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Dynamic Modeling and Control of a Free-flying Space Robot with Flexible-link and Flexible-joints*

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Abstract— **Free-flying space robot's nonlinearity and strong coupling characters make the dynamics and control of such system more complicated than a terrestrial robot system. The Space robots are always built in very light for saving the launch energy. The flexibility of the joints and links is considerable arising from its elasticity. Controlling this manipulator is more complex than controlling one with rigid joints due to the interactions of rigid and flexible motion, in which only a single actuation signal can be applied at each joint and has to control the flexure of both the joint itself and the link attached to it. To discussing an under-actuated flexible-link flexible-joint space manipulator, a free-flying space manipulator with one flexible link and two flexible revolute joints is presented in this paper. The dynamical Lagrange equation is established, and a singularly perturbed model has been formulated and used for designing a reduced-order controller. This controller consists of a rigid control component and two fast control components. Numerical simulations show that the link and joint vibrations have been stabilized effectively with good tracking performance.**

I. Introduction

Outer space is an ultimate field for the application of robotics technology. As outer space is a harsh environment with extreme temperatures, vacuum, radiation, gravity, and great distances, human access is very difficult and hazardous and is therefore limited. To assist human activities in space robotic manipulators have been playing essential roles in orbital operations [1]–[5]. To obtain a good tracking performance of space-based robots, various dynamic modeling and control techniques have been developed during the past several decades. Most of modeling and control methods for space robotic systems assumed that robot arms mounted on their spacecraft are rigid [6]–[9]. But space robots are built very light in order to save expensive launch energy, which results in considerable joint and link flexibilities [10]. Link flexibility is a consequence of the lightweight of manipulator arms. Joint flexibility arises because of the elasticity of the joint. Thus, flexible space robots undergo two types of motion, i.e. rigid and flexible motion. Because of the interaction of these motions, the resulting dynamic equations of flexible space robots are highly complex and, in turn, the control task becomes more challenging compared to that of rigid space robots. Therefore, a first step towards designing an efficient control strategy for these manipulators must be aimed at developing accurate dynamic models that can characterize the above flexibilities along with the rigid dynamics.

The dynamic equation of flexible dual-arm space robot based on the Lagrange method and the assumed mode method was presented and the inversion-dynamic nonlinear compensation and PD feedback is performed to track the desired trajectory [11]. A methodology for designing stable manipulation-variable feedback controls of flexible free-flying space manipulators for positioning control to a static target and continuous path tracking control was proposed [12], which was based on virtual rigid manipulator (VRM) concept and can extend other controls for rigid manipulators to those for flexible free-flying space manipulators. A simple direct adaptive control law was presented which improved the performance of vibration control algorithms effectively [13]. Different controllers based on singular perturbation theory were proposed in [14]–[17], which can track the trajectory and damp out the vibration of the flexible link simultaneously. An experimental study of a control policy for end-effector trajectory tracking of structurally flexible space-based manipulators was made [18].

A passivity approach for the control design of flexible joint robots was applied to the rate control of a three-link arm modeled after the shoulder yaw joint of the Space Shuttle Remote Manipulator System (RMS) [19]. A system consisting of several flexible joint robots mounted on a spacecraft with closed kinematic chain constraints was studied and a coordinated controller was proposed, which consisted of a position and internal force controller for the slow subsystem and a joint elastic force controller for the fast subsystem [20]. An extended Kalman filter strategy to estimate state variables from noisy measurements in flexible joint space manipulators was presented [21]. A method of adaptive fuzzy compensation and backstepping control for the trajectory tracking precision of space robot flexible-joint was proposed [22]. The dynamic model of a space manipulator with flexible joints and friction was established, and the anti-windup nonlinear PD control to avoid the torque saturation of the joints was presented [23]. The dynamic modeling and singular perturbation control of free-floating space robot with flexible joints were made [24-25].

From above, we can see that there are some works on space manipulator considering only link flexibilities or only joint elasticities. Space robots with both joint and link flexibilities are seldom studied. Terrestrial manipulator with both flexible links and elastic joints have been studied.

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Ignoring the flexible effects caused flexible-link flexible-joint robots uncontrolled [26]. The coupling effect of the flexible joint and the flexible link on the dynamic singularity of the flexible manipulator was addressed [27]. Subudhi A dynamic modelling technique for a manipulator with multiple flexible links and flexible joints based on a combined Euler–Lagrange formulation and assumed modes method was proposed [28-29]. Output Feedback Sliding Mode Control architecture for slewing flexible joint and link robot structures was presented [30]. Singular perturbation controller of space manipulator with flexible-link and flexible-joint was investigated [31].

Since communication devices and solar panels are more sensitive to the change of spacecraft's attitude than to that of spacecraft's position, it's important to study space manipulator with attitude controlled spacecraft (that is a free-flying space manipulator)[32]–[33].This paper deals with the problem of controlling a flexible-link flexible-joint free-flying space manipulator. Firstly the dynamic model of a free-flying space manipulator with a flexible link and two flexible revolute joints is established by applying the Lagrange equations and linear momentum conservation theory. Secondly based on singular perturbation theory, the system is transformed into two subsystems: a flexible-link subsystem and a fast flexible-joint subsystem. The flexible-link subsystem is further transformed into two subsystems: a rigid subsystem and a fast flexible-link subsystem. A composite controller which consists of a rigid control component and two fast control components is proposed. The flexible-joint fast controller is designed to stabilize the flexible-joint fast subsystem around the equilibrium trajectory set up by the flexible-link subsystem under the effect of the rigid controller and the flexible-link fast controller. The flexible-link fast subsystem controller will damp out the vibration of the flexible link using optimal Linear Quadratic Regulator (LQR) method. The rigid subsystem controller dominates the trajectory tracking of coordinated motion. Finally the numerical simulation is carried out, which confirms the controller proposed is feasible and effective.

I. DYNAMIC MODELLING

The planar free-flying space robot with one flexible-link and two flexible-joints as shown in Fig.1 is studied. The flexible joint is dynamically simplified as a linear torsion spring that works as a connector between the actuator and the link [34]. The system consists of the base B_0, the rigid link B_1, the flexible link B_2 and two flexible joints. O_{Ci}(i=0,1,2) is the mass center of B_i(i=0,1,2). O_0 coincides O_{C0}, O_i(i=1,2) is the rotational center of the revolute joint between B_{i-1} and B_i. x_1 is the symmetry axis of link B_1 , x_2 is tangent with the symmetry axis of link B_2. (O -xy) is inertial coordinate frame of the system, and (O_i -x_iy_i) is local coordinate frame of B_i (i =0,1,2). Other symbols are defined as follows.

$\boldsymbol{e}_i$ Unit vector along the x_i axis (i =0,1,2);

l_0 Distance from point O to O_1;

d_i Distance from point O_{Ci} to O_i(i =1,2);

l_i Length of B_i (i =1,2) along the x_i axis;

C Mass center of the entire system;

$\boldsymbol{r}_i$ Position vector of the mass center O_{Ci} of B_i (i=0,1) in (O -xy) frame;

$\boldsymbol{r}_P$ Position vector of arbitrary point P of B_2 in (O -xy) frame;

$\boldsymbol{r}_C$ Position vector of the mass center C of the entire system in (O -xy) frame;

θ_0 Attitude angle of the base, which is the angle between the y axis and the x_0 axis;

θ_i The ith link angular position (i =1,2) , i.e. the angle between the x_{i-1} axis and the x_i axis;

θ_{ai} The ith actuator angular position (i =1,2);

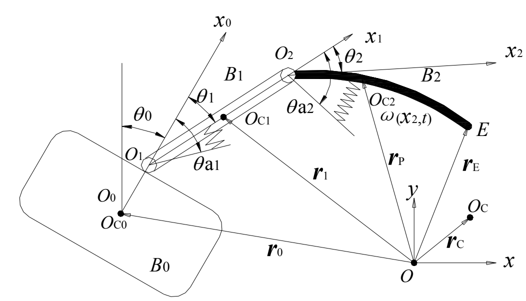

Figure 1. A planar free-flying flexible space robot

$\dot{(\ )}$ The first time derivative of $(\ )$;

$\ddot{(\ )}$ The second time derivative of $(\ )$;

$\bar{(\ )}$ The corresponding variable of $(\ )$ in the slow subsystem.

The flexible Link is modeled as Euler-Bernoulli beams with uniform density ρ and constant flexural rigidity *(EI)*. A finite-dimensional model (of order m) of link flexibility can be obtained by the assumed modes technique. Exploiting separability in time and space [35], the link deflection can be expressed as

$$w(x_2,t)=\sum_{i=1}^{m}\varphi_i(x_2)\delta_i(t)\,, \tag{1}$$

where $w(x_2,t)$ denotes the transversal deflection of link 2 at abscissa x_2 ($0\le x_2\le l_2$), $\varphi_i(x_2)$ are the time-varying variables associated with the assumed spatial mode shapes $\delta_i(t)$ of link 2**.** Here m =2. The translation kinetic energy of the motor rotor can be transferred into the link which is connected with it by using the quality concentrated method. m_i is the corresponding mass of B_i (i =0,1).

$\boldsymbol{r}_0$、$\boldsymbol{r}_1$、 and $\boldsymbol{r}_P$ are

$$\begin{cases} \boldsymbol{r}_0 = \boldsymbol{r}_C + N_0\boldsymbol{e}_0 + N_1\boldsymbol{e}_1 + N_2\boldsymbol{e}_2 + N_3\boldsymbol{e}_3 \\ \boldsymbol{r}_1 = \boldsymbol{r}_C + (N_0 + l_0)\boldsymbol{e}_0 + (N_1 + d_1)\boldsymbol{e}_1 + N_2\boldsymbol{e}_2 + N_3\boldsymbol{e}_3 \\ \boldsymbol{r}_P = \boldsymbol{r}_C + (N_0 + l_0)\boldsymbol{e}_0 + (N_1 + l_1)\boldsymbol{e}_1 + (N_2 + x_2)\boldsymbol{e}_2 \\ \quad + (N_3 - \omega(x_2, t))\boldsymbol{e}_3 \end{cases}, \tag{2}$$

where $N_0 = -(m_1 + \rho l_2)l_0 / M$, $N_1 = -(m_1 d_1 + \rho l_2 l_1)/M$, $N_2 = -\rho l_2^2/(2M)$, $N_3 = N_{31}\delta_1 + N_{32}\delta_2$, $N_{31} = \int_0^{l_2} \rho\varphi_1(x_2) d_{x_2} / M$, $N_{32} = \int_0^{l_2} \rho\varphi_2(x_2) d_{x_2} / M$, $M = m_0 + m_1 + \rho l_2$.

Differentiating (1) with respect to time *t,* we get

$$\begin{cases} \dot{\boldsymbol{r}}_0 = \dot{\boldsymbol{r}}_C + N_0\dot{\boldsymbol{e}}_0 + N_1\dot{\boldsymbol{e}}_1 + N_2\dot{\boldsymbol{e}}_2 + N_{31}\delta_1\dot{\boldsymbol{e}}_3 + N_{32}\delta_2\dot{\boldsymbol{e}}_3 \\ \quad + N_{31}\dot{\delta}_1\boldsymbol{e}_3 + N_{32}\dot{\delta}_2\boldsymbol{e}_3 \\ \dot{\boldsymbol{r}}_1 = \dot{\boldsymbol{r}}_C + (N_0 + l_0)\dot{\boldsymbol{e}}_0 + (N_1 + d_1)\dot{\boldsymbol{e}}_1 + N_2\dot{\boldsymbol{e}}_2 + N_{31}\delta_1\dot{\boldsymbol{e}}_3 \\ \quad + N_{32}\delta_2\dot{\boldsymbol{e}}_3 + N_{31}\dot{\delta}_1\boldsymbol{e}_3 + N_{32}\dot{\delta}_2\boldsymbol{e}_3 \\ \dot{\boldsymbol{r}}_P = \dot{\boldsymbol{r}}_C + (N_0 + l_0)\dot{\boldsymbol{e}}_0 + (N_1 + l_1)\dot{\boldsymbol{e}}_1 + (N_2 + x_2)\dot{\boldsymbol{e}}_2 \\ \quad + (N_{31} - \varphi_1(x_2))\delta_1\dot{\boldsymbol{e}}_3 + (N_{32} - \varphi_2(x_2))\delta_2\dot{\boldsymbol{e}}_3 \\ \quad + (N_{31} - \varphi_1(x_2))\dot{\delta}_1\boldsymbol{e}_3 + (N_{32} - \varphi_2(x_2))\dot{\delta}_2\boldsymbol{e}_3 \end{cases}. \tag{3}$$

The rotation kinematic energy of the actuator about its principal axis of rotation is modeled only, and then the total kinetic energy of the free-flying flexible space robot is

$$T = T_0 + \sum_{i=1}^{2}(T_i + T_{ai}), \tag{4}$$

where $T_0 = \frac{1}{2}(m_0\dot{\boldsymbol{r}}_0^2 + I_0\dot{\theta}_0^2)$, $T_1 = \frac{1}{2}(m_1\dot{\boldsymbol{r}}_1^2 + I_1(\dot{\theta}_0 + \dot{\theta}_1)^2)$, $T_2 = \frac{1}{2}\rho\int_0^{l_2}\dot{\boldsymbol{r}}_P^2 d_{x_2}$, $T_{ai} = \frac{1}{2}I_{ai}\dot{\theta}_{ai}^2$ $(i = 1,2)$, I_i $(i = 0,1,2)$ is inertial moment of B_i with respect to its mass center O_{Ci} , I_{ai} is inertial moment of the ith actuator.

Neglecting the effects of the micro gravity in the space, the potential energy of the system is

$$V = \frac{1}{2}\sum_{i=1}^{2}k_i(\theta_i - \theta_{ai})^2 + \frac{1}{2}EI\int_0^{l_2}\left(\frac{\partial^2\omega(x_2,t)}{\partial^2 x_2}\right)^2 d_{x_2}, \tag{5}$$

where k_i is the stiffness' coefficient of the ith flexible joint.The dynamic Euler-Lagrange equations of the planar flexible space robot are

$$\begin{cases} \boldsymbol{D}(\boldsymbol{q}_{rf})\ddot{\boldsymbol{q}}_{rf} + \boldsymbol{h}(\boldsymbol{q}_{rf}, \dot{\boldsymbol{q}}_{rf})\dot{\boldsymbol{q}}_{rf} + \boldsymbol{K}\boldsymbol{q}_{rf} = \begin{bmatrix} \tau_0 \\ -\boldsymbol{K}_{fj}(\boldsymbol{q}_l - \boldsymbol{q}_a) \\ \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{J}_a\ddot{\boldsymbol{q}}_a = \boldsymbol{K}_{fj}(\boldsymbol{q}_l - \boldsymbol{q}_a) + \boldsymbol{\tau}_a \end{cases}, \tag{6}$$

where $\boldsymbol{q}_{rf} = \begin{bmatrix}\boldsymbol{q}_r^T & \boldsymbol{q}_f^T\end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{q}_r = \begin{bmatrix}\theta_0 & \boldsymbol{q}_l^T\end{bmatrix}^{\mathrm{T}}$, $\boldsymbol{q}_f = [\delta_1 \quad \delta_2]^{\mathrm{T}}$, $\boldsymbol{q}_l = [\theta_1 \quad \theta_2]^{\mathrm{T}}$, $\boldsymbol{q}_a = [\theta_{a1} \quad \theta_{a2}]^{\mathrm{T}}$, $\boldsymbol{D}(\boldsymbol{q}_{rf}) \in \mathbf{R}^{5\times5}$ is the base and links inertia positive-definite matrices, , $\boldsymbol{J}_a \in \mathbf{R}^{2\times2}$ is the actuators inertia positive-definite matrices, $\boldsymbol{h}(\boldsymbol{q}_{rf}, \dot{\boldsymbol{q}}_{rf}) \in \mathbf{R}^{5\times5}$ is the Coriolis/centrifugal matrix, $\boldsymbol{K} = \begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_{fl}\end{bmatrix}$, $\boldsymbol{K}_{fl} \in \mathbf{R}^{2\times2}$ is the flexible link stiffness matrix, $\boldsymbol{K}_{fj}(\boldsymbol{q}_l - \boldsymbol{q}_a) \in \mathbf{R}^2$ is the vector of elastic forces at joints; $\boldsymbol{K}_{fj} \in \mathbf{R}^{2\times2}$ is the joint stiffness diagonal constant matrix, $\boldsymbol{\tau} = \begin{bmatrix}\tau_0 & \boldsymbol{\tau}_a^T\end{bmatrix}^T$ is the control torque vector, τ_0 is the attitude control torque of the base, $\boldsymbol{\tau}_a \in \mathbf{R}^2$ is the joint torque/force delivered by actuators.

II. Two-time scale control

A. Flexible-joint Fast Subsystem and the Corresponding Controller

Define $\boldsymbol{\tau}_l = -\boldsymbol{K}_{fj}(\boldsymbol{q}_l - \boldsymbol{q}_a)$, then $\ddot{\boldsymbol{q}}_a = \boldsymbol{K}_{fj}^{-1}\ddot{\boldsymbol{\tau}}_l + \ddot{\boldsymbol{q}}_l$. (6) can be expressed as

$$\begin{cases} \boldsymbol{D}(\boldsymbol{q}_{rf})\ddot{\boldsymbol{q}}_{rf} + \boldsymbol{h}(\boldsymbol{q}_{rf}, \dot{\boldsymbol{q}}_{rf})\dot{\boldsymbol{q}}_{rf} + \boldsymbol{K}\boldsymbol{q}_{rf} = \begin{bmatrix} \tau_0 \\ \boldsymbol{\tau}_l \\ \boldsymbol{0} \end{bmatrix} \\ \boldsymbol{J}_a\boldsymbol{K}_{fj}^{-1}\ddot{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_l = \boldsymbol{\tau}_a - \boldsymbol{J}_a\ddot{\boldsymbol{q}}_l \end{cases}. \tag{7}$$

Let $\boldsymbol{K}_\mu = \mu^2\boldsymbol{K}_{fj}$, where the small constant μ is the singular perturbation parameter. $\boldsymbol{K}_\mu \in \boldsymbol{R}^{2\times2}$ is a suitably defined positive definite diagonal matrix. The flexible-joint fast subsystem is

$$\mu^2\boldsymbol{J}_a\ddot{\boldsymbol{\tau}}_l + \boldsymbol{K}_\mu\boldsymbol{\tau}_l = \boldsymbol{K}_\mu(\boldsymbol{\tau}_a - \boldsymbol{J}_a\ddot{\boldsymbol{q}}_l). \tag{8}$$

By singular perturbation approach [36], the system is transformed into two subsystems: a flexible-link subsystem and a flexible-joint fast subsystem. The cooresponding composite controller $\boldsymbol{\tau}$ is

$$\boldsymbol{\tau} = \begin{bmatrix}\tau_0 & \boldsymbol{\tau}_a^T\end{bmatrix}^T = \begin{bmatrix}\bar{\tau}_0 & \boldsymbol{\tau}_{al}^T + \boldsymbol{\tau}_{afj}^T\end{bmatrix}^T, \tag{9}$$

where $\bar{\tau}_0$ is the flexible-link subsystem base attitude control component, $\boldsymbol{\tau}_{al}$ is the flexible-link subsystem link control component and $\boldsymbol{\tau}_{afj}$ is the flexible-joint fast control component.

The flexible-joint fast controller is

$$\boldsymbol{\tau}_{afj} = -\mu\boldsymbol{K}_f\dot{\boldsymbol{\tau}}_l, \tag{10}$$

where $\boldsymbol{K}_f \in \boldsymbol{R}^{2\times2}$ is a reasonably defined positive-definite diagonal matrix which can stabilize the subsystem

$$\mu^2\boldsymbol{J}_a\ddot{\boldsymbol{\tau}}_l + \mu\boldsymbol{K}_\mu\boldsymbol{K}_f\dot{\boldsymbol{\tau}}_l + \boldsymbol{K}_\mu\boldsymbol{\tau}_l = \boldsymbol{K}_\mu(\boldsymbol{\tau}_{al} - \boldsymbol{J}_a\ddot{\boldsymbol{q}}_l), \tag{11}$$

B. Rigid Subsystem and the Corresponding Controller

Let $\mu = 0$, Eqs. (7) and (11) are written as

$$
\boldsymbol{D}\left(\bar{\boldsymbol{q}}_{rf}\right)\ddot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{h}\left(\bar{\boldsymbol{q}}_{rf},\dot{\bar{\boldsymbol{q}}}_{rf}\right)\dot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{K}\bar{\boldsymbol{q}}_{rf}=\begin{bmatrix}\bar{\tau}_0\\ \bar{\boldsymbol{\tau}}_l\\ \boldsymbol{0}\end{bmatrix}, \tag{12}
$$

$$
\bar{\boldsymbol{\tau}}_l=\boldsymbol{\tau}_{al}-\boldsymbol{J}_a\ddot{\bar{\boldsymbol{q}}}_l\,. \tag{13}
$$

Substituting Eq.(13) into Eq.(12), the flexible-link subsystem is

$$
\left(\boldsymbol{D}\left(\bar{\boldsymbol{q}}_{rf}\right)+\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{J}_a & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\right)\ddot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{h}\left(\bar{\boldsymbol{q}}_{rf},\dot{\bar{\boldsymbol{q}}}_{rf}\right)\dot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{K}\bar{\boldsymbol{q}}_{rf}=\begin{bmatrix}\bar{\tau}_0\\ \boldsymbol{\tau}_{al}\\ \boldsymbol{0}\end{bmatrix}. \tag{14}
$$

Eq.(14) is written as

$$
\bar{\boldsymbol{D}}\ddot{\bar{\boldsymbol{q}}}_{rf}+\bar{\boldsymbol{h}}\dot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{K}\bar{\boldsymbol{q}}_{rf}=\begin{bmatrix}\bar{\tau}_0\\ \boldsymbol{\tau}_{al}\\ \boldsymbol{0}\end{bmatrix}. \tag{15}
$$

Based on the singular perturbation approach [36], the flexible-link subsystem is further transformed into two subsystems: a rigid subsystem and a flexible-link fast subsystem. The composite controller $\bar{\boldsymbol{\tau}}$ is given by

$$
\bar{\boldsymbol{\tau}}=\begin{bmatrix}\bar{\tau}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}=\bar{\bar{\boldsymbol{\tau}}}+\boldsymbol{\tau}_{fl}=\begin{bmatrix}\bar{\bar{\tau}}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}+\boldsymbol{\tau}_{fl}
$$

where $\bar{\bar{\boldsymbol{\tau}}}$ is the rigid control component and $\boldsymbol{\tau}_{fl}$ is the flexible-link fast control component.

Define the inverse of the mass matrix

$$
\boldsymbol{N}=\begin{bmatrix}\boldsymbol{N}_{rr} & \boldsymbol{N}_{rf}\\ \boldsymbol{N}_{fr} & \boldsymbol{N}_{ff}\end{bmatrix}=\bar{\boldsymbol{D}}^{-1}=\begin{bmatrix}\bar{\boldsymbol{D}}_{rr} & \bar{\boldsymbol{D}}_{rf}\\ \bar{\boldsymbol{D}}_{fr} & \bar{\boldsymbol{D}}_{ff}\end{bmatrix}^{-1}, \tag{16}
$$

where $\bar{\boldsymbol{D}}_{rr}\in\boldsymbol{R}^{3\times3}$, $\bar{\boldsymbol{D}}_{rf}\in\boldsymbol{R}^{3\times2}$, $\bar{\boldsymbol{D}}_{fr}\in\boldsymbol{R}^{2\times3}$, and $\bar{\boldsymbol{D}}_{ff}\in\boldsymbol{R}^{2\times2}$ are the sub-matrices of $\bar{\boldsymbol{D}}$.

Eq. (15) is written as

$$
\boldsymbol{D}\left(\bar{\boldsymbol{q}}_{rf}\right)\ddot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{h}\left(\bar{\boldsymbol{q}}_{rf},\dot{\bar{\boldsymbol{q}}}_{rf}\right)\dot{\bar{\boldsymbol{q}}}_{rf}+\boldsymbol{K}\bar{\boldsymbol{q}}_{rf}=\begin{bmatrix}\bar{\tau}_0\\ \bar{\boldsymbol{\tau}}_l\\ \boldsymbol{0}\end{bmatrix}, \tag{17}
$$

where $\bar{\boldsymbol{h}}_{rr}\in\boldsymbol{R}^{3\times3}$, $\bar{\boldsymbol{h}}_{rf}\in\boldsymbol{R}^{3\times2}$, $\bar{\boldsymbol{h}}_{fr}\in\boldsymbol{R}^{2\times3}$, and $\bar{\boldsymbol{h}}_{ff}\in\boldsymbol{R}^{2\times2}$ are the sub-matrices of $\bar{\boldsymbol{h}}$.

Assuming K_m is the smallest element of the flexible link stiffness matrix $\boldsymbol{K}_{fl}$, then define new variables $\varepsilon^2=1/K_m$, $\varepsilon^2\boldsymbol{\xi}_{\bar{\boldsymbol{q}}_f}=\bar{\boldsymbol{q}}_f$, $\tilde{\boldsymbol{K}}_{fl}=\varepsilon^2\boldsymbol{K}_{fl}$, Eq.(18) is obtained as

$$
\begin{cases}
\ddot{\bar{\boldsymbol{q}}}_r=\boldsymbol{N}_{rr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)\begin{bmatrix}\bar{\tau}_0\\ \boldsymbol{\tau}_{al}\end{bmatrix}-\boldsymbol{N}_{rf}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)\boldsymbol{K}_{fl}\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\\
\qquad-\boldsymbol{N}_{rr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)(\bar{\boldsymbol{h}}_{rr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{rf}\varepsilon^2\dot{\xi}_{\bar{\boldsymbol{q}}_f})\\
\qquad-\boldsymbol{N}_{rf}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)(\bar{\boldsymbol{h}}_{fr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{ff}\varepsilon^2\dot{\xi}_{\bar{\boldsymbol{q}}_f})\\
\varepsilon^2\ddot{\xi}_{\bar{\boldsymbol{q}}_f}=\boldsymbol{N}_{fr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)\begin{bmatrix}\bar{\tau}_0\\ \boldsymbol{\tau}_{al}\end{bmatrix}-\boldsymbol{N}_{ff}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)\boldsymbol{K}_{fl}\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\\
\qquad-\boldsymbol{N}_{fr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)(\bar{\boldsymbol{h}}_{rr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{rf}\varepsilon^2\dot{\xi}_{\bar{\boldsymbol{q}}_f})\\
\qquad-\boldsymbol{N}_{ff}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar{\boldsymbol{q}}_f}\right)(\bar{\boldsymbol{h}}_{fr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{ff}\varepsilon^2\dot{\xi}_{\bar{\boldsymbol{q}}_f})
\end{cases}. \tag{18}
$$

Let ε is zero in Eq.(18) , and Eq.(19) is obtained as

$$
\begin{cases}
\ddot{\bar{\bar{\boldsymbol{q}}}}_r=\bar{\boldsymbol{N}}_{rr}\begin{bmatrix}\bar{\bar{\tau}}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}-\bar{\boldsymbol{N}}_{rf}\tilde{\boldsymbol{K}}_{fl}\bar{\xi}_{\bar{\boldsymbol{q}}_f}-\bar{\boldsymbol{N}}_{rr}\bar{\bar{\boldsymbol{h}}}_{rr}\dot{\bar{\bar{\boldsymbol{q}}}}_r-\bar{\boldsymbol{N}}_{rf}\bar{\bar{\boldsymbol{h}}}_{fr}\dot{\bar{\bar{\boldsymbol{q}}}}_r\\
\boldsymbol{0}=\bar{\boldsymbol{N}}_{fr}\begin{bmatrix}\bar{\bar{\tau}}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}-\bar{\boldsymbol{N}}_{ff}\tilde{\boldsymbol{K}}_{fl}\bar{\xi}_{\bar{\boldsymbol{q}}_f}-\bar{\boldsymbol{N}}_{fr}\bar{\bar{\boldsymbol{h}}}_{rr}\dot{\bar{\bar{\boldsymbol{q}}}}_r-\bar{\boldsymbol{N}}_{ff}\bar{\bar{\boldsymbol{h}}}_{fr}\dot{\bar{\bar{\boldsymbol{q}}}}_r
\end{cases}. \tag{19}
$$

$\bar{\xi}_{\bar{\boldsymbol{q}}_f}$ is solved from (19).

$$
\bar{\xi}_{\bar{\boldsymbol{q}}_f}=\tilde{\boldsymbol{K}}_{fl}^{-1}\bar{\boldsymbol{N}}_{ff}^{-1}\left(\bar{\boldsymbol{N}}_{fr}\begin{bmatrix}\bar{\bar{\tau}}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}-\bar{\boldsymbol{N}}_{fr}\bar{\bar{\boldsymbol{h}}}_{rr}\dot{\bar{\bar{\boldsymbol{q}}}}_r-\bar{\boldsymbol{N}}_{ff}\bar{\bar{\boldsymbol{h}}}_{fr}\dot{\bar{\bar{\boldsymbol{q}}}}_r\right) \tag{20}
$$

The rigid subsystem is computed by combining Eqs. (19) and (20)

$$
\ddot{\bar{\bar{\boldsymbol{q}}}}_r=\bar{\bar{\boldsymbol{D}}}_{rr}^{-1}\left(-\bar{\bar{\boldsymbol{h}}}_{rr}\dot{\bar{\bar{\boldsymbol{q}}}}_r+\bar{\bar{\tau}}\right) \tag{21}
$$

The position error vector is defined as

$$
\boldsymbol{e}=\boldsymbol{q}_{rd}-\bar{\bar{\boldsymbol{q}}}_r, \tag{22}
$$

where $\boldsymbol{q}_{rd}=\begin{bmatrix}\theta_{0d} & \theta_{1d} & \theta_{2d}\end{bmatrix}^{\mathrm{T}}$ is the desirable trajectory, θ_{0d} is desirable attitude angle of the base, θ_{id} *is* the desirable *i*th link angular position (i =1,2).

The controller for the rigid subsystem can be designed according to the computed torque control technique, which is written as

$$
\bar{\bar{\boldsymbol{\tau}}}=\begin{bmatrix}\bar{\bar{\tau}}_0\\ \bar{\boldsymbol{\tau}}_{al}\end{bmatrix}=\left(\bar{\bar{\boldsymbol{D}}}_{rr}+\begin{bmatrix}0 & 0\\ 0 & \boldsymbol{J}_a\end{bmatrix}\right)\left(\ddot{\boldsymbol{q}}_{rd}+\boldsymbol{K}_d\dot{\boldsymbol{e}}+\boldsymbol{K}_p\boldsymbol{e}\right)+\bar{\bar{\boldsymbol{h}}}_{rr}\dot{\bar{\bar{\boldsymbol{q}}}}_r, \tag{23}
$$

where $\boldsymbol{K}_p=\boldsymbol{K}_p^{\,T}>0$ is the controller proportional gain, $\boldsymbol{K}_d=\boldsymbol{K}_d^{\,T}>0$ is the controller derivative gain.

The closed-loop dynamic errors of the system are given as

$$
\left(\bar{\bar{\boldsymbol{D}}}_{rr}+\begin{bmatrix}0 & 0\\ 0 & \boldsymbol{J}_a\end{bmatrix}\right)\left(\ddot{\boldsymbol{e}}+\boldsymbol{K}_d\dot{\boldsymbol{e}}+\boldsymbol{K}_p\boldsymbol{e}\right)=0
$$

Obviously the uniform ultimate boundedness for the tracking errors of the slow subsystem are guaranteed by appropriate $\boldsymbol{K}_d$ and $\boldsymbol{K}_p$.

C. Flexible-link Fast Subsystem and the Corresponding Controller

New states $\zeta_1=\xi_{\bar q_f}-\bar\xi_{\bar q_f}$ and $\zeta_2=\varepsilon\dot\xi_{\bar q_f}$ are selected, and a fast time scale $t_f=\frac{t-t_0}{\varepsilon}$ is introduced. Eq.(24) is obtained as

$$\begin{cases}\dfrac{\mathbf{d}\zeta_1}{\mathbf{d}t_f}=\zeta_2-\varepsilon\dot{\bar\xi}_{\bar q_f}\\ \dfrac{\mathbf{d}\zeta_2}{\mathbf{d}t_f}=\boldsymbol{N}_{fr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar q_f}\right)\begin{bmatrix}\bar\tau_0\\ \boldsymbol{\tau}_{al}\end{bmatrix}-\boldsymbol{N}_{ff}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar q_f}\right)\tilde{\boldsymbol{K}}_{fl}\xi_{\bar q_f}\\ \qquad -\boldsymbol{N}_{fr}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar q_f}\right)(\bar{\boldsymbol{h}}_{rr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{rf}\varepsilon^2\dot\xi_{\bar q_f})\\ \qquad -\boldsymbol{N}_{ff}\left(\bar{\boldsymbol{q}}_r,\varepsilon^2\xi_{\bar q_f}\right)(\bar{\boldsymbol{h}}_{fr}\dot{\bar{\boldsymbol{q}}}_r+\bar{\boldsymbol{h}}_{ff}\varepsilon^2\dot\xi_{\bar q_f})\end{cases}. \tag{24}$$

Assuming $\varepsilon=0$ and using Eq. (20) the flexible-link fast subsystem dynamic equation is obatined as

$$\frac{\mathbf{d}\zeta}{\mathbf{d}t_f}=\boldsymbol{A}\zeta+\boldsymbol{B}\boldsymbol{\tau}_{fl}, \tag{25}$$

where $\boldsymbol{B}=\begin{bmatrix}\mathbf{0}\\ \boldsymbol{N}_{fr}\end{bmatrix}$, $\zeta=\begin{bmatrix}\zeta_1^T & \zeta_2^T\end{bmatrix}^T$, $\boldsymbol{A}=\begin{bmatrix}\mathbf{0} & \boldsymbol{I}\\ -\boldsymbol{N}_{ff}\tilde{\boldsymbol{K}}_{fl} & \mathbf{0}\end{bmatrix}$.

Because the pair $(\boldsymbol{A},\boldsymbol{B})$ of the fast subsystem are completely state controllable, the fast state feedback control can be devised to force its states ζ to zero by using optimal LQR approach control method. The cost function is chosen as

$$E=\frac{1}{2}\int_0^\infty\left(\zeta^T\boldsymbol{Q}\zeta+\boldsymbol{\tau}_{fl}^T\boldsymbol{R}\boldsymbol{\tau}_{fl}\right)dt_f, \tag{26}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are standard LQR weighting matrices. Then

$$\boldsymbol{\tau}_{fl}=-\boldsymbol{K}_{opt}\zeta=-\boldsymbol{R}^{-1}\boldsymbol{B}\boldsymbol{P}\zeta, \tag{27}$$

where $\boldsymbol{P}$ is the solution of the following Ricatti equation.

$$\boldsymbol{P}\boldsymbol{A}+\boldsymbol{A}^T\boldsymbol{P}-\boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}+\boldsymbol{Q}=\boldsymbol{0}$$

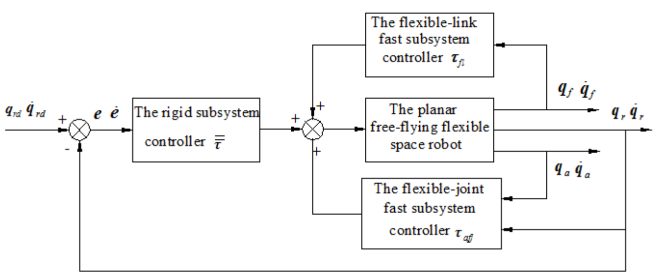

Figure 2. The control strategy

The operational control strategy is shown in Fig. 2.

III. SIMULATION EXAMPLE

The actual plant parameters of the system shown in Fig. 1 are $l_0=l_2=1.0\text{m}$, $l_1=2.0\text{m}$, $d_1=1.0\text{m}$, $m_0=200\text{kg}$, $m_1=8\text{kg}$, $\rho_2=1.6\text{kg/m}$, $I_0=100\text{kg}\cdot\text{m}^2$, $I_1=4.2\text{kg}\cdot\text{m}^2$. The desired trajectories are $\theta_{0d}=0$, $\theta_{1d}=\pi t/18-4\sin(\pi t/4)/3+0.05\pi$, $\theta_{2d}=-\pi t/24+\sin(\pi t/4)+0.4\pi$.

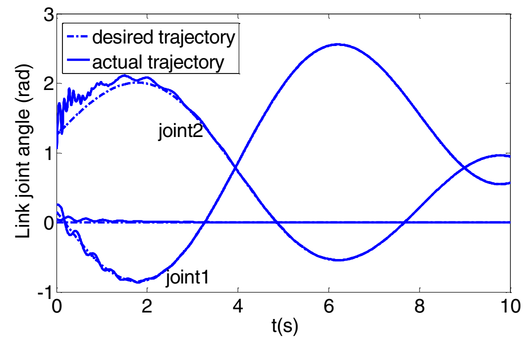

Figure 3. The link joint trajectory and the base attitude trajectory

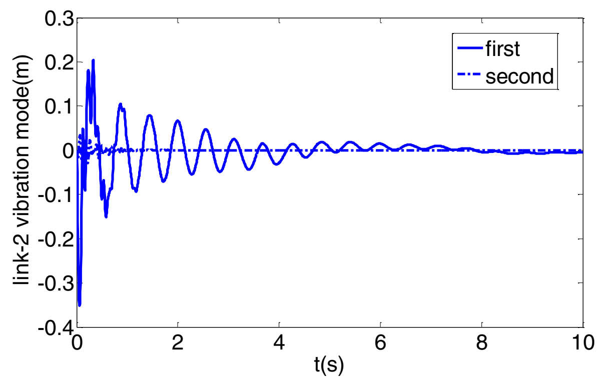

Figure 4. Link-2 vibration mode

The initial state of the space manipulator is $\theta_0(0)=0.05\text{rad}$, $\theta_1(0)=0.26\text{rad}$, $\theta_2(0)=1.06\text{rad}$ .The time taking in the simulation is $t=10\text{s}$.

The controller performances are shown in Figs. 3-4. It can be seen from Fig. 3 that good tracking performance is achieved through the application of the proposed controller. Fig. 4 shows that the first and second mode of vibration of the flexible link are well damped.

IV. CONCLUSION

The planar flexible-link flexible-joint free-flying space manipulator dynamic model has been established by using the Euler–Lagrange principle. The introduced dynamic model first is disassembled into three subsystems: the flexible-joint fast subsystem, the flexible-link fast subsystem and the rigid space manipulator. The general model formulation is exploited to obtain the closed-form dynamic models for practical free-flying space robots with any number of flexible links and flexible joints.

Based on the introduced dynamic model, the composite controller which consists of a rigid control component and two fast control components is proposed. The results of numerical simulation indicate that good tracking is performed and link vibrations are suppressed effectively.

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