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An hp-adaptative pseudospectral method for collision avoidance with multiple UAVs in real-time applications

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Abstract— **This paper proposes the application of an hp-adaptive pseudospectral for trajectory generation in scenarios with multiple aerial vehicles in order to avoid collisions. This method computes an optimal solution numerically. The method assigns a speed profile to each aerial vehicle in real time such that the separation between them is greater than a minimum safety value and the total deviation from the initial trajectories is minimized. The Estimated Time of Arrival (ETA) of each aerial vehicle is also taken into account to solve the conflicts. Its computational load and scalability depending on the main parameters of the method are studied. Many simulations have been performed to analyze the best parameters of the method. Experiments have been also carried out in the multivehicle aerial testbed of the Center for Advanced Aerospace Technologies (CATEC).**

I. INTRODUCTION

Trajectory planning and collision avoidance is a critically important aspect in real-time applications with multiple Unmanned Aerial Vehicles (UAVs) to successfully perform a coordinated mission. Cooperation and coordination of many mobile entities such as aerial vehicles are being performed in the EC-SAFEMOBIL FP7 European Project (http://www.ec-safemobil-project.eu/). This project is developing sufficiently accurate common motion estimation and control methods and technologies in order to reach levels of reliability and safety to facilitate UAV deployment in a broad range of applications. The use of safe trajectory optimisation techniques plays an important role in this project and it is addressed in this work.

This paper addresses the problem of collision avoidance with multiple UAVs to ensure the safety and reliability of the mission. The proposed method is based on speed planning to solve collisions and to meet as much as possible each initial trajectory. This approach has the advantage that the probability of creating new conflicts with other UAVs is low.The proposed method computes an optimal solution and uses an hp-adaptive pseudospectral method. This is an analytical method which main characteristic is the low computational load.

The direct collocation methods compute the solution considering a fixed degree polynomial state approximation in each segment and dividing the problem into segments. Convergence of the numerical discretization is then achieved by increasing the number of segments [1]. On the other hand, pseudospectral methods have been also used in optimal control problems [2]. In contrast to the direct collocation method, a pseudospectral method uses a single segment, and convergence is achieved by increasing the degree of the polynomial. The collocation points are chosen based on accurate quadrature rules and the basis functions are typically Chebyshev or Lagrange polynomials. The more commonly used pseudospectral methods are the Gauss pseudospectral method (GPM) [3], the Radau pseudospectral method [4] (RPM), and the Lobatto pseudospectral method [5] (LPM). Limitations of the pseudospectral methods are presented in [6]. In order to overcome these limitations, an hp-adaptive pseudospectral method has been proposed [6]. This method can increase the number of segments and the degree of the polynomial within a segment to achieve an error less than the tolerance error allowed.

The hp-adaptive pseudospectral method is a good candidate to apply in collision avoidance problems between multiple UAVs. The localisation of the segments and the number of collocation points depend on where the conflicts take place. This method is suitable because of the flexibility to increase segments and/or collocation points.

Pseudospectral and Direct Collocation methods have been applied to compute aircraft trajectories [7] [8] [9]. However, the selection of the numerical parameters that make the system converge to an optimal solution has not been addressed. Several of the parameters may have a strong influence on the numerical convergence of the method and the real time performance. Therefore, these parameters should be analyzed to use the suitable ones depending on the scenario.

Most of published works related to Pseudospectral and Direct Collocation methods consider trajectory generation for a standalone UAV [7] [8]. Other novel aspect of this paper is that trajectory generation for multiple UAVs are considered. Moreover, the solution computed is only valid in the collocation points. That is, the minimum separation among UAVs is maintained in these points. Therefore, an evaluation considering a model of UAV should be carried out in order to ensure that the minimum separation is not violated during the rest of the flight. A study on the separation constraint considered is done depending of the scenario. Work presented in [9] considers three UAVs but error of tolerance, number of nodes or collocation points and separation are not discussed.

The paper is organized into seven sections. Section II presents some works done on this problem. Section III

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describes the problem formulation addressed. The proposed method is explained in Section IV. Simulations and experiments performed are showed in Section V and VI, respectively. Finally, the conclusions are detailed in Section VII.

II. State of the art

UAV planning algorithms and collision avoidance methods have been studied extensively. A detailed survey on the former is presented in [10] and [11] reviews papers on the latter.

Among the different methods can include non-linear programming (NLP) [12], integer programming [13] and collocation methods reducing the number of dimensions of the problem [7] [8] [9] [14], graph search like A* [15] and Rapidly-exploring Random Trees (RRT) [16], particle swarm optimization [17], evolutionary computation methods [18], ant colony optimization methods [19], among many others.

The method in [18] is based on the use of genetic algorithms to solve conflicts by changing the heading. The main drawback of genetic algorithms is that the computation time is not predictable and the convergence to a solution is not ensured in a finite time interval. On the other hand, a stochastic method based on the Monte Carlo approach solves conflicts in air traffic control but the computation time is high [20]. A CDR method based on a mixed-integer linear program (MILP) optimizes the total flight time by modifying velocity or heading [13]. However, this method only allows one speed change for each aircraft. Three different collision detection and resolution methods based on speed planning are presented in [21].

III. Problem description

The problem of collision avoidance of multiple UAVs to perform the coordinated missions proposed by the EC-SAFEMOBIL project is considered in this paper. The project considers a cooperative tracking and surveillance scenario. This scenario considers a situation where a region is to be observed, and multiple targets within that region tracked, in the presence of obstacles. The scenario incorporates the need for cooperative safe trajectory planning to perform the coordinated missions, assuring that the aerial vehicles do not collide with each other. The work presented is related to this task.

The proposed conflict resolution method is based on changing the speed profile of the UAVs involved in the conflict. Note that changes of velocity direction are not considered, so only the speed profile is changed.

The trajectory of each UAV is given by an initial waypoint and a final waypoint. Each waypoint is defined by: 2D coordinates (x,y), speed from that waypoint (v), and the Estimated Time of Arrival (ETA) to the waypoint, t. To meet the ETA is important in many applications. It is assumed that all UAV trajectories are known. We consider that the UAVs maintain the safety separation if they are separated by a minimum distance, D.

The problem would be solved like an optimal control problem where global criterion is the flight cost. This cost is defined by the changes of velocity.

The inputs of the method are the following:

- Initial trajectory of each UAV
- Model of each UAV
- ETA of each UAV

The objective is to find collision-free trajectories that minimize the probability of having a collision while minimizing the changes of speed for each UAV. Moreover, the ETA will be met.

Several definitions are needed to perform the study presented in Section V:

- **Initial mesh**: it is the initial distribution of segments. The initial mesh in the studies will be a only segment with twenty nodes.
- **Tolerance error**: it is the allowed error in each node or collocation point.
- **SplitMult**: it is the index of segmentation of the mesh and is related to the ease to increase the number of segments in every iteration. It also influences the convergence time of the method.
- **Number of nodes per segment**: it is also called collocation points. This number defines the degree of the polynomial of interpolation used in each segment. The number is defined between four and twelve nodes per segment. Initially, each segment has four nodes. This number can increase in each segment in order to meet the tolerance error.
- **Number of iterations**: it is the number of times that the method can iterate to obtain a solution. The number considered is five.

Splitmult and tolerance error have been analyzed from simulations performed in Section V to obtain the best values that minimize computation time.

IV. hp-adaptive pseudospectral method

The hp-adaptive pseudospectral method numerically solves optimal control problems. The basic approach is to transform the optimal control problem into a sequence of nonlinear constrained optimization problems by discretizing the state and control variables.

This method determines the number of segments and the degree of the polynomial in each segment that provides an accurate approximation to the solution of the optimal control problem.

The optimal control problem is considered in Bolza form. The following cost function should be minimized:

$$J = \phi(x(-1), t_0, x(+1), t_f) + \frac{t_f - t_0}{2} \int_{-1}^{1} \mathcal{L}(x(\tau), u(\tau), \tau) d\tau \quad (1)$$

subject to the dynamic constraints:

$$\frac{dx}{d\tau} = \frac{t_f - t_0}{2} f(x(\tau), u(\tau), \tau) \quad (2)$$

Boundary conditions are considered:

$$\phi(x(-1,),t_0,x(+1),t_f)=0 \quad (3)$$

and the inequality path constraints:

$$C(x(\tau),u(\tau),\tau,t_0,t_f)\leq 0 \quad (4)$$

where $x(\tau)$ is the state, $u(\tau)$ is the control, and τ is time. The variable $\tau\epsilon[-1,1]$ and $t\epsilon[t_0,t_f]$ are related as

$$t=\frac{t_f-t_0}{2}\tau+\frac{t_f-t_0}{2} \quad (5)$$

The hp-adaptive pseudospectral method can increase the number of segments and the degree of the polynomial within a segment to achieve an error less than the tolerance error allowed [6].

It is an iterative method and computes the solution until a user-specified tolerance error is met. The solution is determined by the number of segments, the width of each segment and the polynomial degree (number of collocation points) required in each segment.

Therefore, the method computes an accurate solution at the collocation points. Moreover, it should compute an accurate solution between the collocation points in order to ensure that the constraints are met in the whole problem.

The main advantage of this method is that it leads to higher accuracy solutions with less computational load than is required in a global pseudospectral method.

The method could address the 3D problem but in order to clarify the results, the 2D problem has been considered.

A. *Implementation*

A model of aerial vehicle is used in this application with multiple UAVs. The altitude is assumed to be constant. The state vector is defined by the position of the aerial vehicle x_i and the speed of the aerial vehicle v_i. The input control is the speed reference, u_{v_i}, and the heading reference, u_{Ψ_i}.

The model considered is:

$$\dot{x}_i=v_i\cdot cos(\Psi_i) \quad (6)$$

$$\dot{v}_i=\frac{-1}{\tau}(v_i-u_i) \quad (7)$$

where, Ψ_i is the heading of the trajectory and τ is the time needed by the UAV to reach the speed reference u_i.

A multi-UAV system can be defined by concatenating the state of all the UAVs. Therefore, the state vector and control vector are defined as follows:

$$X=[x_1,v_1,\,x_2,v_2,....,x_n,v_n] \quad (8)$$

$$U=[u_{v_1},u_{\Psi_1},u_{v_2},u_{\Psi_2},....,u_{v_n},u_{\Psi_n}] \quad (9)$$

where n is the number of UAVs.

The solution should satisfy constraints taking into account the physical limitations of each UAV and the separation between UAVs. The UAV speed will be constrained:

$$v_{min}<v_{cruise}<v_{max} \quad (10)$$

and the separation between UAV_i and UAV_j should meet:

$$distance(UAV_i,UAV_j)\geq D \quad (11)$$

where D is the safety distance and y is obtained from x:

$$y_i=tan(\Psi_i)x_i+B_i,\ with\ \Psi_i\neq 0 \quad (12)$$

where B_i is the y-intercept of the trajectory.

Moreover, the ETA should be met, so the flight time should be maintained. Finally, the speed in the final waypoint should be the cruise speed, v_{cruise}.

V. Simulations

Many simulations have been carried out in two different scenarios with two, three, four and five UAVs to analyze the best values of: splitmult (S), tolerance error (E_t) and separation between UAVs (D). The problems have been solved by using the open-source pseudospectral optimal control software GPOPS [2]. The algorithms have been run in a PC with a CPU Intel Core i7-3770 @ 3.4 Ghz and 16 GB of RAM. The operating system used in the simulations was Kubuntu Linux 12.10 OS and the code has been implemented in Matlab.

The objective of the study is to find which values of S, E_t and D provide less computation time and the dependency on the number of UAVs. This is a critically aspect in real-time applications. The studies are performed in the scenarios shown in Figure 1). It is relevant the study of D because, although the minimum separation is 1 meter, maybe the value of D in equation (11) could be greater in order to ensure the separation in the whole flight and no only at the collocation points.

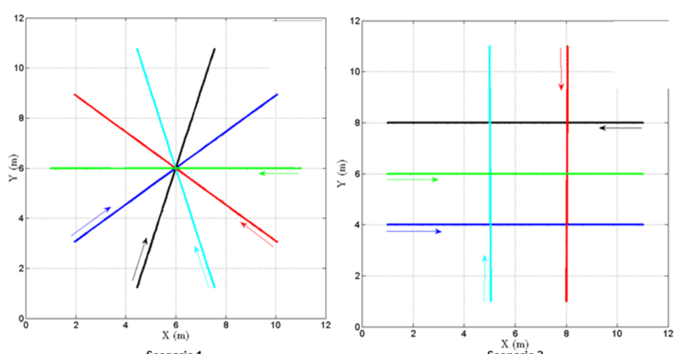

Fig. 1. Scenarios considered in the simulations with five UAVs: UAV1 in blue, UAV2 in black, UAV3 in clear blue, UAV4 in red and UAV5 in green.

The considered values of each parameter are: $S=[1.0,1.1,1.2,1.3]$, $E_t=[0.001,0.005,0.01,0.05,0.1]$ and $D=[1.0,1.1,1.2,1.5,2.0,2.5]$. Therefore, a hundred and twenty possible combinations $\{S,E_t,D\}$ are explored. Moreover, fifty simulations are performed in each combination. Tables I and II show the minimum computation time obtained and the corresponding combination $\{S,E_t,D\}$.

Note that the value of D is greater than 1 meter when the number of UAVs is greater than two. Therefore, the separation constraint should consider a greater separation at each collocation point to ensure the safety distance in the whole flight. The number of nodes, S and E_t depend on the number of UAVs.

TABLE I

VALUES OBTAINED BY CONSIDERING THE SCENARIO S1.

UAVs	Time (s)	Nodes	S	E_t (m)	D (m)
2	0.082	21	2	0.01	1.0
3	0.300	21	1.2	0.01	1.2
4	0.525	21	1.1	0.05	1.2
5	0.880	17	1.1	0.1	1.3

TABLE II

VALUES OBTAINED BY CONSIDERING THE SCENARIO S2.

UAVs	Time (s)	Nodes	S	$E_t(m)$	D (m)
2	0.315	37	2.5	0.005	1.0
3	0.550	29	1.2	0.01	1.2
4	0.7301	21	1.1	0.1	1.2
5	1.1706	25	1.0	0.01	1.3

Two simulations are presented to illustrate the solutions obtained in each scenario. First, scenario S1 with three UAVs is considered. Figure 2 shows the number and localisation of each node or collocation point by considering the best combination (see Table I). Figure 3 depicts the speed profile computed. Note that the initial and final speed are equal, $v_{cruise} = 0.65m/s$. Moreover the ETA is met. Finally, the separation between UAVs is presented in Figure 4. The trajectories are safe. For that, $D = 1.2m$ is considered in equation (11).

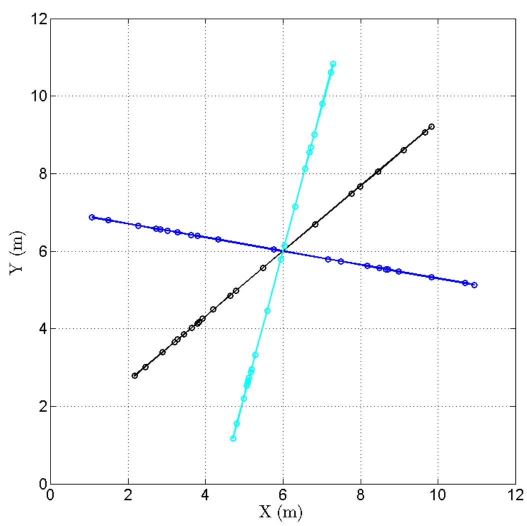

Fig. 2. Number of nodes considering three UAVs in scenario S1: UAV1 in blue, UAV2 in black and UAV3 in clear blue.

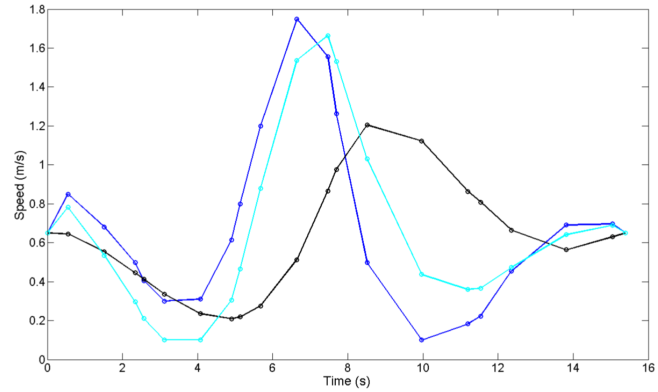

Fig. 3. Speed profile of each UAV considering three UAVs in scenario S1: UAV1 in blue, UAV2 in black and UAV3 in clear blue.

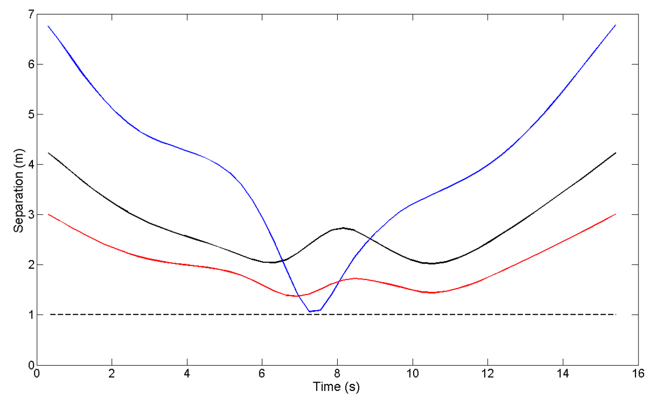

Fig. 4. Separation between UAVs considering three UAVs in scenario S1: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV2-UAV3 in red and the minimum separation in dashed black line.

Second simulation considers scenario S2 with five UAVs. Figure 5 shows the number and localisation of each node by considering the best combination (see Table II), Figure 6 depicts the speed profile computed, and the separation between UAVs is presented in Figure 7. The trajectories are safe. For that, $D = 1.3m$ is considered in equation (11).

VI. EXPERIMENTS

Two experiments have been carried out in the indoor multi-UAV testbed of the CATEC with four Hummingbird quadrotors with 200g payload and up to 20 minutes flight autonomy. The testbed has an indoor localization system based on 20 VICON cameras. This system is able to provide, in real time, the position and attitude of each UAV with centimeter accuracy. The minimum separation is 1.0m in the experiments. The parameters are: $v_{cruise} = 0.65m/s, v_{min} = 0.1m/s, v_{max} = 2m/s$. The experiments can be seen in the attached video.

In the first experiment, a scenario as S1 with four UAVs is considered (see Figure 1). A conflict is detected in the center, and then the hp-adaptive pseudospectral method computes the speed profile for each UAV. Figure 8 shows the UAV

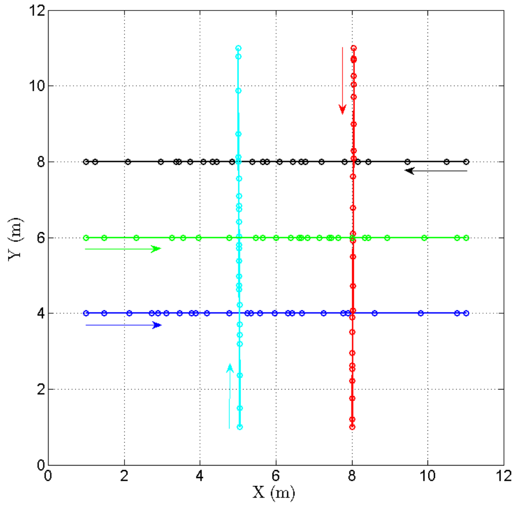

Fig. 5. Number of nodes considering five UAVs in scenario S2: UAV1 in blue, UAV2 in black, UAV3 in clear blue, UAV4 in red and UAV5 in green.

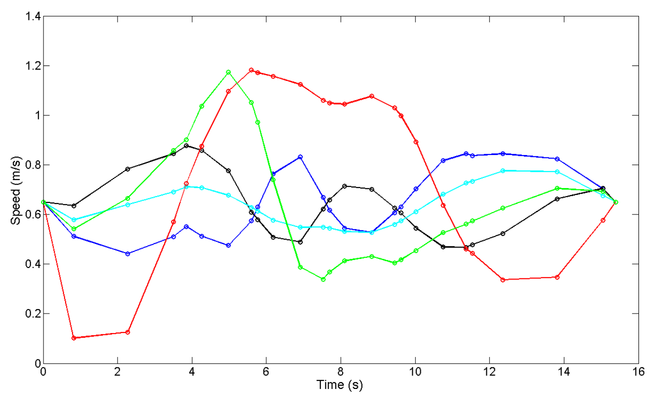

Fig. 6. Speed profile of each UAV considering five UAVs in scenario S2: UAV1 in blue, UAV2 in black, UAV3 in clear blue, UAV4 in red and UAV5 in green.

real trajectories and Figure 9 shows the separation between UAVs. Each UAV maintains its initial trajectory and the minimum separation, and fulfills its ETA.

In the second experiment, a scenario similar to S2 with four UAVs is considered (see Figure 1). Now several conflicts are detected. The hp-adaptive pseudospectral method computes the speed profile for each UAV. Figure 10 shows the UAV real trajectories and Figure 11 shows the separation between UAVs. Again, each UAV maintains its initial trajectory and the minimum separation ($D = 1m$), and fulfills its ETA.

VII. Conclusions

The hp-adaptive pseudospectral method presented addresses problems of collision avoidance with multiple UAVs. An optimal solution is computed and changes of speed are considered to avoid the conflicts. The main advantage of the method is its low computational load and, so its availability to use in real time applications.

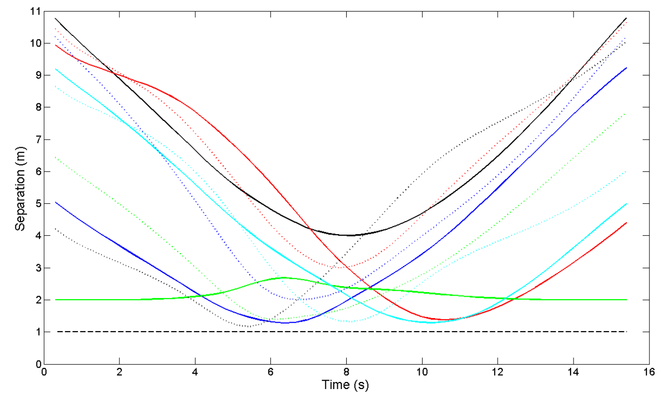

Fig. 7. Separation between UAVs considering five UAVs in scenario S2: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV1-UAV4 in red, UAV1-UAV5 in green, UAV2-UAV3 in clear blue, UAV2-UAV4 in dotted black, UAV2-UAV5 in dotted blue, UAV3-UAV4 in dotted red, UAV3-UAV5 in dotted green, UAV4-UAV5 in dotted clear blue and the minimum separation in dashed black line.

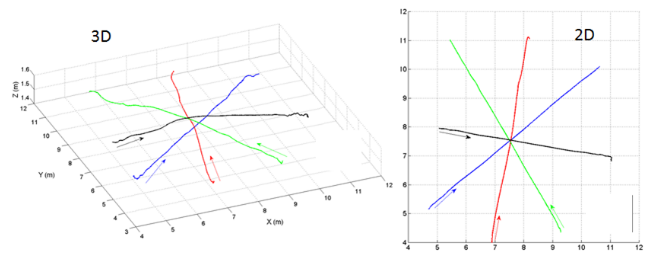

Fig. 8. UAV trajectories in the experiment I: UAV1 in black, UAV2 in blue, UAV3 in red and UAV4 in green.

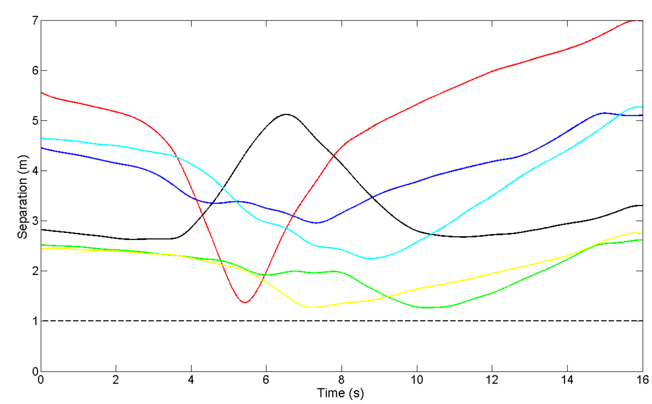

Fig. 9. Separation between UAVs in the experiment I: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV1-UAV4 in red, UAV2-UAV3 in green, UAV2-UAV4 in clear blue, UAV3-UAV4 in yellow and the minimum separation in dashed black line.

The more novel aspects of the paper can be summarized in these points:

- The method considers multiple UAVs. Most of works published on direct collocation for trajectory generation consider one aerial vehicle [7] [8].
- The suitable parameters (splitmult, tolerance error and separation constraint) have been studied to minimize the

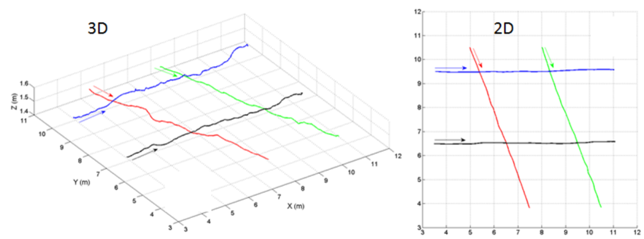

Fig. 10. UAV trajectories in the experiment II: UAV1 in black, UAV2 in blue, UAV3 in red and UAV4 in green.

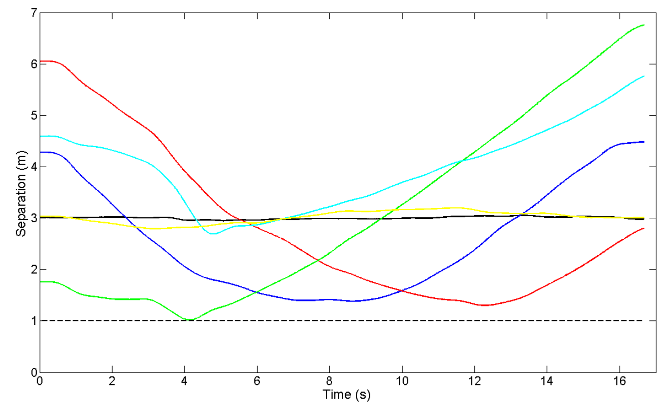

Fig. 11. Separation between UAVs in the experiment II: UAV1-UAV2 in black, UAV1-UAV3 in blue, UAV1-UAV4 in red, UAV2-UAV3 in green, UAV2-UAV4 in clear blue, UAV3-UAV4 in yellow and the minimum separation in dashed black line.

computation time. This study has been omitted in the works published. In this paper it is demonstrated the influence of these parameters on the computation time. The study of the separation constraint, D in equation (11), plays an important role because the method ensures the separation between UAVs in the whole flight and not only at each collocation point computed by the method.

- Several changes of speed can be carried out as in [21] but the proposed method considerably improves the computation time and ensures the optimal solution.
- Experiments to verify the solution computed by the method and its real time application have been carried out.

Maneuvers changing the heading and/or altitude of each UAV will be considered in future works. Also, models to deal with wind will be implemented.

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