Curiosity Based Exploration for Learning Terrain Models

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Abstract—We present a robotic exploration technique in which the goal is to learn a visual model that can be used to distinguish between different terrains and other visual components in an unknown environment. We use ROST, a realtime online spatiotemporal topic modeling framework to model these terrains using the observations made by the robot, and then use an information theoretic path planning technique to define the exploration path. We conduct experiments with aerial view and underwater datasets with millions of observations and varying path lengths, and find that paths that are biased towards locations with high topic perplexity produce better terrain models with high discriminative power.

I. INTRODUCTION

This work presents an exploration technique using a realtime online topic modeling framework, which models the cause of the observations made by a robot with a latent variable (called topic) that is representative of different kinds of terrains or other visual constructs in the scene, and then uses simple gradient ascent in semantic information space to find an exploratory path through the world.

We define curiosity as the unsupervised act of moving through the world in order to seek novel observations with high information content. We posit that observation data collected from such paths that seek novelty and maximize information gain would result in better terrain models. Computing information gain on low level sensor data, which in the case of vision corresponds to pixel colors or edges, might not work in many scenarios where we are interested in modeling more abstract visual constructs. Hence, we propose the use of a topic modeling framework, which have been shown to produce semantic labeling of text [1] and images [2], including satellite maps [3].

In this work we use a realtime online spatiotemporal topic modeling technique called ROST [4] that is suitable for use in the robotic exploration context. ROST allows for topic modeling of streaming data (observations made by a robot over time), while taking into account the spatial and temporal distribution of the data. Moreover, ROST can process the incoming observation data in real time, while providing a very close approximation to traditional batch implementations [4].

At each time step, we add the observations from the current location to the topic model, and compute the perplexity of the observations from the neighboring locations. This perplexity score, along with a repulsive potential from previously visited locations, is then used to bias the probability of next step in the path. Since observations with high

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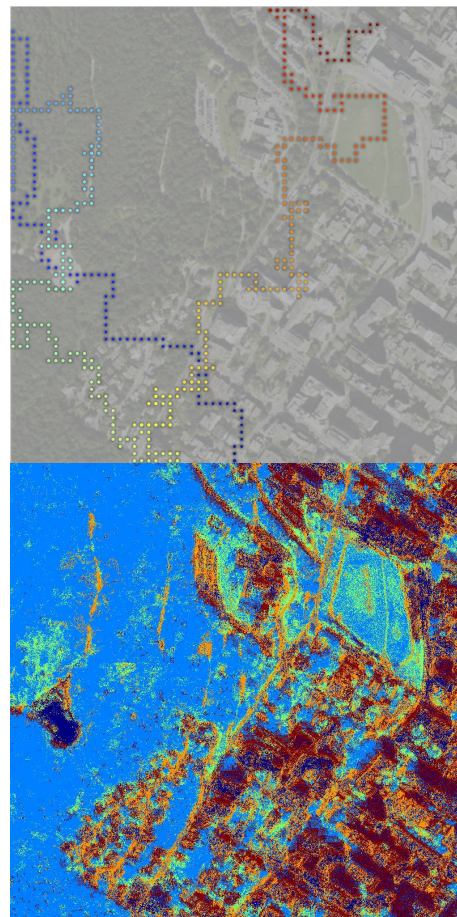
Fig. 1. Example of an exploratory path (top) produced by the proposed technique on a satellite map. The path begins in Blue, and ends in Red. Output of this exploration is a terrain model, which when applied to the observation from entire map produces terrain label for every location(bottom). Different colors represent different terrains.

perplexity have high information gain, we claim that this approach would results in faster learning of the terrain topic model, which would imply shorter exploration paths for the same accuracy in predicting terrain labels for unseen regions.

The main contribution of this work is in demonstrating that first, robots can use online topic modeling to learn a visual terrain model with no supervision, and second, that even with the use of a simple greedy exploration approach, we can achieve improved results.

II. PRIOR WORK

Autonomous exploration is a well studied field in robotics, and there are different variants of the exploration problem.

A. Exploration for Navigation

Navigating a robot through free space is a fundamental problem in robotics. Yamauchi [5] defined exploration as the "act of moving through an unknown environment while building a map that can be used for subsequent navigation". Yamauchi's proposed solution involved moving the robot towards the frontier regions in the map, which were described as the boundary between known free space and the uncharted territories.

If we have an inverse sensor model of the range sensor, it is possible to compute locations in the world which would maximize the utility of the sensor reading in resolving obstacle position and shape. Grabowski [6] proposed such an exploration strategy where the goal is to maximize the understanding of obstacles rather than the exposure to free space. In this approach, the robot identifies the location with next best view in space where a sonar sensor reading would have the greatest utility in improving the quality of representation of an obstacle.

If there is no external localizer available to the robot, then it is desirable that robot explores, maps and localizes in the environment at the same time [7][8][9][10]. Bourgault [2] and Stachniss [11] have proposed an exploration strategy which moves the robot to maximize the map information gain, while minimizing the robot's pose uncertainty.

B. Exploration for Monitoring Spatiotemporal Phenomenon

In underwater and aerial environments, obstacle avoidance is typically not the primary concern, but many different kinds of high level exploration tasks still exist.

Binney [12] has described an exploration technique to optimize the monitoring spatiotemporal phenomena by taking advantage of the submodularity of the objective function. Bender [13] has proposed a Gaussian process based exploration technique for benthic environments, which uses an experiment specific utility function. Das et al. [14] have presented techniques to autonomously observe oceanographic features in the open ocean. Hollinger et al. [15] have studied the problem of autonomously studying underwater ship hulls by maximizing the accuracy of sonar data stream. Smith et al. [16] have looked at computing robot trajectories which maximize information gained, while minimizing the deviation from the planned path.

Thompson et al. [17] have experimented with autonomous planetary surveys using a rover and an orbiter providing non-local information. They used a gaussian process to model sensor data uncertainty, using the data from the orbiter and the rover, and then planned a path maximizing information gain.

III. Terrain Modeling

A. Topic Models

Topic modeling methods were originally developed for text analysis. Probabilistic Latent Semantic Analysis (PLSA) proposed by Hoffman [18], models the probability of observing a word w_i in a given document M as:

$$\mathbf{P}(w_i = v|M) = \sum_{k=1}^{K} \mathbf{P}(w_i = v|z_i = k)\mathbf{P}(z_i = k|M). \quad (1)$$

where v takes a value between $1 \ldots V$, the vocabulary size, and z_i is the hidden variable or topic label for w_i. Topic label z_i takes a value between $1 \ldots K$, where K is a much smaller than V. The central idea here is the introduction of the latent variable z, which models the underlying topic, or the context responsible for generating the word. Latent Dirichlet Allocations [1] improve upon PLSA by placing Dirichlet priors on $\mathbf{P}(w|z)$ and $\mathbf{P}(z|M)$, which bias these distributions to be sparse, preventing overfitting.

Spatial LDA (SLDA) [19] is an extension to LDA allowing it to be useful on data with spatial information, such as images. SLDA models the documents as spatial regions in an image, and models them as another random variable. A word is likely to be assigned to a document if its location is close to other words in the document, and has the same topic label as other words in the document. This approach is however not suitable for realtime implementation because computing the topic distribution of the spatial context (document) of the word requires $O(W_M)$ time, where W_M is the number of words in an image. ROST on the other hand models the spatial and temporal context of the words using only the spatiotemporal information and not the topic label. As a result, it is able to compute the topic distribution of the spatiotemporal context of a word in $O(1)$ time through the use of a cellular decomposition of the spatiotemporal data.

For modeling visual data observed by the robot, instead of text words, we use two different kinds of visual words: Oriented BRIEF (ORB) [20] based visual words [21] that describing local visual features, and texton words [22] in Lab color space to describe texture properties of a region. Moreover, instead of documents, we compute the prior topic distribution for a given word by taking into account the topic distribution in its spatial neighborhood. We posit that the resulting topic labels modeled by the system then represent high level visual patterns that are representative of different terrain types in the world.

B. Generative Process for Observations

We assume the following generative process for observations produced by the spatial region being explored. The world is decomposed into C cells, in which each cell $c \in C$ is connected to its neighboring cells $G(c) \subseteq C$. In this paper, we only experiment with grid decomposition of the world, where each cell is connected to its four nearest neighbors. However, the general ideas presented here are applicable to any topological decomposition of space-time. Note that neighborhoods of neighboring cells are overlapping, which is essential to model continuity in space-time.

Each cell is modeled by a mixture of at most K different kinds of terrain topics, each of which when observed by a robot can result in one of the V different types of visual words. Let $\theta_{G(c)}$ be the distribution of topics in and around the cell. Intuitively, we would like visual words with the

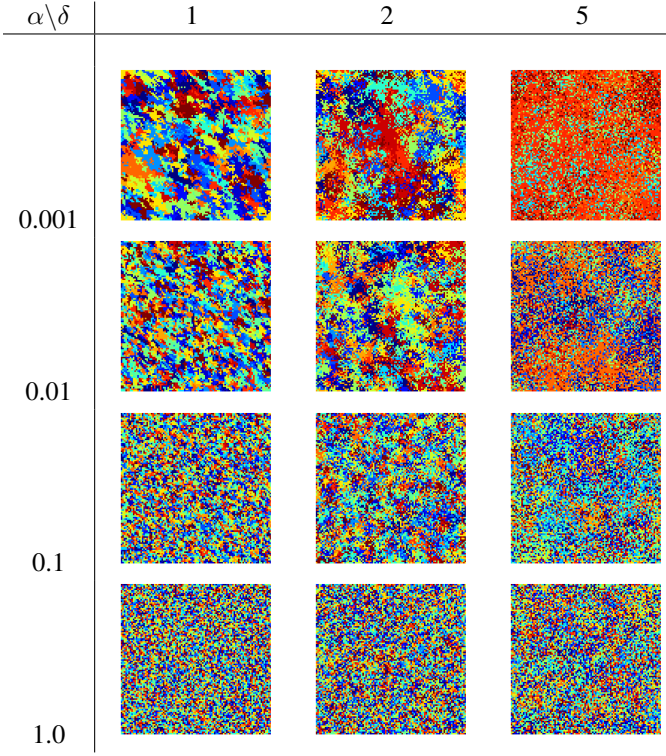| $\alpha\backslash\delta$	1	2	5
0.001			
0.01			
0.1			
1.0			

Fig. 2. Dirichlet Priors for 2D Maps. The table shows random maps sampled from the generative process used to characterize spatial terrain information in 2D worlds. Columns show variation in neighborhood size δ, and rows show variation in Dirichlet concentration parameter α. We see that changes in α control cluster sizes, whereas changes in δ control mixing of adjacent clusters.

same topic to cluster together in space. This phenomena can be modeled by placing a Dirichlet prior on $\theta_{G(c)}$.

Figure 2 shows random samples from the generative process used to describe these terrain topic distribution in a 2D map, with 16 topics. As we vary the Dirichlet concentration parameter α and the neighborhood size δ, we see that smaller α results in fewer topics in the neighborhood of any given cell, and smaller δ results is clusters with less mixing.

Similar to LDA, we describe each topic k by a word distribution ϕ_k over V different types of visual words, and ϕ_k is assumed to have a Dirichlet prior with parameter β. This Dirichlet prior puts a constraint on the complexity of the terrain being described by this topic. Topic model $\Phi = \{\phi_k\}$ is a $K \times V$ matrix that encodes the global topic description information shared by all cells.

The overall generative process for a word w in cell c is thus described as following:

1) Word distribution for each topic k:

$$\phi_k \sim \mathrm{Dirichlet}(\beta)$$

2) Topic distribution for words in neighborhood of c :

$$\theta_{G(c)} \sim \mathrm{Dirichlet}(\alpha)$$

3) Topic label for w:

$$z \sim \mathrm{Discrete}(\theta_{G(c)})$$

4) Word label:

$$w \sim \mathrm{Discrete}(\phi_z)$$

where $x \sim Y$ implies that random variable x is sampled from distribution Y.

C. Gibbs Sampling

At each time step, we add the observed words to their corresponding cells, and use a Gibbs sampler to update and refine the topic labels until the next time step.

Let $P = \{p_1, \ldots, p_T\}$, $p_i \in C$, be the set of cells in the current path at time T.

The posterior topic distribution for a word w_i in cell $p_t = c$ is given by:

$$\begin{aligned}\mathbf{P}(z_i = k | w_i = v, p_t = c) &\propto \frac{n_{k,-i}^v + \beta}{\sum_{v=1}^V (n_{k,-i}^v + \beta)} \cdot \\ &\quad \frac{n_{G(c),-i}^k + \alpha}{\sum_{k=1}^K (n_{G(c),-i}^k + \alpha)},\end{aligned} \quad (2)$$

where $n_{k,-i}^v$ counts the number of words of type v in topic k, excluding the current word w_i, and $n_{G(c),-i}^k$ is the number of words with topic label k in neighborhood of cell c, excluding the current word w_i, and α, β are the Dirichlet hyper-parameters. Note that for a neighborhood size of 0, $G(c) = c$, and the above Gibbs sampler is equivalent to the LDA Gibbs sampler proposed by Griffiths et al. [23].

Several different strategies exist in the literature to do online refinement of the topic label assignment on streaming data [24]. However, in this work, we are interested in the more constrained realtime version of the problem. After each new observation, we only have a constant amount of time to do topic label refinement, hence any online refinement algorithm that has computational complexity which increases with new data is not applicable.

We then must use a refinement strategy which only partially updates the topic labels after each time step. To ensure that the topic labels from the last observation converge before the next observation arrives, at each time step, for each refine iteration, we refine the last observation with probability τ, or a previous observation with probability $(1 - \tau)$. We pick the previous observation using age proportional random sampling. We found $\tau = 0.5$ to work well in most cases, however on faster machines, τ could be set to a lower value, which would encourage better globally optimal topic labels. Algorithm 1 summarizes the proposed realtime topic refinement strategy.

IV. CURIOSITY BASED EXPLORATION

At time t, let the robot be in cell $p_t = c$, and let $G(c) = \{g_i\}$ be the set of cells in its neighborhood. We would like to compute a weight value for each g_i, such that

$$\mathbf{P}(p_{t+1} = g_i) \propto \mathrm{weight}(g_i) \quad (3)$$

In this work we consider a four different weight functions.

```
while true do
    while no new observation do
        a ∼ Bernoulli(τ)
        if  a == 0 then
            (*select last observation*)
            t ← T
        else
            (*pick an observation with probability
            proportional to its timestamp*)
            t ← q, P(q = j) ∝ j, 1 ≤ j ≤ T
        end
        foreach  word wᵢ in pₜ do
            (*update the topic label for word in the
            observation *)
            zᵢ ∼ P(zᵢ = k|wᵢ = v, pₜ = c)
        end
    end
    T ← T + 1
    Add new observed words to their corresponding
    cells.
end
```

Algorithm 1: Keep topic labels up-to-date as new observations arrive.

1) Random Walk - Each cell in the neighborhood is equally likely to be the next step:

$$\text{weight}(g_i) = 1. \tag{4}$$

2) Stochastic Coverage - Use a potential function to repel previously visited locations:

$$\text{weight}(g_i) = \frac{1}{\sum_j n_j/d^2(p_t, c_j)}. \tag{5}$$

where n_j is the number of times we have visited cell c_j, and $d(p_t, c_j)$ is the Euclidean distance between these two cells.

3) Word Perplexity - Bias the next step towards cells which have high word perplexity:

$$\text{weight}(g_i) = \frac{\text{WordPerplexity}(g_i)}{\sum_j n_j/d^2(p_t, g_j)}. \tag{6}$$

4) Topic Perplexity - Bias the next step towards cells which have high topic perplexity:

$$\text{weight}(g_i) = \frac{\text{TopicPerplexity}(g_i)}{\sum_j n_j/d^2(p_t, g_j)}. \tag{7}$$

We compute the word perplexity of the words observed in g_i by taking the inverse geometric mean of the probability of observing the words in the cell, given the current topic model and the topic distribution of the path thus far.

$$\text{WordPerplexity}(g_i) =$$
$$\exp\left(-\frac{\sum_i^W \log \sum_k P(w_i = v|k)P(k|P)}{W}\right), \tag{8}$$

where W is the number of words observed in g_i, $P(w_i = v|k)$ is the probability of observing word v if its topic label

Dataset	width(px)	height(px)	n.cells	n.words
Montreal1	1024	1024	4096	3,239,631
Montreal2	1024	1024	4096	1,675,171
SouthBellairs	2500	2500	6241	1,664,749

TABLE I

DATASET SPECIFICATIONS

is k, and $P(k|P)$ is the probability of seeing topic label k in the path executed by the robot thus far.

To compute topic perplexity of the words observed in g_i, we first compute topic labels z_i for these observed words by sampling them from the distribution in Eq. 2, without adding these words to the topic model. These temporary topic labels are then used to compute the perplexity of g_i in topic space.

$$\text{TopicPerplexity}(g_i) =$$
$$\exp\left(-\frac{\sum_i^W \log P(z_i = k|P)}{W}\right). \tag{9}$$

V. EXPERIMENTS

To validate our hypothesis that biasing exploration towards high perplexity cells will result in a better terrain topic model of the environment, we conducted the following experiment. We considered three different maps, two aerial view, and one underwater coral reef map, the specifications of which is given in Table I. We extracted ORB words describing local features, and texton words describing texture at every pixel (every second pixel for the SouthBellairs underwater dataset). ORB words had a dictionary size of 5000, and texton words had a dictionary size of 1000. The dictionary was computed by extracting features from every 30th frame of a completely unrelated movie[1].

Each of these maps were decomposed into square cells of width 16 (32 for SouthBellairs). Now for each weight function, we computed exploration paths of varying length, with 20 different random restart locations for each case. Each time step was fixed at 200 milliseconds to allow the topic model to converge. We limited the path length to 320 steps, which is about $5\sqrt{|C|}$.

Each of these exploration runs returned a topic model Φ_p, which we then used to compute topic labels for each pixel in the map in batch mode. Let Z_p be these topic labels. We compared this topic labeling with two other labelings: human labeled ground-truth Z_h, and labels computed in batch mode Z_b, with random access to the entire map. An example of this labeling for each of the three dataset is shown in Fig. 3 (j,k,l). Here different colors correspond to different topic labels, and hence regions with the same color, within a map, correspond to similar terrains. Note that there is no relation between colors of topic labels between different maps.

We then computed the mutual information between Z_p and Z_h, Z_p and Z_b, and plotted the results as a function of path length, as shown in Fig. 4.

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[1]We used the documentary movie Baraka(1992) for extracting visual feature, because of its rich visuals from many different contexts
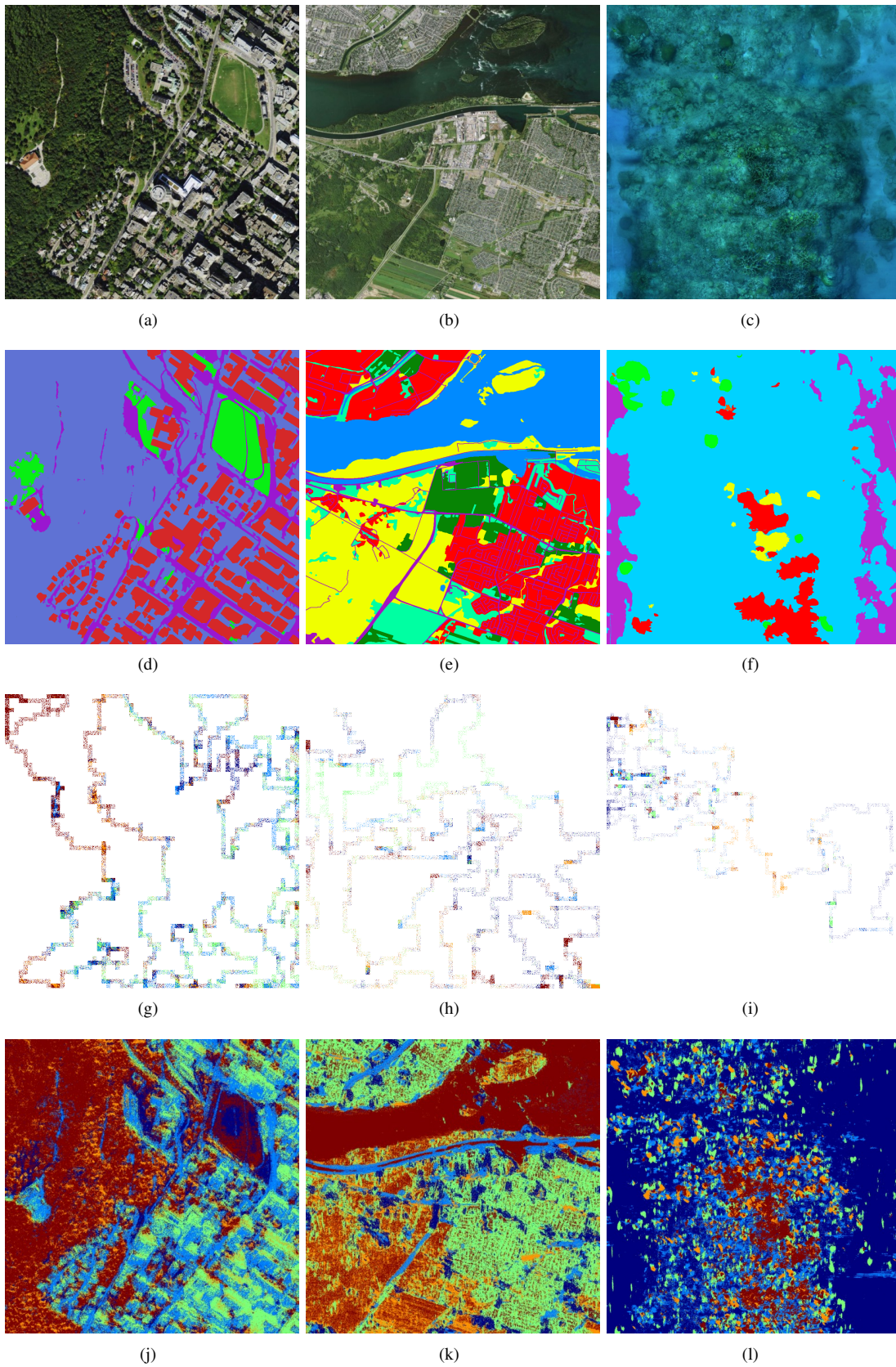
Fig. 3. (a)-(c) Input image used to generate observation data, (d)-(f) Groundtruth labeling. (g)-(i) An example path and the topic labels computed online. Parts of the path with higher density of points is indicative of multiple passes through that cell. (j)-(l) Terrain labeling of the map using the topic model computed on the path. Similar colors in the ground truth and topic labels correspond to similar terrains, however the colors between differnt ground truth and topic label maps are not related.
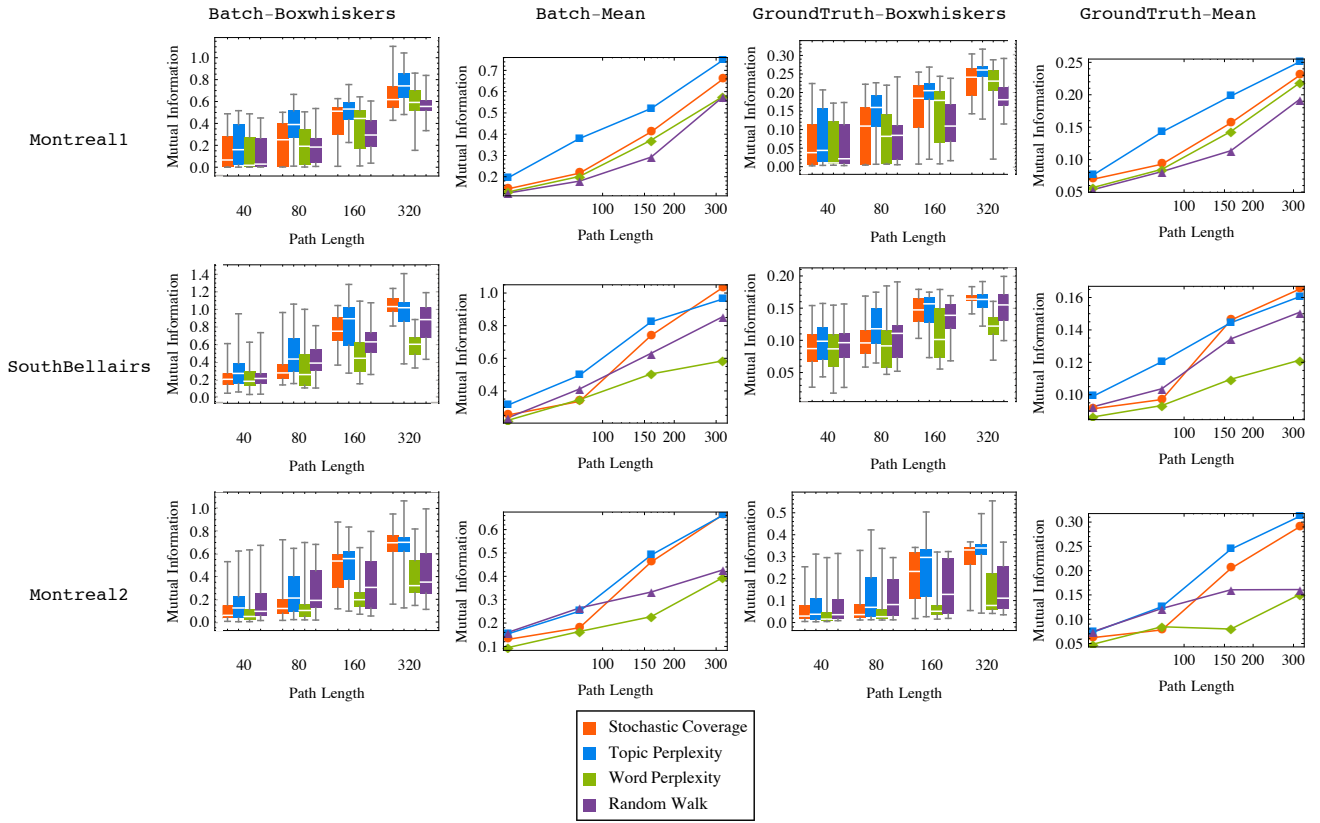
Fig. 4. Evaluation of the proposed exploration techniques. The plots show mutual information between the maps labeling produced using the topic model computed online during the exploration, with maps labeled by a human, and maps labeled by batch processing of the data.

ROST was run with Dirichlet hyper-parameters $\alpha = 0.01$ and $\beta = 0.1$, and number of topics $K = 5$.

VI. Results and Discussion

The results are both encouraging and surprising. As shown in Fig. 4, we see that topic perplexity based exploration (shown with blue squares) performs consistently better than all other weight functions, when compared against ground truth, or the batch results.

For paths of length 80, which is close to the width of the maps, we see that mutual information between topic perplexity based exploration and ground truth is 1.51, 1.20 and 1.05 times higher respectively for the three datasets, compared to the next best performing technique.

For long path lengths (320 steps or more), stochastic coverage (shown with orange circles) based exploration matches the mean performance of topic perplexity exploration. This is expected because the maps are bounded, and as the path length increases, the stochastic coverage algorithm is able to stumble across different terrains, even without a guiding function.

For short path lengths (40 steps or less), we do not see any statistical difference between the performance of different techniques.

Marked with purple triangles, we see the results of exploration using Brownian random motion. Although this strategy

has a probabilistic guarantee of asymptotically complete coverage, but it does so at a lower rate than stochastic coverage exploration startegy. A random walk in two dimensions is expected to travel a distance of $\sqrt{n}$ from start, where n is the number of steps. Hence it is highly likely that it never visits different terrains. The resulting topic models from these paths are hence unable to resolve between these unseen terrains.

The performance of word perplexity exploration (shown with green diamonds) is surprisingly poor in most cases. We hypothesize that this poor performance is due to the algorithm getting pulled towards locations with terrain described by a more complex word distribution. This will cause the algorithm to stay in these complex terrains, and not explore as much as the other algorithms. In comparison, the topic perplexity exploration is not affected by the complexity of the distribution describing the topic, and is only attracted to topic rarity.

VII. Conclusion

We have presented a novel exploration technique that aims to learn a terrain model for the world by finding paths with high information content. The use of a realtime online topic modeling framework allows us to model incoming streams of low level observation data via the use of a latent variable representing the terrain. Given this online model, we measure the utility of the potential next steps in the path. We validated the effectiveness of the proposed exploration

technique over candidate techniques by computing mutual information between the terrain maps generated using the learned terrain model and hand labeled ground truth, on three different datasets.

In our ongoing future work, we are in the process of implementing the proposed approach on Aqua[25] underwater robot and aerial vehicles. We expect the approach to be useful in studying delicate ecosystems by adaptively focusing data collection on regions with higher information content, which in physical world might relate to surprising phenomena.

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