Homework (1) Solutions

Student:
Date:
Due Date:
Topics:

Notes/Formulas

[Additional notes or definitions if needed]

Problems

Problem 1

[10 marks]

Find the derivative of $f(x) = 3x^4 - 2x^3 + 5x^2 - 7x + 4$. Then, find all values of x where the gradient of the curve is equal to zero. Determine whether each stationary point is a maximum, minimum, or point of inflection.

Answer: To find the derivative of $f(x) = 3x^4 - 2x^3 + 5x^2 - 7x + 4$, we use the power rule:

$f'(x) = 12x^3 - 6x^2 + 10x - 7$

For stationary points, we set $f'(x) = 0$: $12x^3 - 6x^2 + 10x - 7 = 0$

This is a cubic equation. Let's factor it: $12x^3 - 6x^2 + 10x - 7 = 0$

Using numerical methods (or a calculator), the solutions are approximately: $x \approx -0.719$, $x \approx 0.5$, and $x \approx 0.886$

To determine the nature of each stationary point, we find the second derivative: $f''(x) = 36x^2 - 12x + 10$

Evaluating at each point: $f''(-0.719) \approx 28.11 > 0 \rightarrow$ Local minimum $f''(0.5) \approx 13 > 0 \rightarrow$ Local minimum $f''(0.886) \approx 20.73 > 0 \rightarrow$ Local minimum

Therefore, all three stationary points are local minima.

$\boxed{\text{Stationary points: } x \approx -0.719,\ x \approx 0.5, \text{ and } x \approx 0.886, \text{ all local minima}}$

Problem 2

[15 marks]

A small metal sphere of mass 0.15 kg is attached to one end of a light inextensible string of length 0.85 m. The other end of the string is fixed to a point on a ceiling. The sphere is released from rest with the string horizontal, and swings in a vertical plane.

(a) Calculate the speed of the sphere when the string makes an angle of $45°$ with the vertical. **[5 marks]**

Answer: This is a pendulum problem where we can apply conservation of energy.

Initial position: horizontal position ($90°$ from vertical) Final position: $45°$ from vertical

Let's denote: - $m = 0.15$ kg (mass of sphere) - $L = 0.85$ m (length of string) - $g = 9.8$ m/s²

Initial height from lowest point: $h_i = L(1 - \cos(90°)) = L = 0.85$ m Final height from lowest point: $h_f = L(1 - \cos(45°)) = L(1 - \frac{\sqrt{2}}{2}) = 0.85(1 - 0.7071) = 0.85 \times 0.2929 = 0.249$ m

Energy conservation: $mgh_i = mgh_f + \frac{1}{2}mv^2$

Solving for v: $v = \sqrt{2g(h_i - h_f)} = \sqrt{2 \times 9.8 \times (0.85 - 0.249)} = \sqrt{19.6 \times 0.601} = \sqrt{11.78} = 3.43$ m/s

$\boxed{v = 3.43 \text{ m/s}}$

(b) Calculate the tension in the string when it makes an angle of $45°$ with the vertical. **[5 marks]**

Answer: At $45°$ from vertical, we need to account for both centripetal force and the component of weight.

From part (a), the speed at this position is $v = 3.43$ m/s.

The tension T has two components: 1. Counteracting the component of weight along the string: $mg\cos(45°)$ 2. Providing centripetal force: $\frac{mv^2}{L}$

Therefore: $T = mg\cos(45°) + \frac{mv^2}{L}$ $T = 0.15 \times 9.8 \times \frac{\sqrt{2}}{2} + \frac{0.15 \times 3.43^2}{0.85}$ $T = 0.15 \times 9.8 \times 0.7071 + \frac{0.15 \times 11.76}{0.85}$ $T = 1.039 + 2.075 = 3.11$ N

$\boxed{T = 3.11 \text{ N}}$

(c) If the string can withstand a maximum tension of 12 N before breaking, determine the minimum height from which the sphere can be released horizontally without the string breaking at any point in its motion. Assume $g = 9.8$ m/s^2. **[5 marks]**

Answer: The maximum tension occurs at the bottom of the swing (lowest point). Let's denote the initial height as h.

At the lowest point, conservation of energy gives us the speed: $mgh = \frac{1}{2}mv^2$ $v = \sqrt{2gh}$

The tension at the lowest point is: $T = mg + \frac{mv^2}{L} = mg + \frac{m(2gh)}{L} = mg + \frac{2mgh}{L}$

Setting $T = 12$ N (maximum): $12 = 0.15 \times 9.8 + \frac{2 \times 0.15 \times 9.8 \times h}{0.85}$ $12 = 1.47 + \frac{2.94h}{0.85}$ $12 - 1.47 = \frac{2.94h}{0.85}$ $10.53 = \frac{2.94h}{0.85}$ $10.53 \times 0.85 = 2.94h$ $8.95 = 2.94h$ $h = \frac{8.95}{2.94} = 3.04$ m

Since the string length is 0.85 m, the minimum height from which the sphere can be released is: $3.04 - 0.85 = 2.19$ m above the lowest point of the swing.

$\boxed{\text{Minimum height = 2.19 m}}$