Cavitation caused by sudden acceleration in multiple liquids

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Abstract

When forced to suddenly accelerate, liquids have a tendency to undergo cavitation, which can prove to be catastrophic for the liquid's container. Whether this cavitation will occur or not can be determined using a formula which takes into account the acceleration of the container as well as the vapor pressure of the liquid, density, and fill height. In this project we investigate the dependence of cavitation on the viscosity, vapor pressure, and surface tension of the liquids. To do this, the cavitation that is produced in deionized water, fructose corn syrup, and isopropyl alcohol is compared. A free falling weight which strikes the container is used to induce cavitation by sudden acceleration. A high speed camera records the created void(s) in the container. From these recordings the size and number of the bubbles induced in the two liquids are compared. We then attempt to find how the viscosity, the vapor pressure, and the surface tension influence cavitation.

Introduction

By striking the top of a bottle containing a liquid, cavitation onsets within the liquid in the form of bubbles. These bubbles are induced and then collapse, which can lead to damage to the bottle, if not total destruction. Cavitation is however not only confined to merely bottles, as it is often something that can occur in various machinery and organic materials. The collapse of the bubbles can lead to the generation of shock waves which can cause damage to the machinery and tissue damage in events such as head injuries, but is also utilized in industry processes, such as rock cutting and acoustic cleaning[1]. Recently it has been theorized that the current way of determining whether or not a liquid will cavitate, through use of the cavitation number, does not accurately predict if a liquid will cavitate when suddenly accelerated[2]. Thus, cavitation may happen at times when it would originally not be thought possible to.

The question then arises in how cavitation occurs in various types of liquids when caused by sudden acceleration. If cavitation is then more prevalent in a certain type of liquid or is dependent upon a certain factor then this could lead to more optimized machinery. An analysis of the bubbles formed in water, fructose corn syrup, and isopropyl alcohol will lead to a relationship that should determine how viscosity, vapor pressure, and surface tension play into the formation of the bubbles that are induced. This is important to explore as the sudden acceleration may lead to an interesting difference in the cavitation formed in each of the liquids.

Theory

Generally, cavitation is a phenomenon that occurs when a liquid is moved at a high velocity but a low acceleration. Whether or not cavitation will occur is predicted by the cavitation number that describes the situation which is a unitless number seen here.

$$C = \frac{P_r - P_v}{\frac{1}{2}\rho v^2} \qquad \boxed{1}$$

In this case P_r is the reference pressure, P_v is the vapor pressure of the liquid, ρ is the liquid density, v is the velocity of the liquid. If the number is less than one then cavitation is likely, and if above one cavitation is not likely.

In 2017, two teams investigated how cavitation can still occur when a liquid is suddenly accelerated[2]. In these instances the liquid is moving at a velocity too low to cause cavitation in the case stated before. The overall velocity is only in the tens of meters per second but the acceleration turns out to be in the thousands of meters per second squared. As such a new cavitation number was formulated to take this acceleration into account and can be seen here:

$$C = \frac{P_r - P_v/\rho h}{a} \qquad \boxed{2}$$

where *h* is the fill height of the liquid and *a* is the acceleration the liquid undergoes.[2] Again, the cavitation number is unitless and if below one, cavitation is likely.

Experimentation

In order to test this cavitation by acceleration, we then performed several tests to see how the cavitation occurs in various different types of liquids. The liquids were chosen based upon the fact that they had differing viscosities, vapor pressures, and surface tension. The three liquids, water, fructose corn syrup, and isopropyl, were kept at a constant temperature of 25^{o} C except for a set of trials where the water was set to a temperature of 2^{o}C as a means to lower the vapor pressure and viscosity of the liquid.

	Water (25^{0}C)	Water (2^{0}C)	Corn Syrup	Isopropyl
Avg. Vapor Pressure (Pa)	3220.5	695.1	133.32	5872.85
Viscosity (mPa*s)	0.89	1.6735	4.9	2.1
Surface Tension (mN/m)	71.99	74.95	92.1	20.93

For the experiment we used a free-fall apparatus in order to induce the cavitation. An Erlenmeyer flask holding 125ml of each of the liquids is placed 120cm below a mass with weight 180grams. The mass is dropped onto the flask and the results are then recorded by a high speed camera shooting at 1250 frames per second. In each of the trials for each of the liquids, the atmospheric pressure was assumed to be constant at 101 kPa and ten trials for each liquid were held. Of those trials, the five high speed videos that were most easily analyzed were chosen for analysis.

From the videos we then looked at the amount of bubbles that were produced in each trial for each liquid and measured the radius of each bubble that could be easily measured. We also measured the acceleration the liquids underwent but this was not used for any relationships found.

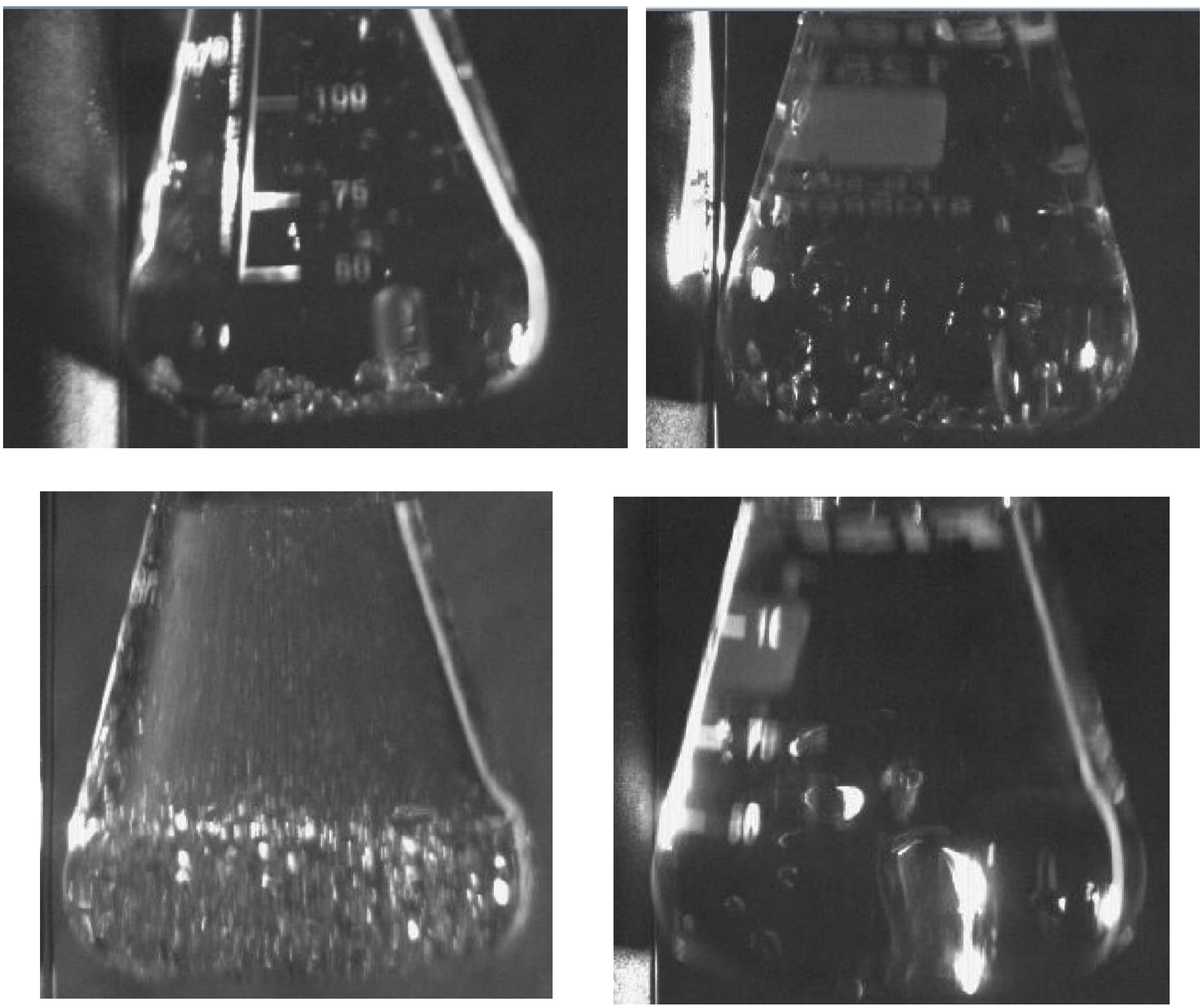

Fig. 1 - Top left: Water 25º C cavitation. Top right: Water 2º C cavitation.
Bottom left: Corn syrup cavitation. Bottom right: Isopropyl cavitation.

Results

From looking at the data we have we can find relationships between the amount of bubbles that were created, the radius of the bubbles, and the vapor pressures and surface tensions of the liquids. First we will look at the vapor pressure and the amount of bubbles created during the trials.

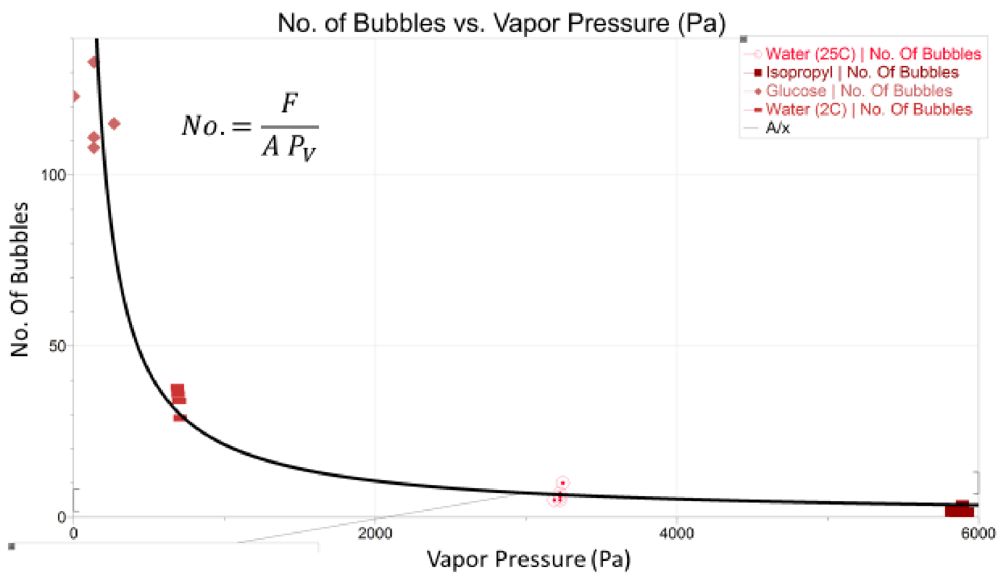

Fig. 2 Graph of the relationship between the number of bubbles produced in each instance of cavitation in each liquid and the vapor pressure of each liquid.

As can be seen from the Fig. 2 the fit is a hyperbolic curve with the specific equation

$$No. = \frac{F}{A\,P_V}. \qquad \boxed{3}$$

where A is the area of the bottom of the flask where the force is applied. What this graph then tells us, besides that a lower vapor pressure should lead to a higher amount of bubbles being created, is that the amount of bubbles formed depends upon the ratio of negative pressure exerted on the bottom of the liquid to the vapor pressure. The curve fit tells that a negative pressure of 2.1kPa is exerted on the bottom of the liquid in each trial since parameters were kept constant. As such, a lower vapor pressure will lead to greater amounts of bubbles formed.

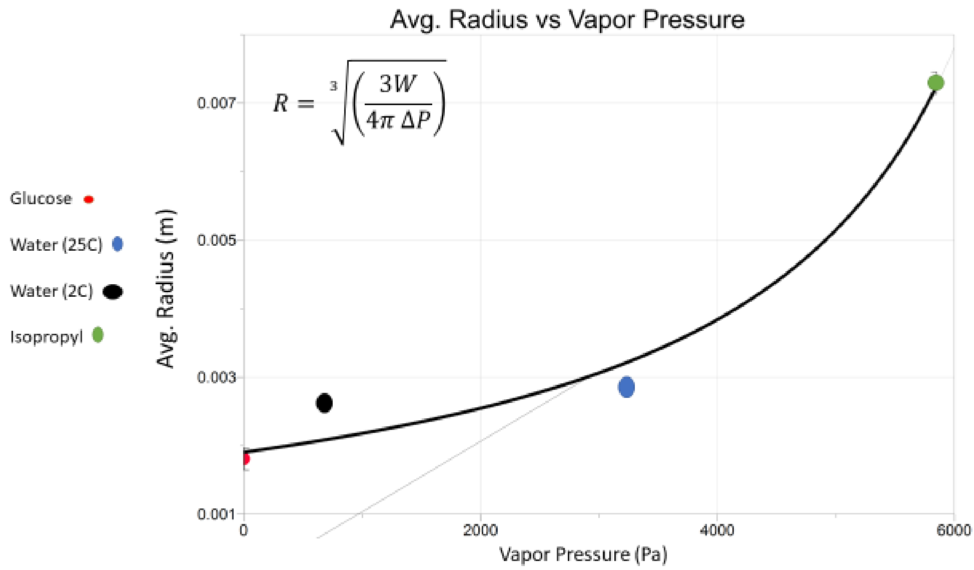

Fig. 3 Graph of the relationship between the average radius of each bubble produced in each instance of cavitation in each liquid and the vapor pressure of each liquid.

From Fig. 3 we can then find the relationship between the average radii of the bubbles and the vapor pressure, which by simply looking at tells us that a larger vapor pressure leads to a larger radius. We take a fit that has the equation

$$R = \sqrt[3]{\left(\frac{3W}{4\pi\,\Delta P}\right)} \tag{4}$$

which we can get from the equation for the energy needed for a bubble to expand[1]. This equation

$$W = \frac{4}{3}\pi R^3 \Delta P \tag{5}$$

where W is the energy needed to create and expand the bubble and ΔP is the difference in the vapor pressure and the reference pressure, can be solved for R which gives us our fit. This fit then gives us a pressure of 79kPa originally inside the bubbles and 185J needed for the bubbles to expand.

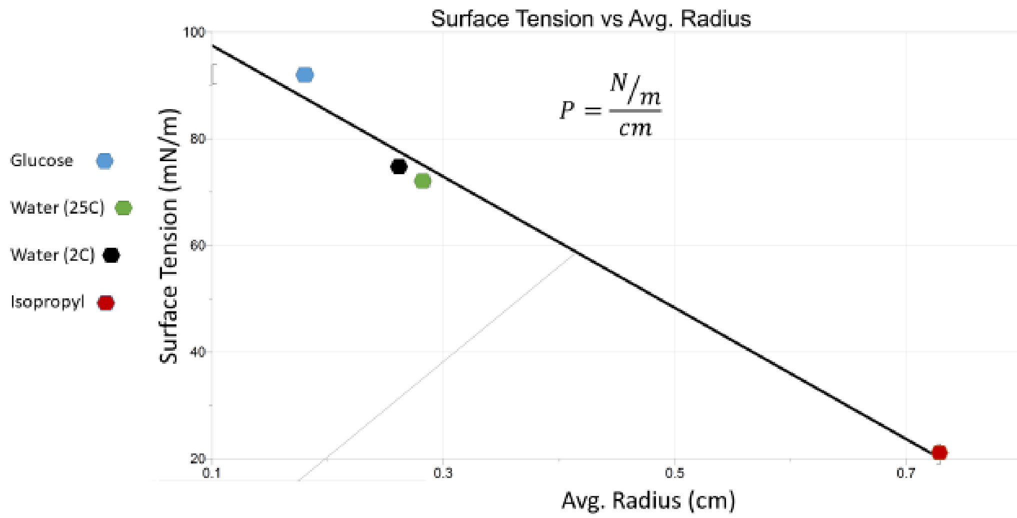

Fig. 3 Graph of the relationship between the average radius of each bubble produced in each instance of cavitation in each liquid and the surface tension of each liquid.

Our final graph, Fig. 4, tells us through a linear fit that there is pressure of 12.3 Pa that is dependent upon the radius of the bubble and the surface tension of the liquid. This pressure is then the pressure difference between the inside of the bubble and outside of it. The equation of this fit is

$$P = \frac{123 * 10^{-3}\, N/m}{cm} \tag{6}$$

which simplifies to 12.3 Pa.

Conclusion

From this experimentation we can then see that the greatest relationships among the physical properties of the liquids and the cavitation is involving the vapor pressure and surface tensions of the liquids. The viscosity during experimentation did not lead to any noticeable trends. We can easily see then that a low vapor pressure and a high surface tension will lead to the largest amount of bubbles with the smallest radius, as the corn syrup had the largest amount of bubbles with the smallest radius. Through these relationships we can also find some insight into the physical processes that the liquid undergoes whenever it is accelerated in this manner but we still do not know entirely why all of these processes occur. In future study we will attempt to find whether or not these results will hold and what the physical processes are in greater detail.

1. Christopher Earls Brennen, *Cavitation and Bubble Dynamics*, 1st Edition (Cambridge University Press, New York, NY, 2013).
2. Pan Z, et al., “Cavitation onset caused by acceleration,” Proceedings of the National Academy of Sciences 114(32):8470–8474 (2017).