Design Optimization and Comparison of Magneto-Rheological Actuators*

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Abstract—In this paper, an optimization method for designing MR clutches is studied. The proposed method optimizes the geometrical dimensions of an MR clutch, hence its mass, for given output torque and electrical input power. The main idea behind this optimization is that the input power and output torque are two parameters that are normally known to the designer prior to the design of an MR clutch and considering these parameters in the optimization as fixed values has a practical significance. Having presented the optimization method, we compare the characteristics of three different MR clutch configurations in order to demonstrate the effectiveness of the proposed method. A comparison between the drum, single-disk and multi-disk configurations of MR clutches is performed. Using the proposed method one can select a suitable configuration as well as the geometrical dimensions for an MR clutch that best suits the requirements of each individual design.

I. INTRODUCTION

Magneto-Rheological (MR) fluids, carrying micrometerscale particles, are a kind of smart material whose viscosity can be changed fast and reversibly using an external magnetic field. This property of MR fluids allows to accurately control the shear stress of the fluid by controlling the intensity of an external magnetic field [1]. As a result, MR fluids devices such as dampers, brakes and clutches have increasingly attracted attentions for various applications in recent years. Regardless of their applications, MR fluid devices hold great potentials in commercial field.

To this effect, a wide range of studies have been proposed within the context of designing good MR devices. The response time of an MR damper is discussed in [2] and the study shows that the response time decreases as the applied current or piston velocity increases. Using the results in [3], the geometrical parameters of a drum-type MR brake including the thickness of MR fluids can be calculated theoretically. After studying the influence of the material and the air gap in an MR clutch, Xu and Zeng proposed a design of an MR clutch for controlling the fan speed of an engine in a cooling system [4]. In order to achieve large output torque, Kwan, et al., proposed a new approach to build an MR brake by using small steel rollers as over-sized magnetic particles [5]. Recently, Finite Element Analysis (FEA) and optimization have been increasingly employed to

design MR devices. A rotary MR brake was designed in [6] for a prosthetic knee optimized for maximum output torque and minimum off-state torque. The number of turns for a electromagnetic coil inside an MR brake was optimized in [7]. Multi objectives optimization techniques were employed in [8], [9], in order to design an MR brake with large output torque and small weight. The geometrical parameters of a disk-type MR brake were optimized in [10]. The objective of the optimization was to maximize the output torque of an MR brake which was constrained with a certain space. Achieving maximum magnetic flux density on MR fluids was selected as the objective of the optimization in [11]. A T-shape MR brake that used magnetic field in the axis and radius directions was proposed in [12]. The geometrical parameters of this brake were optimized considering the output torque, the mass of the brake, and the temperature caused by friction.

In our previous body of work, we developed a new actuation mechanism known as Distributed Active Semi-active Actuator (DASA) using MR clutches [13]. DASA is a novel actuation concept that emphasizes on safe actuation without compromising the performance for Human-Compatible and Human-Friendly robots. The details and advantages of DASA have been reported in [14]-[16]. Unlike the design of MR devices mentioned above, achieving as large output torque as possible in designing MR clutches is not the primary concern. A larger output torque in an MR clutch may lead to more power consumption or lower control resolution which are not desired for human-safe robots. Thus, employing the optimization methods discussed in the aforementioned papers do not provide the most suitable solution in the case of designing an MR clutch. The concept of using an MR clutch in an actuator is presented in Fig. 1. As shown, the output torque of an MR clutch can be controlled using two inputs, namely the input current and the input mechanical power. In practice, the input mechanical power is often kept as a constant and an MR clutch is mainly controlled using the input current. The value of the input current is determined by the resistance of the coil inside the MR clutch and the electrical components outside the MR clutch. The resistance of coil is dependent on the dimensions of the coil which can be optimized. Yet, the electrical components are normally determined prior to designing the MR clutch.

The main contribution of this paper is to propose an optimization method for designing MR clutches. In this optimization, the input electrical power of the MR clutch is considered as a fixed value and the geometrical dimensions of the clutch are optimized so as to make the MR clutch

^{*}This work was supported in part by the Natural Sciences and Engineering Research Council (NSERC) of Canada under Grant RGPIN 346166 and Ontario of Centres of Excellence (OCE) under Grant MR 90545.

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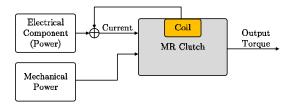


Fig. 1. Use of MR clutch to an actuator.

generate a certain output torque with the least possible mass. As mentioned above, the input electrical power and the output torque are commonly known before designing an MR clutch, hence considering them as fixed values during the optimization is a practical approach. This allows to change the values of the input power and the output torque based on the requirements of the problem so as to obtain the best configurations and geometrical dimensions.

In this paper, we first compare three different MR clutch configurations, i.e., drum, single-disk, and multi-disk clutches with no constraints on the input current and output torque. While similar comparisons for MR brakes have been discussed in [17], [18], such comparisons are not directly applicable to MR clutches as explained previously. To the best of our knowledge, there is no reported study that draw a comparison between the characteristics of different MR clutch configurations. Having discussed the optimization method, we compare the characteristics of different MR clutch configurations for the case of constant input power and output torque. Using this method allows for the selection of a suitable configuration as well as geometrical dimensions for an MR clutch that best suits the requirements of each individual design.

II. MR CLUTCH

As known, with the effect of an external magnetic field, the particles within MR fluids join into columns aligned in the direction of the field. These columns can resist the shearing of the fluid perpendicular to the field. MR clutches are employed as a means of materializing this concept through bounding the amount of transmitted torque based on the intensity of an applied magnetic field. An MR clutch generally consists of input and output mechanical components, the MR fluids which fill the volume between the input and output components, and one or multiple electromagnetic coils used for generating a magnetic field. An MR clutch provides two inputs for control the out of the clutch. One input is a rather constant mechanical power provided, for example, by an electric motor. This input introduces shearing between input and output mechanisms. The other input is the current into the electromagnetic coils to generate a magnetic field. With this magnetic field, the shear stress of the MR fluids would be altered, which allows the MR clutch to transmit the input power to the output as a function of shearing between input and output components.

A. Drum MR clutch

A drum-type MR clutch is one of the earliest proposed configurations. This configuration is often used to demonstrate the design of MR devices [19]. Fig. 2 demonstrates the

cross section of a typical drum-type MR clutch. The output part includes an output shaft and an electromagnetic coil. The input part is a hollow cylindrical casing which surrounds the output part. The volume between the input and output parts is filled with MR fluids. Both output shaft and input parts are made of ferromagnetic materials in order to form a magnetic circuit path inside the MR clutch.

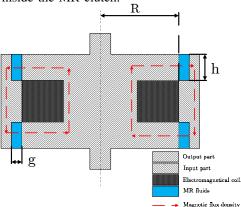


Fig. 2. The cross section of the drum-type MR clutch.

The Bingham visco-plastic model is a good candidate to represent the shear stress of the MR fluids as a function of the applied field and shear rate [20]–[24]. According to the Bingham model, the shear stress can be presented as:

$$\tau = \tau_y(H) + \eta \frac{dv}{dz}, \quad \tau > \tau_y \tag{1}$$

where τ is the shear stress, τ_y is the field dependent yield stress, H is the applied magnetic field intensity, η is the Newtonian viscosity, and $\frac{dv}{dz}$ is the velocity gradient in the direction of the field. Applying the Bingham model to the MR clutch in Fig. 2, results in the following new model:

$$\tau = \tau_u(H) + \eta \dot{\gamma}(\omega, r, g), \quad \tau > \tau_u \tag{2}$$

where the shear rate $\dot{\gamma}$ is defined as

$$\dot{\gamma} = \frac{\omega r}{a} \tag{3}$$

in that ω is the angular velocity between input and output parts, r is the radius from the rotational axis, and g is the thickness of fluid-filled gap between input and output parts. It is interesting to note that the first term on the right-hand side of (2) is only related to the electrical input while the second term is only related to the mechanical input to the MR clutch. In other words, the input current can only affect the output torque via the field dependent yield stress, i.e., τ_y .

In an MR clutch, the output torque can be presented as

$$dT = r\tau dA \tag{4}$$

where r is the radius from the rotational axis, τ is the shear stress of the MR fluids, and A is the area of active MR fluids inside the clutch. Substituting (2) into (4) and integrating across the MR fluids area for a drum-type clutch (shown the Fig. 2), we can obtain the output torque of the clutch as,

$$T_{Drum} = 4\pi \tau_y(H)R^2 h + 4\pi \eta \frac{\omega R^3}{g}.$$
 (5)

It should be noted that in this output torque, the viscosity of the carrier fluid η is typically within a range of 0.1 to 0.3 Pa-s and ω approaches zero as the MR fluids become more solidified. On the other hand, the contribution of $\tau_y(H)$ to the output torque is within a range of kPa-s torque for typical MR fluids. Thus, it is reasonable to ignore the second term in (5) due to its negligible effect on the estimated value of the output torque. As a result, the output torque of a drum-type MR clutch can be approximately rewritten as,

$$T_{Drum} \approx 4\pi \tau_y(H) R^2 h.$$
 (6)

As mentioned earlier, the only parameter affecting $\tau_y(H)$ is the electrical current input. Given that the mechanical input to an MR clutch is often a constant value, one can define the dynamic range of an MR clutch DR with respect to its input current as,

$$DR = \frac{\int_{A} r \tau_{ymax} dA}{\int_{A} r \eta \dot{\gamma} dA} \tag{7}$$

where τ_{ymax} is the maximum attainable shear stress with the MR fluids for a given input current and configuration of an MR clutch. The nominator of (7) represents the maximum output torque that the MR clutch can generate while the dominator is the output torque that the MR clutch can generate with zero input current applied. According to (7), the dynamic range of a drum-type MR clutch can be obtained as,

$$DR_{Drum} = \frac{\tau_{ymax}g}{\eta \omega R_2} \tag{8}$$

B. Single-Disk MR clutch

The disk-type, including single-disk and multi-disk MR clutch is the most common configuration due to its good performance. This configuration has been employed for commercial purposes. A number of studies on disk-type MR devices, including MR clutches, can be found in the literature where incremental modifications on this configuration have been proposed and exercised. In this paper, we consider a typical disk-type MR clutch only, for the purpose of our study.

Fig. 3 shows the cross-section of a single-disk MR clutch. Similar to a drum-type MR clutch, the output part of a single-disk MR clutch also serves as the output shaft. An electromagnetic coil is mounted on the shaft. Additionally, there is a single thin output disk mounted on the output shaft. The input part encloses the output parts, and the volume between the input and the output parts is filled with MR fluids. In this configuration, the magnetic field generated by the coil along the output shaft axis crosses the MR fluids perpendicularly. Providing the mechanical input to rotate the input part of a signal-disk MR clutch introduces shearing on both sides of the output disk when the coil is activated, thereby resulting in an output torque on the shaft. Using (2) and (4), the output torque of a single-disk MR clutch can be obtained in a similar way as before as,

$$T_{SingleDisk} = 2 \int_{R_1}^{R_2} 2\pi \left(\tau_y(H) r^2 + \eta \frac{\omega r^3}{g} \right) dr$$
$$= 4\pi \left(\frac{\tau_y(H) (R_2^3 - R_1^3)}{3} + \frac{\eta \omega (R_2^4 - R_1^4)}{4g} \right) \quad (9)$$

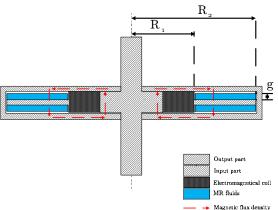


Fig. 3. Cross section of a single-disk-type MR clutch. where R_1 and R_2 are the inner and outer radius of the disk as shown in Fig. 3 and other parameters are as defined previously. Under the same assumptions made in the case of a drum-type clutch, the major contribution to the output torque is made by the first term in (9). In this way, the output torque of a single-disk MR clutch can be rewritten as,

$$T_{SingleDisk} \approx \frac{4\pi\tau(H)(R_2^3 - R_1^3)}{3} \tag{10}$$

Following the same definitions given in (7), the dynamic range of the single-disk MR clutch is obtained as,

$$DR_{SingleDisk} = \frac{4\tau_{ymax}g(R_2^3 - R_1^3)}{3\eta\omega(R_2^4 - R_1^4)}.$$
 (11)

where τ_{ymax} is the maximum shear stress that the MR fluids can achieve in a single-disk MR clutch.

C. Multi-Disk MR clutch

In order to achieve a considerable torque capacity, multi-disk MR clutches can be used to increase the active areas of the MR fluids. Each individual disk inside a multi-disk MR clutch can provide two sides of the disk as the active areas and with N output disks, the active areas inside a multi-disk MR clutch can be expanded up to 2N times larger than the active area of a single-disk MR clutch. Assuming N output disks, the output torque and the dynamic range of a multi-disk clutch can be derived from those for a single-disk MR clutch as,

$$T_{MultiDisk} \approx \frac{4N\pi\tau(H)(R_2^3 - R_1^3)}{3}$$
 (12)

$$DR_{MultiDisk} = \frac{4\tau_{ymax}g(R_2^3 - R_1^3)}{3\eta\omega(R_2^4 - R_1^4)}.$$
 (13)

in that all parameters are as defined previously. As expected, the output torque of an N-disk MR clutch is N times larger than the output torque of a single-disk MR clutch with similar dimensions. However, given that the zero-current output torque of an N-disk MR clutch is also increased by a factor of N, an N-disk MR clutch enjoys the same dynamic range as a single-disk MR clutch.

In this section, the output torque of drum, single-disk, and multi-disk type MR clutches were obtained using Bingham model. The dynamic range of each configuration were also obtained and compared. While using these results it is possible to directly compare different configurations for their achievable torques, to draw a more inclusive comparison, other parameters such as mass, volume, coil size, etc. need to be considered. This will be the subject of the following sections.

III. DESIGN OPTIMIZATION

The three mentioned configurations are optimized in this section, so as to accommodate a fair comparison between different MR clutch configurations. The characteristics considered in this body of work include weight, current capacity, and moment of inertia, all of which are functions of the geometrical parameters for a given configuration. The well-known Finite Element Method (FEM) was hired to estimate the magnetic field and calculate the characteristics for each configuration, using COMSOL Multiphysics software.

A. Material Selection

Before proceeding with the analysis, it is essential to specify the materials considered for each component of an MR clutch, such as the materials for the shaft, disks, casing, and coil. Copper was selected for the coil, imitating the characteristics of typical wires for coiling. It can be argued that using non-magnetic materials is important for outer casing of a disk-type MR clutch to reduce the magnetic field leakage and make magnetic flux perpendicular to the internal disks. To this effect, Aluminum was chosen among other non-magnetic materials. AISI steel 1018 was considered for magnetic circuits inside MR clutches forming magnetic flux. The ferromagnetic components of disk-type configurations are shaft, disks, and inner casing, while they are drum and shaft, in the drum configuration. Lastly, MRF-140CG fluid was selected as the MR fluid inside the clutch.

B. Optimal design of MR clutches

In order to obtain optimal designs for MR clutches the following objective function is proposed,

$$Obj = min(W_1|T_r - T_a| + W_2M)$$
subject to $J_D < 2.5A/mm^2$ (14)

where W_1 and W_2 are weighting coefficients such that $W_1+W_2=1,\,T_r$ is the desired torque, T_a is the predicted output torque of the clutch, M is the mass, and J_D is the current density applied to the coil. The rational behind selecting this objective function is to minimize the clutch weight, yet being capable of providing the desired torque. Optimizing MR clutches weights are important in employment of them as part of robots actuations. Obviously, the objective function results in the maximum torque-to-mass ratio, for a given desired torque. A practical limitation, the temperature tolerance of copper wires for coil, is considered in the proposed objective function. The maximum allowable current density for copper wires is $2.5A/mm^2$ pertinent to their temperature tolerances [25].

In order to find solutions for the optimization problem defined in (14), the coil current density is required. The cross section of a coil in MR clutches is depicted in Fig. 4. To

obtain J_D , the coil resistance produced by a circumferential element at radius r is given by

$$dR = \rho_c \frac{2\pi r}{H_c dr},\tag{15}$$

where ρ_c is the resistivity of the coil wire, H_c is the length of the coil. Then the power required at radius r is given by $dP = (dI)^2 dR$, where $dI = J_D H_c dr$ is the current at radius r. Hence,

$$dP = (J_D H_c dr)^2 \rho_c \frac{2\pi r}{H_c dr}.$$
 (16)

Integrating across the whole cross section gives the required power for a given J_D as follows,

$$P = J_D^2 H_c \rho_c 2\pi \int_{r_1}^{r_2} r dr.$$
 (17)

Given the fact that the available electrical power is limited for a specific application, we define P_r as the available electrical power as an added consideration for the design of an MR clutch. According to (17), an added condition for the current density J_D can be given as

$$J_D \le \left(\frac{P_r}{2\pi H_c \rho_c \int_{r_1}^{r_2} r dr}\right)^{\frac{1}{2}}.$$
 (18)

Note that the application of the optimization results may become controversial without considering the power limitation as well as the temperature tolerance.

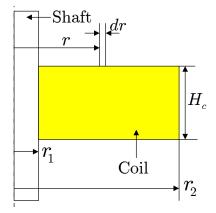


Fig. 4. Cross section of the coil.

IV. RESULTS AND DISCUSSIONS

Optimal designs of single-disk, multi-disk, and drum based MR clutch configurations are presented in this section. To obtain optimal design for each configuration, the objective function given in (14) is considered. To make different configurations comparable, we considered the situation when the desired output toque and the available electrical power for all configurations are identical. Accordingly, P_r was set as 2W and the desired torque T_r was considered as 20Nm. Further, the weighting coefficients W_1 and W_2 in (14) were set as 0.7 and 0.3, respectively. Since our primary goal is to obtain clutch parameters under which the clutch can provide the desired torque, more weight was put on the torque term in the objective function compared to the clutch mass.

Due to the fact that drum and disk-based configurations differ topologically, different optimization variables should be considered. Table I lists the optimization variables for drum configuration, while the optimization variables for disk-type configurations are listed in Table II. Due to manufacturing and implementation considerations, the thickness of disk and the size of MR fluids gaps were fixed to 1 mm and 0.5 mm in both disk-type configurations. Figs. 5 and 6 display the magnetic flux density, respectively, in drum and single-disk configurations. In this study, a 3-output-disk MR clutch is selected as an example of multi-disk configuration.

TABLE I

Variables to be optimized	Corresponding position in Fig. 5
radius of shaft	A
width and height of inner casing	В
width and height of coil	C
width of outer casing	D
height of MR fluids	E

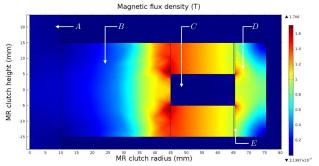


Fig. 5. Magnetic flux density contour map in drum configuration with $2\ W$ input power.

TABLE II

Corresponding position in Fig. 6
A
В
C
D
R_1, R_2

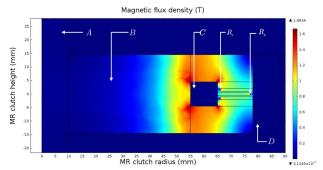


Fig. 6. Magnetic flux density contour map in single-disk configuration with 2 W input power.

A. Analysis Results

The optimization results for drum, single-disk and 3-disk MR clutch configurations are given in Table III. The output torques reported in Table III were calculated based on the Bingham model. As observed, the output torque estimated for all configurations are close to the desired torque, i.e. $T_r=20\ Nm$. Moreover, considering the optimized volume and mass of each configuration, disk-type MR clutches offers

smaller volume and mass comparing with the drum configuration. The maximum current density is obtained for singledisk configuration, indicating that this configuration requires electrical components with higher current capacity compared to the other configurations. As an important result, the 3disk MR clutch features the lowest moment of inertia, while the moment of inertia is significantly higher in drum and single-disk configurations. This is because the optimization for drum and single-disk configurations resulted in heavier components for drum-type and larger radius components for single-disk. Furthermore, assuming the relative velocity between input and output shafts is 1 rad/s, zero-currentapplied output torques were calculated. As can be seen, the calculated torques are insignificant for all configurations, while the drum configuration gives the smallest value. As an additional design consideration, H-R ratio is defined, where H and R stand for the height and radius of the resultant MR clutches. In this respect, the drum configuration features relatively large H-R value, which makes it more suitable for applications requiring small radius clutch, while the height might not be an issue.

TABLE III

COMPARISON	AMONG M	IR CLUT	TCHES IN	DIFFERENT	CONFIGURA	ATIONS

	Drum	1-disk	3-disk
Output torque [Nm]	19.97	19.91	19.78
Volume [mm ³]	556.24	391.18	337.06
Mass [kg]	4.15	2.81	2.36
Current density $[A/mm^2]$	1.32	1.88	1.58
Output moment inertia $[gm^2]$	6.81	4.66	3.84
Zero-current-applied torque [Nm]	0.0138	0.02174	0.02395
H-R ratio	0.55	0.38	0.47

MR fluid gap sizes can be considered as another design consideration specifically for multi-disk configuration. Theoretically, increasing/dreasing the gap sizes results in dramatically increasing/descresing of total reluctance of the magnetic circuit. To observe this effect, the geometrical dimensions of 3-disk MR clutch were optimized considering different fluid gap sizes. The MR fluids gaps for this purpose were chosen as 0.1 mm, 0.5 mm and 0.9 mm respectively. The resultant characteristics for each case are given in Table IV

TABLE IV

COMPARISON AMONG DISK-TYPE MR CLUTCHES WITH DIFFERENT MR

FLUIDS GAP SIZES

	$0.1 \ mm$	$0.5 \ mm$	$0.9 \ mm$
Output torque [Nm]	20.25	19.78	19.62
Volume $[mm^3]$	203.01	337.06	407.98
Mass [kg]	1.38	2.36	3.01
Current density $[A/mm^2]$	1.78	1.58	1.38

As observed, smaller fluid gap size offers lighter and more compact clutch. On the contrary, the MR clutches with smaller gaps require higher input currents. In practice, it is difficult to achieve very small fluids gaps. The main challenges arises when MR fluid needs to be injected through the fluid gaps. In summary, Table V concludes the characteristics of drum, single-disk and multi-disk MR clutches.

TABLE V

CONCLUSION OF COMPARISONS AMONG DIFFERENT CONFIGURATIONS

	Drum	Single-disk	Multi-disk
Compactness	Ordinary	Good	Excellent
Weight	Ordinary	Good	Excellent
Current capacity	Excellent	Ordinary	Good
Zero-current-applied torque	Excellent	Good	Ordinary
Output moment inertia	Ordinary	Good	Excellent
H-R ratio	Excellent	Ordinary	Good
Manufacturing	Excellent	Good	Ordinary

It can be seen that among these three configurations, the drum configuration possesses less compactness, heavier design, and larger output moment of inertia compared to other configurations. However, the drum configuration features the smallest zero-current-applied output torque as well as relatively large H-R ratio. Another advantage of drum configuration is the manufacturing of drum-based clutches is easier in general compared to multi-disk configurations. The singledisk configuration demonstrates middle level characteristics among configurations. The main issue with using single-disk clutches is that they may possess a very large radius, constraining their application in specific environments. They also require the electrical component with larger current capacity. The multi-disk configuration exhibits the best characteristics considering compactness, weight, and moment of inertia. Their H-R ratios are also larger comparing to single-disk configuration, making them an alternative option for tasks requiring higher compactness. Careful attention should be paid however designing MR clutches with high number of disks. Characteristics of MR clutches can dramatically affected by increasing the number of disks, hence the number of fluid gaps. Also, increasing number of disks would result in more complicated assembling. Moreover, the fluid gap sizes can alter characteristics of multi-disk clutches, as increasing gap sizes will result in reduction in magnetic field and output torques in MR clutches.

V. CONCLUSION

In this paper, an optimized design for MR clutches was presented aiming at minimizing the clutch mass for a desired torque. To this intent, several configurations were considered for MR clutches, including drum, single-disk, and multi-disk arrangements. A new objective function was proposed for optimizing the MR clutch parameters. As an important aspect of the new objective function, the physical limitations on the available electrical power and the temperature tolerances of coil wire were considered as well in the design optimization. Thorough analysis were performed to discuss characteristics of each configuration obtained by optimization. Advantages and disadvantages of each configuration were highlighted. In addition, sensitivities of the characteristics to variation of fluid gap sizes were analysed specifically for multi-disk clutches. The analysis provided in this paper could be used in selecting proper configuration for an MR clutch and optimizing MR actuators for robots.

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