

1 Numbers & Operations

1.1 Number Properties

Divisibility Rules

- **2**: last digit **even**
- **3**: sum of digits divisible by 3
- **4**: last two digits divisible by 4
- **5**: last digit 0 or 5
- **6**: divisible by both 2 and 3
- **9**: sum of digits divisible by 9
- **10**: last digit 0

Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$
$$|x| = \sqrt{x^2}$$
$$|x| < a \Leftrightarrow -a < x < a$$
$$|x| > a \Leftrightarrow x < -a \text{ or } x > a$$

Complex Numbers

$$i = \sqrt{-1}$$
$$i^2 = -1$$
$$i^3 = -i$$
$$i^4 = 1$$
$$(a + bi) + (c + di) = (a + c) + (b + d)i$$
$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

Exponents

$$x^a \cdot x^b = x^{a+b}$$
$$\frac{x^a}{x^b} = x^{a-b}$$
$$(x^a)^b = x^{ab}$$
$$x^0 = 1 \quad (x \neq 0)$$
$$x^{-a} = \frac{1}{x^a}$$

Radicals

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$
$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$
$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$
$$\sqrt{a^2} = |a|$$

1.2 Fractions & Decimals

Fraction Operations

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$
$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$
$$\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$$

Common Conversions

$$\frac{1}{2} = 0.5 \quad \frac{1}{3} = 0.333...$$
$$\frac{1}{4} = 0.25 \quad \frac{1}{5} = 0.2$$
$$\frac{1}{8} = 0.125 \quad \frac{1}{9} = 0.111...$$

1.3 Ratios & Proportions

Ratio Basics

- Ratio** $a : b$ means $\frac{a}{b}$ parts.
- If $a : b = c : d$, then $\frac{a}{b} = \frac{c}{d}$
 - **Cross multiply**: $ad = bc$

1.4 Percentages

Percentage Formulas

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\%$$
$$\text{Part} = \text{Percentage} \times \text{Whole}$$
$$\text{Percent change} = \frac{\text{New} - \text{Original}}{\text{Original}} \times 100\%$$

Common Conversions

$$10\% = 0.1 \quad 25\% = 0.25$$
$$20\% = 0.2 \quad 50\% = 0.5$$
$$33\frac{1}{3}\% = 0.333... \quad 75\% = 0.75$$

💡 For **consecutive percent changes**, multiply factors: 10% increase then 10% decrease = $(1.1)(0.9) = 0.99$

2 Algebra

2.1 Linear Equations & Inequalities

Line Forms

Slope-intercept: $y = mx + b$

Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$

Slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Line Relationships

- **Parallel**: same slope ($m_1 = m_2$)
- **Perpendicular**: negative reciprocal ($m_1 \cdot m_2 = -1$)

Inequality Rules

- Add/subtract: direction unchanged
- Multiply/divide by **positive**: unchanged
- Multiply/divide by **negative**: reverse direction

2.2 Systems of Equations

Solving Methods

- **Substitution**: solve for one variable
- **Elimination**: add/subtract equations
- **Graphing**: find intersection point(s)

Solutions

- **One solution**: lines intersect
- **No solution**: parallel lines
- **Infinite solutions**: same line

2.3 Quadratic Equations

Standard Form

$$ax^2 + bx + c = 0$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant

$$\Delta = b^2 - 4ac$$

- $\Delta > 0$: **two real solutions**
- $\Delta = 0$: **one real solution**
- $\Delta < 0$: **two complex solutions**

Factoring Patterns

$$x^2 + bx + c = (x + p)(x + q) \quad \text{where } p + q = b, pq = c$$
$$x^2 - a^2 = (x + a)(x - a)$$
$$x^2 + 2xy + y^2 = (x + y)^2$$
$$x^2 - 2xy + y^2 = (x - y)^2$$

2.4 Polynomials & Functions

Special Products

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$
$$(a + b)(a - b) = a^2 - b^2$$

Function Basics

Function assigns one output to each input.

- **Domain**: valid inputs
- **Range**: possible outputs
- **Notation**: $f(x)$

Function Operations

$$(f + g)(x) = f(x) + g(x)$$
$$(f \cdot g)(x) = f(x) \cdot g(x)$$
$$(f \circ g)(x) = f(g(x))$$

Transformations

- $f(x) + k$: shift **up** k units
- $f(x - k)$: shift **right** k units
- $kf(x)$: **vertical stretch** by k
- $f(kx)$: **horizontal compression** by k
- $-f(x)$: **reflect** across x-axis

2.5 Sequences & Series

Arithmetic Sequences

$$a_n = a_1 + (n - 1)d$$
$$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n - 1)d]$$

Common difference = d

Geometric Sequences

$$a_n = a_1 \cdot r^{n-1}$$
$$S_n = a_1 \cdot \frac{1 - r^n}{1 - r} \quad (r \neq 1)$$

Common ratio = r

2.6 Logarithms

Log Properties

$$\log_b(xy) = \log_b x + \log_b y$$
$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$
$$\log_b(x^n) = n \log_b x$$
$$\log_b 1 = 0$$
$$\log_b b = 1$$

Change of Base

$$\log_b x = \frac{\log_a x}{\log_a b}$$

3 Geometry

3.1 Angles

Angle Relationships

- **Complementary**: sum to 90°
- **Supplementary**: sum to 180°
- **Vertical angles**: equal measures
- **Triangle angles**: sum to 180°
- **Quadrilateral angles**: sum to 360°

Parallel Lines

When **parallel lines** cut by transversal:

- **Corresponding angles** equal
- **Alternate interior angles** equal
- **Consecutive interior** supplementary

3.2 Triangles

Triangle Area

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}ab\sin C$$

Triangle Properties

- Sum of angles = 180°
- **Triangle inequality:** $a + b > c$
- **Pythagorean:** $a^2 + b^2 = c^2$ (right triangles)
- **Isosceles:** two equal sides, two equal angles
- **Equilateral:** all sides equal, all angles 60°

Special Right Triangles

- **30°-60°-90°:** sides $1 : \sqrt{3} : 2$
- **45°-45°-90°:** sides $1 : 1 : \sqrt{2}$

Similar Triangles

- **Corresponding angles** equal
- **Corresponding sides** proportional
- **Area ratio** = (side ratio)²

Triangle Centers

- **Centroid:** intersection of medians
- **Circumcenter:** center of circumscribed circle
- **Incenter:** center of inscribed circle
- **Orthocenter:** intersection of altitudes

3.3 Quadrilaterals

Areas

Rectangle: $A = l \times w$

Square: $A = s^2$

Parallelogram: $A = b \times h$

Trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Rhombus: $A = \frac{1}{2}d_1d_2$

Properties

- **Rectangle:** all angles 90° , opposite sides equal
- **Square:** all sides equal, all angles 90°
- **Parallelogram:** opposite sides equal and parallel
- **Rhombus:** all sides equal, opposite angles equal
- **Trapezoid:** exactly one pair of parallel sides

3.4 Circles

Circle Formulas

Circumference: $C = 2\pi r = \pi d$

Area: $A = \pi r^2$

Arc length: $s = r\theta$ (θ in radians)

Sector area: $A = \frac{1}{2}r^2\theta$

Circle Theorems

- **Inscribed angle** = $\frac{1}{2}$ central angle
- **Angles in same segment** equal
- **Angle in semicircle** = 90°
- **Tangent** perpendicular to radius

Circle Relationships

- **Tangent:** touches circle at one point
- **Secant:** intersects circle at two points
- **Chord:** line segment within circle
- **Central angle:** vertex at center
- **Inscribed angle:** vertex on circle

3.5 3D Geometry

Volume

Rectangular prism: $V = l \times w \times h$

Cube: $V = s^3$

Cylinder: $V = \pi r^2 h$

Cone: $V = \frac{1}{3}\pi r^2 h$

Sphere: $V = \frac{4}{3}\pi r^3$

Surface Area

Rectangular prism: $SA = 2(lw + lh + wh)$

Cube: $SA = 6s^2$

Cylinder: $SA = 2\pi r^2 + 2\pi rh$

Sphere: $SA = 4\pi r^2$

4 Trigonometry

Basic Trig Functions

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta}$$

Common Values

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

Trig Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\sin(-\theta) = -\sin \theta$$
$$\cos(-\theta) = \cos \theta$$
$$\sin(\pi - \theta) = \sin \theta$$

Law of Sines & Cosines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$

5 Statistics & Probability

5.1 Descriptive Statistics

Measures of Center

Mean: $\mu = \frac{\sum x_i}{n}$

Median: = middle value

Mode: = most frequent

Measures of Spread

Range: = max – min

Std dev: $\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{n}}$

Data Interpretation

- **Outliers:** values far from pattern
- **Skewed right:** mean > median
- **Skewed left:** mean < median
- **Symmetric:** mean \approx median

5.2 Probability

Basic Probability

$$P(A) = \frac{\text{favorable}}{\text{total}}$$
$$P(\text{not } A) = 1 - P(A)$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A \text{ and } B) = P(A) \times P(B) \quad (\text{independent})$$

Counting

Perm: $P(n, r) = \frac{n!}{(n - r)!}$

Comb: $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n - r)!}$

Conditional Probability

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Events **independent** if $P(A|B) = P(A)$

6 Coordinate Geometry

Distance & Midpoint

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Circle Equation

Standard: $(x - h)^2 + (y - k)^2 = r^2$

General: $x^2 + y^2 + Dx + Ey + F = 0$

Center = (h, k) , **radius** = r

Transformations

- **Translation:** $(x, y) \rightarrow (x + h, y + k)$
- **Reflection x-axis:** $(x, y) \rightarrow (x, -y)$
- **Reflection y-axis:** $(x, y) \rightarrow (-x, y)$
- **Rotation 90° CCW:** $(x, y) \rightarrow (-y, x)$
- **Dilation by k :** $(x, y) \rightarrow (kx, ky)$

7 Data Analysis

7.1 Graphs & Charts

Graph Types

- **Line graphs:** trends over time
- **Bar graphs:** compare categories
- **Pie charts:** parts of a whole
- **Scatter plots:** relationship between variables
- **Box plots:** distribution and outliers

Interpreting Graphs

- Look for **trends, patterns, outliers**
- Note **scale** and **units**
- Compare **slopes** for rate of change
- Check **axis labels** and **titles**

7.2 Linear Regression

Line of Best Fit

$$y = mx + b$$

m = **slope**, b = **y-intercept**

Correlation

- **Positive:** as x increases, y increases
- **Negative:** as x increases, y decreases
- **No correlation:** no clear pattern
- **Correlation coefficient r :** $-1 \leq r \leq 1$
- $|r|$ close to 1: **strong correlation**
- $|r|$ close to 0: **weak correlation**

⚠ Correlation does not imply causation!