AP Calculus AB | Complete Review Cheat Sheet

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How to Use This Cheat Sheet

- Find your topic in the section headers
- Look for boxed formulas and key concepts
- Review examples and common mistakes
- Use the tips for exam strategy

1 Limits & Continuity

Limit Definition

$\lim_{x\to a} f(x) = L$ means that $f(x)$ can be made arbitrarily close to L by taking x sufficiently close to a.

Key Limit Properties:

- $\lim_{x\to a}[f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
- $\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, if $\lim_{x\to a} g(x) \neq 0$
- $\lim_{x\to a}[f(x)]^n = [\lim_{x\to a} f(x)]^n$

Special Limits

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$
$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$$
$$\lim_{x\to 0}\frac{e^x-1}{x} = 1$$
$$\lim_{x\to 0}(1+x)^{1/x} = e$$
$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

Continuity

A function f is continuous at $x = a$ if:

1. $f(a)$ is defined
2. $\lim_{x\to a} f(x)$ exists
3. $\lim_{x\to a} f(x) = f(a)$

Types of Discontinuities

- Removable: $\lim_{x\to a} f(x)$ exists but $\neq f(a)$
- Jump: Left and right limits exist but are different
- Infinite: Limit approaches $\pm\infty$
- Essential: Limit doesn't exist

When finding limits, watch for indeterminate forms - expressions whose values cannot be determined by direct substitution. These include $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0\cdot\infty$, $\infty-\infty$, 0^0, 1^∞, ∞^0. For example, $\lim_{x\to 0}\frac{\sin x}{x}$ gives $\frac{0}{0}$ but equals 1. Such forms require L'Hôpital's Rule or algebraic manipulation to evaluate.

2 Derivatives

Definition

$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

Alternative Definition

$f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$

2.1 Basic Derivative Rules

Power Rule

$\frac{d}{dx}[x^n] = nx^{n-1}$

Product Rule

$\frac{d}{dx}[f(x)\cdot g(x)] = f'(x)\cdot g(x) + f(x)\cdot g'(x)$

Quotient Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$

Chain Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x))\cdot g'(x)$

2.2 Derivative Table

Common Derivatives

$$\frac{d}{dx}[c] = 0 \quad \text{(constant)}$$
$$\frac{d}{dx}[e^x] = e^x$$
$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$
$$\frac{d}{dx}[\sin x] = \cos x$$
$$\frac{d}{dx}[\cos x] = -\sin x$$
$$\frac{d}{dx}[\tan x] = \sec^2 x$$
$$\frac{d}{dx}[\sec x] = \sec x\tan x$$
$$\frac{d}{dx}[\csc x] = -\csc x\cot x$$
$$\frac{d}{dx}[\cot x] = -\csc^2 x$$
$$\frac{d}{dx}[a^x] = a^x\ln a$$
$$\frac{d}{dx}[\log_a x] = \frac{1}{x\ln a}$$
$$\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$$

2.3 Implicit Differentiation

Steps

1. Differentiate both sides with respect to x
2. When differentiating y terms, multiply by $\frac{dy}{dx}$
3. Solve for $\frac{dy}{dx}$

Example: $x^3 + y^3 = 6xy$

$$3x^2 + 3y^2\frac{dy}{dx} = 6y + 6x\frac{dy}{dx}$$
$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$
$$\frac{dy}{dx} = \frac{6y-3x^2}{3y^2-6x}$$

2.4 Related Rates

Set up an equation relating the quantities, then differentiate with respect to time.

Steps for Related Rates:

- Draw a diagram if helpful
- Identify known and unknown rates
- Write equation relating variables
- Differentiate with respect to time
- Substitute known values and solve

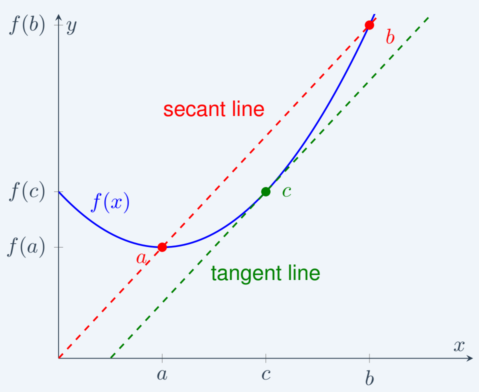
3 Applications of Derivatives

3.1 Mean Value Theorem

MVT

If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one $c \in (a, b)$ such that:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$



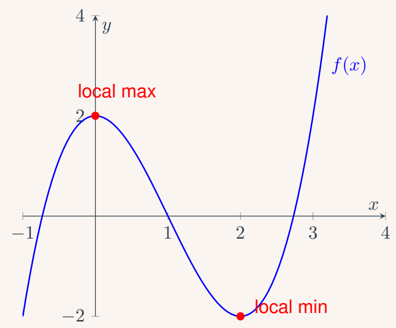
Rolle's Theorem

If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then there exists at least one $c \in (a, b)$ such that $f'(c) = 0$.

3.2 Curve Analysis

Critical Points

Critical points occur where $f'(x) = 0$ or $f'(x)$ is undefined.



Increasing/Decreasing

- $f'(x) > 0 \Rightarrow f(x)$ is increasing
- $f'(x) < 0 \Rightarrow f(x)$ is decreasing

Concavity

- $f''(x) > 0 \Rightarrow f(x)$ is concave up
- $f''(x) < 0 \Rightarrow f(x)$ is concave down
- Inflection points occur where $f''(x) = 0$ or $f''(x)$ is undefined and $f''(x)$ changes sign

Extrema Tests

- First Derivative Test: Check sign of $f'(x)$ around critical point
- Second Derivative Test: If $f'(c) = 0$ and $f''(c) > 0$, then local min; if $f''(c) < 0$, then local max
- Always check endpoints for absolute extrema

3.3 Optimization

Steps for Optimization:

- Identify the quantity to optimize
- Express as a function of one variable
- Find critical points
- Determine max/min using first or second derivative test
- Check endpoints if applicable

Example: Rectangle with max area

Find rectangle with perimeter 20 that maximizes area.
Let x = width, then height = $10 - x$
$A(x) = x(10-x) = 10x - x^2$
$A'(x) = 10 - 2x = 0 \Rightarrow x = 5$
Max area = $A(5) = 25$ square units

3.4 L'Hôpital's Rule

L'Hôpital's Rule

If $\lim_{x\to a}\frac{f(x)}{g(x)}$ gives $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then:

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

provided: (1) f, g are differentiable near a (except possibly at a), (2) $g'(x) \neq 0$ near a, and (3) the right limit exists.

Example: $\lim_{x\to 0}\frac{e-1-x}{x}$

$\frac{0}{0}$ form, apply L'Hôpital:
$\lim_{x\to 0}\frac{e-1}{2x} = \frac{0}{0}$
Apply again: $\lim_{x\to 0}\frac{e}{2} = \frac{1}{2}$

You may need to apply L'Hôpital's Rule multiple times.

4 Integration

Indefinite Integral

$\int f(x)\,dx = F(x) + C$ where $F'(x) = f(x)$

Definite Integral

$\int_a^b f(x)\,dx = F(b) - F(a)$ where $F'(x) = f(x)$

4.1 Fundamental Theorem of Calculus

FTC Part 1

If f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then $F'(x) = f(x)$.

FTC Part 2

If f is continuous on $[a, b]$ and F is any antiderivative of f, then:

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

4.2 Basic Integration Rules

Power Rule

$\int x^n\,dx = \frac{x}{n+1} + C, \quad n \neq -1$

Common Integrals

$$\int e^x\,dx = e^x + C$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$
$$\int \sin x\,dx = -\cos x + C$$
$$\int \cos x\,dx = \sin x + C$$
$$\int \sec^2 x\,dx = \tan x + C$$
$$\int \csc^2 x\,dx = -\cot x + C$$
$$\int \sec x \tan x\,dx = \sec x + C$$
$$\int \csc x \cot x\,dx = -\csc x + C$$
$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$
$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$$
$$\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}|x| + C$$

4.3 Integration Properties

Properties

$$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$
$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$
$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$
$$\int_a^a f(x)\,dx = 0$$
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

4.4 Integration Techniques

Substitution

Let $u = g(x)$, then:

$$\int f(g(x))g'(x)\,dx = \int f(u)\,du$$

Example: $\int xe^{x^2}\,dx$

Let $u = x^2$, then $du = 2x\,dx$
$\int xe^{x^2}\,dx = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Choose u and dv using LIATE priority (a helpful guideline, not a strict rule):

- L: Logarithmic functions
- I: Inverse trigonometric functions
- A: Algebraic functions
- T: Trigonometric functions
- E: Exponential functions

Some texts use ILATE or ALTI – all work similarly. Choose the leftmost function for u.

Example: $\int x \ln x\,dx$

Let $u = \ln x$, $dv = x\,dx$
Then $du = \frac{1}{x}\,dx$, $v = \frac{x}{2}$
$\int x\ln x\,dx = \frac{x}{2}\ln x - \int \frac{x}{2} \cdot \frac{1}{x}\,dx$
$= \frac{x}{2}\ln x - \frac{x}{4} + C$

Partial Fractions

For rational functions $\frac{P(x)}{Q(x)}$ where $\deg P < \deg Q$:

- Factor denominator $Q(x)$
- For each linear factor $(ax + b)$, add term $\frac{A}{ax+b}$
- For each quadratic factor $(ax^2 + bx + c)$, add term $\frac{Ax+B}{ax+bx+c}$
- Solve for constants

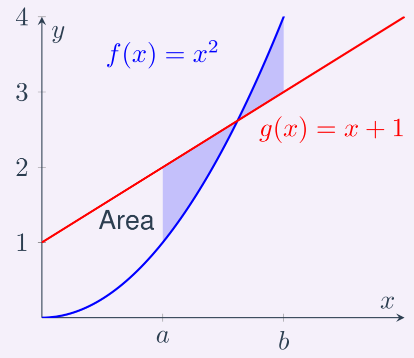
5 Applications of Integration

5.1 Area

Area Between Curves

Area between $f(x)$ and $g(x)$ from $x = a$ to $x = b$:

$$A = \int_a^b |f(x) - g(x)|\,dx$$



Average Value

Average value of f on $[a, b]$:

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

5.2 Volume

Disk Method

Volume of solid of revolution around x-axis:

$$V = \pi\int_a^b [f(x)]^2\,dx$$

Washer Method

Volume of solid of revolution with hole:

$$V = \pi\int_a^b [(R(x))^2 - (r(x))^2]\,dx$$

where $R(x)$ is outer radius and $r(x)$ is inner radius.

Shell Method

Volume of solid of revolution around y-axis:

$$V = 2\pi\int_a^b |x| \cdot h(x)\,dx$$

where x is the radius of each shell and $h(x)$ is the height.
For rotation around x-axis, use $V = 2\pi\int_a^b y \cdot h(y)\,dy$

5.3 Arc Length

Arc Length Formula

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

6 Differential Equations

First-Order ODE

$\frac{dy}{dx} = f(x, y)$

6.1 Separation of Variables

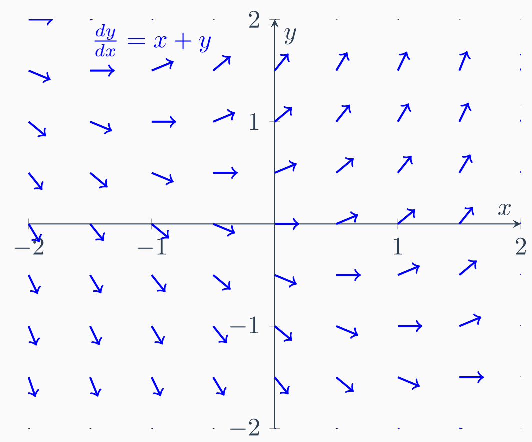
Method

For equations of form $\frac{dy}{dx} = g(x)h(y)$:

1. Rewrite as $\frac{1}{h(y)}\,dy = g(x)\,dx$
2. Integrate both sides
3. Solve for y

6.2 Slope Fields

A slope field is a graphical representation of a differential equation, showing the slope at various points in the xy-plane.



6.3 Exponential Growth/Decay

Model

$$\frac{dy}{dt} = ky \quad \Rightarrow \quad y = Ce^{kt}$$

- $k > 0$: exponential growth
- $k < 0$: exponential decay

Logistic Model

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{L}\right) \quad \Rightarrow \quad y = \frac{L}{1 + Ae^{-kt}}$$

where L is carrying capacity.

7 AP Exam Tips

Free-Response Questions:

- Show all work clearly
- Include units when appropriate (some answers are dimensionless)
- Use correct mathematical notation
- Justify answers when asked
- Interpret results in context of the problem

Common Mistakes

- Forgetting to include $+C$ in indefinite integrals
- Errors in chain rule application
- Incorrect use of L'Hôpital's Rule
- Not checking endpoints in optimization problems
- Forgetting to check for absolute extrema at endpoints
- Confusing average rate of change with instantaneous rate of change
- Not using absolute value when finding area between curves

When in doubt on a multiple-choice question, try substituting values or checking limiting cases.