Polynomial Interpolation

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Book Problems

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$$p(1) = 5(1)^3 - 27(1)^2 + 45(1) - 21 = 5 - 27 + 45 - 21 = 2$$

$$p(2) = 5(2)^3 - 27(2)^2 + 45(2) - 21 = 40 - 108 + 90 - 21 = 1$$

$$p(3) = 5(3)^3 - 27(3)^2 + 45(3) - 21 = 135 - 243 + 135 - 21 = 6$$

$$p(4) = 5(4)^3 - 27(4)^2 + 45(4) - 21 = 320 - 432 + 180 - 21 = 47$$

$$q(1) = (1)^4 - 5(1)^3 + 8(1)^2 - 5(1) + 3 = 1 - 5 + 8 - 5 + 3 = 2$$

$$q(2) = (2)^4 - 5(2)^3 + 8(2)^2 - 5(2) + 3 = 16 - 40 + 32 - 10 + 3 = 1$$

$$q(3) = (3)^4 - 5(3)^3 + 8(3)^2 - 5(3) + 3 = 81 - 135 + 72 - 15 + 3 = 6$$

$$q(4) = (4)^4 - 5(4)^3 + 8(4)^2 - 5(4) + 3 = 256 - 320 + 128 - 20 + 3 = 47$$

6

0	2			
2	4	1		
3	-4	-8	-3	
5	82	43	17	4

$$p(x) = 2 + x - 3x(x-2) + 4x(x-2)(x-3)$$

10

0	7			
2	11	2		
3	28	17	5	
4	63	35	9	1

$$p(x) = 7 + 2x + 5x(x-2) + x(x-2)(x-3)$$

$$p(x) = 7 + x(2 + (x-2)(5 + (x-3))$$

2

0	0
0.5	1.25
1	0
1.5	-11.25

$$\ell_0(x) = \left(\frac{x-0.5}{0-0.5}\right)\left(\frac{x-1}{0-1}\right)\left(\frac{x-1.5}{0-1.5}\right) = -\frac{3}{4}(x-0.5)(x-1)(x-1.5)$$

$$\ell_1(x) = \left(\frac{x-0}{0.5-0}\right)\left(\frac{x-1}{0.5-1}\right)\left(\frac{x-1.5}{0.5-1.5}\right) = \frac{1}{4}(x-0)(x-1)(x-1.5)$$

$$\ell_2(x) = \left(\frac{x-0}{1-0}\right)\left(\frac{x-0.5}{1-0.5}\right)\left(\frac{x-1.5}{1-1.5}\right) = -\frac{1}{4}(x-0)(x-0.5)(x-1.5)$$

$$\ell_3(x) = \left(\frac{x-0}{1.5-0}\right)\left(\frac{x-0.5}{1.5-0.5}\right)\left(\frac{x-1}{1.5-1}\right) = \frac{3}{4}(x-0)(x-0.5)(x-1)$$

$y_0, y_2 = 0$ so...

$$p(x) = 1.25\ell_1(x) - 11.25\ell_3(x)$$

3

0	0			
0.5	1.25	2.5		
1	0	-2.5	-5	
1.5	-11.25	-22.5	-20	-10

$$p(x) = 2.5x - 5x(x-0.5) - 10x(x-0.5)(x-1)$$

$$p(x) = x(2.5 - (x-0.5)(5 + 10(x-1)))$$

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See Figure 1 at the bottom of this document.

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At 1 and 2?? Okay...

$$f'(x) = 10x\sin(\pi x) + 5\pi x^2\cos(\pi x)$$

$$f'(1) = -15.70796$$

$$f'(2) = 62.83185$$

0	0						
0.5	1.25	2.5					
1	0	-2.5	-5				
1	0	-15.70796	-26.41592	-21.41592			
1.5	-11.25	-22.5	-13.58408	12.83184	22.83184		
2	0	22.5	45	58.58408	30.50149	3.834825	
2	0	62.83185	80.6637	35.6637	-22.92038	-35.61458	-19.7247

$$p(x) = 2.5x - 5x(x-0.5) - 21.41592x(x-0.5)(x-1) + 22.83184x(x-0.5)(x-1)(x-1)$$
$$+3.834825x(x-0.5)(x-1)(x-1)(x-1.5) - 19.7247x(x-0.5)(x-1)(x-1)(x-1.5)(x-2)$$

My plot is in Figure 2. My interpolationg polynomial is in green, the function is in red, and the derivative is in blue.

6

1	-2			
1	-2	-3		
1	-2	-3	-2	
2	-5	-3	0	2

$$p(x) = -2 - 3(x-1) - 2(x-1)(x-1) + 2(x-1)(x-1)(x-1)$$

```
plot({5*x^2*sin(Pi*x), x·(2.5 − (x − 0.5)·(5 + 10·(x − 1)))}, x=0..2)
```

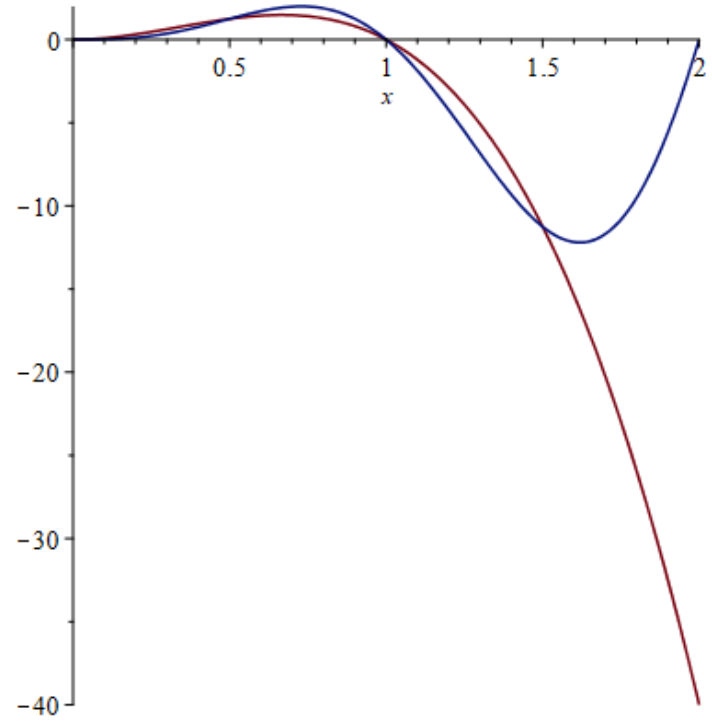

Figure 1: Interpolation Plot

```
plot({5*x^2*sin(Pi*x), -(267699*(x-1)*(x-1/2)*x)/12500-(197247*(x-2)*(x-3/2)*(x-1)^2*(x-1/2)*x)/10000 + (153393*(x
-3/2)*(x-1)^2*(x-1/2)*x)/40000 + (142699*(x-1)^2*(x-1/2)*x)/6250-5*(x-1/2)*x + (5*x)/2, 10*x*sin(Pi*x) + 5*Pi
*x^2*cos(Pi*x)}, x=0..2)
```

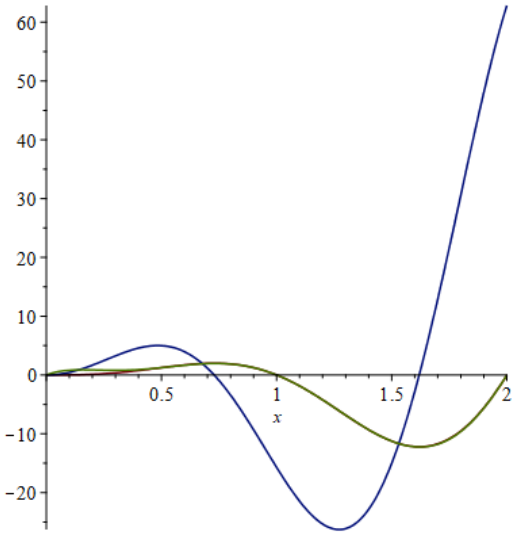

Figure 2: Interpolation Plot 2