2014 IEEE International Conference on Robotics & Automation (ICRA)
Hong Kong Convention and Exhibition Center
May 31 - June 7, 2014. Hong Kong, China

A Globally Stabilizing Hybrid Control Algorithm for Mobile Manipulation Subject to Joint-Space Constraints

Johan Markdahl and Xiaoming Hu

Abstract— **This paper proposes a hybrid control algorithm for mobile manipulation subject to joint-space constraints. More specifically, we consider the problem of making the end-effector of a planar manipulator attached to a nonholonomic mobile platform reach a set-point or equivalently follow a given path in workspace by means of kinematic actuation. A switched control strategy allows the robot to avoid singular joint configurations and execute a pseudo-inverse based feedback using bounded and continuous control signals. The switching scheme is also utilized to maintain feasible joint configurations. Numerical examples provide an intuitive understanding of the algorithm's workings.**

I. Introduction

A mobile manipulator combines the unlimited workspace of a mobile platform with the multiple functionalities of a manipulator, thus providing a flexible system with a high degree of manipulability [1]. Wheeled mobile platforms are generally preferred to those using other means of locomotion since wheeled robots tend to move faster while consuming less energy. The wheels of a nonholonomic mobile platform are of a simpler and more reliable mechanical design than those of a mobile platform capable of omnidirectional motion. The nonholonomic motion constraints do however pose challenges to a control engineer. For example, a consequence of Brockett's theorem is that the pose of nonholonomic vehicles cannot be stabilized to a point by means of a continuous time-invariant feedback [2]. Still, there is a considerable literature on the subject of control of nonholonomic vehicles and mobile manipulators with nonholonomic platforms, see *e.g.* [3] and the references therein.

This paper proposes an inverse kinematics based feedback linearization approach to the problem of end-effector (EE) path following for mobile manipulators subject to joint-space constraints. The problem of inverse kinematics based control has a long history of study, a sample of works are [4], [5], [6], and [7]. A related problem is that of redundancy resolution as studied in *e.g.* [6], [8], [9], [10], and [11]. Redundancy arises when a task requires less degrees of freedom than those possessed by the manipulator. Some degrees of freedom are thus left unspecified and can be utilized to achieve objectives of secondary (or lower) priority. The key idea is to project the control laws corresponding to those objectives on the nullspace of the manipulator Jacobian [3] (or a block Jacobian in the case of mobile manipulation [7], [8], [10]). Multiple objectives of successively lower priority can be accommodated by repeating this procedure [9], [11].

Issues that have been addressed concerning inverse kinematics based algorithms and priority based methods include: (i) the pseudo-inverse becoming unbounded as the manipulator approach a singular configuration, (ii) a low priority objective does not belong to the nullspace and cannot be met exactly, and (iii) there may be a conflict between objectives of different priority. The case of (i) can be addressed using optimization techniques such as damped least-squares [4], [6], but is sometimes left outside the scope of the work even though it is of critical importance [5], [7]. Issue (ii) can be addressed approximately by use of a least squares approach [7], or be resolved by a technique that avoids inversion [6], or (for joint-limits avoidance) by defining a weighted pseudo-inverse using nonlinear optimization [5]. Sufficient conditions for convergence using a high gain approach in the case of (iii) is given in [11]. However, the combined approach of exact cancellation and gains of different magnitudes may be sensitive to noise and modeling errors, making lower priority objectives conflict with higher ones in practice.

The algorithm we propose is an inverse-kinematic based control law for stabilization of the EE position that also guarantees the achievement of two additional objectives, namely singularity-avoidance and joint-space constraint satisfaction. The chosen approach separates the inverse kinematics based control from singularity avoidance and joint-space constraint satisfaction in time by means of a switched control strategy. One advantage of this separation is a kind of inverse optimality: if, at any time, the desired set-point can be reached along a line in workspace without any joint self-motion (although the joint configurations must be feasible and at a sufficient distance from singularities), then, for the remainder of time, the set-point will be approached along a geodesic curve in workspace. Moreover, if the initial condition do not satisfy these requirements, then the mobile manipulator will reach a state which does.

Our main motivation for developing the proposed algorithm is to solve the cooperative manipulation problem of stabilizing the pose of an object that has been grasped by multiple mobile manipulators as addressed in our previous works [12], [13]. The paper [12] mainly considers the aspect of distributed control, using standard inverse kinematics based manipulator control techniques to establish asymptotical stability of the object pose. Its main contribution is to provide a distributed algorithm for object pose control that scales linearly with the number of agents. The work [13] uses a singularity-free but discontinuous switched control

This work was supported by the Swedish Foundation for Strategic Research (SSF) through the Centre for Autonomous Systems (CAS) at KTH.

Johan Markdahl and Xiaoming Hu are with the Division of Optimization and Systems Theory, KTH Royal Institute of Technology, Stockholm, Sweden, markdahl@kth.se, hu@kth.se

approach to establish global asymptotical stability. Its main contribution is to rigorously prove convergence results that are valid on a global scale.

This paper extends the algorithm of [12] and [13] to the case of (i) a larger class of mobile manipulators, (ii) continuous feedback control, and (iii) the satisfaction of rather general joint-space constraints. The problem of EE path following for a single mobile manipulator is, in a way, a special case of the cooperative manipulation problem. As such, the proposed algorithm differs from that of [5], [7], by providing a guarantee that the inverse kinematics are well defined. It also differs from works such as [6], [7], in guaranteeing the satisfaction of joint-space constraints without requiring the system to be redundant (note however that the proposed approach is compatible with priority-based task assignment).

NOMENCLATURE

The unit circle is denoted by $\mathbb{S}$ and the n-torus $\mathbb{S}\times\ldots\times\mathbb{S}$ by $\mathbb{T}(n)$. A k-dimensional box in $\mathbb{R}^k$ is denoted $\mathbb{P}(k)$. The set of 2×2 rotation matrices and skew-symmetric matrices are denoted $\mathsf{SO}(2)$ and $\mathfrak{so}(2)$ respectively. The standard basis for $\mathbb{R}^n$ is denoted $\{\mathbf{e}_1,\ldots,\mathbf{e}_n\}$. The interior, closure, and complement of a set S are denoted by $\mathrm{int}(S)$, $\mathrm{cl}(S)$, and S^{c} respectively. A closed ball with radius r around a point $\mathbf{p}$ is denoted $B_r(\mathbf{p})$. The distance between a set S and a point $\mathbf{p}$ is denoted $d(\mathbf{p},S)$. The inner product between two vectors of equal length is denoted $\langle\cdot,\cdot\rangle$. The norm of a vector or a matrix is denoted $\|\cdot\|$ and refers either to the Euclidean vector norm, the induced matrix 2-norm or the Frobenius norm (the results we obtain hold for any matrix norm that is compatible with the Euclidean vector norm). We also write

$$\mathbf{S} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \in \mathfrak{so}(2). \tag{1}$$

Consider a system consisting of a manipulator and a mobile platform. The position of the end-effector (EE) is denoted by $\mathbf{p}_e$, the position of the first manipulator joint by $\mathbf{p}_j$, and the position of the mobile platform by $\mathbf{p}_m$. The joint-space variables are denoted by $\mathbf{q}\in\mathbb{T}(n)\times\mathbb{P}(k)$. The mobile platform orientation is expressed using a rotation matrix $\mathbf{R}\in\mathsf{SO}(2)$ that rotates the inertial frame by θ radians into the mobile platform's body frame. Control signals are denoted by $\mathbf{u}_i^\sigma$, where $i\in\{e,j,m\}$ indicates the entity being actuated and $\sigma\in\{\mathtt{e},\mathtt{m},\mathtt{p},\mathtt{t}\}$ indicates the value of the switching signal.

Relative positions are denoted by double subindices, $\mathbf{p}_{ik}=\mathbf{p}_i-\mathbf{p}_k$, $i,k\in\{e,j,m\}$. This can be illustrated by

$$\mathbf{p}_{ej} = \mathbf{f}(\mathbf{q}) \tag{2}$$

where $\mathbf{f}:\mathbb{T}(n)\times\mathbb{P}(k)\to\mathbb{R}^2$ is the forward kinematic map. In relation to $\mathbf{f}$ we define the manipulator Jacobian matrix, $\mathbf{J}:\mathbb{T}(n)\times\mathbb{P}(k)\to\mathbb{R}^{2\times n+k}$, by

$$\mathbf{J}(\mathbf{q}) = \left[\frac{\partial \mathbf{f}_i}{\partial q_j}\right]. \tag{3}$$

The superindex $*$ indicates a desired value for some quantity. For example, $\mathbf{p}_{jm}^*$ could denote a vector in workspace such that all $\mathbf{q}^*\in\mathbf{f}^{-1}(\mathbf{p}_{jm}^*)$ are joint configurations characterized by a high manipulability index [3]. See Fig. 1 for a graphic illustration of the notation describing the robotic system.

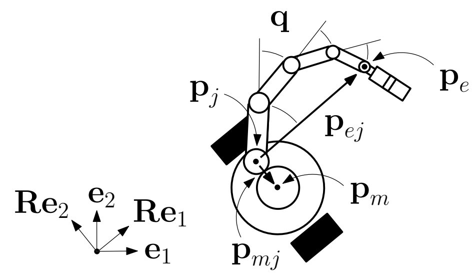

Fig. 1: A planar mobile manipulator.

II. SYSTEM DESCRIPTION

We make the following assumptions regarding the robotic system.

Assumption 1 (Robotic System): The manipulator has a known, open kinematic chain and is planar. Its joints are fully actuated. The first joint is revolute and is situated on the line that connects the midpoint of the platform's both drive wheels. The feasible joint configurations belong to the set

$$F = \{\mathbf{g}(\mathbf{q})\le \mathbf{0} \,|\, \mathbf{q}\in\mathbb{T}(n)\times\mathbb{P}(k)\}, \tag{4}$$

where $\mathbf{g}$ is such that F is connected, $\mathrm{cl}(\mathrm{int}(F)) = F$, q_1 is unconstrained, and there exists a desired operating configuration $\mathbf{q}^*\in F$ which is characterized by a high manipulability index [3]. Also, any prismatic joint q_i is constrained by $q_i\in[a_i,b_i]$ with $a_i,b_i\in\mathbb{R}$. The platform is nonholonomic of unicycle type, an example of which is the differential-drive vehicle [3].

Remark 1: Assumption 1 is more general than the corresponding assumption in [13] and may include dual-arm mobile manipulators like the one illustrated in Fig. 2. In [13] we only consider nR manipulators but the introduction of (4) allows us to include prismatic joints without having to address the theoretical issue of their extension becoming unbounded.

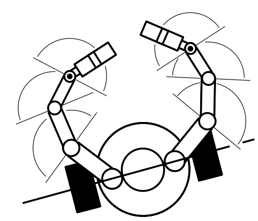

Fig. 2: A dual-arm mobile manipulator. The line referred to in Assumption 1 runs diagonally through the figure. The joints are subject to linear constraints

$$-\frac{1}{2}\pi \le q_i \le \frac{1}{2}\pi,\; i\in\{2,3,4\}.$$

Before presenting the system model we make a key assumption regarding the control system.

Assumption 2 (Control System): Control of the manipulator and the platform is carried out on a kinematic level and is a bounded, continuous function of time.

Remark 2: Assumption 2 is more restrictive than the corresponding assumption in [13] but also more realistic since we do not allow discontinuous feedback.

System 1: The first-order differential kinematics of a planar mobile manipulator,

$$\dot{\mathbf{p}}_{em} = \dot{\theta}\mathbf{S}\mathbf{p}_{em} + \mathbf{J}\dot{\mathbf{q}}, \tag{5}$$

express the relative linear velocity between the platform and the EE, $\dot{\mathbf{p}}_{em} = \dot{\mathbf{p}}_e - \dot{\mathbf{p}}_m$, as a function of the platform angular velocity $\dot{\theta}$ and the angular velocities of the joints $\dot{\mathbf{q}}$ (see [12] for a derivation of (5)). Assumption 2 gives

$$\dot{\mathbf{q}} = \mathbf{u}_q, \tag{6}$$

where $\mathbf{u}_q$ is a control input. The mobile platform kinematics are described by

$$\dot{\mathbf{p}}_m = u_v\mathbf{R}\mathbf{e}_1, \quad \dot{\mathbf{R}} = u_\omega\mathbf{S}\mathbf{R} \tag{7}$$

where $\mathbf{R}$ represent the platform orientation, u_v is the linear velocity and equals $\|\dot{\mathbf{p}}_{em}\|$ up to sign, and $u_\omega = \dot{\theta}$ is the angular velocity. Assumption 2 also implies that the input signals are subject to constraints of the type

$$|u_{q_i}| \leq U_{q_i}, \quad |u_v| \leq U_v, \quad |u_\omega| \leq U_\omega, \tag{8}$$

where the bounds may be interpreted as input saturation levels.

In this paper we will be satisfied with establishing bounded control signals without specifying the actual bounds (8).

III. Problem Statement

Consider System 1 under Assumption 1–2. Our goal is to design a bounded, continuous feedback control that steers an operational point, which we chose to be the EE, to a desired set-point from any initial condition while at the same time avoiding any singular manipulator configurations. There is no loss of generality in assuming the set-point to be the origin. Note that the seemingly more general problem of EE path following can also be solved by such a control law under rather mild additional assumptions using a feedforward technique.

IV. Control Algorithm

The proposed solution to the problem described in §III will first be outlined in §IV-A and then be given an in-depth description in §IV-B–IV-E.

A. Switched control strategy

Inverse kinematics based task space control is a form of feedback linearization and suffers from the limitations associated with such an approach. Stability results can be established on a local level but the system may not be globally asymptotically stable and can even display finite escape time. To avoid the input norm growing unboundedly as the manipulator approaches a singular configuration we modify the inverse kinematics based approach by making use of a switched control strategy.

The discrete state σ of the system switches between four modes which we refer to as the EE mode ($\mathtt{e}$), manipulator mode ($\mathtt{m}$), mobile platform mode ($\mathtt{p}$), and transition mode ($\mathtt{t}$). The following switching sequences are allowed $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$, $\mathtt{m} \to \mathtt{t} \to \mathtt{e}$, $\mathtt{m} \to \mathtt{t} \to \mathtt{p}$, and $\mathtt{p} \to \mathtt{t} \to \mathtt{m}$. The switching modes are grouped as $\mathtt{T} = \{\mathtt{e}\}$, $\mathtt{J} = \{\mathtt{m}, \mathtt{p}\}$ where $\mathtt{T}$ corresponds to task space control and $\mathtt{J}$ to joint-space control in the sense that the EE position is fixed in $\mathtt{J}$. The switching mode $\mathtt{t}$ is transient since it only serves as a passage between other modes. An illustration of the overall switching scheme is provided in Fig. 3.

The switching is state and time dependent. The mode $\mathtt{e}$ is where the robot performs the main task of stabilizing the EE position. A robot executes the switch $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$ when its manipulator is close to a singular configuration or an unfeasible configuration and the reverse switch $\mathtt{m} \to \mathtt{t} \to \mathtt{e}$ when said manipulator is at sufficient distance from such configurations to resume its work. The mode $\mathtt{m}$ is hence about improving the manipulability of the manipulator configuration. The mode $\mathtt{t}$ is used to connect the control signals $\mathbf{u}_i^\sigma$ for $i \in \{e, j, m\}$, $\sigma \in \{\mathtt{e}, \mathtt{m}, \mathtt{p}\}$ in a continuous fashion, thereby circumventing the discontinuity that a direct switch between these modes could result in. The switch $\mathtt{m} \to \mathtt{t} \to \mathtt{p}$ is executed when the nonholonomic motion constraints of the platform prevent the robot from approaching a manipulator configuration that is characterized by a good manipulability index, *i.e.* is far from a singularity. A robot in $\mathtt{p}$ will readjust the orientation of its mobile platform whereafter it executes the switch $\mathtt{p} \to \mathtt{t} \to \mathtt{m}$.

In light of the recently introduced notation, we rewrite (5) as follows

$$\mathbf{u}_e^\sigma = u_v^\sigma\mathbf{R}\mathbf{e}_1 + u_\omega^\sigma\mathbf{S}\mathbf{p}_{em} + \mathbf{J}\mathbf{u}_q^\sigma, \tag{9}$$

where the superindex $\sigma \in \{\mathtt{e}, \mathtt{m}, \mathtt{p}, \mathtt{t}\}$ is the switching signal, $\mathbf{u}_e^\sigma$ may be considered as a virtual input signal to the EE, u_v^σ and u_ω^σ are input signals to the mobile platform, $\mathbf{R}\mathbf{e}_1$ represents the mobile platform orientation, and $\mathbf{u}_q^\sigma$ is an input signal to the joints of the arm. In (9) we use that $\dot{\mathbf{p}}_e^\sigma = \mathbf{u}_e^\sigma$, $\dot{\mathbf{p}}_m^\sigma = u_v^\sigma\mathbf{R}\mathbf{e}_1$, and $\dot{\mathbf{q}}^\sigma = \mathbf{u}_q^\sigma$.

B. The end-effector mode

In end-effector mode ($\mathtt{e}$) we want to achieve the main task of moving the EE to its desired set-point position. This is done using pseudo-inverse based control in work space. Introduce the constraints

$$\|\mathbf{J}^\dagger\| \leq \rho, \tag{10}$$

$$d(\mathbf{q}, F^{\mathsf{c}}) \geq \varepsilon^{\mathsf{e}}. \tag{11}$$

where ρ and ε^{e} are strictly positive constants and $\dagger$ denote the Moore-Penrose inverse operator. Note that condition (11) is stricter than the requirement on joint configuration feasibility made in Assumption 1. Condition (10) is an important step towards formalizing the requirement of bounded control

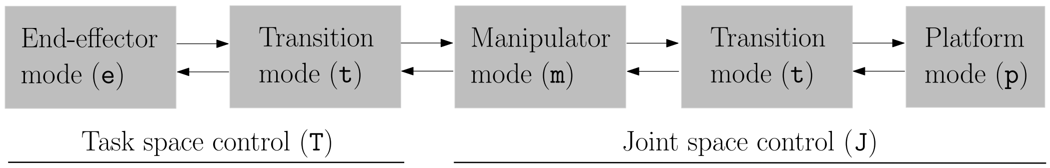

Fig. 3: The switching modes.

inputs made in Assumption 2. The choice of $\varepsilon^{\mathtt{e}}$ and ρ cannot be done arbitrarily, but must take $\|\mathbf{J}^{\dagger}\|$ and F into account.

The switched control of [13] is autonomous but depends discontinuously on the states. The use of an instantaneous switch simplifies the switching criteria. In this paper we need to predict the possibility of (10) being violated and initiate the switch prior to this happening.

A robot switches $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$ whenever

$$\|\mathbf{J}^{\dagger}\| \geq \rho' \quad \text{or} \quad d(\mathbf{q}, F^{\mathrm{c}}) \leq \varepsilon^{\mathtt{e}\prime}, \tag{12}$$

an illustration of the former case is provided in Fig. 4. Condition (12) implies that the robot leaves $\mathtt{e}$ whenever the norm of its manipulator Jacobian grows close to an upper bound ρ or when the joints are close to within a lower distance $\varepsilon^{\mathtt{e}\prime}$ of an infeasible configuration. Note that condition (12) covers both boundary and internal singularities [14].

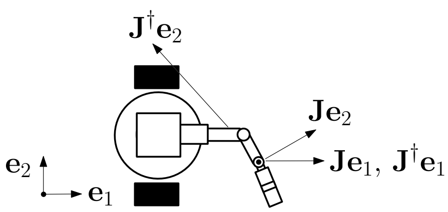

Fig. 4: A PR manipulator for which $\|\mathbf{J}^{\dagger}\|$ is large.

The constraints (10), (12) are soft in the sense that they can be violated when $\sigma \in \{\mathtt{m}, \mathtt{p}\}$ (note however that the constraint (11) is hard for all σ). To make sure that (10) is not violated in the transition phase $\mathtt{t}$ of the switch $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$ when pseudo-inverse based control is still being applied, we set $0 < \rho' < \rho$ so that $\|\mathbf{J}^{\dagger}\|$ may exceed ρ' without exceeding ρ. Likewise we set $0 < \varepsilon^{\mathtt{e}} < \varepsilon^{\mathtt{e}\prime}$ so that $d(\mathbf{q}, F^{c})$ may decrease in $\mathtt{t}$ without falling below $\varepsilon^{\mathtt{e}}$.

The time spent in $\mathtt{t}$ is designed to be a constant number $\Delta\tau^{\mathtt{t}}$ (see §IV-E). Hence $\Delta\tau^{\mathtt{t}}, \rho'$, and $\varepsilon^{\mathtt{e}\prime}$ should be chosen so that the maximum rate of change of $\mathbf{q}$ cannot lead to a violation of (10)–(11) before $\Delta\tau^{\mathtt{t}}$ time units have passed after that the switch has been initiated. More specifically, ρ' and $\varepsilon^{\mathtt{e}\prime}$ are such that $\|\mathbf{J}^{\dagger}(\mathbf{q}(\tau))\| \leq \rho'$ and $d(\mathbf{q}(\tau), F^{\mathrm{c}}) \geq \varepsilon^{\mathtt{e}\prime}$ respectively implies that

$$\|\mathbf{J}^{\dagger}\| \leq \rho, \quad d(\mathbf{q}, F^{\mathrm{c}}) \geq \varepsilon^{\mathtt{e}}, \quad \forall\, \mathbf{q} \in B_r(\mathbf{q}(\tau)),$$

where $r = \Delta\tau^{\mathtt{t}} \max_{\mathbf{q}, u_v^{\mathtt{e}}, u_\omega^{\mathtt{e}}, \mathbf{u}_e^{\mathtt{e}}} \mathbf{u}_q^{\mathtt{e}}$ subject to $\mathbf{q}, u_v^{\mathtt{e}}, u_\omega^{\mathtt{e}}$, and $\mathbf{u}_e^{\mathtt{e}}$ being feasible.

The switching logic guarantees that $\|\mathbf{J}^{\dagger}\| < \rho$ holds at any time when $\sigma = \mathtt{e}$ or $\mathtt{e} \to \mathtt{t}$. Hence $\mathbf{J}^{\dagger}$ is well-defined in $\mathtt{e}$ and we may set

$$\mathbf{u}_q^{\mathtt{e}} = \mathbf{J}^{\dagger}(-u_v^{\mathtt{e}}\mathbf{R}\mathbf{e}_1 - u_\omega^{\mathtt{e}}\mathbf{S}\mathbf{p}_{em} + \mathbf{u}_e^{\mathtt{e}}), \tag{13}$$

which satisfies (9) for any $\mathbf{u}_e^{\mathtt{e}}$. Stated another way, we have used a feedback linearization to obtain

$$\dot{\mathbf{p}}_e^{\mathtt{e}} = \mathbf{u}_e^{\mathtt{e}}, \tag{14}$$

where $\mathbf{u}_e^{\mathtt{e}}$ is a bounded, continuous virtual input signal that does not directly correspond to any physical actuator. The input $\mathbf{u}_e^{\mathtt{e}}$ can be chosen to make the EE move towards the desired set-point using standard techniques.

Remark 3: In order to solve the path following problem, the path needs to be parametrized by a variable that only evolves when $\sigma = \mathtt{e}$ and in the first half of the switch $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$ so that the error vector connecting the end-effector with its desired position remains constant when $\sigma \in \{\mathtt{m}, \mathtt{p}\}$.

C. The manipulator mode

In manipulator mode ($\mathtt{m}$) we want to restore (12) (with some additional margin) in finite time without moving the EE. Introduce the constraint

$$\dot{\mathbf{p}}_e = \mathbf{u}_e^{\mathtt{p}} = \mathbf{0}. \tag{15}$$

The purpose of (15) is to avoid having the control in $\mathtt{m}$ interfere with the progress in $\mathtt{e}$ towards the goal of making the EE reach its desired set-point.

A robot switches $\mathtt{m} \to \mathtt{t} \to \mathtt{e}$ when

$$\mathbf{q} \in B_{\delta^{\mathtt{m}\prime}}(\mathbf{q}^*), \tag{16}$$

where $\mathbf{q}^* \in F$ is a joint configuration characterized by a high manipulability index [3] and $\delta^{\mathtt{m}\prime}$ is a small positive number. The $\delta^{\mathtt{m}\prime}$ is introduced so that $\mathbf{q}$ can come sufficiently close to $\mathbf{q}^*$ in finite time using a large class of control laws $\mathbf{u}_q^{\mathtt{m}}$ (not necessarily finite time convergent control).

The idea is to pick a $\mathbf{q}^*$ and bring $\mathbf{q}$ sufficiently close to it before switching to $\mathtt{e}$ so that the next switch $\mathtt{e} \to \mathtt{t} \to \mathtt{m}$ do not occur until after at least a time $\Delta\tau^{\mathtt{e}} > 0$ have passed wherefore a minimum of progress in the objective of moving the EE can be made after every switching sequence of the form $\mathtt{e} \to \mathtt{t} \to \mathtt{m} \to \ldots \to \mathtt{m} \to \mathtt{t} \to \mathtt{e}$. Stated in another way, the system has a dwell time $\Delta\tau^{\mathtt{e}}$ in $\mathtt{e}$. Almost global asymptotical stability of the desired objective can then be established by formulating the switching logic for $\sigma \in \{\mathtt{m}, \mathtt{p}\}$ such that the system always returns to $\mathtt{e}$ [13], *i.e.* $\sigma = \mathtt{e}$ for infinitely long time.

This requires that $\mathbf{q}^*$ of the condition (16) is chosen such that $\|\mathbf{J}^{\dagger}(\mathbf{q}^*)\| < \rho'$ and $d(\mathbf{q}^*, F^{\mathrm{c}}) > \varepsilon^{\mathtt{e}\prime}$ holds with some margin since there will then be a dwell time $\Delta\tau^{\mathtt{e}}$ in $\mathtt{e}$. More precisely, for some $\Delta\tau^{\mathtt{e}} > 0$ we need a $\mathbf{q}^*$ such that

$\|\mathbf{J}^{\dagger}(\mathbf{q})\| < \rho'$ and $d(\mathbf{q}, F^{\mathtt{c}}) > \varepsilon^{\mathtt{e}\prime}$ for all $\mathbf{q} \in B_{\delta^{\mathtt{m}}}(\mathbf{q}^*)$ where $\delta^{\mathtt{m}}$ satisfies

$$\delta^{\mathtt{m}} \geq \delta^{\mathtt{m}\prime} + \Delta\tau^{\mathtt{e}} \max_{\substack{\mathbf{q}\in B_{\delta^{\mathtt{m}}}(\mathbf{q}^*),\\ u_v^{\mathtt{e}}, u_\omega^{\mathtt{e}}, \mathbf{u}_e^{\mathtt{e}}}} \|\mathbf{u}_q^{\mathtt{e}}\|.$$

The fact that $\|\mathbf{J}^{\dagger}(\mathbf{q}^*)\| < \rho'$ allows us to bound $\|\mathbf{u}_q^{\mathtt{e}}\|$ in a neighborhood of $\mathbf{q}^*$ and ensure the existence of a $\delta^{\mathtt{m}}$ for sufficiently small $\Delta\tau^{\mathtt{e}}$ in a manner similar to that of [13].

The switch $\mathtt{m} \to \mathtt{t} \to \mathtt{p}$ occurs when

$$|\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle| \leq \varepsilon^{\mathtt{m}\prime}, \tag{17}$$

where $\mathbf{Re}_1$ is a vector parametrization of the mobile platform orientation and $\varepsilon^{\mathtt{m}\prime}$ is a small positive number. Note that as $|\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle|$ becomes zero, the terms including $\mathbf{Re}_1$ and $\mathbf{Sp}_{em}$ in (9) will be parallel and the input signals u_v^σ and u_ω^σ will hence have the same instantaneous effect on the EE velocity, *i.e.* the mapping from u_v^σ and u_ω^σ to $\mathbf{u}_e^{\mathtt{e}}$ will become singular (an illustration is provided in Fig. 5).

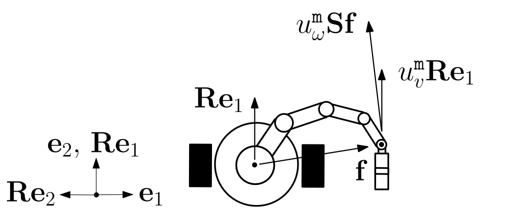

Fig. 5: A robot for which $\mathbf{p}_{jm} = \mathbf{0}$ and $|\langle \mathbf{Re}_1, \mathbf{f}\rangle|$ is small.

Knowing that (17) is not fulfilled in $\mathtt{m}$ tells us there is an $\varepsilon^{\mathtt{m}}$ such that $0 < \varepsilon^{\mathtt{m}} < \varepsilon^{\mathtt{m}\prime}$ and $|\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle| \geq \varepsilon^{\mathtt{m}}$ for $\sigma = \mathtt{m}$. This allows us to find a bounded solution of (9) with $\mathbf{u}_e^{\mathtt{m}} = \mathbf{0}$ and $u_v^{\mathtt{m}}$, $u_\omega^{\mathtt{m}}$ expressed in terms of $\mathbf{u}_q^{\mathtt{m}}$,

$$\begin{bmatrix} u_v^{\mathtt{m}} \\ u_\omega^{\mathtt{m}} \end{bmatrix} = \frac{1}{\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle} \begin{bmatrix} \langle \mathbf{p}_{em}, \mathbf{J}\mathbf{u}_q^{\mathtt{m}}\rangle \\ \langle \mathbf{Re}_2, \mathbf{J}\mathbf{u}_q^{\mathtt{m}}\rangle \end{bmatrix}. \tag{18}$$

Recall that there is a feasible path from any feasible $\mathbf{q}$ to $\mathbf{q}^*$ due to the characterization of F in Assumption 1. Since (6) is a linear system we can make $\mathbf{q}$ move towards $\mathbf{q}^*$ along such a path using standard techniques.

Finally, we need to assure that $|\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle| \geq \varepsilon^{\mathtt{m}}$ holds also in the transition phase $\mathtt{m} \to \mathtt{t} \to \mathtt{p}$. This is fulfilled if we chose $\varepsilon^{\mathtt{m}}$ and $\varepsilon^{\mathtt{m}\prime}$ such that

$$\varepsilon^{\mathtt{m}\prime} = \varepsilon^{\mathtt{m}} + \Delta\tau^{\mathtt{t}} \max_{\mathbf{q}, u_v^{\mathtt{m}}, u_\omega^{\mathtt{m}}} |\langle \dot{\mathbf{R}}\mathbf{e}_1, \mathbf{p}_{em}\rangle + \langle \mathbf{Re}_1, \dot{\mathbf{p}}_{em}\rangle|,$$

where the optimization is subject to $\mathbf{q}, u_v^{\mathtt{m}}$, and $u_\omega^{\mathtt{m}}$ being feasible.

D. Mobile platform mode

In mobile platform mode ($\mathtt{p}$) we want to change the platform pose in finite time so that the system can return to performing manipulator mode control. Introduce the constraints

$$\dot{\mathbf{p}}_e = \mathbf{u}_e^{\mathtt{p}} = \mathbf{0}, \tag{19}$$

$$\dot{q}_i = 0,\ i \in \{2, 3, \ldots, n\} \tag{20}$$

The purpose of (19)–(20) is to avoid having the control in $\mathtt{p}$ interfere with the progress in $\mathtt{e}$ and $\mathtt{m}$ towards the goal of making the EE reach its desired set-point and the joint variables approach $\mathbf{q}^*$ respectively.

The switch $\mathtt{p} \to \mathtt{t} \to \mathtt{m}$ occurs when

$$|\langle \mathbf{Re}_1, \mathbf{p}_{em}\rangle| \geq \delta^{\mathtt{p}}, \tag{21}$$

where $\delta^{\mathtt{p}}$ satisfies $\delta^{\mathtt{p}} > \varepsilon^{\mathtt{m}\prime}$ and $\delta^{\mathtt{p}} < \|\mathbf{f}\|$. The switch is achieved by changing $\mathbf{Re}_1$ (*i.e.* the platform orientation) as explained below. The fact that $\delta^{\mathtt{p}} > \varepsilon^{\mathtt{m}\prime}$ assures us that there is a dwell time $\Delta\tau^{\mathtt{m}}$ in $\mathtt{m}$ (or at least during switching sequences that do not include $\mathtt{e}$).

The control signals in $\mathtt{p}$ are chosen such that (9) holds with $\dot{q}_2, \ldots, \dot{q}_k = 0$, and $\mathbf{f} = \mathbf{p}_{em} - \mathbf{p}_{jm}$ constant, *i.e.*

$$\dot{\mathbf{f}} = u_\omega^{\mathtt{p}}\mathbf{S}(\mathbf{p}_{jm} + \mathbf{f}) + \mathbf{J}\dot{q}_1\mathbf{e}_1 - u_\omega^{\mathtt{p}}\mathbf{S}\mathbf{p}_{jm} = \mathbf{0}. \tag{22}$$

Note that $\mathbf{J}\dot{q}_1\mathbf{e}_1 = \dot{q}_1\mathbf{Sf}$ since the first joint is revolute by Assumption 1. The control in $\mathtt{p}$ is given by

$$\begin{aligned} u_\omega^{\mathtt{p}} &= \operatorname{sgn}(\langle \mathbf{p}_{em}, \mathbf{Re}_2\rangle)\langle \mathbf{p}_{em}, \mathbf{Re}_2\rangle, \\ u_v^{\mathtt{p}} &= \operatorname{sgn}(\langle \mathbf{p}_{jm}, \mathbf{Re}_2\rangle)\|\mathbf{p}_{jm}\| u_\omega^p, \\ \mathbf{u}_q^{\mathtt{p}} &= -u_\omega^p \mathbf{e}_1, \end{aligned}$$

where the last equality implies $\dot{q}_1 = -u_\omega^p$. Note that (22) is satisfied regardless of $u_v^{\mathtt{p}}$. What is more, plugging the control into (9) we obtain $\mathbf{u}_e^{\mathtt{p}} = \mathbf{0}$ as required. The choice of control signals in the case of $\mathbf{p}_{jm} = \mathbf{0}$ is illustrated in Fig. 6.

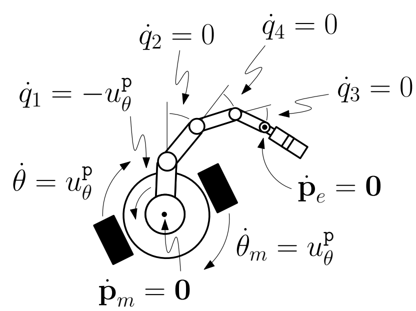

Fig. 6: A mobile robot with $\mathbf{p}_{mj} = \mathbf{0}$ in platform mode.

E. Transition mode

In transition mode ($\mathtt{t}$) we want to connect the input signals of the previous mode (σ) with that of the next (ς) in a continuous fashion. This is done by multiplying all signals by a time-dependent gain $g(t)$, essentially ramping down the previous signal to zero and then ramping up the next signal to its value at the current states.

Consider the case of a switching sequence $\sigma \to \mathtt{t} \to \varsigma$ where the switch $\sigma \to \mathtt{t}$ takes place at time τ. Then

$$\mathbf{u}_i^{\mathtt{t}} = \begin{cases} g(t-\tau)\mathbf{u}_i^{\sigma} & \text{if } t - \tau \leq \Delta\tau^{\mathtt{t}} \\ g(t-\tau)\mathbf{u}_i^{\varsigma} & \text{if } t - \tau > \Delta\tau^{\mathtt{t}}, \end{cases} \tag{23}$$

where $\Delta\tau^{\mathtt{t}}$ is a positive constant corresponding to half the time spent in $\mathtt{t}$ and $g : [0, 2\Delta\tau^{\mathtt{t}}] \to [0, 1]$ is a Lipschitz continuous function that satisfies, $g(0) = 1$, $g(\Delta\tau^{\mathtt{t}}) = 0$, and $g(2\Delta\tau^{\mathtt{t}}) = 1$.

V. NUMERICAL EXAMPLES

The proposed algorithm as simulated in MATLAB for the task of moving the EE position of a 3R mobile manipulator from $\mathbf{p}_e(0)$ to the origin while maintaining the constraints

$$\|\mathbf{J}^{\dagger}\| \leq 5,\ \forall t \text{ s.t. } \sigma(t) = \mathtt{e} \text{ or } \sigma(t) = \mathtt{t},\ \sigma(t - \Delta\tau^{\mathtt{t}}) = \mathtt{e},$$
$$\mathbf{q} \in F = \left\{ \mathbf{q} \in \mathbb{S}^3 \,\middle|\, -\frac{2\pi}{3} \leq q_i \leq \frac{2\pi}{3},\ i \in \{2, 3\} \right\}.$$

We used the control laws $\mathbf{u}_e^{\mathtt{e}}$, $\mathbf{u}_q^{\mathtt{m}}$ from [13] and put $u_v^{\mathtt{e}}$, $u_\omega^{\mathtt{e}}$ to zero. The linear control $\mathbf{u}_q^{\mathtt{m}}$ is bounded since $\mathbf{q} \in \mathbb{S}^3$ and respects the constraints due to F being convex. The algorithm performed as required.

Consider the case of an initial condition $\mathbf{p}_e(0) = [2\ 2]^\top$ and $\mathbf{q}(0) = \mathbf{q}^* = [0.8\,1.5\,0.5]^\top$. The parameters were tuned, in particular we set $\rho' = 4$. Fig. 7 shows the manipulator first reaching out until $\|\mathbf{J}^{\dagger}\| = 4$ at time 1.2 (see Fig 8). The platform then moves until $\mathbf{q} = \mathbf{q}^*$ again. Note from Fig. 8 that $\|\mathbf{J}^{\dagger}\|$ approaches the value 5 at time 1.5. This does not risk a violation of the norm constraint since $\sigma = \mathtt{m}$ at that time (the joint constraints are never close to being violated).

The motion sequence consisting of the manipulator reaching out followed by the platform repositioning itself is repeated three times (the number of separate intervals on which $\sigma = \mathtt{e}$ in Fig 8). It is occurs due to the choice of $u_v^{\mathtt{e}} = u_\omega^{\mathtt{e}} = 0$. Setting the mobile platform inputs to zero is sometimes reasonable in practice, for example if the EE is to follow a path which lies close to the platform most of the time. For most tasks it will however be beneficial to use nonzero controls $u_v^{\mathtt{e}}$, $u_\omega^{\mathtt{e}}$. Also note the two manipulator configurations in Fig. 7 that are similar to the one in Fig. 5. They occur at times instances that correspond to the two occasions $\sigma = \mathtt{p}$ in Fig. 8.

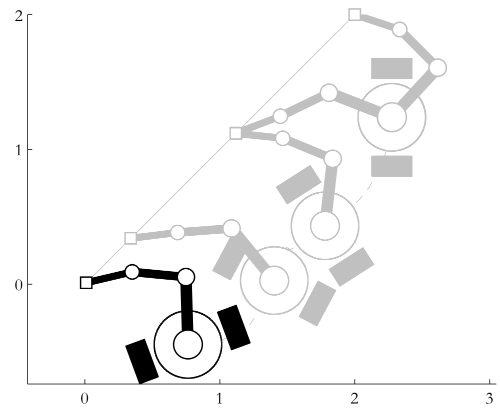

Fig. 7: The robot trajectory in work space at the current time (black) and three previous time instances (grey).

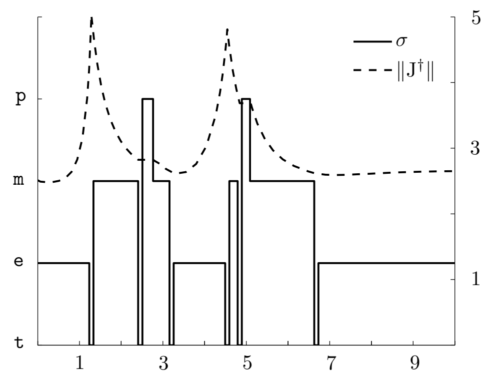

Fig. 8: The values of σ (left axis) and $\|\mathbf{J}^{\dagger}\|$ (right axis) as functions of time.

REFERENCES

[1] B. Bayle, J.Y. Fourquet, and M. Renaud, "Manipulability of wheeled mobile manipulators: Application to motion generation," *The International Journal of Robotics Research*, vol. 22, no. 7-8, p. 565, 2003.

[2] R.W. Brockett, "Asymptotic stability and feedback stabilization," in *Differential Geometric Control Theory*, R.W. Brockett, R.S. Millman, and H.J. Sussmann, Eds. Birkauser, 1983.

[3] B. Siciliano and O. Khatib, *Springer handbook of robotics*. Springer-Verlag, 2008.

[4] C. W. Wampler, "Manipulator inverse kinematic solutions based on vector formulations and damped least-squares methods," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 16, no. 1, pp. 93–101, 1986.

[5] T. F. Chan and R. V. Dubey, "A weighted least-norm solution based scheme for avoiding joint limits for redundant joint manipulators," *Robotics and Automation, IEEE Transactions on*, vol. 11, no. 2, pp. 286–292, 1995.

[6] S. Chiaverini, "Singularity-robust task-priority redundancy resolution for real-time kinematic control of robot manipulators," *Robotics and Automation, IEEE Transactions on*, vol. 13, no. 3, pp. 398–410, 1997.

[7] B. Bayle, J.-Y. Fourquet, F. Lamiraux, and M. Renaud, "Kinematic control of wheeled mobile manipulators," in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 2, 2002, pp. 1572–1577.

[8] J. Tan, N. Xi, and Y. Wang, "Integrated task planning and control for mobile manipulators," *The International Journal of Robotics Research*, vol. 22, no. 5, pp. 337–354, 2003.

[9] L. Sentis and O. Khatib, "Synthesis of whole-body behaviors through hierarchical control of behavioral primitives," *International Journal of Humanoid Robotics*, vol. 2, no. 04, pp. 505–518, 2005.

[10] A. De Luca, G. Oriolo, and P. R. Giordano, "Kinematic modeling and redundancy resolution for nonholonomic mobile manipulators," in *IEEE International Conference on Robotics and Automation*, 2006, pp. 1867–1873.

[11] G. Antonelli, "Stability analysis for prioritized closed-loop inverse kinematic algorithms for redundant robotic systems," *Transactions on Robotics*, vol. 25, no. 5, pp. 985–994, 2009.

[12] J. Markdahl, Y. Karayiannisdis, X. Hu, and D. Kragic, "Distributed cooperative object attitude manipulation," in *Proceedings of the IEEE International Conference on Robotics and Automation*, 2012, pp. 2960–2965.

[13] J. Markdahl, Y. Karayiannisdis, and X. Hu, "Cooperative object path following control by means of mobile manipulators: a switched systems approach," in *Proceedings of the IFAC Symposium on Robot Control*, 2012, pp. 773–778.

[14] B. Siciliano, L. Sciavicco, and L. Villani, *Robotics: Modelling, Planning and Control.* Springer Verlag, 2009.