CS 477: Homework #7

Due on December 8th, 2016 at 2:30pm

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Problem 1

[20 points](U & G Required)

Answer the questions below regarding the following graph:

```
              (e)            (f)
               |              |
               |              |
               |17            |15
               |              |
       3       |      10      |     1
(A)------------(c)------------(g)---------(h)
 |             |              |          /
 |             |              |         /
 |7            |9             |11     /5
 |             |              |     /
 |     12      |      14      |   /
(b)------------(d)------------(i)
```

a.) **[5 points]** In what order are edges added to the Minimum Spanning Tree (MST) using Kruskal's Algorithm? List the edges by giving their endpoints.

b.) **[5 points]** In what order are edges added to the MST using Prim's Algorithm starting from vertex A? List the edges by giving their endpoints.

Solution

a.) g-h, A-c, h-i, A-b, c-d, c-g, f-g, c-e

b.) A-c, A-b, c-d, c-g, g-h, h-i, f-g, c-e

Problem 2

[30 points](U & G Required) Exercise 22.2-9 (page 602).

22.2-9) Let $G = (V, E)$ be a connected, undirected graph. Give an $\mathcal{O}(V + E)$-time algorithm to compute a path in G that traverses each edge in E exactly once in each direction. Describe how you can find your way out of a maze if you are given a large supply of pennies.

Problem 3

[30 points](U & G Required) Exercise 22.5-6 (page 621).

22.5-6) Given a directed graph $G = (V, E)$, explain how to create another graph $G' = (V, E')$ such that

a.) G' has the same strongly connected components as G

b.) G' has the same component graph as G

c.) E' is as small as possible.

Describe a fast algorithm to compute G'.

Problem 4

[20 points](U & G Required) Exercise 24.3-2 (page 663).

24.3-2) Give a simple example of a directed graph with negative-weight edges for which Dijkstras algorithm produces incorrect answers. Why doesnt the proof of Theorem 24.6 go through when negative-weight edges are allowed?

Problem 6

[20 points](Extra Credit) Exercise 24.3-6 (page 663).

24.3-6) We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a rela number in the range $0 \leq r(u, v) \leq 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.