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Cooperative Positioning/Orientation Control of Mobile Heterogeneous Anisotropic Sensor Networks for Area Coverage

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***Abstract*—This article examines the coordination problem of the nodes' motion in a heterogeneous anisotropic mobile sensor network for area coverage purposes. The mobile agents are assumed to have non–uniform with varying scaling sensing ability around themselves. The nodes' sensor footprint is allowed to be any arbitrary compact planar set, while the coordination scheme accounts for rotation of the latter. The domain sensed by the swarm is partitioned via the proposed distributed scheme that differentiates for standard Voronoi–alike distance–based metrics. The distributed cooperative scheme developed manages to lead the group towards an area–optimal configuration via proper control of the movement and rotation of each sensing node. Numerical results are provided in order to indicate the efficiency of the proposed technique.**

***Index Terms*—Distributed optimization, area coverage, cooperative control, heterogeneous swarms, non–uniform sensory**

I. INTRODUCTION

Cooperative control of swarms has received increased interest by the scientific community in the last decade. Although each member of the group may act independently in order to contribute to an objective cost, the benefits acquired by coordinated action as a team is largely increased [1, 2]. In these cases the nodes act by taking into account information from neighbouring members, while efficiently designed cooperative strategies can lead the network in an optimal state, dependent on the aggregate criterion [3, 4].

Robotic swarms have been utilized in coverage/surveillance applications. Mobility offers the advantage of allowing to the members of the team to evolve in space through time autonomously and seek the optimum configuration that would serve the common purpose [5]. In either case, the mobile platforms are equipped with RF antennas for agent–to–agent communication, along with sensing devices such as microphones, cameras, humidity/temperature sensors, smoke detectors and others, dependent on the application. Thus, maximal potential of the group's abilities can be achieved via optimum deployment schemes (area coverage) or optimal patrolling trajectories (region surveillance) [6, 7].

The majority of the research in coordination of networks examine the case where the nodes have uniform radial sensing performance (i.e. circular sensing domains). There exist several types of sensors that this model represents suitably concerning their perceptional abilities (i.e. omnidirectional microphones, smoke detectors). Extensions have also been presented for accounting for unequal ranges of the disc sensor model. The distributedness of the proposed control strategies relies on spatially distributed evaluation of the corresponding Euclidean/Laguerrean Voronoi cell [6, 8–10]. Specifically, each node being able to evaluate its own region of responsibility can decide the direction to move at that will lead towards the network's (local) area optimum configuration.

The authors in [11] have proposed a distributed control strategy for the wedge–shaped case, relying on standard Euclidean Voronoi tessellation of the space. Extension to arbitrary patterns has been introduced by the authors in [12], where partitioning was performed in a pattern–based manner rather than a distance–based one. However, in that case the sensors were considered to have common orientation, while no rotation of the sensing domain was taken into account. Apart from that the sensory of the nodes, although arbitrary, was assumed to be a compact convex set, derived as the maximal inscribed convex set in the original footprint (in case of non–convex domains). The reason for that demand was the utilization of specific Helly–type theorems for planar convex curves [13, 14], during the development of the space partitioning.

This article is an extension of the authors' work [12], by allowing the rotation of the patterns, thus dropping the conservative demand for common orientation. Furthermore, the sensing footprint for the nodes is not required to be a convex set (since the homothetic requirement [14] is dropped), while heterogeneity in the network lies in the scaling factor of the (common) pattern among the nodes. The proposed control scheme is based on an appropriate partitioning of the sensed space, while in the sequel account for both translational and rotational motion of the nodes in order to reach an area–optimum topological state.

The rest of the article is organized as follows. In section II the main assumptions concerning the network's sensing performance are introduced, while the group's optimization objective is formally stated. The proposed algorithm for partitioning the sensed space via the core footprint is presented in section III. In section IV the distributed coordination scheme based on the aforementioned region–assignment is developed, emphasizing in the additional rotational degree of freedom. Simulation results are presented in section V in order to indicate the efficiency of the proposed scheme, followed by comparative discussions on its advantages over existing circular–approximation–based approaches (in coverage–based terms), while concluding remarks along with future extensions are discussed in the last section.

II. PROBLEM STATEMENT

Assume n mobile robotic agents responsible for the sensing coverage of a compact convex planar region of interest D. Let

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the coordinates of each node in the Euclidean configuration space be denoted as x_i, $i \in I_n$. The set I_n includes all natural numbers up to n, i.e. $I_n = \{1, 2, \dots, n\}$. Assume that over the domain D an importance function $\phi : \mathbb{R}^2 \to \mathbb{R}_+$ is defined, determining an a–priori knowledge/estimation of the importance of its interior points.

Let each node be equipped with a sensing device appropriate for the current swarm–objective. In contrast to the majority of the works in coverage control, the sensing patterns considered in this work are not required to be node–centred circular ones (standard disc model). In regard to the extensions provided in comparison with the authors' work [12], in this article:

- The approach of under–approximating the original pattern (non–convex in the general case) with the maximal–area inscribed node–centred disc (Fig. 1a) or convex set (Fig. 1b) is extended by taking into account the original footprints in the coordination scheme (convex or not).
- The demand for common orientation among the nodes' convex approximated footprints (Fig. 1b) is dropped, allowing for different ones, not only during the initial configuration, but throughout the network evolution, by adding extra control inputs for orientation control.
- Although all nodes are assumed to be equipped with the same kind of sensor (i.e. same sensing pattern), the latter is allowed to be scaled among the nodes (Fig. 1c), incorporating the network's heterogeneity.

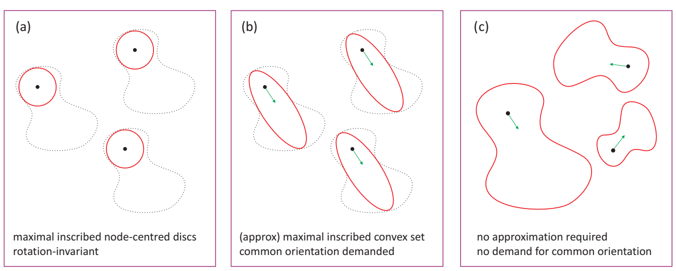

Fig. 1. Graphical representation of the evolution of suggested techniques for non–uniform sensor footprints.

Let C° be the common pattern of the nodes, the pinch–point (base–node) of which is assumed, without loss of generality, at the origin. The sensing pattern of the node i is determined by the base–pattern C°, translated at the node's position, while rotated and scaled accordingly. That is

$$C_i := x_i + \mathbf{R}(\theta_i)\, \lambda_i C^\circ, \tag{1}$$

where x_i are node's–i spatial coordinates, θ_i, λ_i its orientation and scaling compared to the base–pattern, while $\mathbf{R}(\cdot)$ is the rotation matrix corresponding to the angle argument, that is

$$\mathbf{R}(\cdot) = \begin{bmatrix} \cos(\cdot) & -\sin(\cdot) \\ \sin(\cdot) & \cos(\cdot) \end{bmatrix}.$$

Although different orientations are allowed among the nodes in this work, they are assumed to be controlled via an extra control input at each nodes' kinodynamics, that is

$$\begin{aligned} \dot{x}_i &= u_i, \quad x_i \in D \subset \mathbb{R}^2, \ u_i \in \mathbb{R}^2, & (2) \\ \dot{\theta}_i &= \omega_i, \quad \theta_i, \omega_i \in \mathbb{R}, & (3) \end{aligned}$$

where (2) represents the node's translation–part and (3) the part concerning the pattern's rotation.

Having defined the nodes' footprints, one can formally associate the network's performance with them. Let $f_i \colon D \mapsto \{0, 1\}$ be the descriptor function of node's–i sensing performance, where $f_i(x) = 1(0)$ iff $x \in (\notin) C_i$. Consequently, the overall performance of the swarm can be interpreted as the union of all the sensed parts of D, algebraically formulated as

$$\mathscr{H} = \int_D \max_{i \in I_n} f_i \ \phi, \tag{4}$$

where the functions' arguments and integration variables have been omitted to avoid notation–complexity in the sequel of the article. It should be noted that although the indicator function f_i is parametrized/dependent on x_i in previous works, i.e. $f_i(x; x_i)$, in this work is also dependent on the non–common orientation and scaling among the nodes, $f_i(x; x_i, \theta_i, \lambda_i)$.

The decision for the motion is taken based on own self–localization information along with that acquired from the neighbouring nodes. In regard to the need for information exchange, it is assumed that:

Assumption 1. *Each node has sufficient transceiver RF–range so that it can exchange information at any time with any node that it shares sensed parts with.*

III. Proposed Pattern–based Partitioning

Having in mind the need for distributed coordination scheme towards the optimization of the overall coverage performance of the network, a region of responsibility is assigned at each node in a local manner, assisting in lowering the computational complexity in the optimization problem via proper space decomposition.

A. Related Approaches

The most common approach followed in swarm coordination for defining the way that the configuration space is tessellated among the nodes is the well–known Voronoi partitioning [15]. More specifically, each node is set responsible for the parts of the area under consideration that are closer to that node compared to any other in the network. The resulting Voronoi diagram is proven to be the optimal way of tessellating the space in the Euclidean metric in a distance–based manner, based only on the nodes' coordinates, while suitably resolves the standard case of swarms with disc–patterns. Alternatively, considering heterogeneity among the members of the network via unequal disc–radii, the power diagram can be utilized for the space partitioning purpose, which can be considered as the Voronoi diagram in the Laguerrean metric [10, 16].

The main issue with the previous approach lies mainly in the disc–model approximation rather than the partitioning method itself. Although almost any sensing pattern can be approximated with its maximal inscribed node–centred disc, resulting in rotation–invariant schemes, the approximation cost is in some cases extremely conservative, comparing the real and approximated coverage performances of the network.

An extension to this issue has been recently proposed by the authors [12], bypassing the standard uniform model. More specifically, a partitioning scheme (followed by the proper coordination algorithm) has been proposed, that takes into account the maximal inscribed convex set, rather than the circular one. However, the demand for common orientation of the sensors' footprints, arising from utilization of particular Helly–type theorems [14], imposes a hard constraint from an application point of view.

This article, considers the case where the nodes have non–uniform, not necessarily convex, unequally scaled sensing footprints, allowing for different pattern–orientations as well. The partitioning is performed in the network's sensed space, rather than the whole region of interest.

B. Proposed Partitioning

Let $C_i,\ i\in I_n$ be the sensing patterns of the n nodes of a network, which are assumed: a) non-uniform and non–convex (in the general case), b) same in shape, though unequally scaled, c) non–commonly oriented (in the general case). These features have been graphically summarized in Fig. 1c. Each footprint is characterized by a pinch–point, which is the node itself; its positioning is denoted as x_i, while θ_i stands for the orientation of the pattern/sensing–node.

Unlike distance–based tessellation schemes that are based on the nodes' positioning alone and uniform disc sensing models, the approach presented in this article relies solely on the footprints. Let W_i be the region of responsibility of node i, restricted in the domain of interest D. The former is defined via comparison of that node's footprint with the pattern of any other neighbouring node, as the part of the sensed space that is sensed by that node only, that is

$$W_i :\ = D\cap C_i \setminus \cup_{j\neq i} C_j,\ i\in I_n. \tag{5}$$

Apparently, the set–family $\{W_i,\ i\in I_n\}$ does not consist a complete tessellation of the sensed space, since the common parts of the sensing domains are not yet characterized. Let us denote them as W_c, that is

$$W_c :\ = \bigcup_{i\neq j} C_i\cap C_j\cap D, \tag{6}$$

which are not assigned at any node by definition.

Remark 1. *The set W_c in (6) can alternatively be defined via the sets W_i as $W_c = \cup_{i\in I_n} C_i \setminus \cup_{i\in I_n} W_i$. In a straightforward manner, the set $\{W_1, W_2, \ldots W_n, W_c\}$, consisted of mutually disjoint sets, tessellates completely the sensed space.*

Remark 2. *Unlike the cells resulting from distance–based partitioning techniques [15] or certain pattern–based ones [12], the sets defined in (5) are, in the generic case, non–compact.*

Remark 3. *In the case where a nodes's pattern is completely included in the union of the footprints of the rest of the network, that is $C_i \subset \bigcup_{j\neq i} C_j$, no region of responsibility is assigned at it ($W_i = \emptyset$), as derived directly from definition.*

In order to provide a visual aspect of the proposed partitioning scheme, fig. 2 presents visually the way that the sensed space is partitioned and assigned among the nodes via the proposed scheme. The non–coloured parts of the sensed

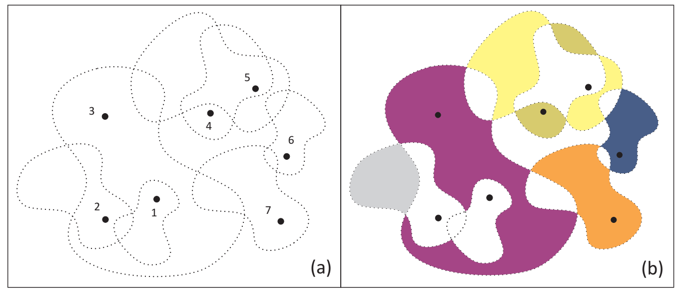

Fig. 2. Illustrative example indicating the partitioning of the sensed space via the proposed technique.

domain belong to the set W_c and correspond to the parts of the network that are sensed by more than one node at the same time. It should be noted that the sensed domain of the bottom small node (node 1) belongs to that set since it lies exclusively in the sensing region of a "larger" node (in terms of pattern–scale; purple–coloured) (Remark 3). As defined in (5), one can verify that the assigned parts are the ones that are sensed exclusively by one node, while the issue stated in Remark 2 can be examined via the nodes 4–5, indicating the (possibly) disjoint nature of the resulting cells.

Utilizing the formerly defined partitioning scheme, the network's coverage performance index $\mathscr{H}$ in (4) can be decomposed as

$$\mathscr{H} = \sum_{i\in I_n} \int_{W_i} \phi + \int_{W_c} \phi, \tag{7}$$

resulting directly from Remark 1. Note that W_i, W_c are already defined as subsets of the domain D.

IV. Distributed Motion Coordination

Assuming self–organizing robotic swarms, each mobile agent should be able to plan its motion so that it contributes to the aggregate swarm objective, while based on local information rather than requiring knowledge of the global network's state. To this end, the proposed partitioning definition presented in the previous section is utilized in order to efficiently resolve the control design problem concerning the nodes' motion.

Unlike the majority of works presented in the existing literature on the field of distributed swarm coordination, this work incorporates rotation of the sensing pattern, thus allowing an extra degree of freedom, and demanding an extended control design. The need for rotation arises from the fact of a non–radial anisotropic sensing performance assumption, providing however the ability to achieve far better optimal topological configurations.

Let us further introduce some notations at this point. For any set S with well–defined exterior, let $n(x)$ stand for the outward unit normal vector pointing towards the exterior of S for any $x \in \partial S$, where ∂S is the boundary of the set. For the sake of notations' simplicity, the former definition is extended in the case where the set S is non–compact, too, as long as the exterior of every sub–part of it is well–defined. These properties hold for both W_i and W_c, as defined in (5)–(6), presented also visually in Fig. 2. We will refer to $n(x)$, $x \in \partial W_i$ and $n(x)$, $x \in \partial W_c$ simply as n_i and n_c, respectively. At this point let us state the main result.

Theorem 1. *In a mobile sensor network governed by (2)–(3) with arbitrary non–radial nodes' sensing performance, the control law*

$$u_i = \alpha_{i,u} \int_{\partial W_i \cap \partial C_i} n_i \phi, \tag{8}$$

$$\omega_i = \alpha_{i,\omega} \int_{\partial W_i \cap \partial C_i} \mathbf{R}(90^{\mathrm{o}})(x - x_i) \cdot n_i \phi, \tag{9}$$

where $\alpha_{i,u}, \alpha_{i,\omega} > 0$, leads the network monotonically towards a locally area–optimal configuration.

Proof: We start by evaluating the time derivative of the criterion under optimization $\mathscr{H}$ as

$$\frac{d\mathscr{H}}{dt} = \sum_{i \in I_n} \left\{ \frac{\partial \mathscr{H}}{\partial x_i} \cdot \dot{x}_i + \frac{\partial \mathscr{H}}{\partial \theta_i} \dot{\theta}_i \right\}.$$

Interested in the design of gradient–based control action, that is

$$u_i = \alpha_{i,u} \frac{\partial \mathscr{H}}{\partial x_i}, \quad \omega_i = \alpha_{i,\omega} \frac{\partial \mathscr{H}}{\partial \theta_i},$$

we begin with the translational part of the proposed law, u_i, by evaluating the partial derivative of the decomposed performance index w.r.t. x_i, followed by the corresponding rotational part, ω_i.

Falling along the lines of [17, 18], utilization of the Leibniz integral rule [19] in (7) results in

$$\begin{aligned} \frac{\partial \mathscr{H}}{\partial x_i} &= \sum_{j \in I_n} \int_{\partial W_j} \frac{\partial x}{\partial x_i} n_j \phi + \int_{\partial W_c} \frac{\partial x}{\partial x_i} n_c \phi \\ &= \sum_{j \neq i} \int_{\partial W_j} \frac{\partial x}{\partial x_i} n_j \phi + \int_{\partial W_i} \frac{\partial x}{\partial x_i} n_i \phi + \int_{\partial W_c} \frac{\partial x}{\partial x_i} n_c \phi. \end{aligned}$$

Considering the second integral, the boundary ∂W_i can be decomposed into a) parts that lay on the border of the area of interest, $\partial W_i \cap \partial D$, b) parts that are shared by ∂W_c, $\partial W_i \cap \partial W_c$, and c) parts that orient towards the uncovered space, $\partial W_i \cap \partial C_i$. This decomposition is depicted graphically in Fig. 3 [upper part].

However, at $x \in \partial D$ it holds that $\frac{\partial x}{\partial x_i} = \mathbf{0}$, since we assume a static surveillance domain. In addition, since at the common boundaries it holds that $\frac{\partial x}{\partial x_i} n_j = -\frac{\partial x}{\partial x_i} n_c, j \in I_n$ (see Fig. 3 [bottom part]), the former expression simplifies into

$$\frac{\partial \mathscr{H}}{\partial x_i} = \int_{\partial W_i \cap \partial C_i} \frac{\partial x}{\partial x_i} n_i \phi. \tag{10}$$

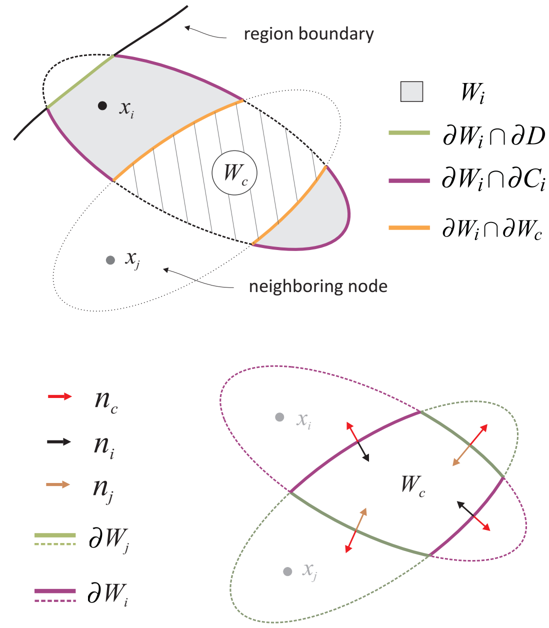

Fig. 3. Decomposition of ∂W_i into mutually disjoint sets.

Exactly the same concept is repeated for the rotational part, by evaluating the partial derivative of $\mathscr{H}$ w.r.t. θ_i as

$$\frac{\partial \mathscr{H}}{\partial \theta_i} = \int_{\partial W_i \cap \partial C_i} \frac{\partial x}{\partial \theta_i} \cdot n_i \phi. \tag{11}$$

What remains is the evaluation of the partial derivatives $\frac{\partial x}{\partial x_i}, \frac{\partial x}{\partial \theta_i}$ at $x \in \partial W_i \cap \partial C_i$. However, each point lying on the boundary of the a node's sensing domain can be expressed as

$$x = x_i + \rho(x) \begin{bmatrix} \cos(\xi(x) + \theta_i) \\ \sin(\xi(x) + \theta_i) \end{bmatrix}, \tag{12}$$

where it is evident that the parameters ρ, ξ for each x are not dependent on the configuration of the sensor, i.e. x_i, θ_i, but exclusively on the pattern itself.

Consequently, $\frac{\partial x}{\partial x_i} = \mathbb{I}_2$ as the identity matrix, resulting directly from (12), simplifying the control action via (10) into (8). Similarly, utilizing the rotation matrix notation and substituting back the trigonometric terms via (12) results in

$$\frac{\partial x}{\partial \theta_i} = \rho(x) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \cos(\xi(x) + \theta_i) \\ \sin(\xi(x) + \theta_i) \end{bmatrix} = \mathbf{R}(90^{\mathrm{o}})(x - x_i),$$

leading the gradient–based rotational control into (9).

Updating the time derivative of the sensed area via the proposed control law results in

$$\frac{d\mathscr{H}}{dt} = \sum_{i \in I_n} \left\{ \alpha_{i,u} \left\| \frac{\partial \mathscr{H}}{\partial x_i} \right\|^2 + \alpha_{i,\omega} \left| \frac{\partial \mathscr{H}}{\partial \theta_i} \right|^2 \right\} \geq 0,$$

for $\alpha_{i,u}, \alpha_{i,\omega} > 0$, guaranteeing monotonic convergence of the network towards the optimum state. This completes the proof. ∎

Remark 4. *The proposed control scheme degenerates into the one presented by the authors in [18], when the sensing domains are assumed circular node–centred ones, providing* *so an extension for arbitrarily non–radial patterns. Evidently, in the former case, due to the rotation–invariance of the disc–model, the rotational part of the control action is not taken into account.*

It should be noted that the proposed control law does not require global knowledge of the network's state, but instead only by those within an a–priori bounded range equal to $\sup\left\{\left\|x_i - x_j\right\|: \ \left|\partial C_i \cap \partial C_j\right| = 1\right\}$.

V. Simulation Studies

Numerical results derived from simulations studies conducted are presented in this section. The domain under surveillance D is identical to the one used in [17] for consistency purposes. The proposed control law (8)–(9) based on the partitioning scheme (5) presented in section III is compared for evaluation purposes with the control proposed in [17], appropriate for disc–modelled networks.

Specifically, the proposed control action was

$$u_i = \alpha_i \int_{\partial V_i^r \cap \partial B_i} n_i \phi, \ \alpha_i > 0 \tag{13}$$

where $B_i := \{x \colon \|x - x_i\| \leq r\}$ is the disc–modelled sensing set of a node and V_i^r corresponds to its range–limited Voronoi cell [15], while has been proven that it leads to area–optimal configurations for uniform homogeneous networks. Extensions for the heterogeneous case can also be found in [18, 20, 21].

For numerical evaluations, a network of 8 nodes is examined. Their sensing footprints are assumed ellipsoidal ones with their axes–parameters set as $a = 0.5$ units, $b = 0.3$ units, while the relative position of the node to the footprint is set at half–way along the long axis. All nodes are initially randomly placed in a small region of the interior of D, with random patterns' orientations.

Initially, the control scheme (13) is applied in order to control the nodes' motion. The ellipsoidal model is approximated as the maximal inscribed node–centred disc, while standard Voronoi partitioning is utilized during the coordination stage in order to perform region–assignment and evaluate the control action. The nodes' initial configuration, their paths followed during transition, along with the final network's configuration are presented from left to right in the top row of Fig. 4, in this order.

Judging from the end–configuration one can observe at first that the nodes have indeed positioned themselves in an area–optimal topology, considering the disc models, as expected. Apparently, due to the rotation–invariance of the disc model, there is no control to amend for orientation–regulation of the original footprints. Examining, though, the real network's performance evaluated via the original ellipsoidal patterns, it is evident that the network can achieve far better sensing coverage ratio of the space under consideration. More specifically, the network starting from an initial coverage percentage of D equal to 35.26% converges to 52.05%, as shown with the blue line in Fig. 5, which is the optimal network's performance if someone takes into account only the approximated discs. It should be stated clearly that this deficiency is due to the disc–approximation of the nodes' performance rather than the control law itself, while comparative studies are performed mainly for evaluation purposes.

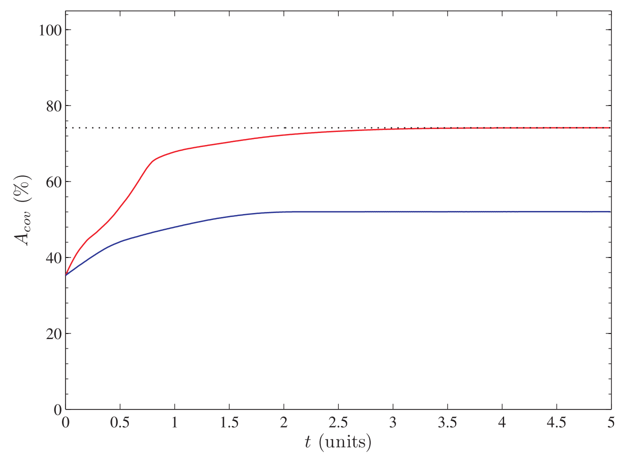

Fig. 5. Evolution of covered area percentage w.r.t. time evaluated via the non–uniform original patterns. Blue (Red) line corresponds to top (bottom) row of Fig. 4.

On the other hand, utilizing the proposed control scheme for the nodes' motion coordination, that takes into account both the patterns' anisotropy and the need for their orientation regulation, produces the results depicted in the bottom row of Fig. 4. It is apparent that the nodes have been organized in a way that optimum coverage is attained, equal to 74.16% (red line in Fig. 5), while their orientation is controlled properly via (9), throughout the whole evolution.

It should be noted that the proposed scheme has increased complexity in its evaluation when dealing with arbitrary non–convex sensing domains, especially when compared with approaches that are based on disc models. Particularly, the demand arises from the difficulty in parametrizing complex boundary curves, when compared to evaluations that are based on circle–arcs. In fact, even if numerical approaches are utilized in order to avoid complex curves' descriptions, extra computational demand is needed at evaluating the corresponding boundary–parts appearing in the proposed control law. However, the overall network's performance in our case is by far superior to that of disc–approximating techniques, arising so a trade–off to balance: complexity vs. performance.

VI. Conclusions

Coordination of a group of mobile agents with heterogeneous arbitrary sensory for optimum area deployment is examined in this article. The network is consisted of nodes whose sensing regions differ in regard to scaling and orientation, highly differentiating from standard circular–case assumptions appearing in the existing literature. Efficient partitioning of the sensed domain is proposed in order for the nodes to distributively evaluate the corresponding parts that should determine their motion for coverage–increase. The presented scheme guarantees monotonic increase of the area sensed by the network while accounts not only for the nodes' positioning,

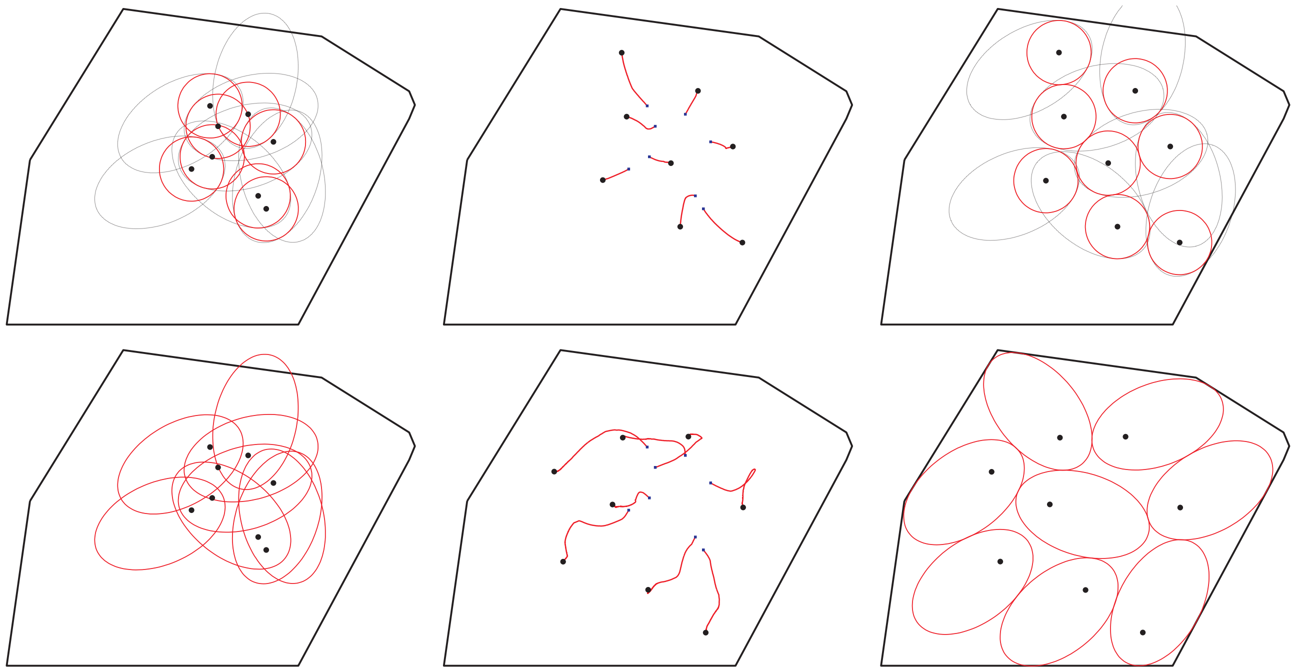

Fig. 4. Coordination results derived via control schemes (13) [top row] and (8)–(9) [bottom row], respectively. [Left column] Initial network configuration. [Middle column] Network evolution through time. The black circles (blue squares) represent the nodes' final (initial) positions. [Right column] Final network state.

but sensor orientation as well. Numerical results presented indicate the efficiency of the control scheme over standard circular–approximation ones.

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