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Polynomial Reconstruction of 3D Sampled Curves Using Auxiliary Surface Data

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***Abstract*— This paper proposes a method for structural enhancement of a 3D sampled curve. The curve is assumed to be organized, but corrupted with low frequency noise. The proposed method approaches the notion of curve reconstruction in a novel way, where information about the structure in a scanned surface is used to reconstruct the curve. Principal Component Analysis is carried out on successive neighborhoods along the curve to estimate reduced dimensionality spaces, which allows polynomial reconstruction. The effectiveness of the proposed method is verified by both simulations and experiments.**

***Index Terms*— Polynomial reconstruction, 3D sampled curve, point cloud, smoothing**

I. INTRODUCTION

Industrial robots have traditionally been used for pick and place tasks, where absolute accuracy performance is not crucial. In recent years however, industrial manipulators have been introduced to contact tasks and machining, where they can offer a flexible, lower cost alternative to CNC (Computer Numerical Control) machines [1] [2]. Traditionally, programming of industrial robots have been done using *online* programming approaches such as *Lead through* or *Walk through* [3]. This method requires an operator to manually guide the robot to a number of poses along the desired trajectory, which will then be remembered and repeated. In the context of Small and Medium-sized Enterprises (SMEs), high-mix low-volume operations where the robot program is executed a few times, is common. The workload related to online programming will, in such operations, amount to a significant part of the total cost related to operating the robot. This often makes online programming economically infeasible, thus raising the need for new, rapid, programming methods.

Offline programming is an alternative programming method that involves planning of a robot trajectory using a CAD model of the work piece. The work piece CAD model however may not always be accessible in practice, especially if the work piece was used or modified since design. To enable offline programming, the operator may resort to reverse engineering of the work piece which is both time consuming and costly. Nevertheless, the field of reverse engineering of industrial work pieces has seen increasing popularity along with the introduction of the offline programming methodology. The availability of high performance 3D scanners from companies such as GOM [4] and Leica [5] has further increased the interest in techniques for CAD model construction based on point cloud data [6].

An important part of a workpiece model is natural features, such as edges. Several authors have considered reconstruction of point sampled surfaces using feature reconstruction [7] [8] [9] [10]. In [11], the author used a moving least-squares technique to obtain a thinner version of an unorganized point cloud representing a curve sampled under heavy noise and in [12], knowledge of a kinematic process used to generate a surface were used to reconstruct the surface itself and its features.

To alleviate the offline programming procedure from the costs of workpiece reconstruction, a new programming approach is proposed. As can be seen, the market is still open for a rapid robot programming method that requires no prior knowledge of the work piece geometry and does not rely on perfect work piece reconstruction, which is based on costly and time consuming reverse engineering. In this work, we introduce the need for reconstruction of a sampled curve in an application of intuitive robot programming, where a user indicates a robot path using a device, which location in space can be accurately tracked. The user indicated path is assumed to be following a surface with unknown structure and is prone to errors due to hand shake and mistakes made by the user, reducing the accuracy and raising the need for curve reconstruction.

After recording, the user indicated path is augmented using information from a 3D scan of the work piece along the indicated path. The point cloud representing the surface of the work piece will contain structural information, which can be exploited in order to reconstruct the recorded path and correct errors caused by the user.

The above mentioned approaches all provide reconstruction of curves present in a point cloud representa-

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tion of a surface. No method exits that considers path reconstruction where the curve is reconstructed using structural information from an auxiliary, related data set.

The area of curve reconstruction is a well explored field outside the field of workpiece reconstruction. Savitzky and Golay pioneered the field of polynomial reconstruction of curves [13]. Their method fits a polynomial to points within some distance from a center point. The center point is then projected onto the fitted function. This approach applies to two-dimensional data and does unfortunately not extend directly to higher dimensions.

To proceed to three dimensions, one approach is to fit a three dimensional function to the recorded data. This has been explored in [14], where the authors use Principal Component Analysis (PCA) to locally estimate the tangent vector of a point sampled curve. A piece wise linear curve is then obtained using the estimated tangents. The data sets under consideration were unorganized noisy point clouds and no information regarding the sequence in which the points occur was available, thus raising problems with self-intersecting curves.

As can be seen, existing literature concentrate either on reconstruction of curves from unorganized point clouds, or on reconstruction of curves that are believed to be present in a point cloud, such as edges. Reconstruction of a curve related to a point cloud, but not belonging to it, has not been treated.

The piecewise linear curve obtained in [14] is not suitable for representation of a curve following an arbitrary, smooth surface. To deal with this shortcoming, a new method is proposed. The proposed approach considers the surface point cloud as an auxiliary data set, used as a mean to enhance the structure of a curve which is sharing features with it, in a way that may not have been considered before. A novel approach is therefore investigated, where no CAD model reconstruction is done and only relevant parts of the surface, from a small neighborhood around the curve, is used. This results in an algorithm suitable for reconstruction of noisy curves indicated by hand, aimed at execution by industrial robots for manufacturing and finishing tasks.

First, a novel procedure for extension of the Savitzky-Golay filter to three dimensional curves is developed. The filter is then further augmented with information in the auxiliary data set, producing a reconstructed curve which more closely resembles the structure in a surface it is related to. Since the reconstruction amounts to the offline calculation of an improved estimate of a point p_i in a sequence $p_{1:N}$, given the entire sequence, the problem falls in the category of smoothing [15].

The curve and the surface point cloud are dissimilar in nature, but related to each other. In the application considered, no restriction is put on what kind of surface feature the curve is following. A reconstruction algorithm must thus be able to handle a large variety of cases. For high flexibility, the 3D scanner is mobile, which introduces uncertainty in the kinematic calibration between the tracking system and the scanner. This fact increases the difficulty of the exploitation of the auxiliary data.

The paper is organized as follows. Initially, a brief review of theory related to the proposed solution is presented in Sec. II, followed by the proposed approach in Sec. IV. Simulations and experimental results are presented in Sec. V and VI, followed by an ending discussion.

II. Preliminaries - Principal Component Analysis

Principal Component Analysis (PCA), originally introduced by Pearson [16], is often used as a way to reduce the dimensionality of high-dimensional data. A dimensionality reduction may enable easier visualization of the data or a lower complexity representation. It will be used here as a way to allow analytical functions on the form $f : \mathbb{R} \to \mathbb{R}$ to be fitted to the originally three-dimensional data.

To allow dimensionality reduction, PCA finds a vector $\in \mathbb{R}^n$, along which the data exhibit the greatest amount of variance. This vector will be called the first principal component of the data set. The second principal component will be the direction which describes as much as possible of the variance that is not described by the first principal component, under the restriction that the two components are orthogonal (Fig. 1). The figure illustrates how, in this case, the small third principal component can be neglected in order to obtain a lower-dimensional representation of the data, which is mostly contained in a two-dimensional subspace.

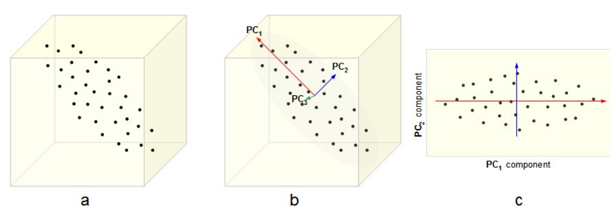


Fig. 1. Illustration of dimensionality reduction using PCA [17]

Formally, PCA finds the eigenvalue, eigenvector pairs (σ, V) of the covariance matrix Σ of the data, such that

$$\Sigma V = \sigma V \tag{1}$$

The eigenvector V_X corresponding to the greatest eigenvalue σ_x will be the first principal component.

III. Problem Statement

A path, consisting of a sequence of points sampled with equidistant spacing, is following a surface with unknown geometry. Given a point cloud representation of the surface, a reconstruction of the curve is to be performed so as to both reduce the amount of noise present in the path, and impose upon the path, structure in the surface not already present in the path.

Existing literature considers the problem of reconstruction of curves present in a surface cloud or pure reconstruction of point sampled curves. No method exists that reconstructs a curve and allows for exploitation of an auxiliary, related surface data set.

IV. PROPOSED APPROACH

In this section, a method for polynomial reconstruction of a 3D curve using a related point sampled surface is developed. The curve is assumed to be organized, corrupted with low frequency noise and following the surface of a workpiece with unknown geometry.

After pre-processing, outlined in Algorithm 1, the path is assumed to consist of a single segment with equidistant points and without discontinuities. Reconstruction starts by a choice of a parameter r called the reconstruction radius. For each point p_i on the path, neighboring points within distance r will be used in the reconstruction of p_i. This parameter will allow the user to control the amount of smoothing and must be chosen according to the amount of detail present in the work piece. The set of points within the reconstruction radius will be termed the path reconstruction neighborhood N_P. Each point will also have a set of nearest neighbors in the surface. If a large fraction of the nearest neighboring surface points to the current path segment is considered to be edge points (points with high curvature), the path segment is considered to be following an edge. Nearby edge points will then be chosen as the surface reconstruction neighborhood N_S. If no edge is present in the surface, N_S will consist of the k nearest neighbors in the surface to all points $\in N_P$.

Algorithm 1 Pre-processing outline

Re-sample path to ensure points are equidistant;
Divide path into segments if sharp features are present;
for all *Segments* **do**
  Determine if *segment* follows edge based on surface curvature;
  for all points p in *segment* **do**
    Establish N_P and N_S;
  end for
end for

Using the established N_P and N_S, the reconstruction proceeds as follows. A PCA will be carried out on N_P and N_S respectively. The analysis will yield two orthonormal rotation matrices C_P and $C_E \in SO(3)$, whose columns correspond to the principal components of the respective data set. The PCA will through the matrices C_P and C_E transform each data point p_i to the principal component space $\hat{p}_i$ as

$$\hat{p}_i = C^{-1}(p_1 - \mu) \tag{2}$$

where μ is the center of mass for the considered data set.

The transformed points are then orthogonally projected to the subspace spanned by the first two principal components. Under the assumption that the data lives in a planar subspace, the third principal component holds no information and only noise is lost in the projection.

A polynomial of low degree is fitted to the projected points. For a single data set, a reconstructed point $\tilde{p}_i$ is formed by the projection of $\hat{p}_i$ onto a fitted polynomial f as

$$\tilde{p}_i = (\tilde{p}_i^x, \tilde{p}_i^y, \tilde{p}_i^z) = \mu + C^x \hat{p}_i^x + C^y f(\hat{p}_i^x) \tag{3}$$

where the super-script $p^x \in \mathbb{R}^1$ denotes the first component of $p = (p^x, p^y, p^z)^\mathsf{T}$ and $C^x \in \mathbb{R}^{3\times 1}$ denotes the first column of $C = (C^x \ C^y \ C^z)$. The operation $p_i = C\hat{p}_i + \mu$ transforms a point in principal component space back to Cartesian space. The term $C^y f(\hat{p}_i^x)$ forms the second component of the reconstructed point by evaluating the polynomial and transforming the result back to the original space.

To allow the use of information from the surface in the reconstruction, Eq (3) is extended to

$$\tilde{p}_i = \mu_P + C_P^x \hat{p}_i^x + C_P^y f_P(\hat{p}_i^x)\alpha\beta + C_P^y f_S(\hat{p}_i^x)(1-\alpha)\beta \tag{4}$$

where μ_P is the center of mass of the points $\in N_P$. The parameter $\alpha \in [0, 1]$ is a weight that balances the influence of information from the path and the surface and $\beta \in [0, 1]$ determines the total amount of reconstruction. The procedure is illustrated in Fig. 2.

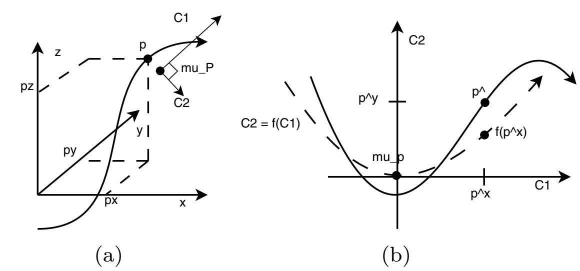


Fig. 2. Path in Cartesian space 2(a) and in principal component space, together with a fitted polynomial 2(b). The point $\hat{p}$ will be projected onto the polynomial to form the reconstructed point $\tilde{p} = f(\hat{p}^x)$

Notice how separate principal component analyses are done on the two data sets. This is motivated by the desire to use only structural information in the surface cloud, without using any absolute location or directional information. This way, the impact of a slight misalignment of the two data sets is reduced. A reconstructed point is transformed back to the original space of the path using only the principal components of the path C_P.

By choosing the parameter α close to one, little or no structural information from the surface is used, and all reconstruction is done from information in the path. This choice resembles a two dimensional Savitzky-Golay filter, extended to work in three dimensions. If α is chosen closer to zero, only information from the surface is used in the reconstruction. The reconstruction procedure is summarized in Alg. 2.

Algorithm 2 Reconstruction algorithm summary

for all Points p in *segment* **do**

$C_P, C_S \leftarrow$ Perform PCA on N_P and N_S separately;

$\hat{N}_P$, $\hat{N}_S \leftarrow$ Transform N_P, N_S using C_P^{-1}, C_S^{-1};

Discard third component of transformed points;

$f_P, f_S \leftarrow$ Fit functions to two-dimensional points;

Project $\hat{p}$ onto f_P and f_S;

Balance influence of projected points using α;

$\tilde{p} \leftarrow$ Transform projected points back to original path space using C_P;

end for

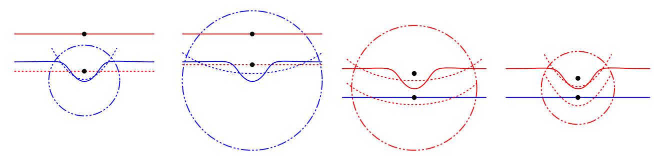
The choice of functions onto which the points are projected is not limited to low-order polynomials. If the curve is known to follow some specific type of function, this function may be used instead.

To assess the rationality of the proposed reconstruction equation, Eq. (4), consider the case depicted in Fig. 3(a). The path depicted exhibits structure not present in the surface. The reconstruction radius is chosen so small that the fitted polynomial (blue) will render the reconstructed point almost equal to the original point. The surface, however, is free from the defect in the curve. When the polynomial fitted to the surface (red) is used in the frame of the path's principal components, it is no longer a good fit. When a path point is projected to the surface polynomial it will therefore render a reconstructed point far from the original one, more consistent with the structure in the surface. Projecting the path point onto the surface polynomial in this case will have a smoothing effect, reducing the unwanted structure in the path. For both reconstruction radii shown, the smoothing effect will be greater using the surface polynomial than using the path polynomial.

In Fig. 3(b), the surface exhibits structure not present in the path. The center point on the path will be orthogonally projected to a point on the surface polynomial, thus introducing to the path, some of the structure present in the surface. By using a higher-degree polynomial, a larger reconstruction radius can be used while maintaining a good fit on features in the surface like the one in Fig. 3(b). Using a larger r in this case would shift the center of mass closer to the straight part of the surface, while the higher-degree polynomial would be able to closely follow the deep feature. The reconstructed points in this case would closer resemble the structure in the surface. For the application considered however, the case depicted in Fig. 3(a), where the path exhibits unwanted structure, is more likely to occur. For this task, a lower-degree polynomial will allow the amount of smoothing to be dependent on the reconstruction radius chosen.

V. Simulation

In this section, the functionality of the proposed algorithm is verified using artificial data sets. The synthetic data were corrupted with noise to simulate the output



(a) (b)

Fig. 3. Surface (red) and path (blue) together with fitted polynomials (dashed). The center point of the respective data sets within the reconstruction radius (circle) is shown as a black dot. Both cases are depicted with two different reconstruction radii.

from real systems, such as low-frequency noise induced by the operators hand motions and noise in the 3D scanner generating the surface point cloud.

Initially, the intended usage of the algorithm is illustrated. Fig. 4 shows a simulated point cloud surface (red) with a noisy path (blue) following an edge in the surface. The simulation compares the output of the algorithm using both pure polynomial smoothing of the path, ($\alpha = 1$, magenta), and reconstruction using edge information ($\alpha = 0$, green). For this experiment, colored noise was added to the path before reconstruction to allow comparison between the reconstruction result and a noise free, ground truth path. The noisy path was aligned to the edge points of the surface point cloud using the Iterative Closest Point algorithm [18]. This resembles a real scenario where alignment between the two data sets may be inaccurate due to the uncertainty in the location of the 3D scanner. The surface cloud was synthetically constructed and corrupted with noise with standard deviation of 2% of the model scale. Moving least-squares filtering and curvature estimation was done using Point Cloud Library [19].

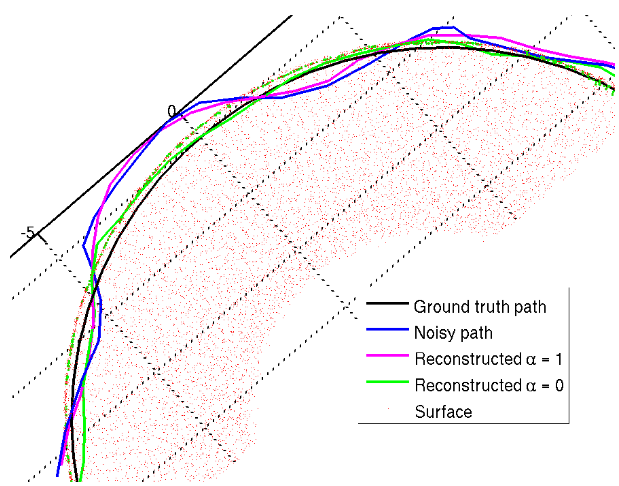


Fig. 4. Reconstructed curved path. The path is following the circular edge of a cylinder. Results are shown using $\alpha = 1$, $\alpha = 0$ and radius 4.

A simulated path allows a point-wise error to be calculated between the reconstructed path and the original, ground truth path, before it was corrupted by noise. Fig. 5 shows a residual plot for an experiment using the same configuration as when the result in Fig. 4 was produced. The errors produced by the formula

$$e = \sum_{i=1}^{N} \left\| p_i^{desired} - p_i \right\| \tag{5}$$

was $e_0 = 13.0$ for the reconstructed path using $\alpha = 0$, $e_1 = 16.8$ for the reconstructed path using $\alpha = 1$, and $e_n = 21.9$ before reconstruction. The absolute magnitude of these numbers is of secondary interest, the ratio however indicates that the average error decreased after reconstruction.

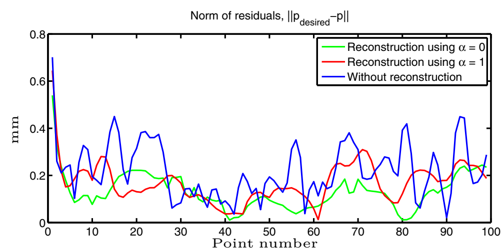


Fig. 5. Norm of residuals for the configuration shown in Fig. 4. Residuals are calculated from the ground truth path.

We refer to Fig. 6 for a comparison between cases where varying quality of both path and surface is used. All data sets were constructed from a second-order polynomial with added colored noise to produce sets of varying quality. The results are discussed in section VII.

VI. Experimental results

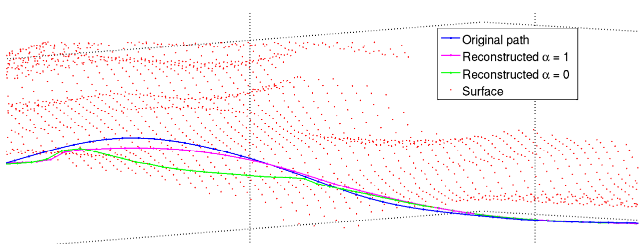


Fig. 7. Test results using a scan from the Kinect sensor and path recorded using OptiTrack Flex 13 optical tracking system.

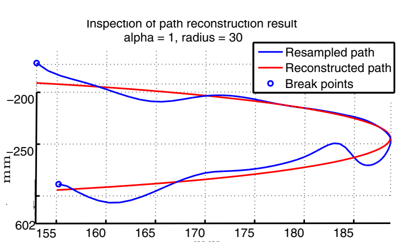


Fig. 8. Reconstruction of a path using no surface information. Path recorded using OptiTrack Flex 13 optical tracking system.

To verify the functionality of the proposed approach on real world data, an experimental result using a surface point cloud obtained from the Kinect sensor is shown in Fig. 7. The Kinect [20] sensor is a low-cost 3D scanner which is used to prove the concept of the proposed intuitive robot programming approach. The Kinect estimates suffer from correlated noise, described in [21]. When used at its minimum working distance, this noise has approximately the same magnitude as the typical errors in the path caused by the operator, with a standard deviation of approximately 1mm.

The path used in these experiments was recorded using the OptiTrack Flex 13 optical tracking system [22]. The recorded path shows typical errors related to indicating a path by hand. Experience has shown that indication of an edge by hand typically introduces a maximum error of 2-5mm and a standard deviation of 0.5-1mm.

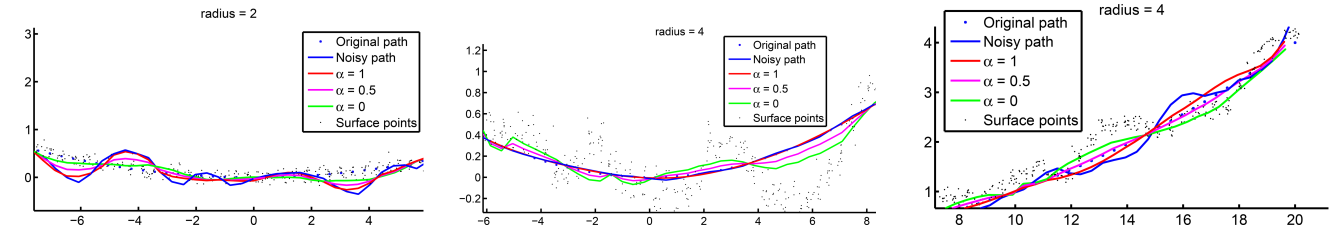
In the experiment, the path is supposed to follow a flat surface. In the center of the segment, the user has made a significant error, causing the path to rise above the surface. If the surface is used in the reconstruction ($\alpha = 0$), the reconstructed path will be following the surface more closely, indicated by the green path.

If no surface information is available, the proposed method can be used using only information from the path. Figure 8 illustrates a result where a curved path is reconstructed without the use of surface information. If no auxiliary data is available, a large reconstruction radius may be used to enhance the structure of the path using pure polynomial smoothing, adapted to three dimensions. This indicates that the proposed method is useful even for cases where the auxiliary data is absent or of poor quality.

VII. Discussion

In Secs. V and VI, it has been shown that the proposed algorithm can indeed enhance the structure of the point sampled curve. Figure 4 indicates that involving edge information yield a better result, where the reconstructed path follows the edge better compared to when pure polynomial smoothing of the path is used for reconstruction. This verifies that reconstruction using structural information from the auxiliary data may increase the performance of the algorithm, provided that the surface cloud used has low noise content. However, if the surface used is of low quality, the reconstruction is likely to be better using only information in the path.

For a comparison between cases where varying quality of both path and surface is used, refer to Fig. 6. The figure indicates that the parameter α can be used to balance the amount of information used from the different data sources. If the surface data contain less noise than the path, a value of α close to 0 yields a better reconstruction. If the path data, however, are of higher quality, a value of α closer to 1 is more likely to yield a good result. A value of $\alpha \approx 0.5$ can be used if both the surface and the path are noisy. The noise in the two data sets are then more likely to average out and yield a better estimate. All data sets were constructed from a second-order polynomial with added colored noise



(a) High-quality surface and noisy path (b) Noisy surface and high-quality path (c) Both surface and path are of low quality

Fig. 6. Results using different levels of α are shown for varying quality of path and surface.

to produce sets of varying quality. The results indicate that the tunable parameter α can be used to balance the relative confidence in quality between the two data sets.

The simulated results shown earlier indicate that the performance of the algorithm will improve with better quality surface data. Still, Fig. 7 shows that the data from a very low-end scanner can be used for correction of errors in the user indicated path. This result indicates that *low-cost* equipment such as the Kinect is indeed useful for reconstruction for manufacturing purposes. An application where the curve under consideration have errors of greater magnitude than the ones presented here, may thus benefit greatly from structural enhancement using such low cost equipment.

The errors typically introduced by the user in an unprocessed path, in particular the high maximum error, make the indicated path unsuitable as trajectory for an industrial robot in the considered application. After reconstruction, a path can typically be indicated and executed with a maximum error of 1mm, making it suitable for finishing tasks such as polishing and deburring using force control [23].

VIII. Conclusions

An algorithm has been proposed which can significantly improve the structure of a point sampled curve in three dimensions, making it more suitable as trajectory for execution on an industrial robot. This is achieved using a novel approach, where structural similarities in a related point cloud surface is exploited. The alignment between the surface and the curve may be assumed to be imperfect and the amount of reconstruction desired is tunable. The effectiveness of the algorithm has been verified in both simulations, using a synthetic ground truth path and in experiments, using data from a real 3D scanner and optical tracking system.

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