Assignment #1:
Dynamics and Statics for a Simple Language

Fundamentals of Programming Languages

Out: Tuesday, Sept 6th, 2016
Due: Thursday, Sept 15th, 2016 11:59pm EST

The tasks in this homework ask you to prove ("meta theoretical") properties about the language **E**, defined in Ch. 4 and Ch. 5 of *PFPL*. They are "meta theoretical" in that they give a theory about the theory of **E**, and are true about a large set of **E** programs, not specific, individual programs.

Tasks

There is a lot of error checking going on in the dynamics in Appendix A. But as we've been discussing in class, we can eliminate much or all of this by equipping our language with a static type system (see Ch. 6 of PFPL)!

The type checking rules for **E** are reproduced in Appendix B for your reference.

Grading criteria: To receive full credit for any proof below, you must *at least* do the following:

- At the beginning of your proof, specify over what structure or derivation you are performing induction (i.e., which structure's inductive principle are you using?)
- In the inductive cases of the proof, specify how you are applying the inductive hypothesis, and what result it gives you.

If you omit these steps and/or do not make them explicit, you will receive zero credit for your proof. If you attempt to do these steps, but you make a mistake, you may still receive some partial credit, depending on your proof.

Note on omitting redundant proof cases: In the proofs below, some cases are very similar to other cases, e.g., the cases for `plus` and `times` in the proofs below are likely to be analogous, in that (nearly) the same proof steps are used in each. When this happens, you can omit the redundant cases as follows: If you do one case, say for `plus`, you may (optionally) write in the other case for `times` that it is "analygous to the case above, for `plus`". You must make this omission explicit, to show that you have thought about it. Further, this shortcut is only applicable when the cases really are analygous, and (nearly) the same steps apply in the proof. **When in doubt, do not omit the proof case.**

Task 1 (10 pts). (Unicity of Typing) State and prove that for every typing context Γ and expression e, there exists at most one τ such that $\Gamma \vdash e : \tau$.

Task 2 (10 pts). (Canonical Forms) Prove that if e val, then

1. if $\Gamma \vdash e : \texttt{num}$ then $e = \texttt{num}[n]$ for some number n.
2. if $\Gamma \vdash e : \texttt{str}$ then $e = \texttt{str}[s]$ for some string s.

Task 3 (20 pts). (Substitution) State and prove the *substitution lemma* for $\mathsf{\textbf{E}}$.

Task 4 (35 pts). (Progress) State and prove the *progress theorem* for $\mathsf{\textbf{E}}$. To receive full credit, you must additionally use the canonical forms lemma correctly, when appropriate.

Task 5 (35 pts). (Preservation) State and prove the *preservation theorem* for $\mathsf{\textbf{E}}$. To receive full credit, you must additionally use the substitution lemma correctly, when appropriate.

A Dynamics of E

$\boxed{e\ \mathsf{val}}$

$$\frac{}{\texttt{num}[n]\ \mathsf{val}} \qquad \frac{}{\texttt{str}[s]\ \mathsf{val}}$$

$\boxed{e \mapsto e'}$

$$\frac{}{\texttt{plus}(\texttt{num}[n_1];\texttt{num}[n_2]) \mapsto \texttt{num}[n_1 + n_2]} \qquad \frac{e_1 \mapsto e_1'}{\texttt{plus}(e_1;e_2) \mapsto \texttt{plus}(e_1';e_2)}$$

$$\frac{e_2 \mapsto e_2'}{\texttt{plus}(\texttt{num}[n_1];e_2) \mapsto \texttt{plus}(\texttt{num}[n_1];e_2')} \qquad \frac{}{\texttt{times}(\texttt{num}[n_1];\texttt{num}[n_2]) \mapsto \texttt{num}[n_1 * n_2]}$$

$$\frac{e_1 \mapsto e_1'}{\texttt{times}(e_1;e_2) \mapsto \texttt{times}(e_1';e_2)} \qquad \frac{e_2 \mapsto e_2'}{\texttt{times}(\texttt{num}[n_1];e_2) \mapsto \texttt{times}(\texttt{num}[n_1];e_2')}$$

$$\frac{}{\texttt{cat}(\texttt{str}[s_1];\texttt{str}[s_2]) \mapsto \texttt{str}[s_1\,\hat{}\,s_2]} \qquad \frac{e_1 \mapsto e_1'}{\texttt{cat}(e_1;e_2) \mapsto \texttt{cat}(e_1';e_2)}$$

$$\frac{e_2 \mapsto e_2'}{\texttt{cat}(\texttt{str}[s_1];e_2) \mapsto \texttt{cat}(\texttt{str}[s_1];e_2')} \qquad \frac{}{\texttt{len}(\texttt{str}[s]) \mapsto \texttt{num}[|s|]} \qquad \frac{e \mapsto e'}{\texttt{len}(e) \mapsto \texttt{len}(e')}$$

$$\frac{e_1\ \mathsf{val}}{\texttt{let}(e_1;x.e_2) \mapsto [e_1/x]e_2} \qquad \frac{e_1 \mapsto e_1'}{\texttt{let}(e_1;x.e_2) \mapsto \texttt{let}(e_1';x.e_2)}$$

$\boxed{e\ \mathsf{err}}$

$$\frac{}{\texttt{plus}(\texttt{str}[s];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{plus}(\texttt{num}[n];\texttt{str}[s])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{plus}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{plus}(\texttt{num}[n];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{times}(\texttt{str}[s];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{times}(\texttt{num}[n];\texttt{str}[s])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{times}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{times}(\texttt{num}[n];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{cat}(\texttt{num}[n];e_2)\ \mathsf{err}} \qquad \frac{}{\texttt{cat}(\texttt{str}[s];\texttt{num}[n])\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{cat}(e_1;e_2)\ \mathsf{err}} \qquad \frac{e_2\ \mathsf{err}}{\texttt{cat}(\texttt{str}[s];e_2)\ \mathsf{err}}$$

$$\frac{}{\texttt{len}(\texttt{num}[n])\ \mathsf{err}} \qquad \frac{e\ \mathsf{err}}{\texttt{len}(e)\ \mathsf{err}} \qquad \frac{e_1\ \mathsf{err}}{\texttt{let}(e_1;x.e_2)\ \mathsf{err}}$$

B Statics of E

$\boxed{\Gamma \vdash e : \tau}$

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{}{\Gamma \vdash \texttt{num}[n] : \texttt{num}} \qquad \frac{}{\Gamma \vdash \texttt{str}[s] : \texttt{str}} \qquad \frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{plus}(e_1; e_2) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{num} \qquad \Gamma \vdash e_2 : \texttt{num}}{\Gamma \vdash \texttt{times}(e_1; e_2) : \texttt{num}} \qquad \frac{\Gamma \vdash e_1 : \texttt{str} \qquad \Gamma \vdash e_2 : \texttt{str}}{\Gamma \vdash \texttt{cat}(e_1; e_2) : \texttt{str}} \qquad \frac{\Gamma \vdash e : \texttt{str}}{\Gamma \vdash \texttt{len}(e) : \texttt{num}}$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \qquad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \texttt{let}(e_1; x.e_2) : \tau_2}$$