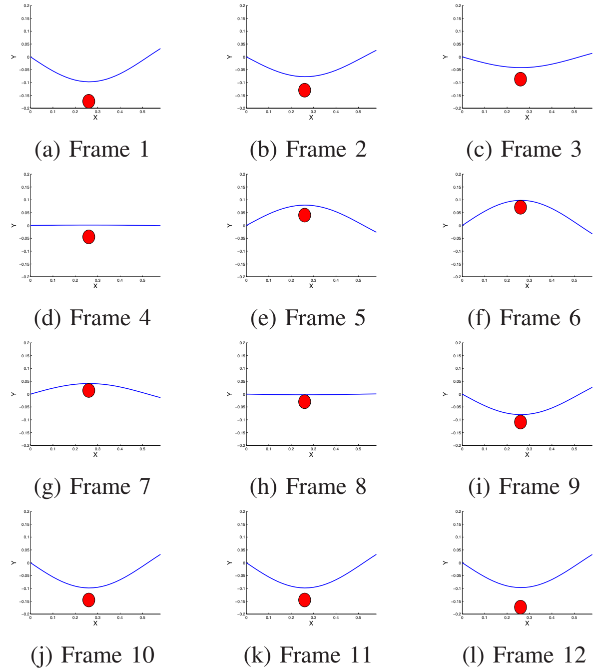
V. Conclusion and Future work

This paper presented design and analysis of an elastic end-effector for active juggling of a ball. The dynamics of an elastic beam clamped at one end with a 3 DOF robotic arm was analyzed. This method is used for dribbling and juggling a ball by the proper excitation and control of the beam. The simulations show how a variety of tasks like catching, throwing and juggling can be achieved by a single end-effector. Although the juggling experiment was sensitive to initial conditions, our theory with an actual implementation



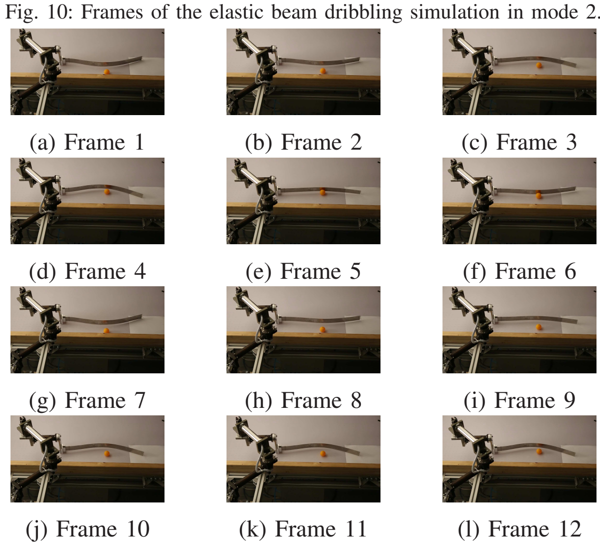
(a) Frame 1 (b) Frame 2 (c) Frame 3 (d) Frame 4 (e) Frame 5 (f) Frame 6 (g) Frame 7 (h) Frame 8 (i) Frame 9 (j) Frame 10 (k) Frame 11 (l) Frame 12

Fig. 9: Frames of the elastic beam juggling simulation in mode 2.



(a) Frame 1 (b) Frame 2 (c) Frame 3 (d) Frame 4 (e) Frame 5 (f) Frame 6 (g) Frame 7 (h) Frame 8 (i) Frame 9 (j) Frame 10 (k) Frame 11 (l) Frame 12

Fig. 10: Frames of the elastic beam dribbling simulation in mode 2.



(a) Frame 1 (b) Frame 2 (c) Frame 3 (d) Frame 4 (e) Frame 5 (f) Frame 6 (g) Frame 7 (h) Frame 8 (i) Frame 9 (j) Frame 10 (k) Frame 11 (l) Frame 12

Fig. 11: Frames of the elastic beam dribbling experiment in mode 2.

of dribbling showing very high repeatability and robustness was experimentally justified.

In the future, juggling techniques with lateral ball movements like shower, waterfall and cascade for juggling multiple balls will be implemented. Moreover, the investigation on the variation of amplitudes, frequencies and accelerations at impact are responsible for switching between various dynamic actions is planned.

Acknowledgment

The authors would like to thank Ken Friedl for filming and editing video for the paper. This work is supported in part within the ERC Advanced Grant SHRINE Agreement No. 267877 (www.shrine-project.eu).

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