Assignment #1:
Dynamics and Statics for a Simple Language

Fundamentals of Programming Languages

Out: Tuesday, Sept 6th, 2016
Due: Thursday, Sept 15th, 2016 11:59pm EST

1 Language

Now we have all the infrastructure we need to implement a language. We will be working with the language $\mathcal{L}\{\texttt{num str}\}$, defined in Ch. 4 of *PFPL*.

1.1 Statics

There is a lot of error checking going on in the dynamics in Appendix A. But as we've been discussing in class, we can eliminate much or all of this by equipping our language with a static type system (see Ch. 6 of PFPL)! The type checking rules for $\mathcal{L}\{\texttt{num str}\}$ are reproduced in Appendix B for your reference.

Task 1 (10%). (Unicity of Typing) Prove that for every typing context Γ and expression e, there exists at most one τ such that $\Gamma \vdash e : \tau$.

Task 2 (10%). (Canonical Forms) Prove that if e val, then

1. if $\Gamma \vdash e : \texttt{num}$ then $e = \texttt{num}[n]$ for some number n.
2. if $\Gamma \vdash e : \texttt{str}$ then $e = \texttt{str}[s]$ for some string s.

A Dynamics of $\mathcal{L}\{\texttt{num str}\}$

$\boxed{e \text{ val}}$

$$\frac{}{\texttt{num}[n] \text{ val}} \qquad \frac{}{\texttt{str}[s] \text{ val}}$$

$\boxed{e \mapsto e'}$

$$\frac{}{\texttt{plus}(\texttt{num}[n_1];\texttt{num}[n_2]) \mapsto \texttt{num}[n_1 + n_2]} \qquad \frac{e_1 \mapsto e_1'}{\texttt{plus}(e_1;e_2) \mapsto \texttt{plus}(e_1';e_2)}$$

$$\frac{e_2 \mapsto e_2'}{\texttt{plus}(\texttt{num}[n_1];e_2) \mapsto \texttt{plus}(\texttt{num}[n_1];e_2')} \qquad \frac{}{\texttt{times}(\texttt{num}[n_1];\texttt{num}[n_2]) \mapsto \texttt{num}[n_1 * n_2]}$$

$$\frac{e_1 \mapsto e_1'}{\texttt{times}(e_1;e_2) \mapsto \texttt{times}(e_1';e_2)} \qquad \frac{e_2 \mapsto e_2'}{\texttt{times}(\texttt{num}[n_1];e_2) \mapsto \texttt{times}(\texttt{num}[n_1];e_2')}$$

$$\frac{}{\texttt{cat}(\texttt{str}[s_1];\texttt{str}[s_2]) \mapsto \texttt{str}[s_1 \hat{} s_2]} \qquad \frac{e_1 \mapsto e_1'}{\texttt{cat}(e_1;e_2) \mapsto \texttt{cat}(e_1';e_2)}$$

$$\frac{e_2 \mapsto e_2'}{\texttt{cat}(\texttt{str}[s_1];e_2) \mapsto \texttt{cat}(\texttt{str}[s_1];e_2')} \qquad \frac{}{\texttt{len}(\texttt{str}[s]) \mapsto \texttt{num}[|s|]} \qquad \frac{e \mapsto e'}{\texttt{len}(e) \mapsto \texttt{len}(e')}$$

$$\frac{e_1 \text{ val}}{\texttt{let}(e_1;x.e_2) \mapsto [e_1/x]e_2} \qquad \frac{e_1 \mapsto e_1'}{\texttt{let}(e_1;x.e_2) \mapsto \texttt{let}(e_1';x.e_2)}$$

$\boxed{e \text{ err}}$

$$\frac{}{\texttt{plus}(\texttt{str}[s];e_2) \text{ err}} \qquad \frac{}{\texttt{plus}(\texttt{num}[n];\texttt{str}[s]) \text{ err}} \qquad \frac{e_1 \text{ err}}{\texttt{plus}(e_1;e_2) \text{ err}} \qquad \frac{e_2 \text{ err}}{\texttt{plus}(\texttt{num}[n];e_2) \text{ err}}$$

$$\frac{}{\texttt{times}(\texttt{str}[s];e_2) \text{ err}} \qquad \frac{}{\texttt{times}(\texttt{num}[n];\texttt{str}[s]) \text{ err}} \qquad \frac{e_1 \text{ err}}{\texttt{times}(e_1;e_2) \text{ err}} \qquad \frac{e_2 \text{ err}}{\texttt{times}(\texttt{num}[n];e_2) \text{ err}}$$

$$\frac{}{\texttt{cat}(\texttt{num}[n];e_2) \text{ err}} \qquad \frac{}{\texttt{cat}(\texttt{str}[s];\texttt{num}[n]) \text{ err}} \qquad \frac{e_1 \text{ err}}{\texttt{cat}(e_1;e_2) \text{ err}} \qquad \frac{e_2 \text{ err}}{\texttt{cat}(\texttt{str}[s];e_2) \text{ err}}$$

$$\frac{}{\texttt{len}(\texttt{num}[n]) \text{ err}} \qquad \frac{e \text{ err}}{\texttt{len}(e) \text{ err}} \qquad \frac{e_1 \text{ err}}{\texttt{let}(e_1;x.e_2) \text{ err}}$$

B Statics of $\mathcal{L}\{\texttt{num str}\}$

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{}{\Gamma \vdash \texttt{num}[n] : \texttt{num}} \qquad \frac{}{\Gamma \vdash \texttt{str}[s] : \texttt{str}} \qquad \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{plus}(e_1; e_2) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{times}(e_1; e_2) : \texttt{num}} \qquad \frac{\Gamma \vdash e_1 : \texttt{str} \qquad \Gamma \vdash e_2 : \texttt{str}}{\Gamma \vdash \texttt{cat}(e_1; e_2) : \texttt{str}} \qquad \frac{\Gamma \vdash e : \texttt{str}}{\Gamma \vdash \texttt{len}(e) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{let}(e_1; x.e_2) : \tau_2}$$