2014 IEEE International Conference on Robotics & Automation (ICRA)
Hong Kong Convention and Exhibition Center
May 31 - June 7, 2014. Hong Kong, China

Set-Point Control of a Musculoskeletal Arm by the Complementary Combination of a Feedforward and Feedback Manner

Yuki Matsutani[1], Kenji Tahara[2], Hitoshi Kino[3], Hiroaki Ochi[3] and Motoji Yamamoto[2]

***Abstract*— This paper proposes a novel set-point control method of a musculoskeletal system by combining a feedforward and feedback manner to complement each drawback each other. In our previous work, a feedforward positioning method of the musculoskeletal arm model was proposed which does not need any realtime sensory information. Its performance, however, depends on a muscular arrangement and an attitude of the arm, and thereby a large initial muscular internal force is necessary to make a good performance. On the other hand, it is well-known that a visual servoing is effective and versatile for the set-point control. However, there is a considerable time-delay due to a computational burden to acquire useful information from an image and an insufficient sampling period to capture each image when using a video frame rate camera. Thus in this paper, the feedforward and feedback signal are linearly combined into one in order to mutually complement each drawback. The combined control signal is newly designed and then numerical simulation results are shown to demonstrate the effectiveness and usefulness of the proposed method.**

I. Introduction

There is no doubt that human movements are still much smoother and more sophisticated than those of present robotic systems. It is said that these sophisticated movements are conducted by the appropriate combination of two main control strategies, the feedforward control using proprioceptive information and the feedback control using a lot of sensory information such as visual or tactile information. It is known that the sensory feedback control loop of a human body contains a considerable time-delay. In particular, the visual feedback loop using eyes has more than 100 ms delay from just a sensing moment to its response movement. Therefore, it is hard to accomplish a fast movement conducted within shorter than dozens of milli seconds by only using the sensory feedback control manner. Meanwhile, making only use of the feedforward control manner must be difficult to perform an accurate and robust positioning in unknown environments. Namely, both the feedforward and feedback control manners play each crucial role in the strategy of a human body movement [1].

Up to now, many related works have been done not only in robotics, but also in physiological field. These have proposed several control strategies by combining a feedforward with a feedback control manner. For instance, Arimoto [2] proposed an iterative learning control scheme that does not need any dynamical model in advance, and Slotine [3] proposed a model based adaptive control scheme that is able to compensate the model error in real-time. Also Blana *et al.* [4] proposed a control scheme based on an inverse dynamics model, and Katayama *et al.* [5] proposed a parallel-hierarchical neural network model expanding the feedback-error-learning scheme proposed by Kawato [1]. Blana's and Katayama's methods can accomplish a highly accurate trajectory tracking. There are, however, still several drawbacks in these previous methods that both the iterative learning control scheme and Katayama's method require a number of trials to obtain a desired result, and the model based adaptive control scheme and Blana's method need an accurate dynamical model in advance, and they all have only applied to a trajectory tracking task. We also have proposed another feedforward positioning method in which the particular structure of the musculoskeletal system is employed effectively [6], [7]. In this method, an arbitrary constant muscular internal force, which is able to balance at a desired position, is applied to the system as a feedforward input. The arbitrary constant muscular internal force makes a kind of potential field, and the system is able to converge on the desired position according to the potential field if it owns a unique equilibrium at the desired position. The most remarkable advantage of this method is that no sensing information is necessary to compose the controller. However, its control performance in terms of response and trajectory strongly depends on the shape of the potential field. Moreover, this field depends not only on an initial and desired position, but also on the magnitude of the constant muscular internal force which can be chosen arbitrarily, and its muscular arrangement which is hard to change once fixed. Thus, the muscular internal force must be chosen as large to make a good performance wherever an initial and desired position are.

This paper proposes a novel set-point control method for the musculoskeletal system which combines the feedforward control method with a visual feedback control method including a considerable time-delay. This method allows the musculoskeletal system to make a fast movement with much smaller muscular internal force than the feedforward control in our previous study, and also to make a robust positioning against a considerable time-delay in the visual feedback. The visual feedback control method is quite effective to regulate the end-effector to the desired position accurately.

[1]Y. Matsutani is with the Graduate School of Engineering, Kyushu University, 744 Moto'oka, Nishi-ku, Fukuoka 819-0395, Japan matsutani@hcr.mech.kyushu-u.ac.jp
[2]K. Tahara and M. Yamamoto are with the Faculty of Engineering, Kyushu University, 744 Moto'oka, Nishi-ku, Fukuoka 819-0395, Japan tahara@ieee.org, yama@mech.kyushu-u.ac.jp
[3]H. Kino and H. Ochi are with the Department of Intelligent Mechanical Engineering, Faculty of Engineering, Fukuoka Institute of Technology, 3-30-1 Wajiro-higashi, Higashi-ku, Fukuoka 811-0295, Japan kino@fit.ac.jp, ad13001@bene.fit.ac.jp

978-1-4799-3684-7/14/$31.00 ©2014 IEEE

Meanwhile, it needs to take into account a considerable time-delay due to a computational cost to extract meaningful information from a raw image data, and an inadequate sampling period to capture an image when using a commonly utilized video frame rate camera. Accordingly, it is hard to choose any feedback gain to be large enough because it induces unstable phenomena. This fact leads to an insufficient performance in terms of agility and accuracy of the set-point control. The visual feedback control using a high speed camera has been being studied [8]; the high speed camera is still not common, and we can perform a fast motion by the combination of the feedforward and feedback control manner although we do not possess such a high speed visual sensor. Namely, this combination makes it possible to overcome these drawbacks so that the feedforward manner plays a role in suppressing an adverse effect of the time-delay in the visual feedback signal, and the feedback manner plays a role in reducing the necessary muscular internal force with enough performances.

In what follows, the kinematics of the two-link six-muscle musculoskeletal planar arm model used in this study is shown in Section II. The new positioning method, which combines the feedforward control signal with the visual feedback control signal including a considerable time-delay, is designed in Section III. Finally, numerical simulation results are shown in Section V to demonstrate the effectiveness of the new positioning method. Additionally, the stability of the overall system with the considerable time-delay using Lyapunov-Krasovskii method [9], [10] is given briefly in Appendix.

II. Musculoskeletal System

Figure 1 shows a kinematic model of a musculoskeletal arm model used in this study. This model is composed of two joints and six muscles. Each joint is driven by related muscles which can generate only a tensile force. In order to facilitate the analysis, the muscles are assumed to be a massless line, are ignored muscular viscoelasticity, and are attached to a corresponding link directly. It means that a moment arm of each muscle is changed depending on the joint angle. Besides, each joint torque is also changed depending on the joint angles even if the muscular tensile force is constant. In this model, offset parameters in the muscular arrangement b_j and d_j $(j = 1, \ \cdots, \ 4)$ are important to satisfy the stability condition of the feedforward positioning [6]. Its movement is limited within the horizontal plane (xy-plane) and thereby the gravity effect can be ignored. In addition, assume that the visual sensor used in the system is a commonly used video frame rate camera.

A. Kinematics between the Task Space and the Joint Space

The relation between the end-point position vector $\boldsymbol{x} = [x, \ y]^{\mathrm{T}} \in \mathbb{R}^2$ of the musculoskeletal arm model and the joint angle vector $\boldsymbol{\theta} = [\theta_1, \ \theta_2]^{\mathrm{T}} \in \mathbb{R}^2$ can be expressed as follows:

$$\boldsymbol{x} = \begin{bmatrix} L_1 C_1 + L_2 C_{12} \\ L_1 S_1 + L_2 S_{12} \end{bmatrix}, \tag{1}$$

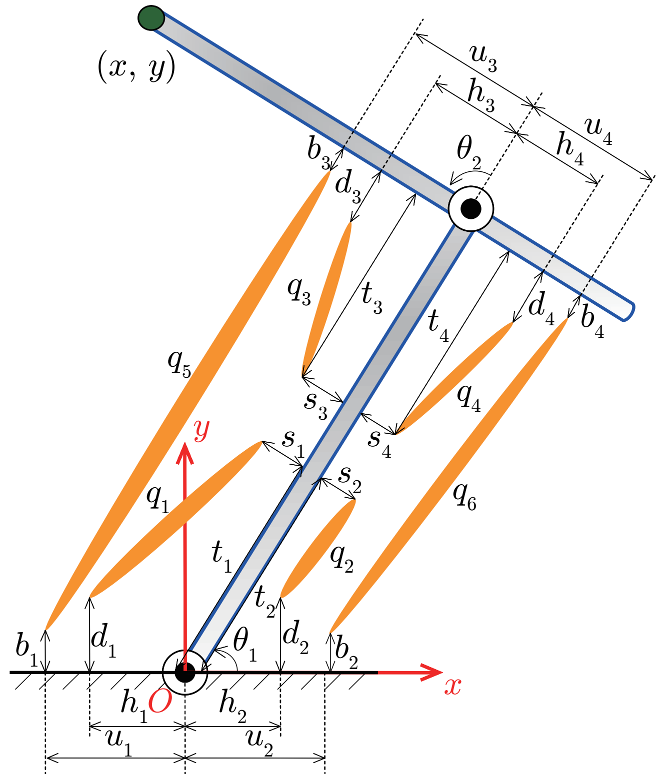


Fig. 1. Two-link six-muscle musculoskeletal planar arm model

where $S_k = \sin\theta_k$, $C_k = \cos\theta_k$ $(k = 1, 2)$, $S_{12} = \sin(\theta_1 + \theta_2)$ and $C_{12} = \cos(\theta_1 + \theta_2)$, and L_1, L_2 are the 1st link length and the 2nd link length respectively. Taking a time derivative of (1) yields:

$$\dot{\boldsymbol{x}} = \boldsymbol{J}\dot{\boldsymbol{\theta}}, \tag{2}$$

where $\dot{\boldsymbol{x}} \in \mathbb{R}^2$ is the end-point velocity vector, $\dot{\boldsymbol{\theta}} \in \mathbb{R}^2$ is the joint angular velocity vector and $\boldsymbol{J} \in \mathbb{R}^{2\times 2}$ is the Jacobian matrix for the end-point velocity with respect to the joint angular velocity. In addition, by means of the principle of virtual work, the relation between the joint torque vector $\boldsymbol{\tau} \in \mathbb{R}^2$ and the force vector $\boldsymbol{f} \in \mathbb{R}^2$ of the end-point can be given as follows:

$$\boldsymbol{\tau} = \boldsymbol{J}^{\mathrm{T}}\boldsymbol{f}. \tag{3}$$

B. Kinematics between the Muscle Space and the Joint Space

The relation between the muscle length vector $\boldsymbol{q} \in \mathbb{R}^6$ and the joint angle vector $\boldsymbol{\theta} = [\theta_1, \ \theta_2]^{\mathrm{T}} \in \mathbb{R}^2$ is expressed as follows:

$$\boldsymbol{q} = \left[q_1,\ q_2,\ q_3,\ q_4,\ q_5,\ q_6\right]^{\mathrm{T}}, \tag{4}$$

$$\left[\begin{aligned}
q_1 &= \sqrt{(h_1 + t_1 C_1 - s_1 S_1)^2 + (d_1 - t_1 S_1 - s_1 C_1)^2} \\
q_2 &= \sqrt{(h_2 - t_2 C_1 - s_2 S_1)^2 + (d_2 - t_2 S_1 + s_2 C_1)^2} \\
q_3 &= \sqrt{(h_3 + t_3 C_2 - s_3 S_2)^2 + (d_3 - t_3 S_2 - s_3 C_2)^2} \\
q_4 &= \sqrt{(h_4 - t_4 C_2 - s_4 S_2)^2 + (d_4 - t_4 S_2 + s_4 C_2)^2} \\
q_5 &= \Big\{ (u_1 + u_3 C_{12} - b_3 S_{12} - L_1 C_1)^2 \\
&\qquad + (b_1 - u_3 S_{12} - b_3 C_{12} - L_1 S_1)^2 \Big\}^{1/2} \\
q_6 &= \Big\{ (u_2 + u_4 C_{12} + b_4 S_{12} - L_1 C_1)^2 \\
&\qquad + (b_2 + u_4 S_{12} - b_4 C_{12} - L_1 S_1)^2 \Big\}^{1/2},
\end{aligned}\right.$$

where, note that $h_j,\ d_j,\ s_j,\ t_j,\ u_j$ and b_j $(j = 1,\ \cdots,\ 4)$ are defined in the muscular arrangement as shown in Fig. 1. Taking a time derivative of (4) yields:

$$\dot{\boldsymbol{q}} = -\boldsymbol{W}^{\mathrm{T}} \dot{\boldsymbol{\theta}}, \tag{5}$$

where, $\dot{\boldsymbol{q}} \in \mathbb{R}^6$ is the muscle contractile velocity vector, and $\boldsymbol{W}^{\mathrm{T}} \in \mathbb{R}^{6\times 2}$ denotes the Jacobian matrix for the muscle contractile velocity with respect to the joint angular velocity. Additionally, the relation between the joint torque vector $\boldsymbol{\tau} \in \mathbb{R}^2$ and the muscular tensile force vector $\boldsymbol{\alpha} \in \mathbb{R}^6$ can be expressed as follows using the principle of virtual work.

$$\boldsymbol{\tau} = \boldsymbol{W}\boldsymbol{\alpha}. \tag{6}$$

Taking an inverse relation of (6), it follows that the muscular tensile force vector $\boldsymbol{\alpha}$ can be expressed as follows:

$$\boldsymbol{\alpha} = \boldsymbol{W}^{+}\boldsymbol{\tau} + \boldsymbol{v}, \tag{7}$$

where, $\boldsymbol{W}^{+} \in \mathbb{R}^{6\times 2}$ signifies the pseudo-inverse matrix defined as $\boldsymbol{W}^{+} = \boldsymbol{W}^{\mathrm{T}}(\boldsymbol{W}\boldsymbol{W}^{\mathrm{T}})^{-1}$. The vector $\boldsymbol{v} \in \mathbb{R}^6$ lies on the null-space of the matrix $\boldsymbol{W}$ and its physical meaning is an internal force which does not generate any joint torque. It is given as follows:

$$\boldsymbol{v} = \left(\boldsymbol{I}_6 - \boldsymbol{W}^{+}\boldsymbol{W}\right)\boldsymbol{k_e}, \tag{8}$$

where, $\boldsymbol{I}_6 \in \mathbb{R}^{6\times 6}$ is the identity matrix, $\boldsymbol{k_e} \in \mathbb{R}^6$ is an arbitrary vector and assume that it can be chosen so as to satisfy $\boldsymbol{v} > 0$ in this study. Furthermore, substituting (3) into (7), the relation between the muscular tensile force vector $\boldsymbol{\alpha}$ and the end-point force vector $\boldsymbol{f}$ can be expressed as follows:

$$\boldsymbol{\alpha} = \hat{\boldsymbol{J}}\boldsymbol{f} + \boldsymbol{v}, \tag{9}$$

where,

$$\hat{\boldsymbol{J}} := \boldsymbol{W}^{+}\boldsymbol{J}^{\mathrm{T}}. \tag{10}$$

Namely, the matrix $\hat{\boldsymbol{J}} \in \mathbb{R}^{6\times 2}$ plays a role as a transformation matrix from the task space to the muscle space. Assume that the matrix $\hat{\boldsymbol{J}}$ is of full rank during movement in this study.

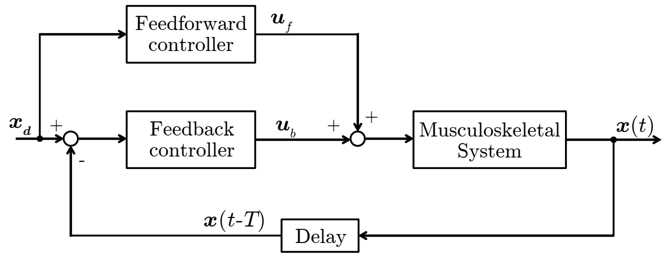


Fig. 2. Block diagram of the overall system including the proposed controller

III. Control Design

In this section, a new control signal is designed which linearly combines the feedforward control manner for the musculoskeletal system with a visual feedback control manner including a considerable time-delay. Figure 2 shows a block diagram of the overall system. The new control signal $\boldsymbol{\alpha}$ proposed here is given by the linear combination of the feedforward and feedback signals as follows:

$$\boldsymbol{\alpha} = \boldsymbol{u_b} + \boldsymbol{u_f} \tag{11}$$

where $\boldsymbol{u_b} \in \mathbb{R}^6$ and $\boldsymbol{u_f} \in \mathbb{R}^6$ are the feedback and the feedforward input vector, respectively. It is known that the muscle is able to generate only a tensile force, that is, the muscle cannot output a push force no matter how the control signal $\boldsymbol{\alpha}$ demands any push force for the muscle. In order for the tensile force to be a positive value, the control signal must satisfy $\alpha_i > 0$ $(i = 1,\ \cdots,\ 6)$ during movement. Each element of the feedforward input vector is given as a positive value which will be shown in the following section. Thanks to the feedforward input large enough, the control signal α_i is able to keep $\alpha_i > 0$ during movement.

A. Feedforward Control Part

The feedforward input $\boldsymbol{u_f}$ is given as follows [6]:

$$\boldsymbol{u_f} = \boldsymbol{v}_d = \left(\boldsymbol{I}_6 - \boldsymbol{W}_d^{+}\boldsymbol{W}_d\right)\boldsymbol{k_e}, \tag{12}$$

where $\boldsymbol{W}_d \in \mathbb{R}^{2\times 6}$ is the constant Jacobian matrix from the muscle space to the joint space at a desired position. It is remarkable to note that the feedforward control input makes the constant internal force $\boldsymbol{v}_d \in \mathbb{R}^6$ which balances at the desired position. Also note that the feedforward control signal has nothing to do with any time-delay in sensing because of no sensory information. In this paper, an arbitrary vector of the feedforward input $\boldsymbol{k_e}$ as shown in (12) is defined as follows:

$$\boldsymbol{k_e} = \gamma\left[1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0\right]^{\mathrm{T}}, \tag{13}$$

where, $\gamma > 0$ signifies a feedforward gain. The given constant desired internal force balancing at the desired position generates a potential field, and the overall system is possible to converge on the desired state according to the shape of the potential field if it has a unique stable equilibrium at the desired state from the quasi-static viewpoint [6]. The potential energy P is a nonlinear function depending on a muscular arrangement and a given muscular internal force.

TABLE I
PHYSICAL PARAMETERS OF THE MUSCULOSKELETAL ARM MODEL

Mass	(m_1, m_2)=(1.68, 0.950) [kg]
Length	(L_1, L_2)=(0.315, 0.234) [m]
Inertia moment	(I_1, I_2)=(0.011, 0.004) [kgm^2]
Joint viscosity	(D_1, D_2)=(1.00, 1.00) [Nms/rad]

TABLE II
MUSCULAR ARRANGEMENT

j	1	2	3	4
h_j [mm]	20	20	20	20
d_j [mm]	2	2	2	2
s_j [mm]	4	4	4	4
t_j [mm]	157	157	157	157
u_j [mm]	28	28	28	28
b_j [mm]	3	3	3	3

It needs further analysis in order to demonstrate whether the potential possesses a unique equilibrium at the desired position. Since it has been analyzing our another ongoing study [7] and is not a main issue of this paper, we assume that the system generates a potential which possesses a unique equilibrium at the desired position. Namely, the potential P satisfies the following equations in this study.

$$P\left(\boldsymbol{q}, \boldsymbol{q}_d, \boldsymbol{v}_d\right) \geq 0 \tag{14}$$

$$\dot{P}\left(\boldsymbol{q}, \boldsymbol{q}_d, \boldsymbol{v}_d\right) \leq 0, \tag{15}$$

$$\dot{P} = 0 \Longleftrightarrow \{\boldsymbol{q} = \boldsymbol{q}_d,\ \dot{\boldsymbol{q}} = \boldsymbol{0}\}$$

where, $\boldsymbol{q}_d \in \mathbb{R}^6$ is the vector of a desired length of the muscle.

B. Visual Feedback Control Part

The visual feedback control signal $\boldsymbol{u_b}$ is given here. It is a traditional task space PD feedback control method except including a considerable time-delay. Assume that the system treated here does not have any sensor other than a single video frame rate camera. Namely, any information used in this study is based on the end-point position $\boldsymbol{x}(t)$ including the time-delay T obtained by the camera. The visual feedback control signal including the time-delay T is given as follows:

$$\boldsymbol{u_b} = -\hat{\boldsymbol{J}}(t-T)\left\{\boldsymbol{K_p}\boldsymbol{\Delta x}(t-T) + \boldsymbol{K_v}\dot{\boldsymbol{x}}(t-T)\right\}, \tag{16}$$

where, $\boldsymbol{K_p} \in \mathbb{R}^{2\times 2} > \boldsymbol{0}$ and $\boldsymbol{K_v} \in \mathbb{R}^{2\times 2} > \boldsymbol{0}$ are the gain matrices which are positive-definite and diagonal, $\boldsymbol{\Delta x} = \boldsymbol{x} - \boldsymbol{x}_d \in \mathbb{R}^2$ is the end-point position error vector, $\boldsymbol{x}_d \in \mathbb{R}^2$ is a desired position vector.

The total control signal is eventually given by the summation of (12) and (16). The stability of the overall system is given briefly in the Appendix.

IV. NUMERICAL SIMULATIONS

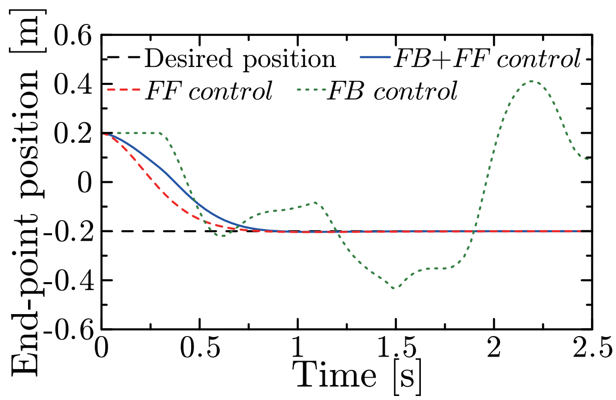
Several numerical simulation results are shown to demonstrate the effectiveness of the proposed method. Physical parameters of the musculoskeletal arm model, muscular arrangement parameters and the initial and desired positions of the end-point used in the simulations are shown in Tables I $\sim$ III. The time-delay of the visual information in the simulations is set to be $T = 300$ [ms] which is almost similar to humans' worst case. Three types of controllers are compared in the simulations. The first one is the proposed combined controller indicated as *FB+FF control*, the second one is the feedforward controller for the musculoskeletal system indicated as *FF control*, and the final one is the visual feedback controller indicated as *FB control*. All gains of each controller are chosen as optimum values under the condition of the simulation time $t_f = 2.5$ [s].

TABLE III
INITIAL AND DESIRED POSITION OF THE END-POINT

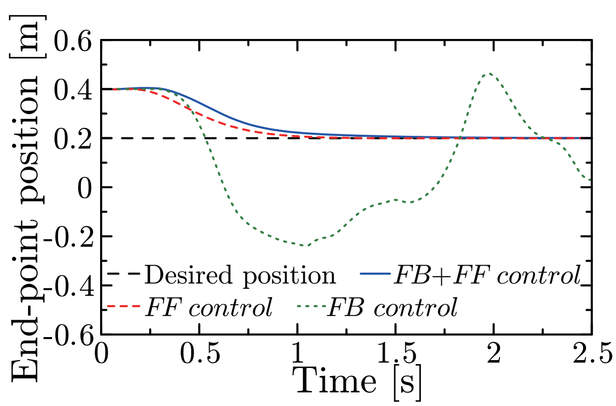
Initial position [m]	$\boldsymbol{x}_0 = [0.2,\ 0.4]^{\mathrm{T}}$
Desired position [m]	$\boldsymbol{x}_d = [-0.2,\ 0.2]^{\mathrm{T}}$

TABLE IV
FEEDBACK AND FEEDFORWARD GAINS

	$\boldsymbol{K_p}$	$\boldsymbol{K_v}$	γ
FB+FF control	$12\boldsymbol{I}_2$	$2.5\boldsymbol{I}_2$	200
FF control	-	-	350
FB control	$80\boldsymbol{I}_2$	$10\boldsymbol{I}_2$	-



(a) x-component



(b) y-component

Fig. 3. Transient responses of the end-point position

TABLE V
EACH GAIN FOR THE SIMULATION WITH HIGH FEEDBACK GAINS

	$\boldsymbol{K_p}$	$\boldsymbol{K_v}$	γ
FB+FF control	$100\boldsymbol{I}_2$	$10\boldsymbol{I}_2$	700
FB control	$100\boldsymbol{I}_2$	$10\boldsymbol{I}_2$	-

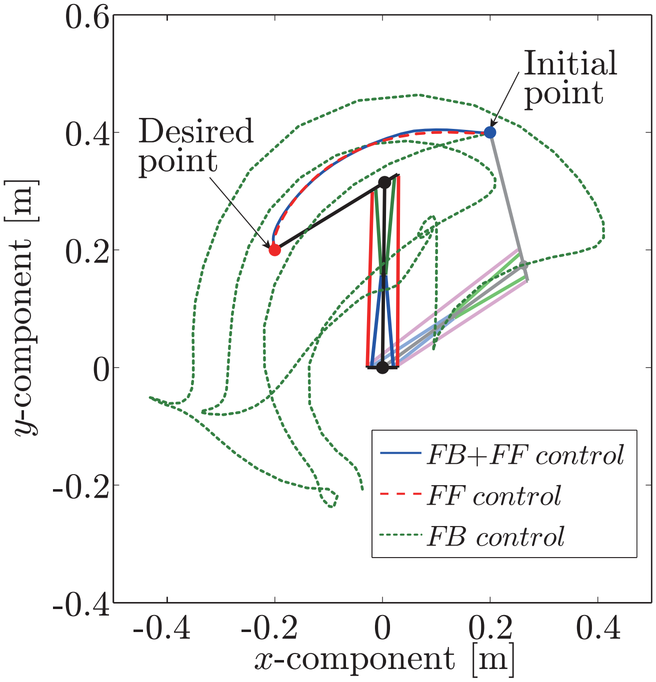
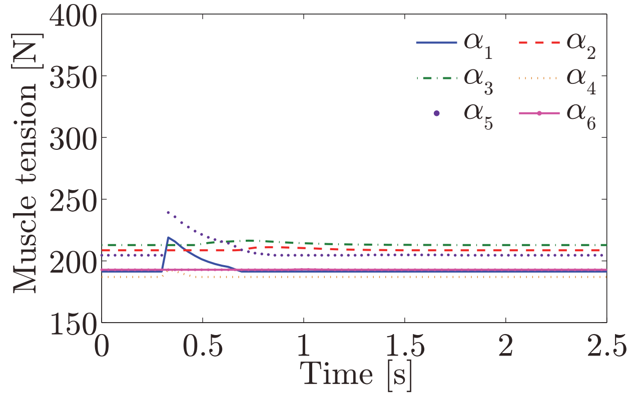
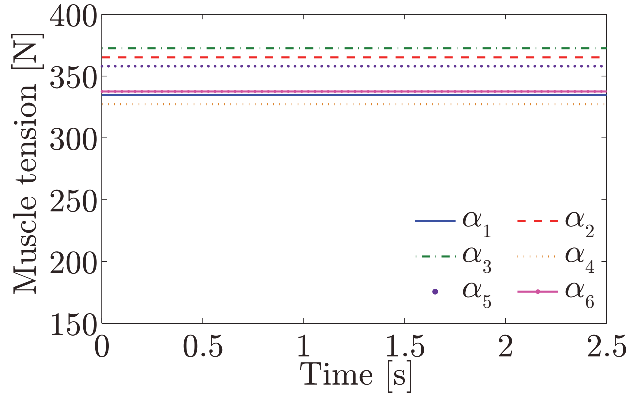


Fig. 4. Loci of the end-point position on xy-plane



(a) *FB+FF control*



(b) *FF control*

Fig. 5. Comparison of the transient responses of the muscular tensile forces; (a) In the case of using *FB+FF control*, (b) In the case of only using *FB control*

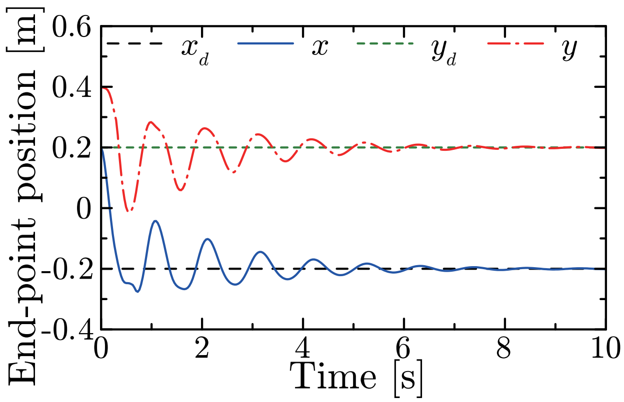
Each simulation result is shown in Figs. 3 $\sim$ 5. Figure 3 shows the transient responses of each x and y component of the end-point position, and Fig. 4 shows the loci of the end-point position in xy-plane. It can be seen from these figures that the system is obviously getting unstable because of the time-delay when using *FB control*. Of course by choosing the feedback gains smaller, the end-point position is able to converge on the desired position using *FB control*, but in that case, it makes very low performance in terms of response. In contrast, the end-point position converges on the desired position within at least 1.5 [s] when using *FB+FF control* or *FF control*. Additionally, it can be seen from Fig. 4 that both resulted end-point trajectories in xy-plane are quite similar, and for that matter, *FF control* is slightly better than *FB+FF control* in terms of response performance because *FF control* has nothing to do with the time-delay. However, Fig. 5 showing a comparison of the transient responses of each muscular tensile force α_i, $(i = 1, \cdots, 6)$ indicates that the necessary muscular tensile force in *FF control* is almost twice larger than that in *FB+FF control*. Namely, *FB+FF control* is able to make a good performance as well as *FF control* though its necessary muscular tensile force is almost a half of *FF control*'s one.

We also perform other simulations in order to demonstrate the effectiveness of the feedforward control input that plays a role in suppressing unstable phenomena when using high feedback gains. Each gain used in the simulations is shown in Table V, and the results are shown in Fig. 6. It can be seen from the figure that the end-point trajectory when using only *FB control* with high feedback gains is obviously getting unstable because of the large feedback gains. In contrast, the end-point eventually converges on the desired position when using *FF+FB control* with high feedback gains though there still remains an oscillating component. This fact obviously indicates that *FF control* plays a role in suppressing unstable phenomena attributed to the large time-delay and high feedback gains in *FB control*.

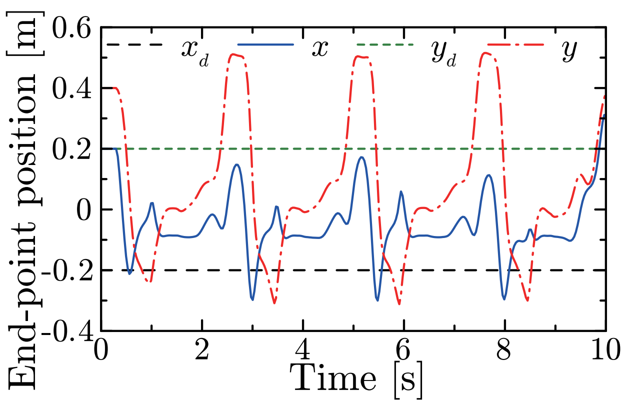
Through all the simulation results, we can conclude that the proposed method is able to perform a stable and fast reaching movement using less muscular forces even though there exists the large time-delay.

V. Conclusions

In this paper, the set-point control method of the musculoskeletal system was newly proposed which combines the visual feedback control with the considerable time-delay and the feedforward control using the desired muscular internal force. It was shown through numerical simulations that the



(a) *FB+FF control*



(b) *FB control*

Fig. 6. Comparison of the end-point trajectories with high feedback gains; (a) In the case of using *FB+FF control*, (b) In the case of only using *FB control*

proposed method is robust against the large time-delay, and quite efficient in terms of a low necessary muscular tensile force.

In our future works, an asymptotic stability of the overall system has to be given, and the combination of the feedback and feedforward manner would be optimized during movement for the improvement of its performance, stability, and robustness. Also its practical usefulness would be demonstrated through experiments.

APPENDIX

The dynamic stability of the overall system including the time-delay is discussed here by using Lyapunov-Krasovskii method [9], [10]. The dynamics of the musculoskeletal arm model in the joint space is given as follows:

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})\dot{\boldsymbol{\theta}} + \boldsymbol{D}\dot{\boldsymbol{\theta}} = \boldsymbol{\tau}, \tag{17}$$

where, $\boldsymbol{M}(\boldsymbol{\theta}) \in \mathbb{R}^{2\times 2}$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) \in \mathbb{R}^{2\times 2}$ is the non-linear matrix including Coriolis and centrifugal forces, $\boldsymbol{D} \in \mathbb{R}^{2\times 2}$ is the joint viscosity matrix, $\ddot{\boldsymbol{\theta}} \in \mathbb{R}^2$ is the joint angular acceleration vector. On the other hand, taking the time derivative of (2) yields:

$$\ddot{\boldsymbol{x}} = \boldsymbol{J}\ddot{\boldsymbol{\theta}} + \dot{\boldsymbol{J}}\dot{\boldsymbol{\theta}}, \tag{18}$$

where, $\ddot{\boldsymbol{x}} \in \mathbb{R}^2$ is the end-point acceleration vector. Substituting (2), (3), (6) and (18) into (17) yields:

$$\boldsymbol{M_x}\ddot{\boldsymbol{x}} + \boldsymbol{C_x}\dot{\boldsymbol{x}} + \boldsymbol{D_x}\dot{\boldsymbol{x}} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{W}\boldsymbol{\alpha}, \tag{19}$$

$$\left[\begin{array}{l} \boldsymbol{M_x} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{J}^{-1} \in \mathbb{R}^{2\times 2} \\ \boldsymbol{C_x} = \boldsymbol{J}^{-\mathrm{T}}\{\boldsymbol{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) - \boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{J}^{-1}\dot{\boldsymbol{J}}\}\boldsymbol{J}^{-1} \in \mathbb{R}^{2\times 2} \\ \boldsymbol{D_x} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{D}\boldsymbol{J}^{-1} \in \mathbb{R}^{2\times 2}, \end{array}\right.$$

where, we assume that $\boldsymbol{J}$ is of full-rank during movement, and $\boldsymbol{J}^{-\mathrm{T}}$ denotes the inverse matrix of $\boldsymbol{J}^{\mathrm{T}}$. Equation (19) stands for the dynamics of the musculoskeletal arm model in the task space. By substituting (11), (12), and (16) into (19), the overall dynamics considering the time-delay is given as follows:

$$\begin{aligned} &\boldsymbol{M_x}\ddot{\boldsymbol{x}} + \boldsymbol{C_x}\dot{\boldsymbol{x}} + \boldsymbol{D_x}\dot{\boldsymbol{x}} = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{W}\boldsymbol{v}_d \\ &\quad - \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{W}\hat{\boldsymbol{J}}(t-T)\left\{\boldsymbol{K_p}\boldsymbol{\Delta x}(t-T) + \boldsymbol{K_v}\dot{\boldsymbol{x}}(t-T)\right\}. \end{aligned} \tag{20}$$

In addition, (20) be rewritten about $\ddot{\boldsymbol{x}}$ and let $\boldsymbol{z}_1(t)$ be $\dot{\boldsymbol{x}}$, $\boldsymbol{z}_2(t)$ be $\boldsymbol{\Delta x}(= \boldsymbol{x} - \boldsymbol{x}_d)$, then we finally obtain a state equation of the musculoskeletal arm model including the time-delay. It is given as follows:

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{z}(t-T) + \boldsymbol{c}, \tag{21}$$

where,

$$\left[\begin{array}{l} \boldsymbol{z}(t) = \begin{bmatrix} \boldsymbol{z}_1^{\mathrm{T}}(t) & \boldsymbol{z}_2^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^4 \\ \boldsymbol{A} = \begin{bmatrix} -\boldsymbol{M_x}^{-1}(\boldsymbol{C_x} + \boldsymbol{D_x}) & \boldsymbol{O}_{2\times 2} \\ \boldsymbol{I}_2 & \boldsymbol{O}_{2\times 2} \end{bmatrix} \in \mathbb{R}^{4\times 4} \\ \boldsymbol{B} = \begin{bmatrix} -\boldsymbol{M_x}^{-1}\boldsymbol{B}_1\boldsymbol{K_v} & -\boldsymbol{M_x}^{-1}\boldsymbol{B}_1\boldsymbol{K_p} \\ \boldsymbol{O}_{2\times 2} & \boldsymbol{O}_{2\times 2} \end{bmatrix} \in \mathbb{R}^{4\times 4} \\ \boldsymbol{c} = \begin{bmatrix} (\boldsymbol{M_x}^{-1}\boldsymbol{J}^{-\mathrm{T}}\boldsymbol{W}\boldsymbol{v}_d)^{\mathrm{T}} & \boldsymbol{O}_{1\times 2} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^4 \\ \boldsymbol{B}_1 = \boldsymbol{J}^{-\mathrm{T}}\boldsymbol{W}\hat{\boldsymbol{J}}(t-T) \in \mathbb{R}^{2\times 2}. \end{array}\right.$$

Let us consider a scalar function V as follows:

$$V = \boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{z}(t) + \frac{1}{T}\int_0^T \boldsymbol{z}^{\mathrm{T}}(t-\beta)\boldsymbol{Q}\boldsymbol{z}(t-\beta)\mathrm{d}\beta + P, \tag{22}$$

where,

$$\boldsymbol{Q} = \begin{bmatrix} \frac{1}{2}\boldsymbol{M_x} & \boldsymbol{O}_{2\times 2} \\ \boldsymbol{O}_{2\times 2} & \boldsymbol{K_p} \end{bmatrix} \in \mathbb{R}^{4\times 4}. \tag{23}$$

The time derivative of V is given as follows:

$$\begin{aligned} \dot{V} = &- \boldsymbol{z}_1^{\mathrm{T}}(t)\boldsymbol{D_x}\boldsymbol{z}_1(t) - \boldsymbol{z}_1^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{K_v}\boldsymbol{z}_1(t-T) \\ &+ 2\boldsymbol{z}_2^{\mathrm{T}}(t)\boldsymbol{K_p}\boldsymbol{z}_1(t) - 2\boldsymbol{z}_1^{\mathrm{T}}(t)\boldsymbol{B}_1\boldsymbol{K_p}\boldsymbol{z}_2(t-T) \\ &- \tfrac{1}{2T}\boldsymbol{z}_1^{\mathrm{T}}(t)\boldsymbol{M_x}\boldsymbol{z}_1(t) + \tfrac{1}{2T}\boldsymbol{z}_1^{\mathrm{T}}(t-T)\boldsymbol{M_x}\boldsymbol{z}_1(t-T) \\ &- \tfrac{1}{T}\boldsymbol{z}_2^{\mathrm{T}}(t)\boldsymbol{K_p}\boldsymbol{z}_2(t) + \tfrac{1}{T}\boldsymbol{z}_2^{\mathrm{T}}(t-T)\boldsymbol{K_p}\boldsymbol{z}_2(t-T), \end{aligned} \tag{24}$$

Furthermore, we obtain

$$\begin{aligned}\dot{V} \leq &- \|\boldsymbol{z}_1(t)\|^2 \left\{\lambda_{\min}\left[\boldsymbol{D_x}\right] + \tfrac{1}{2T}\lambda_{\min}\left[\boldsymbol{M_x}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right]\right. \\ &\left.+\tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B}_1\right]|\left(\lambda_{\min}\left[\boldsymbol{K_v}+\boldsymbol{K_p}\right]\right)\right\} \\ &- \|\boldsymbol{z}_2(t)\|^2\left(\tfrac{1}{T}\lambda_{\min}\left[\boldsymbol{K_p}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right]\right) \\ &-\|\boldsymbol{z}_1(t-T)\|^2\left(\tfrac{1}{2T}\lambda_{\max}\left[\boldsymbol{M_x}\right] + \tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B_1K_v}\right]|\right) \\ &-\|\boldsymbol{z}_2(t-T)\|^2\left(\tfrac{1}{T}\lambda_{\max}\left[\boldsymbol{K_p}\right] + \tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B_1K_p}\right]|\right), \end{aligned} \tag{25}$$

where, $\lambda_{\min}[\cdot]$ and $\lambda_{\max}[\cdot]$ denote the minimum and maximum eigenvalue of the matrix respectively. Assume that the feedback gain $\boldsymbol{K_p}$ and $\boldsymbol{K_v}$ can be chosen so as to satisfy the following inequality.

$$\begin{aligned}\lambda_{\min}\left[\boldsymbol{D_x}\right] + \tfrac{1}{2T}\lambda_{\min}\left[\boldsymbol{M_x}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right] \\ +\tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B}_1\right]|\left(\lambda_{\min}\left[\boldsymbol{K_v}+\boldsymbol{K_p}\right]\right) > 0,\end{aligned} \tag{26}$$

$$\tfrac{1}{T}\lambda_{\min}\left[\boldsymbol{K_p}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right] > 0. \tag{27}$$

Accordingly, we finally obtain the following inequality.

$$\dot{V} \leq -\delta\|\boldsymbol{z}(t)\|^2 + \eta\|\boldsymbol{z}(t-T)\|^2, \tag{28}$$

where,

$$\left[\begin{aligned} \delta = \min\{&\lambda_{\min}\left[\boldsymbol{D_x}\right] + \tfrac{1}{2T}\lambda_{\min}\left[\boldsymbol{M_x}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right] \\ &+\tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B}_1\right]|\left(\lambda_{\min}\left[\boldsymbol{K_v}+\boldsymbol{K_p}\right]\right), \\ &\tfrac{1}{T}\lambda_{\min}\left[\boldsymbol{K_p}\right] - \lambda_{\max}\left[\boldsymbol{K_p}\right]\} \\ \eta = \max\{&\tfrac{1}{2T}\lambda_{\max}\left[\boldsymbol{M_x}\right] + \tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B_1K_v}\right]|, \\ &\tfrac{1}{T}\lambda_{\max}\left[\boldsymbol{K_p}\right] + \tfrac{1}{2}|\lambda_{\min}\left[\boldsymbol{B_1K_p}\right]|\}. \end{aligned}\right.$$

The scalar function V decreases with respect to time t if (28) satisfies. Thus, it needs to satisfy the following inequality in order to satisfy (28).

$$\|\boldsymbol{z}(t)\|^2 > \tfrac{\eta}{\delta}\|\boldsymbol{z}(t-T)\|^2. \tag{29}$$

If (29) is satisfied, $\boldsymbol{\Delta x}(t)$ and $\dot{\boldsymbol{x}}(t)$ approach to zero as $t \to \infty$. In contrast, $\dot{V}$ becomes positive and then V increases with time t if (29) is not satisfied. In such a case, $\boldsymbol{z}(t)$ and $\dot{\boldsymbol{x}}(t)$ also increase and this eventually may induce some unstable phenomenon. The increase of V, however, means that the end-point leaves far from the desired position $\boldsymbol{x}_d$. In that case, $\boldsymbol{z}(t)$ and $\boldsymbol{z}(t-T)$ intuitively satisfy the following inequality.

$$\|\boldsymbol{z}(t)\| > \|\boldsymbol{z}(t-T)\|. \tag{30}$$

Eventually, V decreases again since (29) satisfies. Namely, the state is ultimately bounded. In this paper, the diagonal elements of the feedback gain $\boldsymbol{K_p}$ are set to be the same value. That is, $\lambda_{\min}[\boldsymbol{K_p}]$ is equal to $\lambda_{\max}[\boldsymbol{K_p}]$, and thereby the allowable time-delay for maintaining the ultimately boundedness must be $T < 1.0$ [s] from (27).

References

[1] M. Kawato and H. Gomi, "A computational model of four regions of the cerebellum based on feedback-error learning," Biological Cybernetics, vol. 68, pp. 95–103, 1992.

[2] S. Arimoto, "Control theory of non-linear mechanical system: a passivity-based and circuit-theoretic approach," Oxford, U. K.: Clarendon, 1996.

[3] J.J. Slotine and W. Li, "On the adaptive control of robot manipulators," Int. J. Robot. Res., vol. 6, no. 3, pp.49–59, 1987.

[4] D. Blana, R. F. Kirsch, E. K. Chadwick, "Combined feedforward and feedback control of a redundant, nonlinear, dynamic musculoskeletal system," Med. Biol. Eng. Comput., vol. 47, pp. 533–542, 2009.

[5] M. Katayama and M. Kawato, "Learning trajectory and force control of an artificial muscle arm by parallel-hierarchical neural network model," Proc. Conf. Advances in neural information processing systems 3, pp. 436–442, 1990.

[6] H. Kino, T. Yahiro S. Taniguchi and K. Tahara, "Sensorless position control using feedforward internal force for completely restrained parallel-wire-driven systems," IEEE Trans. on Robotics, vol. 25, no. 2, pp. 467–474, 2009.

[7] H. Kino, S. Kikuchi, Y. Matsutani, K. Tahara and T. Nishiyama, "Numeracal analysis of feedforward position control for non-pulley musculoskeletal system: a case study of muscular arrangements of a two-link planar system with six muscles," Advanced Robotics, vol. 27, pp. 1235–1248, 2013.

[8] A. Namiki and M. Ishikawa, "Sensory-motor fusion architecture based on high-speed sensory feedback and its application to grasping and manipulation," Proc. the 32nd Int. Symp. Robotics, 2001.

[9] J.K. Hale and S.M.V. Lunel, "Introduction to functional differential equations," Springer, 1993.

[10] K. Gu, V.L. Kharitonov and J. Chen, "Stability of time-delay system," Birkhauser, 2003.