A Dynamical Characterization of Internally-Actuated Microgravity Mobility Systems

Adam W. Koenig, Marco Pavone, Julie C. Castillo-Rogez, Issa A. D. Nesnas

Abstract— The in-situ exploration of small Solar System bodies (such as asteroids or comets) is becoming a central objective for future planetary exploration. Such bodies are characterized by very weak gravitational fields, which make hopping mobility platforms one of the preferred mobility strategies for microgravity surface exploration, as recognized by space agencies worldwide. However, little is known about the dynamical behavior of hopping platforms in low gravity environments, where small bodies' rotational dynamics can have a critical effect. Accordingly, the objective of this paper is to study in detail the "dynamic envelope" of hopping microgravity rovers, with a focus on internal actuation. Specifically, we first perform a static analysis with the goal of determining regions of a small body where an internally-actuated hopping rover can stably remain at rest. Then, we perform a dynamic analysis and discuss the actuation and instrument pointing performance of hopping microgravity platforms as a function of a number of system and environmental parameters (e.g., rover shape, body rotation rate). Finally, we tailor our analysis to a potential mission to Mars' moon Phobos. Collectively, our results show that internally-actuated rovers, from an actuation standpoint, are a viable mobility solution for a vast class of small Solar System bodies. Also, our analysis represents a key first step to develop path planning algorithms for microgravity explorers to safely explore dynamically feasible regions.

I. INTRODUCTION

The *in-situ* exploration of small Solar System bodies (such as asteroids, comets, or irregular satellites) is becoming a central objective for future planetary exploration [1], [5]. The primary scientific return of such missions would be the characterization of planetary material composition and chemistry, which holds the promise of shedding light on the origin and dynamical evolution of the Solar System [5]. Furthermore, such missions, by characterizing regolith mechanical properties, dust dynamics, and electrostatic charging, could pave the way for Human Exploration in the Solar System [17]. However, weak gravitational fields (micro-g to milli-g), which are characteristic of small bodies, make the development of microgravity robotic explorers a challenge,

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and the selection of the "best" mobility mechanism for *controlled* mobility on small bodies is an open problem.

Essentially, mobility mechanisms for micro-gravity environments can be broadly divided into four classes, namely, mobility via thrusters, wheeled mobility, legged mobility, and hopping mobility. In the absence of strong traction forces, wheeled vehicles are limited to extremely low speeds ($1.5\,\mathrm{mm/s}$ per previous JPL studies [10]). Use of thrusters poses risk of surface contamination, negatively affecting the science return of the mission. Additionally, thrusters are characterized by mechanical and operational complexity, and their lifetime is critically limited by propellant availability. Legged mobility systems are also characterized by mechanical complexity. Furthermore, contact forces between legs and the surface vary widely with surface characteristics, which cannot be accurately determined beforehand. Hopping systems use the low gravity levels to their advantage. Prototypes of hopping systems have been developed by several agencies including NASA [10], [7], RKA [14], ESA [6], and JAXA [2]. There are two types of hopping designs: (#1) the rover drives an external contact point into the ground [7], [16], and (#2) the hopper develops internal momentum to augment contact forces [18], [11]. Key advantages of hopping platforms include relatively simple actuation, ability to cover a large surface, and relative insensitivity to soil characteristics. Additionally, internally-actuated systems possess the key advantages of immunity to contamination from surface dust and simplified thermal control.

Despite the apparent consensus about the advantages of hopping platforms for small bodies exploration, little is known about their dynamical behavior in low gravity environments, where small bodies' rotational dynamics can have a critical effect. Consider, for example, a small spherical body of uniform density. Depending on the class of asteroid, density can range from approximately $1\,\mathrm{g/cm^3}$ to $6\,\mathrm{g/cm^3}$. Using Newton's formula for gravitational acceleration, one can estimate the gravity on the surface of a uniform spherical body according to $a_g = 4\,G\,\pi\rho\,r/3$, where ρ is the (constant) object's density, G is the gravitational constant, and r is the radius of the body. Now, assume that the body is rotating. At the equatorial position, the observed surface acceleration at the equator is reduced by the centripetal acceleration, denoted by a_c. To ensure that the rover stays in contact with the body, one needs $a_c \leq a_g$ which translates into a bound for the body's angular velocity reading as $\omega \leq \sqrt{4\,G\,\pi\rho/3}$. Considering the aforementioned typical values for asteroids' density, one obtains maximum values for the angular velocity in the range $5 \cdot 10^{-4}\,\mathrm{rad/s}$ to $10^{-3}\,\mathrm{rad/s}$, which have been

observed for known bodies. These considerations call for an in-depth assessment of the "dynamic envelope" of hopping microgravity explorers. To the best of our knowledge, only two studies have (partially) addressed this problem. In [9], the authors study the stability of different points on a rotating ellipsoid, while in [4] the authors consider the effect of friction on a point mass bouncing on a rotating ellipsoid. These studies, however, do not treat the rovers as rigid bodies and do not take into account actuation mechanisms, and therefore their relevance to rover's design and path planning appears limited.

Accordingly, the objective of this paper is to study in detail the "dynamic envelope" of hopping microgravity rovers, with a focus on internal actuation (part of our analysis, however, is also applicable to other types of rovers not equipped with gripping mechanisms, such as externally-actuated hoppers and wheeled vehicles). In particular, the contribution of this paper is threefold. First, we present a *static analysis* that allows determination of the regions of a small body where an internally-actuated hopping rover (and indeed any rover not equipped with thrusters or gripping mechanisms) can stably remain at rest, e.g., to recharge the batteries or to acquire measurements. Second (this is the main contribution of the paper), we present a *dynamic analysis* of internally-actuated hopping platforms and discuss their hop actuation and instrument pointing performance as a function of a number of system and environmental parameters (e.g., body rotation rate, rover shape). Finally, we tailor our analysis to a potential mission to Mars' moon Phobos. Collectively, our results show that internally-actuated rovers, from an actuation standpoint, are a viable mobility solution for a vast class of small Solar System bodies. Also, our analysis represents a key first step to develop path planning algorithms for microgravity explorers to safely explore dynamically feasible regions.

The rest of the paper is structured as follows. In Section II we discuss the mathematical models for the hopping rover and the environment. In Section III we present the static analysis, while in Section IV we present the dynamic analysis. Then, in Section V we discuss the results for a case study for Mars' moon Phobos and briefly consider similar bodies. Finally, in Section VI we summarize our findings and discuss direction for future work.

II. MODELS FOR THE ROVER AND THE ENVIRONMENT

In this section we present the mathematical model for the rovers we wish to study and our assumptions about the environment.

A. Geometrical and Actuation Model for the Rover

We model a microgravity hopper as a rigid body with mass m, to which a set of n mass-less spikes are attached. It is assumed that the tips of the spikes constitute the vertices of a convex polyhedron. The tip of each spike is located a distance l_i from the center of mass and is oriented at an angle θ_i from the surface normal vector $\hat{n}$, where i is the index of of the spike.

The rover is *internally-actuated* through a set of actuators that develop internal *linear* or *angular* momentum to temporarily augment contact forces. *Linear actuators* consist of a mass constrained to move along an axis actuated by a linear motor. To reduce power requirements of the motor, the system may use the motor to stretch a spring, which is then released to actuate the rover. *Angular actuators* consist of a flywheel and a rotary motor. This system can be implemented using a very small motor to spin up the flywheel, after which a braking mechanism or similar system quickly decelerates the flywheel, actuating the rover. MINERVA, the Stanford/JPL Hedgehog, and Cubli (see Figure 1), are examples of this type of system. Both linear and angular systems can be implemented with multiple actuators on different axes used together or a single actuator on a rotating frame to develop momentum in any direction.

B. Environmental Model

A small body is modeled as a (possibly non-convex) polyhedron, where the local normal to a face is indicated by $\hat{n}$. We make the natural assumption that its angular velocity vector $\vec{\omega}$ and angular acceleration vector $\vec{\alpha}$ have independent magnitudes lying in a bounded set. We also assume that the density of the object is uniform, so that the local gravity field can be computed knowing only its mass and shape when empirical models are not available.

Contact forces from each spike in contact with the surface can be divided into two components: the component normal to the surface, $\vec{F_{Ni}}$, and the component tangent to the surface, $\vec{F_{f_i}}$. In this paper we consider a Coulomb friction model to calculate contact forces on each spike, which reads as:

$$\begin{cases} \|\vec{F}_{f_i}\| \leq \mu_s \|\vec{F_{Ni}}\| & \text{if } \vec{v_i} = 0, \\ \|\vec{F}_{f_i}\| = \mu_k \|\vec{F_{Ni}}\|, & \text{if } \vec{v_i} \neq 0, \end{cases}$$

where v_i is the relative velocity between the tip of the spike and the surface, μ_s is the coefficient of static friction, and μ_k is the coefficient of kinetic friction.

C. Model Generality

Our model of internally-actuated rovers captures a large variety of existing prototypes, including MINERVA, the Hedgehog, MASCOT, and Cubli (developed for Earth gravity applications), see Figure 1.

As discussed, our model assumes that the small body has uniform density; indeed, our results can be easily generalized to the nonuniform case whenever the gravitational field is known. Our assumption of a Coulomb friction model is more problematic, since the Coulomb model is known to be quite inaccurate for slipping interactions with small particulate surfaces. We note, however, that this model usually yields first-order analyses of rover mobility well matched by experiments [19], so we believe this is sufficient for the first-order static and dynamic analyses performed in this paper. Finally, our assumption of a locally flat surface can be scaled to the limits of our knowledge of a target small body. We recognize

(a) JAXA's MINERVA [18].

(b) Stanford/JPL Hedgehog [11].
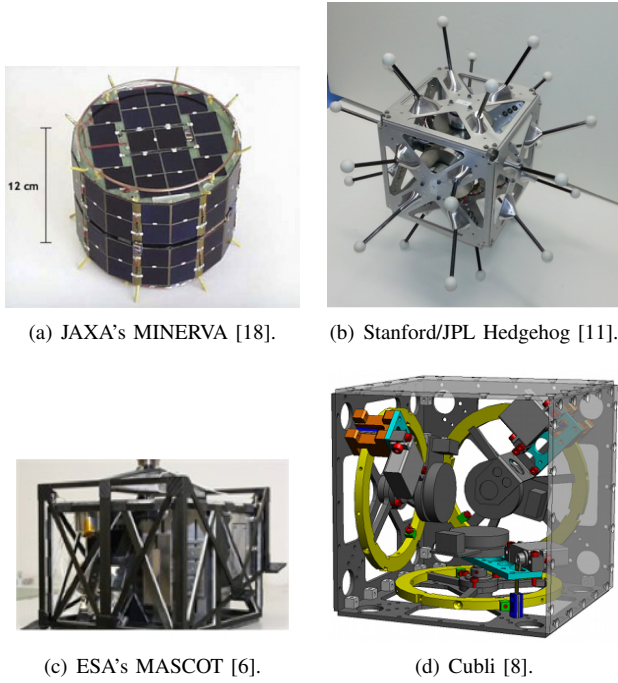

(c) ESA's MASCOT [6].

(d) Cubli [8].

Fig. 1. Examples of internally-actuated rovers that can be studied within the framework presented in this paper. All four platforms have angular actuators inside the enclosure.

that the existence of small geographic features may impact mobility, but these features have little impact on a global feasibility study as presented here.

III. STATIC ANALYSIS

In this section we study the passive stability of internally-actuated rovers on small Solar System bodies (potentially undergoing a rotational motion). This analysis allows us to identify regions where such rovers could safely land and/or stably stay at rest for either battery re-charging or measurement acquisition.

A. Governing Equations

According to Newton's laws, the three conditions that must be satisfied by a rover to be at rest on the surface of a rotating body read as follows:

1) $\Sigma \vec{F_{Ni}} + \Sigma \vec{F_{f_i}} + m\vec{g} = m(\vec{\omega} \times \vec{\omega} \times \vec{r} + \vec{\alpha} \times \vec{r})$,
2) $\|\Sigma \vec{F_{f_i}}\| / \|\Sigma \vec{F_{Ni}}\| \leq \min(\mu_s, \tan(\phi))$,
3) $\vec{F_{Ni}} \cdot \hat{n} \geq 0$,

where $\vec{r}$ is the vector from the body center of mass to the rover center of mass and ϕ is the maximum angle the rover can be tipped before it will continue to roll. This angle is defined in any direction by the polygon formed by spikes in contact with the surface, hereafter called the support polygon. The first equation defines the unique total contact force required to stay in place. The second condition requires that the rover does not slip or roll. The third relation enforces the no-grip stipulation.

B. Numerical Procedure

The aim of the static analysis is to determine whether the three aforementioned conditions are met for a given rover shape and rotating body. The shape of the body is assumed to be known from radar or other measurements and is treated as a polyhedron (see Section II). If no empirical gravity field is available, one can be calculated from the mass and shape of the body under the assumption of uniform density. Specifically, one discretizes the mass into a grid of specified resolution, gives each interior point in the grid an equal mass, and sums the contributions of each mass at each point of interest. To account for the effects of rotation, the boundary of the set of possible angular velocities and accelerations is sampled and used to compute centripetal and Euler acceleration. Because the transformations from angular velocity and acceleration to their corresponding Euclidean accelerations are continuous, if any rotational state exists that causes these conditions to be violated, there must exist a state on the boundary of the set that violates the same conditions. We take advantage of this by only sampling the boundary of the set of rotational values, saving computation time. To determine passive stability, the necessary contact force to achieve static equilibrium is calculated at all points of interest on the body for each rotational state considered. If the face is stable, this contact force will satisfy the above conditions.

C. Numerical Experiments

In this section we aim to illustrate the effects of body geometry and rotation on the existence of passively stable regions. The aforementioned analysis has been implemented on a spherical body (selected solely to illustrate rotation effects), and the asteroid Itokawa. The sphere has density $5\,\text{g/cm}^3$, is rotating at $0.0012\,\text{rad/sec}$, and has a presumed static friction coefficient of 0.7. We consider a Hedgehog shaped rover (spikes defining orthogonal octagons) with mass $10\,\text{kg}$ and spike length of $0.2\,\text{m}$ [11]. The results are shown in Figure 2(a). One can distinguish four different regions. Specifically, the green region represents the part of the asteroid where the system can stably stay at rest and the orange region denotes regions where the rover would roll without slipping, which depends *only* on the selected geometry. The red region represents the part of the asteroid where the rover will slip, but remain on the surface and the black region is where the rover would be thrown from the surface. No system without gripping or thruster capability could stay at rest in the red or black regions.

Figure 2(b) shows a similar analysis for the asteroid Itokawa, which has mass $3.56 \cdot 10^{10}\,\text{kg}$, a presumed static friction coefficient of 0.7, and is rotating with a period of 12 hours. One can observe that a hopping rover could stably stay at rest on most regions on Itokawa.

D. Implications for System Design and Mission Feasibility

The results in this section and presented later in Table I suggest that internally-actuated rovers can stably operate on scientifically interesting small bodies, such as Itokawa.

(a) Static analysis on a spheric asteroid

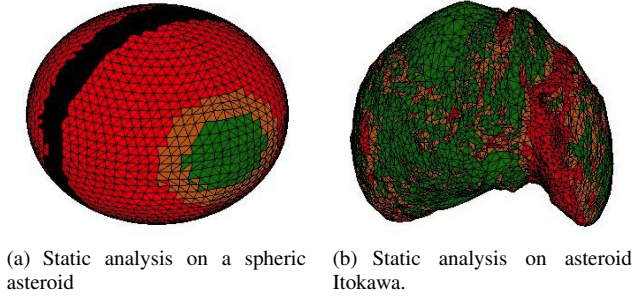(b) Static analysis on asteroid Itokawa.

Fig. 2.   Static analysis for two representative small bodies.

The presence of the orange regions suggests that more regions on these bodies would be explorable if the selected geometry were more stable. Therefore, the explorable region is maximized by choosing a system geometry that will slip before it will roll. We note that the spin rate of the sphere is rather large compared to most asteroids with rotation rates in the range $1 - 12 \, \mathrm{rev/day}$ [13], but it is still within the observed range. Indeed, some monolithic asteroids have rotation speeds several times larger [12]. In such cases, exploration without use of gripping systems or thrusters is impossible.

IV. DYNAMIC ANALYSIS

In this section we study the hopping initiation and instrument pointing aspects for the operation of a hopping microgravity explorer.

A. Hopping Initiation

In this section, we intend to use first-order analysis techniques to demonstrate that the feasible trajectories for hopping rovers are almost exclusively defined by the contact force model and chosen geometry. This, in turn, allows us to draw several general conclusions about ideal rover design.

The rover behavior during actuation varies widely depending on the type of actuator used and on the command profile given to the actuator. While in contact with the ground, the rover can either be at rest on three or more contact points, pivot about an axis defined by one or more contact points without slipping, or slide on the surface. The following analysis captures all these possibilities. For clarity, linear actuation is considered separately from angular actuation.

1) Equations of motion under linear actuation: The governing equations in this case read as follows:

1) $\Sigma \vec{F}_{N\,i} + \Sigma \vec{F}_{f_i} + m\vec{g} = m(\vec{\omega} \times \vec{\omega} \times \vec{r} + \vec{\alpha} \times \vec{r}) + \dot{\vec{p}}$,
2) $\|\Sigma \vec{F}_{f_i}\| / \|\Sigma \vec{F}_{N\,i}\| \leq \min(\mu_s, \tan(\phi))$,
3) $\Sigma \vec{F}_N \cdot \hat{n} \geq 0$,

where $\dot{\vec{p}}$ is the time rate of change of the linear momentum of the actuator. In general, on a small body the rover should be operated at well below orbital speed. However, one seeks to maximize horizontal speed to increase size of the explored region, thereby improving the scientific return of the mission. Therefore, it is desirable to actuate the rover as quickly as possible using the lowest achievable trajectory. For simplicity, we consider the desired hopping speed of

the rover to be some fraction of orbital speed. Accordingly, a reasonable actuation model for linear systems would be: (1) the rover has a total mass m and an actuator mass m_a which can be moved a distance d in any direction; (2) the mass accelerates uniformly over its movable distance with an acceleration whose magnitude is a; (3) the actuator mass collides inelastically and instantaneously with the wall of the rover; (4) the rover is on a region in which passive static equilibrium is possible; (5) the surface is locally flat; (6) the rover cannot roll during actuation; and (7) the rover is intended to move at some fraction c of local orbital velocity.

Under these assumptions, the final momentum of the mass is $p_{mass} = m_a\sqrt{2ad}$. After the inelastic collision, the speed of the rover is given by $v = m_a\sqrt{2ad}/m$. Since the commanded velocity is a fraction of the orbital velocity, one obtains: $m_a\sqrt{2ad}/m = c\sqrt{g\,r}$. Hence, solving for the ratio of actuator force to gravitational force, $m_a\,a/mg$, one obtains:

$$m_a\,a/mg = c^2\,m\,r/(2\,m_a\,d).$$

We assume that the rover can operate safely for some fraction range $c \geq 0.1$ of orbital velocity; also, typical values of r range between $100 \, \mathrm{m}$ and $100 \, \mathrm{km}$ for bodies of interest. It follows that $m_a\,a/mg >> 1$ in most cases. Therefore, if the system is to develop speeds comparable to orbital speed, the actuating force vector dominates the behavior of the system. Accordingly, we reasonably neglect the small terms in the governing force equation, leaving: $\Sigma \vec{F}_{N\,i} + \Sigma \vec{F}_{f_i} = \dot{\vec{p}}$. Thus, according to the Coulomb friction model, an actuation force is achievable if its resulting contact forces satisfy the previous no-slip, no-roll, and no-grip conditions.

The above considerations give rise to the following conclusions regarding hopping initiation for internally-actuated rovers under linear actuation. For reliability, the actuator should be restricted to commanded forces that satisfy the aforementioned conditions or the rover will slide or roll, which may not be controllable. For ideal performance, the system should have a wide support base to allow the rover to achieve low hop angles (limited by static friction). To address viability, hop directional control is assured because no stipulations besides a minimum elevation are imposed on the actuator force. Additionally, moving at some fraction of orbital speed at a hop angle defined by the static friction coefficient is sufficient to achieve meaningful scientific information from exploration missions

2) Equations of motion for angular systems: In the same way, we make the following assumption for a "reasonable" actuation model under angular actuation (here, for simplicity, we consider a 2D model): (1) the rover has a mass m, an actuator moment of inertia j, a constant spike length l, and a remaining moment of inertia J; (2) the rover applies a uniform torque T to the flywheel during actuation; (3) the rover is on a region in which passive static equilibrium is possible; (4) the surface is locally flat; (5) the rover does not slip during actuation; (6) the local gravity is normal to the surface; (7) the rover is intended to move at some fraction $c \geq 0.1$ of local orbital velocity.

Enforcing the non-slip constraint requires the following equations must be satisfied throughout the actuation process:

1) $T - m\,g\,l\sin(\psi) = (J + 1/2ml^2)\ddot{\psi}$,
2) $F_N - m\,g = m\,l\,\ddot{\psi}\sin(\psi)$,
3) $F_f = m\,l\,\ddot{\psi}\cos(\psi)$,
4) $\|F_f\|/\|F_N\| \leq \mu_s$,
5) $g \geq \dot{\psi}^2\,l\cos(\psi)$,

where ψ defines the angle between the vector from the axis of rotation to the center of mass and the surface normal vector, as shown in Figure 3. The first three equations are required by Newton's 2nd law. The fourth equation is the Coulomb static friction condition, and the fifth equation requires gravity to provide sufficient centripetal acceleration while pivoting. The limiting case for the friction condition to be satisfied is then given by:

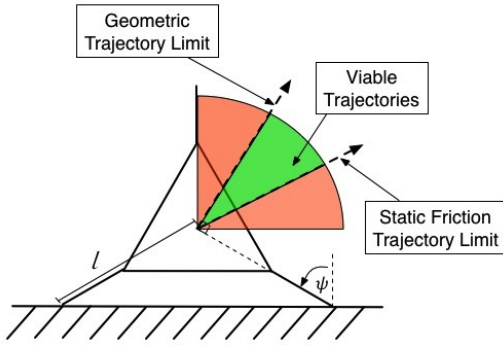$$(l\,\ddot{\psi}\cos(\psi))/(l\,\ddot{\psi}\sin(\psi) + g) = \mu_s.$$

Fig. 3.   Limitations on initial hopping angles for angular systems.

Again, the target velocity and gravity are related by the equation $v = c\sqrt{g\,r}$, where the linear speed v is related to the rotation speed by $v = \dot{\psi}\,l$. As the rover rotates about the axes the angle between the velocity of the center of mass and the horizontal surface decreases. The maximum value of this angle, denoted ψ_0 (where $\psi_0 > 0$), is defined by the chosen geometry. Since the velocity vector cannot dip below horizontal and still cause the rover to hop, the center of mass of the rover follows a path no longer than $l\,\psi_0$. This requires an average linear acceleration of:

$$a = v^2/(2\,l\,\psi_0) = c^2\,g\,r/(2\,l\,\psi_0).$$

Solving for a/g, one obtains $a/g = c^2\,r/(2\,l\,\psi_0)$. Again, we assume that r ranges between 100 m and 100 km. Also, $l\,\psi_0$ is clearly on the order of meters or less. One then concludes that $a/g >> 1$ in most cases. The result is again that the lowest hopping angle achievable is limited by the coefficient of static friction. Additionally, we must consider the consequences of the last condition. Since the target takeoff condition is $v = c\sqrt{gr}$, one can eliminate g and obtain:

$$1 \geq c^2\,r\,\cos(\psi)/l$$

Since r will exceed l by several orders of magnitude, this last condition cannot be satisfied at the target velocity.

Consequently, the rover will either hop immediately at an angle of approximately ψ_0, or start to slip. It is therefore impossible to achieve hop angles larger than the greater of ψ_0 and $\arctan(\mu_k)$. Slipping takeoffs have simple dynamics since the direction of the contact force is defined by the kinetic friction coefficient. Consequentially, the slipping hop angle is *fixed* by the kinetic friction coefficient.

For geometries with large ψ_0, we may achieve a larger hop angle envelope by implementing a more advanced control method. One could apply a small torque to slowly adjust the attitude of the rover to the desired orientation, at which point the actuating torque is applied to initiate the hop. Such fine balancing attitude adjustment has been already demonstrated on the Cubli robot [8]. This strategy allows us to develop any hop angle between ψ_0 and the angle defined by the static friction coefficient as shown in Figure 3.

Finally, we consider the effect of local surface topology. Because the cone satisfying the static friction coefficient is defined by the surface norm vector, any change in the surface norm at the point of contact can substantially change the range of viable hop angles. This behavior is ignored in this analysis because it is entirely dependent on the local surface texture, which is largely unknown. Determination of the significance of these variances is left to future research.

This analysis has the following consequences for system design. On flat surfaces, rover geometries with large ψ_0 (fewer faces) will perform well so long as attitude control is implemented in the actuation method. Conversely, geometries with low ψ_0 values will always slip when hopping, resulting in a *fixed* hop angle. In addition, the chosen geometry imposes an upper limit to the hop angle, which may be problematic if the rover were to fall into a hole. There are no limitations on the hop direction and the horizontal speed limitations are identical to those of linear systems, which demonstrates viability of using angular actuators to achieve hopping.

B. Instrument Pointing

Scientific missions to small bodies will entail repeated use of scientific instruments to characterize the surface. Mobility systems must be able to reliably point these instruments to meet mission requirements. In this section we analyze the feasibility of each type of actuator system to meet these requirements.

1) Linear systems: Attitude adjustment for systems using linear actuators is a nontrivial problem. A common method used for these systems is to use the actuator to shift the center of mass over the supported region, causing the rover to roll. We will now demonstrate that the geometric requirements for this implementation and the aforementioned ideal geometry for low takeoff angles are in *direct contradiction* through 2-D analysis. Let the ensemble of spikes form a regular polygon with n vertices and length l from the center to each vertex. The internal angle spanned by each side is $2\pi/n$ radians. Now let us assume the system is composed of two masses, the shell mass m_s located at the center, and the actuator mass m_a that can be placed anywhere within the polygon.

To roll, we require the center of mass of the system to be over the edge of the support face. Assuming that the actuator mass can be moved a distance l from the center of mass (a slight simplification), the required mass ratio to move the center of mass a distance $l \, sin(\pi/2)$ follows the relation $m_a/m_s \geq \sin(\pi/n)/(1 - \sin(\pi/n))$ when $n > 4$. Because we already established that minimizing the number of faces is a design priority, using this system would clearly compromise system performance. On the other hand, if a system with a single actuator with an orientation mechanism is used, the orientation system can be used to control attitude adjustment.

2) Angular systems: Fine attitude adjustment for torque actuated systems has already been demonstrated by the Cubli robot [8] in Earth gravity. This technique would easily satisfy the instrument pointing requirement. Additionally, it is possible to achieve instrument pointing in regions where the rover would roll without active control. In such cases the actuator torque can be set to directly oppose the gravitational torque, though this is only sustainable until the flywheel motor reaches its saturation speed. Furthermore, it is possible to maneuver on these surfaces by small adjustments of the torque to achieve controlled rolling motion. If the rover is traveling downhill, minimal torque will be needed since the behavior will be similar to that of a rimless wheel [15], which is known to have a stable rolling speed. Using a reference gravitational acceleration of $1 \, \mathrm{mm/s^2}$ and current specifications for the Hedgehog rover [3], which includes a mass of $10 \, \mathrm{kg}$, a leg length of $0.3 \, \mathrm{m}$, a flywheel moment of inertia of $0.02 \, \mathrm{kg \cdot m^2}$ and a saturation speed of $1000 \, \mathrm{rad/s}$, simple Buckingham Pi analysis suggests that the rover could hold the maximum gravitational torque for over $6000 \, \mathrm{s}$. The conservative nature of this method suggests that the time limit is actually much higher. This control system would allow an angular actuator system to maintain pointing in any region where the contact force is consistent with Coulomb friction even if the rover would roll with no active control. This is at least as versatile as any other system without gripping or thruster capability.

V. ANALYSIS FOR REFERENCE MISSION TO PHOBOS

In this section, we demonstrate an analysis of a reference mission to Phobos as outlined in [3] to demonstrate that internally actuated systems are a feasible option for exploration missions to small bodies.

The difference between this procedure and the static analysis is the condition to allow the motor to apply a minimum torque, T_{min}, to spin up the flywheel, which reads as:

$$F_n \, l \Big(\sin(\theta) - F_f/\|F_n + F_f\| \Big) \geq T_{min}.$$

This condition requires that that the line of action of the contact force be sufficiently far from the edge of the polygon of support to allow the flywheel to spin up without the rover rolling. It is important to note that this equation imposes a directional sensitivity to the analysis (i.e. the rover may still be able to actuate in some directions). For simplicity, in

(a) Phobos. The region within the white circle indicates surface material that is representative of Phobos bulk interior and is of high scientific value

(b) Phobos feasibility map for a rover with an octahedral shape.

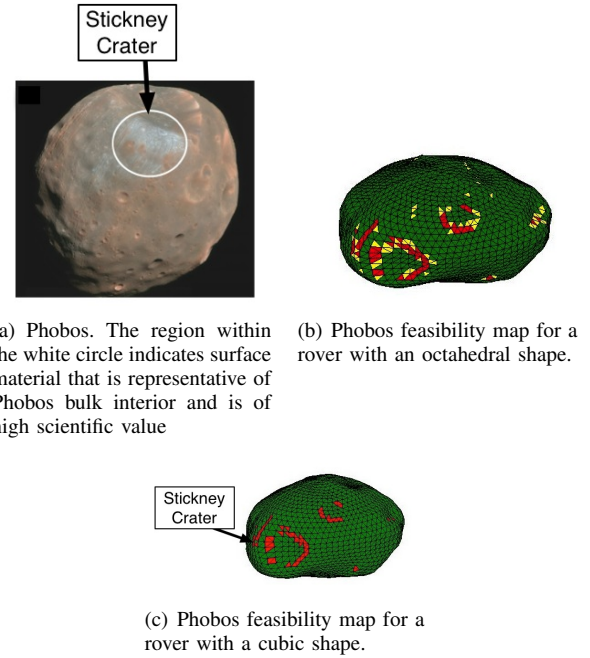(c) Phobos feasibility map for a rover with a cubic shape.

Fig. 4. Dynamical feasibility map for the Hedgehog rover on Phobos as a function of its spikes' geometrical envelope.

this method we require that this condition be satisfied in all directions.

To demonstrate the consequences and relevance of this analysis, we consider the reference mission proposed in [3]. The gravity field is calculated using the previously described method, and we assume a static friction coefficient of 0.7. The rover has a mass of 10kg and spike length of 0.2m. We assume a motor torque resolution of $1\mathrm{mN \cdot m}$. Lastly, we consider octagonal and cubic shapes. The results of these studies are shown in Figures 4(b) and 4(c). The yellow regions present in Figure 4(b) indicate regions where the spin-up restriction prevents the rover from actuating correctly. Note that the cube geometry eliminates these regions due to increased stability. To further demonstrate the viability of these systems, we show summary results from analysis of the a cubic rover with no minimum torque requirement on bodies of scientific interest in Table I. We acknowledge a degree of uncertainty in these calculations due to the dependence on the accuracy of the shape models.

TABLE I
EXPLORABLE REGIONS FOR A SET OR REPRESENTATIVE TARGETS.

Body Name	Mass (kg)	Period (h)	Explorable Regions
Phobos	$1.072 \cdot 10^{16}$	7.66	97%
Deimos	$1.48 \cdot 10^{15}$	30.3	96%
Itokawa	$3.51 \cdot 10^{10}$	12.1	75%
Hartley 2	$4.7 \cdot 10^{11}$	18.3	99%

VI. DISCUSSION AND CONCLUSIONS

In this paper we have studied the "dynamic envelope" of hopping microgravity rovers with a focus on internal

actuation. Our findings are summarized below together with a discussion about directions for future research.

A. Summary

This analysis has shown that internally actuated systems are, from an actuation standpoint, a viable option for exploration missions to small bodies of scientific interest. In Section III, we established the requirements for passive stability. In Section IV we established rover geometry as a driving parameter for actuation and instrument pointing performance. Lastly, in Section V we showed that Phobos and many other bodies of scientific interest meet the aforementioned requirements. Performance factors driven by actuator type are detailed in the following sections.

1) Linear Actuator Systems: Linear actuator systems are restricted to regions where the contact force satisfies the contact force model *and* the no-roll condition with no active control. Controlling the hop angle is very simple with these systems so long as the change in contact force is consistent with the contact force model and the no-roll condition. While the rover will have better hopping behavior if a wide support base is used, meeting the instrument pointing requirement would be more difficult. Using a single linear actuator on a rotating frame can alleviate this problem since the frame control can be used to meet the pointing requirement using the same principle as the angular actuator.

2) Angular Actuator Systems: When using an angular actuator, the system can explore any region where the required contact force is allowed under the contact force model if the aforementioned controls are implemented. The *only* way to improve this performance is to include a gripping or thruster system. The hop angle for these systems have an upper limit defined by the geometry of the system, which may be problematic if the rover falls into a hole. Using a wide support base simultaneously increases this upper limit and reduces the need for active control to prevent rolling.

B. Future Work

Recognizing that this analysis depends strongly on the contact force model, efforts to characterize the contact force behavior of regolith and loose material surfaces is warranted. For exploration missions to be successful, it is essential to have functional contact force models for these surfaces in order to determine any further mobility restrictions. To achieve this, we plan to perform an experimental validation of these conclusions on these surface types.

This analysis has direct consequences to the path planning process for exploration missions, which represents an important future research avenue. The dynamic environment introduces a number of complications in this process. The existence of regions in which rovers cannot achieve static equilibrium requires path planning algorithms capable of identifying and avoiding such regions. These regions may vary with time on tumbling bodies due to the explicit dependence on angular velocity and acceleration. Use of the aforementioned slow spin-up procedure or active stabilization controls on slopes introduces directional mobility

restrictions, making determination of feasible paths more difficult. Path planning algorithms for these missions must account for all of these limitations. The numerical method for determination of explorable regions presented in this paper can already account for most of these restrictions, and could easily be modified to identify the rest of these constraints. This method may be developed into a path planning system in the future.

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