1. The physical (real) electric field in a monochromatic wave is given by

$$
\begin{aligned}
\boldsymbol{\mathcal{E}}\left(\boldsymbol{r}, t\right) &= \mathrm{Re}\left\{\boldsymbol{E}_0\, e^{(i\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\right\} \\
&= \sum_{i=1,2} \boldsymbol{\epsilon}_i \mathrm{Re}\left\{E_{0_i}\, e^{(i\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\right\} \\
&= \sum_{i=1,2} \boldsymbol{\epsilon}_i\, \mathcal{E}_i\left(\boldsymbol{r}, t\right),
\end{aligned}
$$

in terms of linear polarization vector $\boldsymbol{\epsilon}_{1,2}$, satisfying $\boldsymbol{\epsilon}_i \cdot \boldsymbol{\epsilon}_j = \delta_{ij}$, $\boldsymbol{\epsilon}_i \times \boldsymbol{\epsilon}_j = \hat{\boldsymbol{k}}$. Show that the physical electric field, $\boldsymbol{\mathcal{E}}\left(\boldsymbol{r}, t\right)$, is constrained by

$$
\sum_{i,j=1}^{2} \mathcal{E}_i\left(\boldsymbol{r}, t\right) \underline{\underline{A}}_{ij} \mathcal{E}_j\left(\boldsymbol{r}, t\right) = 1.
$$

This is a double sum over polarizations. Find the 2×2 matrix $\underline{\underline{A}}$ and show that the tip of the electric field vector traces out an ellipse as a function of time at each point $\boldsymbol{r}$ in space.