CS440 - Artificial Intelligence
Assignment 1 (3 Credit Hours)

Jon Reynolds (jdrynld2)
Matthew Krikorian (krikorn2)
Patrick McMahon (pfmcmah2)

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Implementation Described

State Representation

The states in our project are determined by the following attributes:

- x, y position in maze
- number of dots collected thus far
- which dots were collected

In order to optimize performance, we represent which dots have been collected as an unsigned 32-bit integer, where the i-th bit represents whether the i-th dot has been collected or not. We can use bitwise operators to create these numbers quickly.

In order to allow fast lookup of neighbors, we store all maze states visited in a 3D array of hashmaps. The first argument to the array is x-coordinate, the second is y-coordinate, the third is the number of dots collected, and the key to the hashmap is the unsigned integer representing which dots have been collected thus far. This has the benefit of storing **only** the combinations of dots that have been collected thus far, instead of allocating space to account for all $2^{numberOfDots}$ combinations for every possible position and number of dots collected.

Frontier Representation

Our frontier is a doubly linked list implemented as a deque, or *double-ended queue*, and synchronized with a min-priority queue. Double-ended means push and pop operations work on both the front and back of the frontier, allowing it to act as a stack or a queue. Moreover, since the frontier is backed by a min-priority queue, the frontier node with the lowest cost can be found and removed in $O(\lg n)$ time.

It follows that we can use the following functionalities of the frontier for the four different search methods:

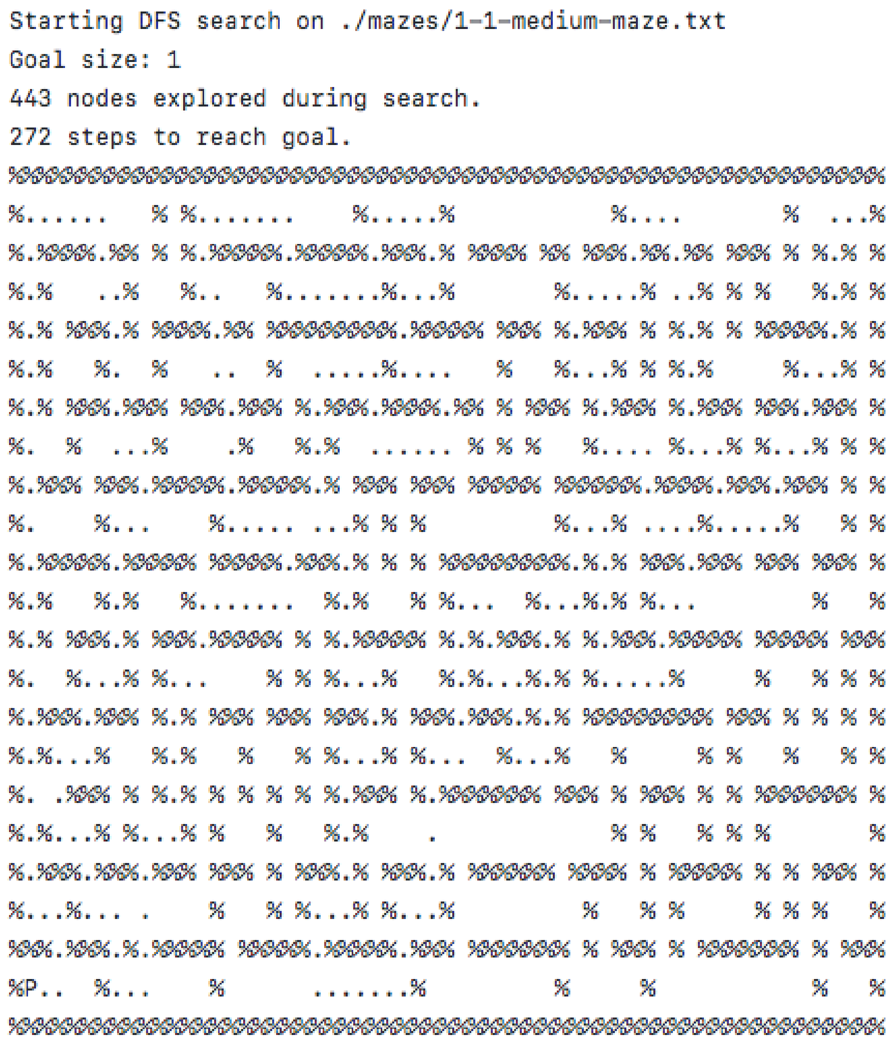
- DFS - uses frontier as a stack
- BFS - uses frontier as a queue
- Greedy - uses frontier as a min-priority queue on heuristic cost
- A* - uses frontier as a min-priority queue on heuristic cost + path cost

Section 1.1, Basic Pathfinding

Medium Maze Search Outputs

Below are the outputs we generated for the big maze using the four different search algorithms.

DFS



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Starting DFS search on ./mazes/1-1-medium-maze.txt
Goal size: 1
443 nodes explored during search.
272 steps to reach goal.
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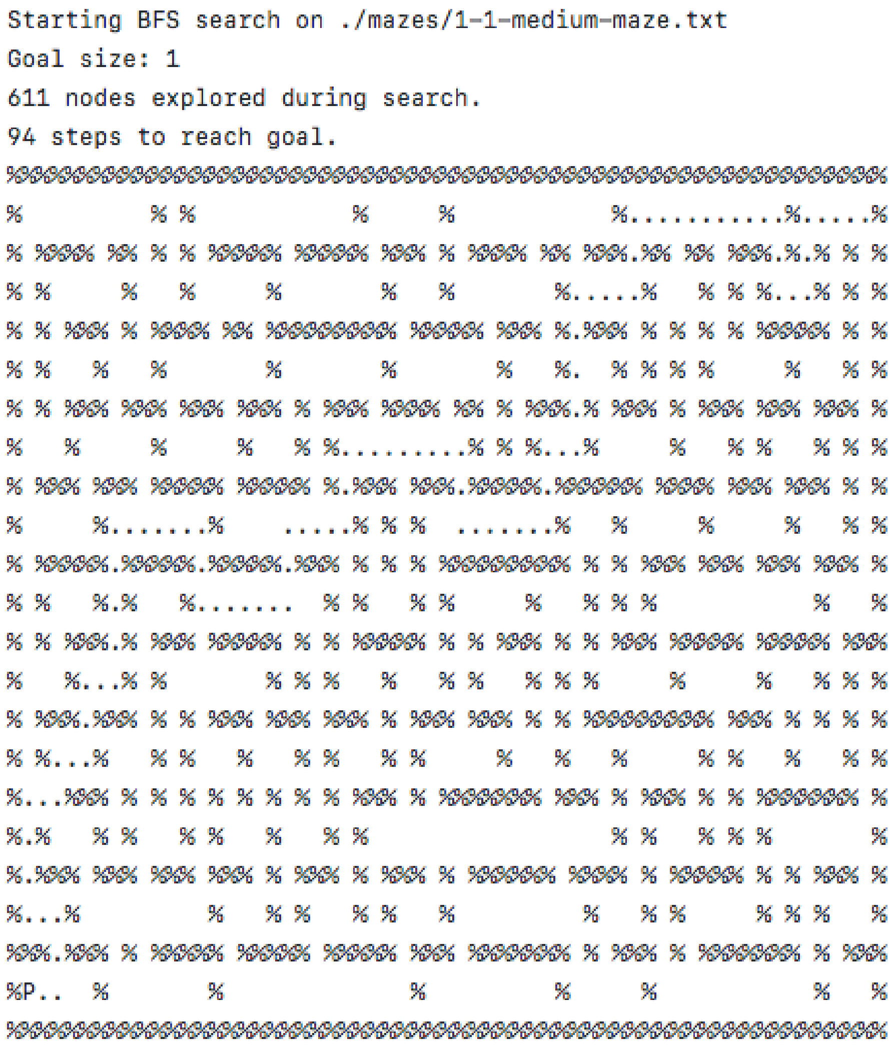
Figure 1: Depth-first search on the medium maze

BFS



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Starting BFS search on ./mazes/1-1-medium-maze.txt
Goal size: 1
611 nodes explored during search.
94 steps to reach goal.
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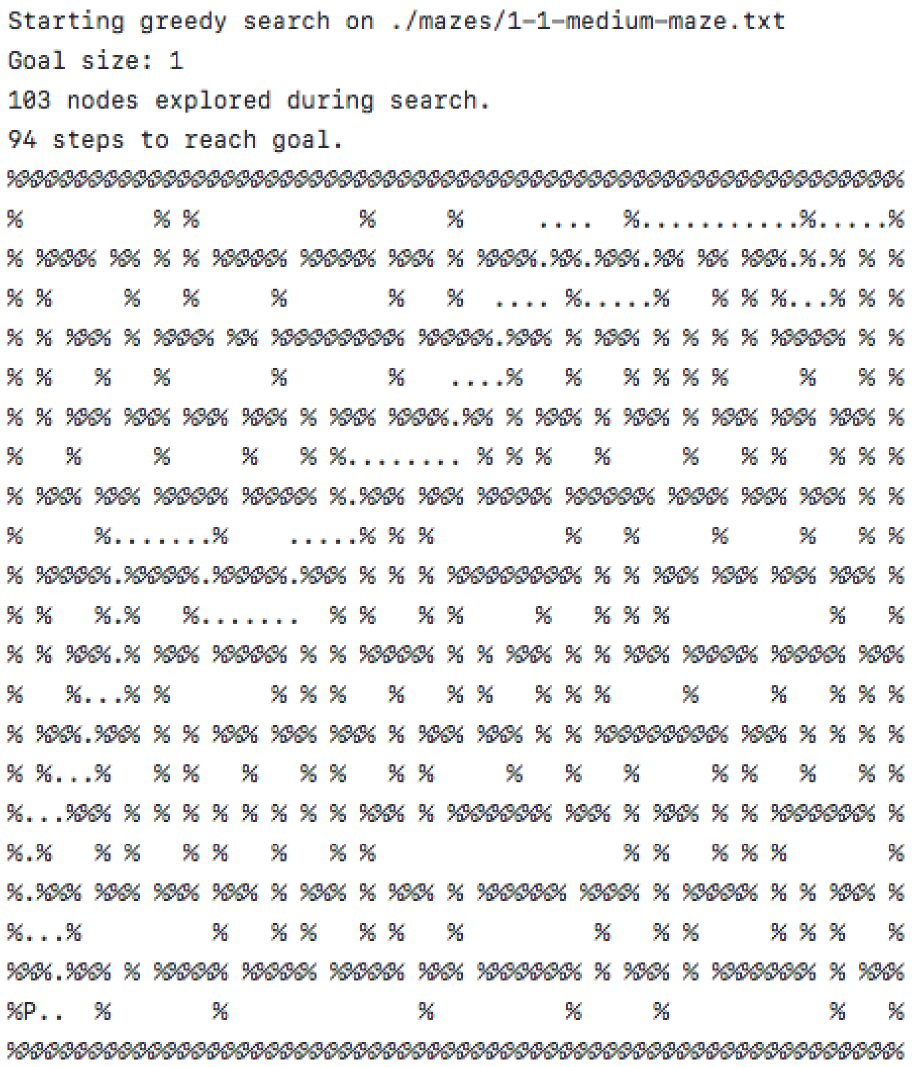
Figure 2: Breadth-first search on the medium maze

Greedy



```
Starting greedy search on ./mazes/1-1-medium-maze.txt
Goal size: 1
103 nodes explored during search.
94 steps to reach goal.
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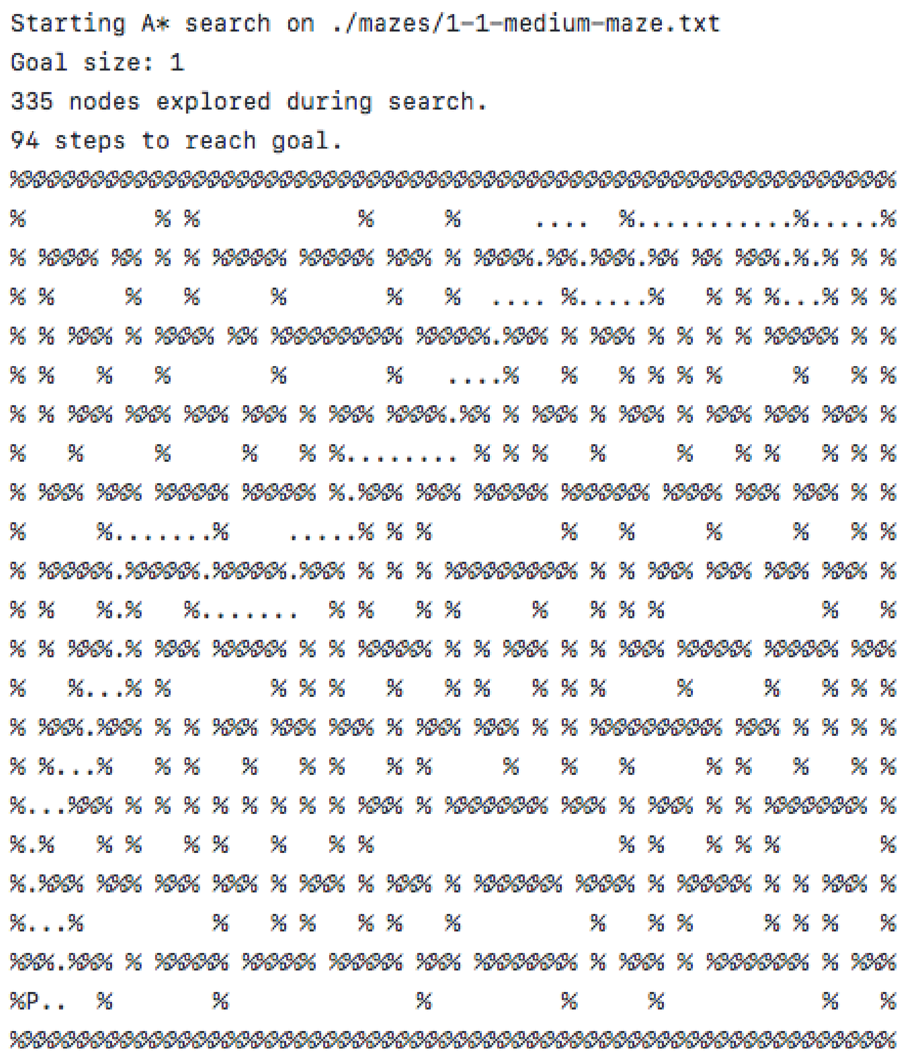
Figure 3: Greedy search on the medium maze

A*



```
Starting A* search on ./mazes/1-1-medium-maze.txt
Goal size: 1
335 nodes explored during search.
94 steps to reach goal.
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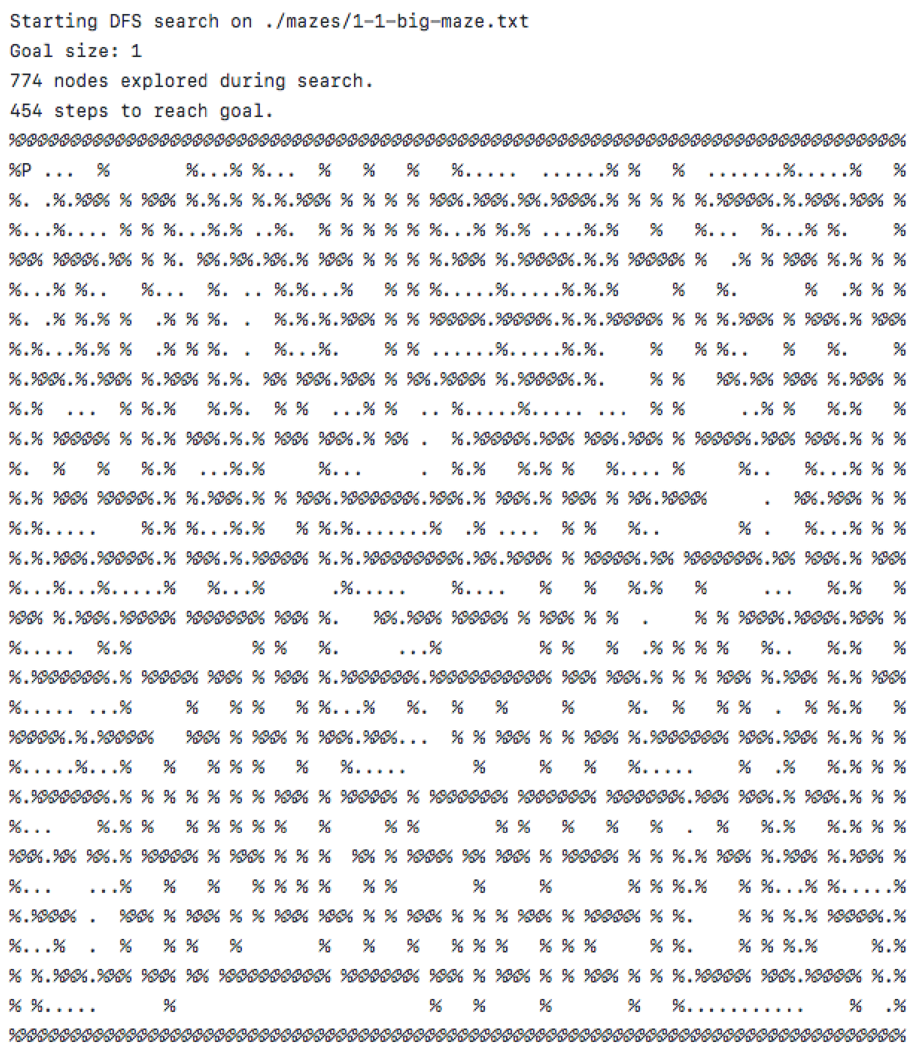
Figure 4: A* search on the medium maze

Big Maze Search Outputs

Below are the outputs we generated for the big maze using the four different search algorithms.

DFS



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Starting DFS search on ./mazes/1-1-big-maze.txt
Goal size: 1
774 nodes explored during search.
454 steps to reach goal.
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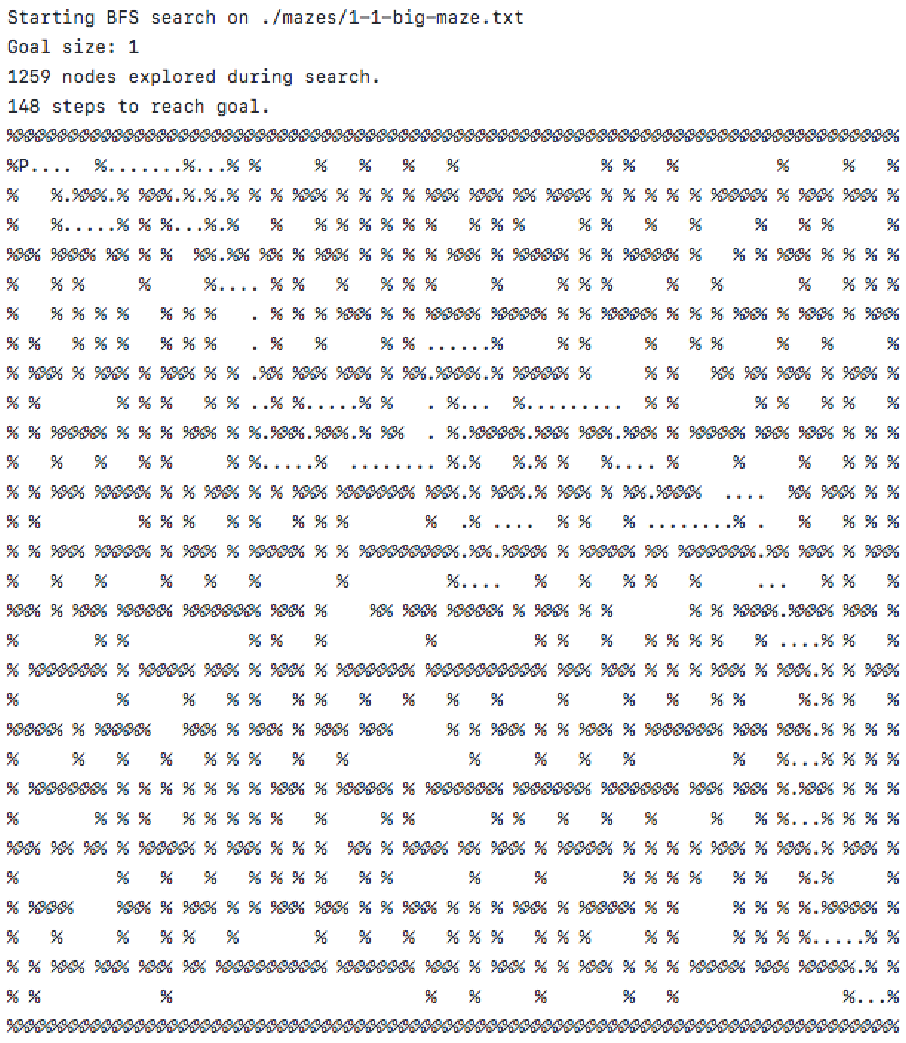
Figure 5: Depth-first search on the big maze

BFS



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Starting BFS search on ./mazes/1-1-big-maze.txt
Goal size: 1
1259 nodes explored during search.
148 steps to reach goal.
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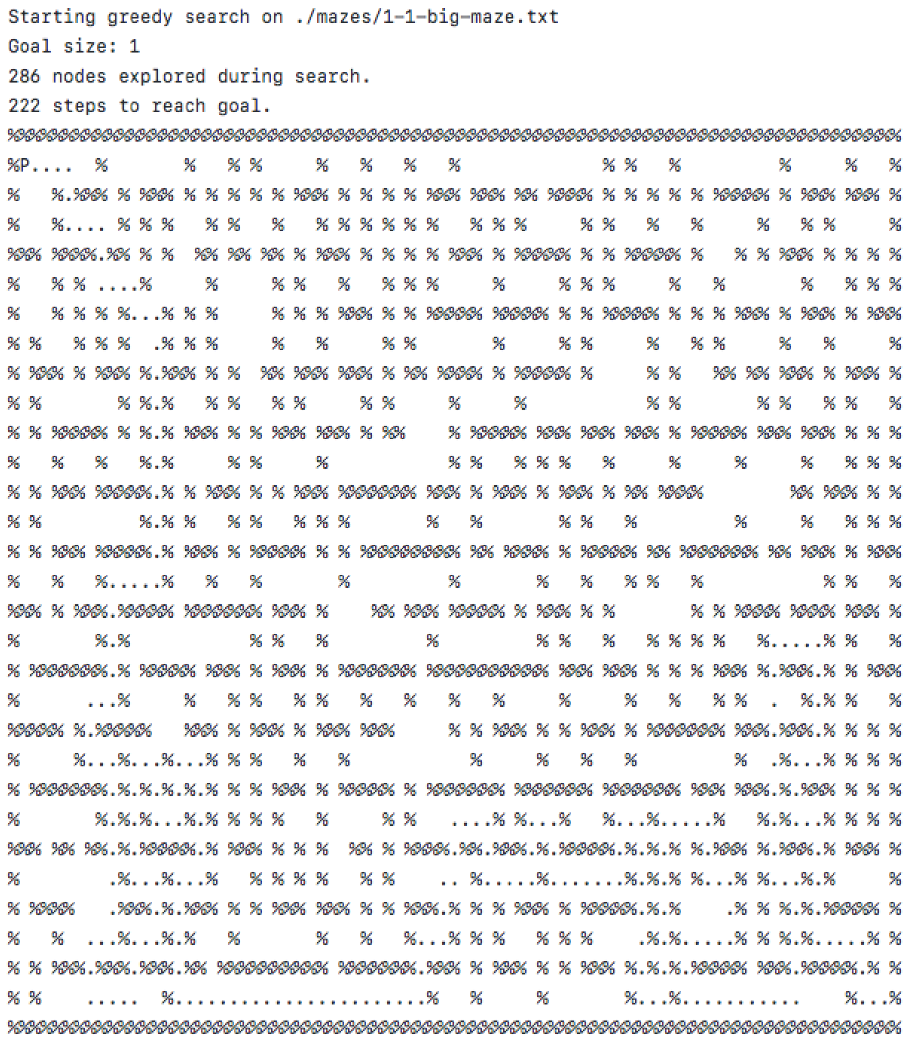
Figure 6: Breadth-first search on the big maze

Greedy



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Starting greedy search on ./mazes/1-1-big-maze.txt
Goal size: 1
286 nodes explored during search.
222 steps to reach goal.
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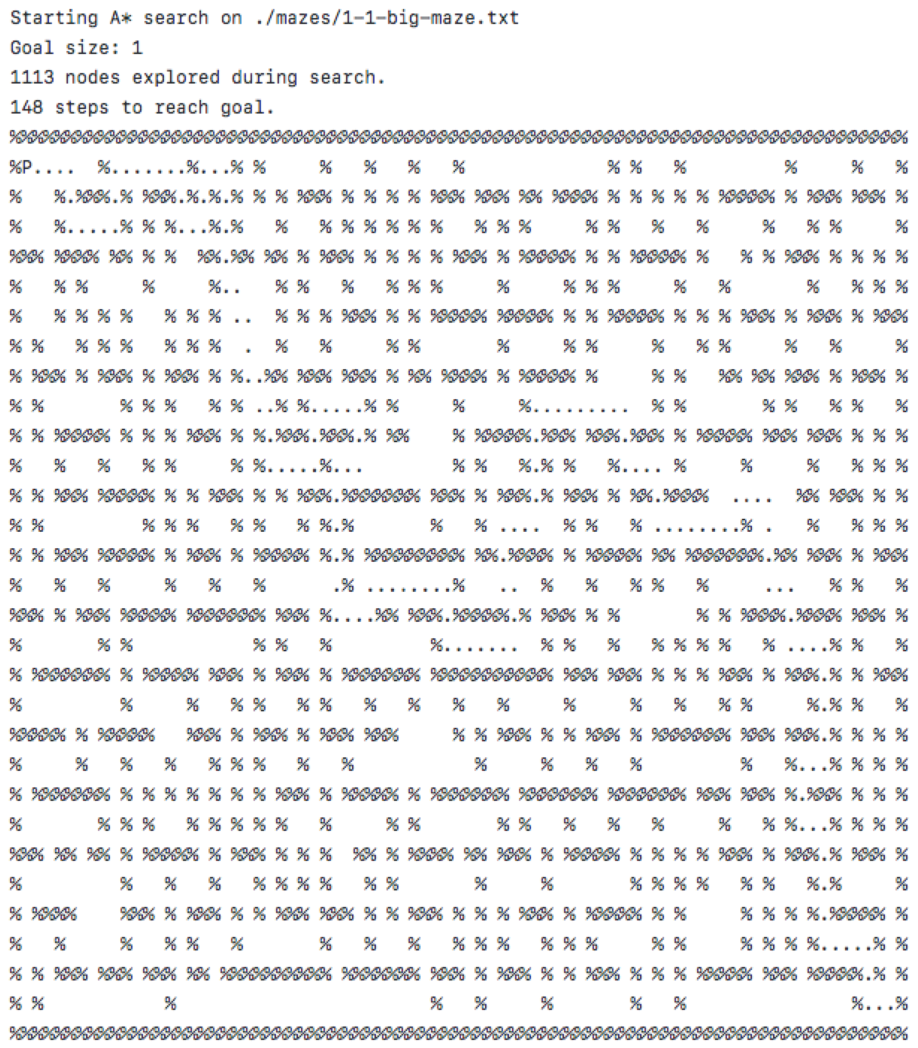
Figure 7: Greedy search on the big maze

A*



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Starting A* search on ./mazes/1-1-big-maze.txt
Goal size: 1
1113 nodes explored during search.
148 steps to reach goal.
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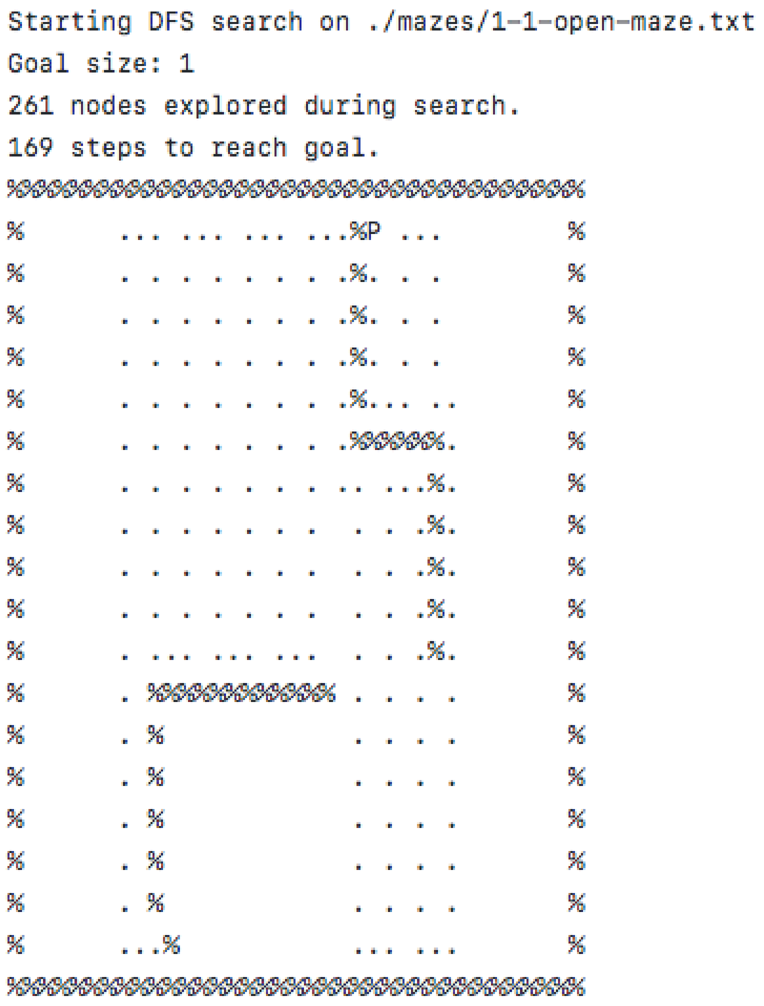
%       % %           % %   %         %.......  % %   %   % % % %   % ....% %   %
% %%%%%%% % %%%%% %%% % %%% % %%%%%%% %%%%%%%%%%% %%% %%% % % % %%% % %%%.% % %%%
%         %     %   % %   % %   %   %   %   %     %     %   %   % %     %.% %   %
%%%%% % %%%%%   %%% % %%% % %%% %%%     % % %%% % % %%% % %%%%%%% %%% %%%.% % % %
%     %   %   %   % % %   %   %           %     %   %   %         %   %...% % % %
% %%%%%%% % % % % % % % %%% % %%%%% % %%%%%%% %%%%%%% %%%%%%% %%% %%% %.%%% % % %
%       % % %   % % % % %   %     % %       % %   %   %   %     %   %   %...% % % %
%%% %% %% % %%%%% % %%% % % %  %% % %%%% %% %%% % %%%%% % % % % %%% % %%%.% %%% %
%         %   %   %   % % % %   % %       %     %       % % % %   % %   %.%     %
% %%%%    %%% % %%% % % %%% %%% % % %%% % % % %%% % %%%%% % %     % % % %.%%%%% %
%   %     %   % %   %       %   %   %   % % %   % % %     % %     % % % %.....% %
% % %%% %%% %%% %% %%%%%%%%%% %%%%%%% %%% % %%% % % %%% % % % %%%%% %%% %%%%%.% %
% %           %                       %   %     %       %   %               %...%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 8: A* search on the big maze

Open Maze Search Outputs

Below are the outputs we generated for the open maze using the four different search algorithms.

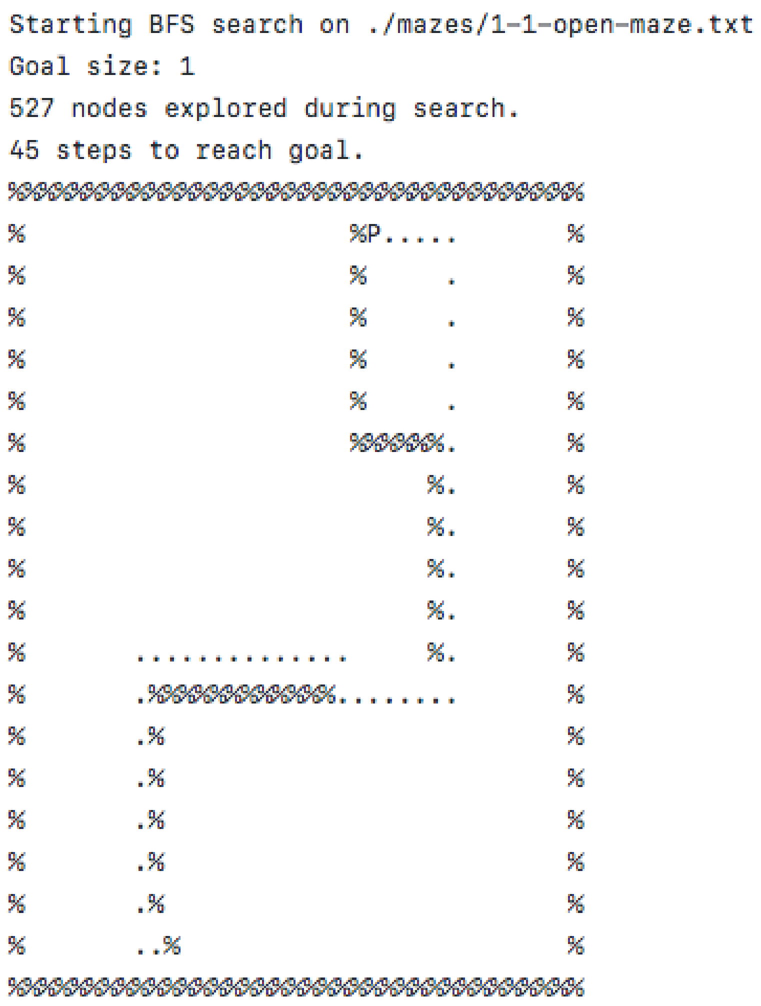
DFS



```
Starting DFS search on ./mazes/1-1-open-maze.txt
Goal size: 1
261 nodes explored during search.
169 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      ... ... ... ...%P ...        %
%      . . . . . . . .%. . .        %
%      . . . . . . . .%. . .        %
%      . . . . . . . .%. . .        %
%      . . . . . . . .%... ..       %
%      . . . . . . . .%%%%%%.       %
%      . . . . . . . .. ...%.       %
%      . . . . . . .  . . .%.       %
%      . . . . . . .  . . .%.       %
%      . . . . . . .  . . .%.       %
%      . ... ... ...  . . .%.       %
%      . %%%%%%%%%%%% . . . .       %
%      . %            . . . .       %
%      . %            . . . .       %
%      . %            . . . .       %
%      . %            . . . .       %
%      . %            . . . .       %
%      ...%           ... ...       %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 9: Depth-first search on the open maze

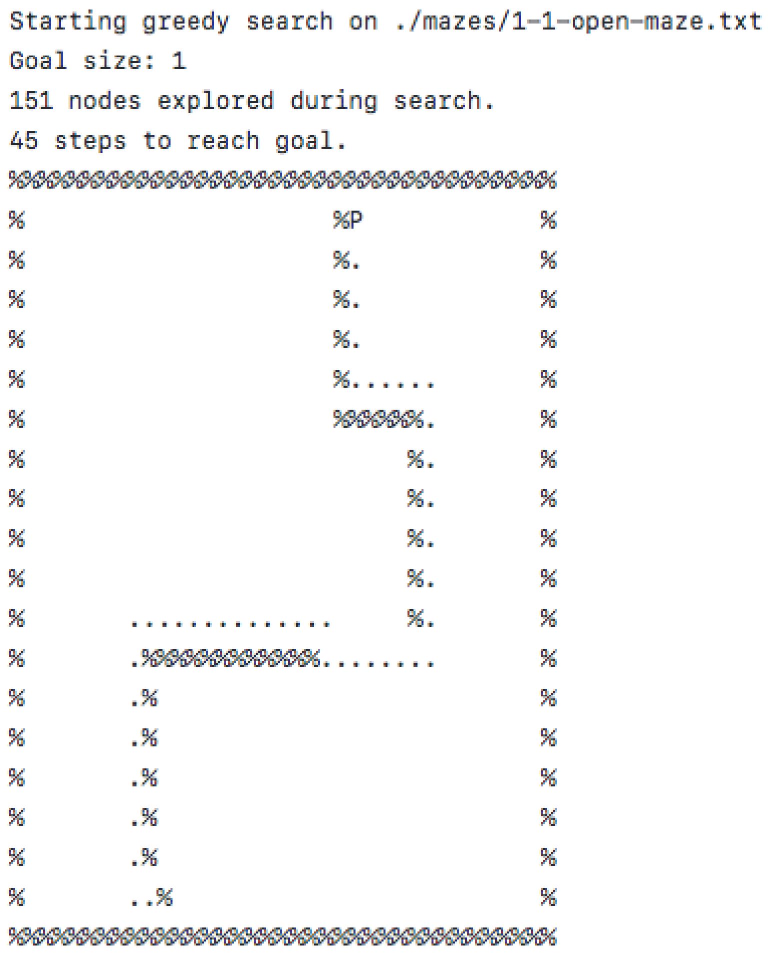
BFS



```
Starting BFS search on ./mazes/1-1-open-maze.txt
Goal size: 1
527 nodes explored during search.
45 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                     %P.....       %
%                     %     .       %
%                     %     .       %
%                     %     .       %
%                     %     .       %
%                     %%%%%%.       %
%                          %.       %
%                          %.       %
%                          %.       %
%                          %.       %
%       ..............     %.       %
%       .%%%%%%%%%%%%........       %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       ..%                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 10: Breadth-first search on the open maze

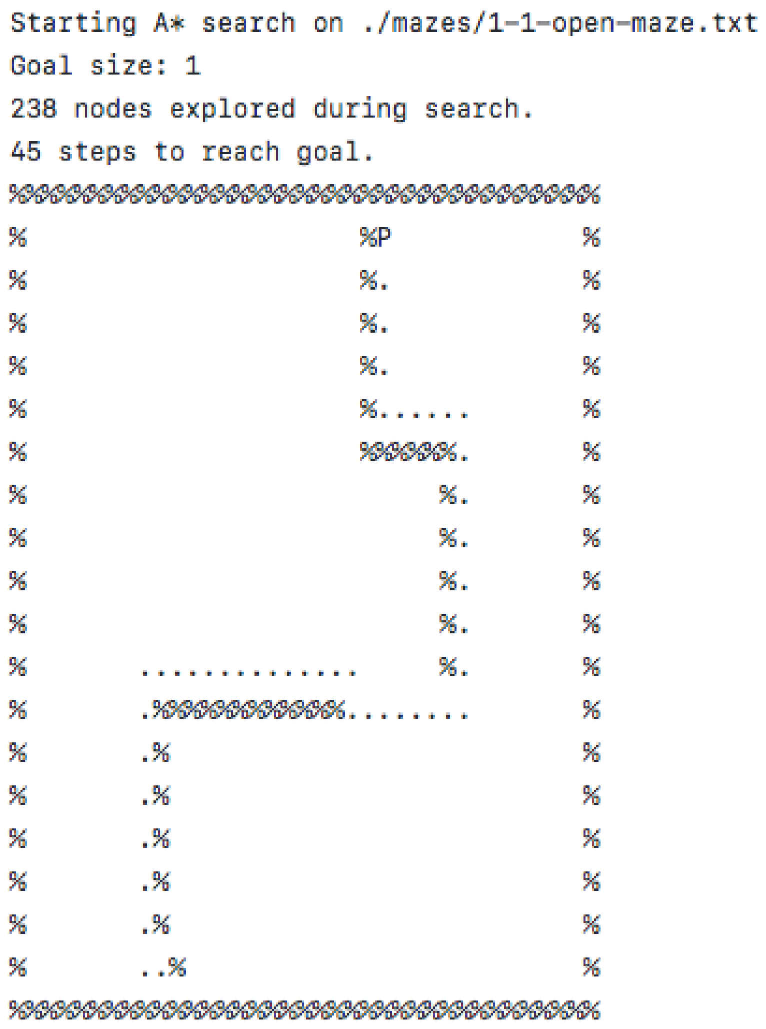
Greedy



```
Starting greedy search on ./mazes/1-1-open-maze.txt
Goal size: 1
151 nodes explored during search.
45 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                     %P            %
%                     %.            %
%                     %.            %
%                     %.            %
%                     %......       %
%                     %%%%%%.       %
%                          %.       %
%                          %.       %
%                          %.       %
%                          %.       %
%       ..............     %.       %
%       .%%%%%%%%%%%%........       %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       ..%                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 11: Greedy search on the open maze

A*



```
Starting A* search on ./mazes/1-1-open-maze.txt
Goal size: 1
238 nodes explored during search.
45 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                     %P            %
%                     %.            %
%                     %.            %
%                     %.            %
%                     %......       %
%                     %%%%%%.       %
%                          %.       %
%                          %.       %
%                          %.       %
%                          %.       %
%       ..............     %.       %
%       .%%%%%%%%%%%%........       %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       .%                          %
%       ..%                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

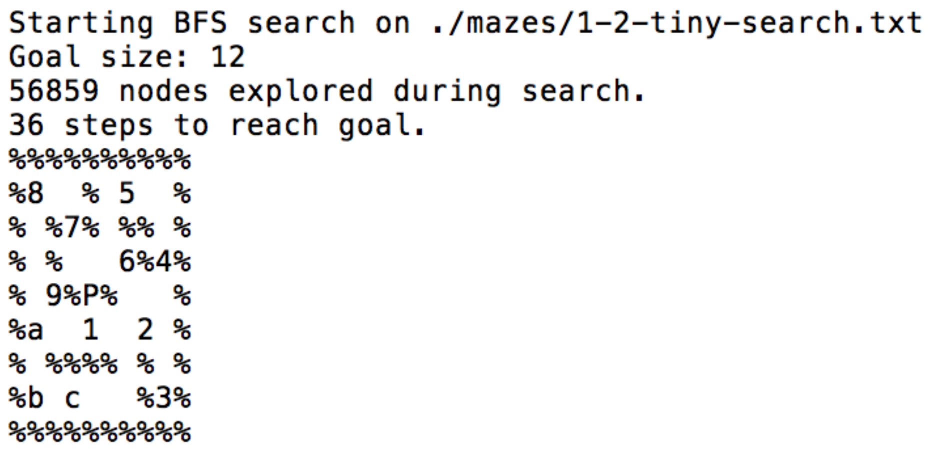
Figure 12: A* search on the open maze

Section 1.2, Search With Multiple Dots

Tiny Maze Search Outputs

Below are the outputs we generated for the tiny maze using the BFS and A* search algorithms.

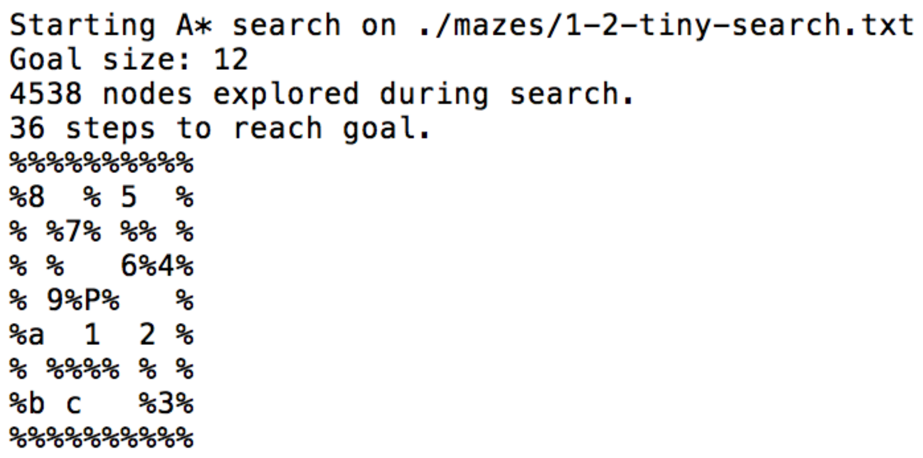
BFS



```
Starting BFS search on ./mazes/1-2-tiny-search.txt
Goal size: 12
56859 nodes explored during search.
36 steps to reach goal.
%%%%%%%%%%
%8  % 5  %
% %7% %% %
% %   6%4%
% 9%P%   %
%a  1  2 %
% %%%% % %
%b c   %3%
%%%%%%%%%%
```

Figure 13: Breadth-first search on the tiny maze

A*



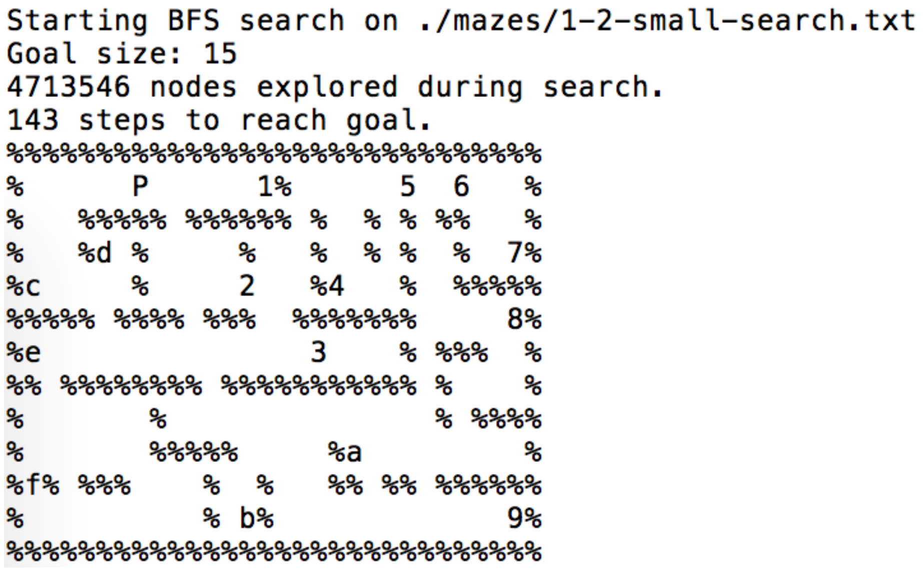
```
Starting A* search on ./mazes/1-2-tiny-search.txt
Goal size: 12
4538 nodes explored during search.
36 steps to reach goal.
%%%%%%%%%%
%8  % 5  %
% %7% %% %
% %   6%4%
% 9%P%   %
%a  1  2 %
% %%%% % %
%b c   %3%
%%%%%%%%%%
```

Figure 14: A* search on the tiny maze

Small Maze Search Outputs

Below are the outputs we generated for the small maze using the BFS and A* search algorithms.

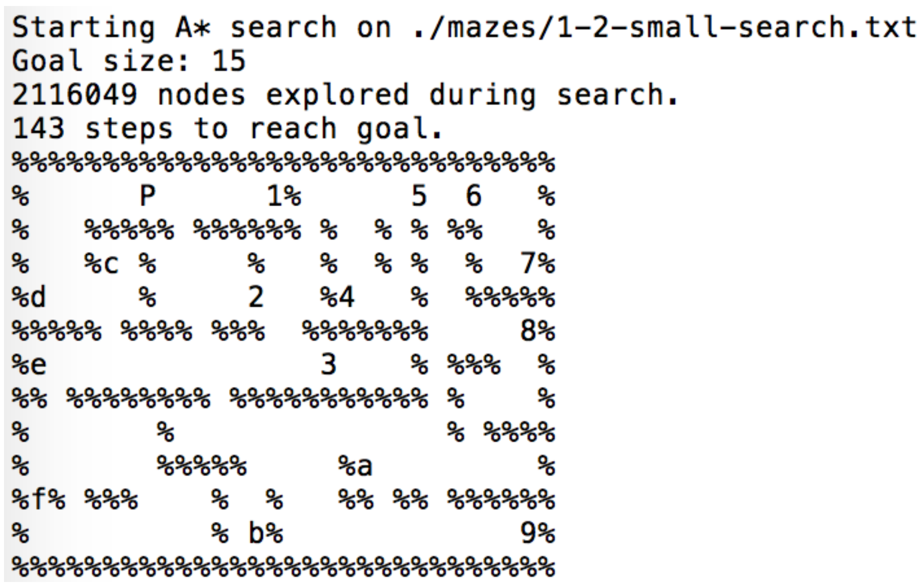
BFS



```
Starting BFS search on ./mazes/1-2-small-search.txt
Goal size: 15
4713546 nodes explored during search.
143 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      P      1%      5  6   %
%   %%%%% %%%%%% %  % % %%   %
%   %d %     %   %  % %  %  7%
%c     %     2   %4   %  %%%%%
%%%%% %%%% %%%  %%%%%%%     8%
%e               3    % %%%  %
%% %%%%%%%% %%%%%%%%%%% %    %
%       %               % %%%%
%       %%%%%     %a         %
%f% %%%    %  %   %% %% %%%%%%
%          % b%             9%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 15: Breadth-first search on the small maze

A*



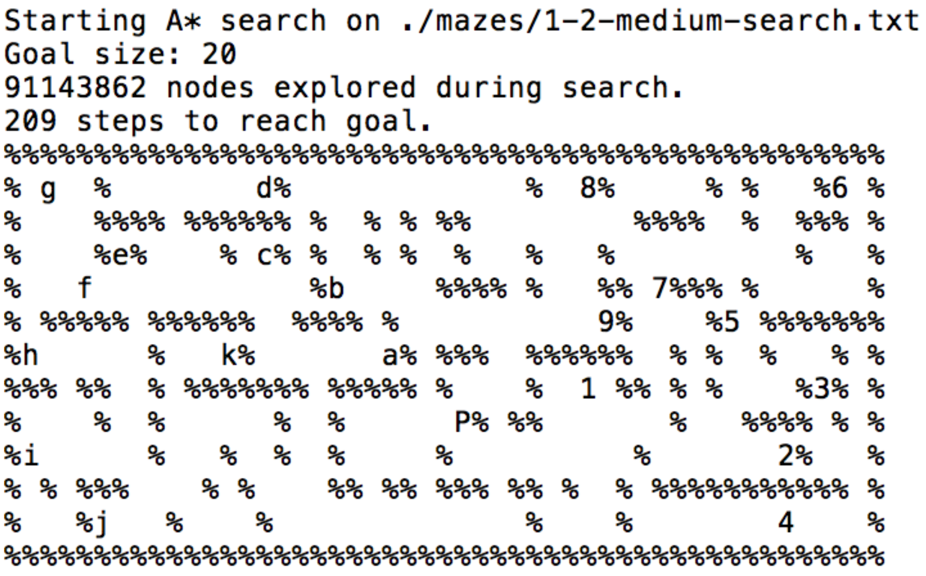
```
Starting A* search on ./mazes/1-2-small-search.txt
Goal size: 15
2116049 nodes explored during search.
143 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      P      1%      5  6   %
%   %%%%% %%%%%% %  % % %%   %
%   %c %     %   %  % %  %  7%
%d     %     2   %4   %  %%%%%
%%%%% %%%% %%%  %%%%%%%     8%
%e               3    % %%%  %
%% %%%%%%%% %%%%%%%%%%% %    %
%       %               % %%%%
%       %%%%%     %a         %
%f% %%%    %  %   %% %% %%%%%%
%          % b%             9%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 16: A* search on the small maze

Medium Maze Search Outputs

Below are the outputs we generated for the medium maze using the BFS and A* search algorithms.

A*



```
Starting A* search on ./mazes/1-2-medium-search.txt
Goal size: 20
91143862 nodes explored during search.
209 steps to reach goal.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% g  %        d%             %  8%     % %   %6 %
%    %%%% %%%%%% %  % % %%         %%%%  %  %%% %
%    %e%    % c% %  % %  %   %   %          %   %
%   f            %b     %%%% %   %% 7%%% %      %
% %%%%% %%%%%%  %%%% %           9%    %5 %%%%%%%
%h      %   k%       a% %%%  %%%%%%  % %  %   % %
%%% %%  % %%%%%%% %%%%% %    %  1 %% % %    %3% %
%    %  %      %  %      P% %%       %   %%%% % %
%i      %   %  %  %     %          %       2%   %
% % %%%    % %    %% %% %%% %% %  % %%%%%%%%%%% %
%   %j   %    %              %    %        4    %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure 17: A* search on the medium maze

Our Heuristic, Explained

The heuristic we chose to use for our A* search is to calculate the distance from the current node of the algorithm to each of the goal states and create a minimum spanning tree. Using this minimum spanning tree and the A* traversal technique we developed in section 1.1, we were able to solve all three of the different sized mazes. Our heuristic is also admissible because the sum of the path cost from the starting position to the current node added to the Manhattan Distance edge of the Minimum Spanning Tree we create from the current node to each of the goal nodes will always under-estimate the true cost to each goal state. Since our heuristic will always under-estimate the true cost of reaching each node, our heuristic passes as being admissible. Our heuristic also never over-estimates the distance to any goal state, because if there are walls in between the current node and the goal state, our heuristic will always calculate a much smaller distance than the distance of the path the algorithm will actually have to take, thus confirming once again that we never over-estimate the cost to reach the goal, proving admissibility. Finally, on the fundamental level, the Minimum Spanning Tree heuristic is essentially the cost subgraph that connects all the nodes together. The solution path is also used to form a subgraph of nodes, the minimum spanning tree formed in the solution path graph cannot weigh less than the minimum spanning tree our algorithm uses to generate a solution, so our minimum spanning tree heuristic can never over-estimate the solution. Also, the generated subgraph by the solution will re-use edges that will get added to the overall path cost multiple times, whereas our minimum spanning tree heuristic will never re-use edges in path-cost calculation, proving even further that our minimum spanning tree heuristic will again only under-estimate the solution path cost, proving admissibility.

Minimum Spanning Tree Implementation, Explained

To implement our A* search in Section 1.2, we used a Minim Spanning Tree to store the edges between nodes in the frontier and each goal state. We use the Disjoint Sets data structure to implement the minimum spanning tree for efficiency purposes and to properly store and manage the data we continually are editing for each node.

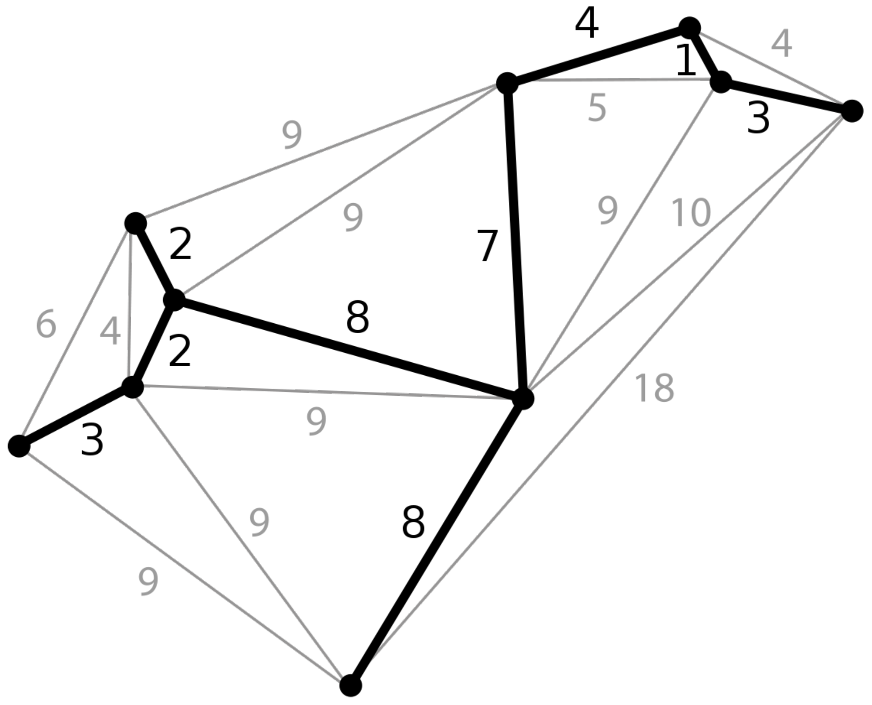


Figure 18: A minimum spanning tree with weighted edges

The minimum spanning tree's role in the A* search algorithm is to calculate a values for the heuristic that is used in calculating values for the node choosing function. For each neighbor of the current node being analyzed, we use the distance of each neighbor to the nearest remaining dot on the board, adding it to the minimum spanning tree cost of that node. This allows us to put the minimum spanning tree cost of the neighbor node and the remaining dots in a single minimum spanning tree, which then gets used to calculate which node we traverse to next.

Team Involvement

Jon Reynolds was responsible for implementing the maze class, and the node class as well as implementing the minimum spanning tree in section 1.2, as well as documenting his respective work in the report. Matthew Krikorian was responsible for writing the maze class and the main file which processed the mazes, as well as documenting the respective work in the report. Patrick McMahon was responsible for coming up with the heuristic for section 1.2, as well as implementing the disjoint sets class and the frontier class, as well as documenting his respective work in the report.