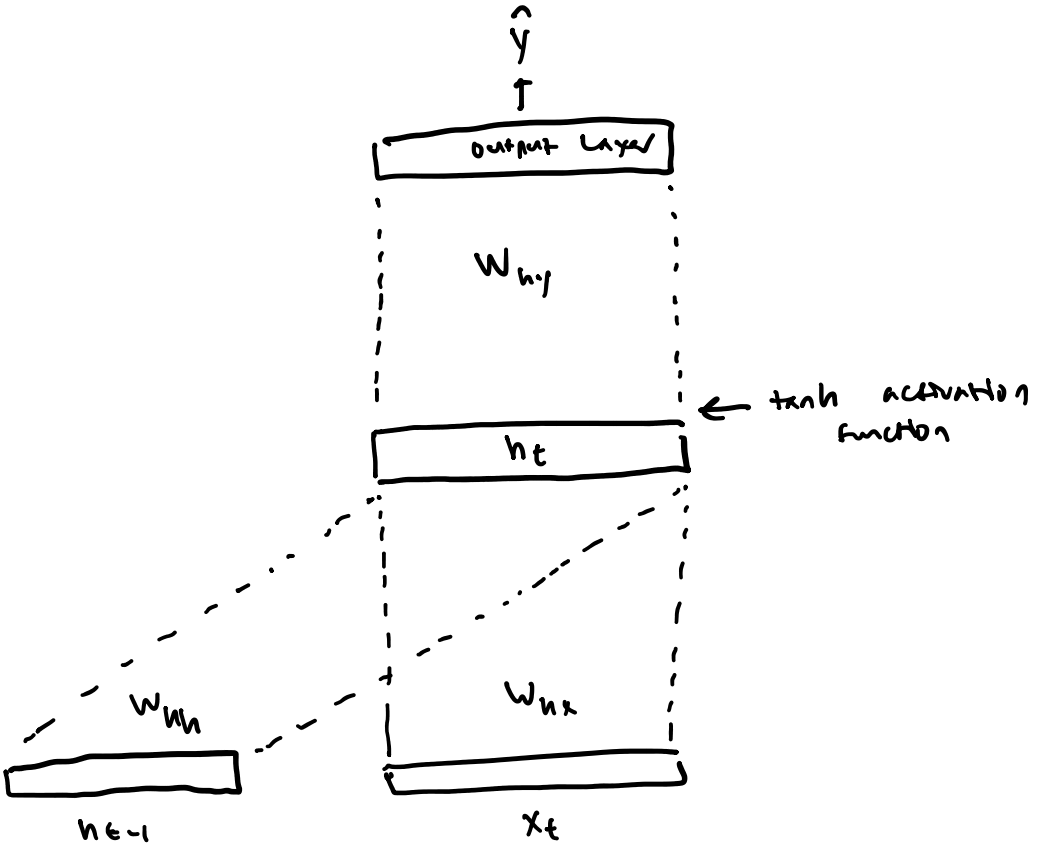
Model Illustration

Friday, April 12, 2024 3:40 PM



Forward & Back Propagation

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$W_{hh} \in \mathbb{R}^{m \times m}$

$W_{hx} \in \mathbb{R}^{n \times m}$

$W_{hy} \in \mathbb{R}^{m \times 1}$

m = size of hidden layer

n = # features

$$h_t = x_t W_{hx} + h_{t-1} W_{hh} + b_h$$

$$a_t = \tanh(h_t)$$

$$\hat{y}_t = a_t W_{hy} + b_y$$

$$L = \frac{1}{2}(y_t - \hat{y}_t)^2$$

$$\frac{\partial L}{\partial W_{hy}} = \frac{\partial L}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial W_{hy}} = -1 \cdot (y_t - \hat{y}_t) \cdot a_t$$

$$\frac{\partial L}{\partial b_y} = \frac{\partial L}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial b_y} = -1 \cdot (y_t - \hat{y}_t)$$

$$\frac{\partial L}{\partial W_{hx}} = \frac{\partial L}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial a_t} \cdot \frac{\partial a_t}{\partial h_t} \cdot \frac{\partial h_t}{\partial W_{hx}}$$

$$= -1 \cdot (y_t - \hat{y}_t) \cdot W_{hy} \cdot (1 - (\tanh(h_t))^2) \cdot x_t$$

$$\frac{\partial L}{\partial W_{hh}} = \frac{\partial L}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial a_t} \cdot \frac{\partial a_t}{\partial h_t} \cdot \frac{\partial h_t}{\partial W_{hh}}$$

$$= -1 \cdot (y_t - \hat{y}_t) \cdot W_{hy} \cdot (1 - (\tanh(h_t))^2) \cdot h_{t-1}$$

$$\frac{\partial L}{\partial b_h} = \frac{\partial L}{\partial \hat{y}_t} \cdot \frac{\partial \hat{y}_t}{\partial a_t} \cdot \frac{\partial a_t}{\partial h_t} \cdot \frac{\partial h_t}{\partial b_h}$$

$$= -1 \cdot (y_t - \hat{y}_t) \cdot W_{hy} \cdot (1 - (\tanh(h_t))^2)$$