Assignment 2

Table of Contents

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Question 2: Conductive Elements

In this question, a grid is set up, with the left sided boundary set to 1V. Two bottle-necks of conductivities different than the rest of the grid are introduced. Using the maxtrix form of the problem (GV=F), the electrostatic potential within the region is found and plotted. Additionally, the conducitivity, electric field and currents are plotted as well.

```
% Clear all previous variables, figures, etc, to ensure that the
 workspace
% is clean.
clear all
clearvars
clearvars -GLOBAL
close all

%Repeat the calculations 16 times. The first time is for a), times 2-6
 are
%for b), times 7-11 are for c) and 12-16 are for d).
for repeat = 1:16

    %Set the length and width of the grid.
    L=160;
    W=120;
    %b) - Vary the size of the grid
    if(repeat>1 && repeat <7)
        L = (repeat-1)*40;
        W = (repeat-1)*30;
    end
    Lb = L/5;
    Wb = W/5;
    %c) - vary the size of the bottle-necks
    if(repeat>6&&repeat<12)
        Lb = ceil(L/(repeat-4));
        Wb = ceil(W/(repeat-4));
    end

    %Initialize the G,B and conductivity matrices.
    G = sparse(L*W,L*W);
    B=zeros(L*W,1);
    B(1:W,1)=1;
```

```
    condMap = zeros(W,L);

    %Populate the conductivity matrix.
    for lCount = 1:L
        for wCount = 1:W
            if(lCount < L/2 + Lb/2 && lCount > L/2 - Lb/2 &&...
                    (wCount > W-Wb || wCount < Wb))
                condMap( wCount,lCount) = 10^-2;

                %d) - Vary the conductivity of the bottle-necks
                if(repeat>11)
                    condMap( wCount,lCount) =10^(repeat-15);
                end
            else
                condMap(wCount,lCount) = 1;
            end
        end
    end

    % Set the diagonal of the G matrix to 1. This value will be
 overwritten
    % later if it is not a boundary condition.
    for count = 1:L*W
        G(count,count)=1;
    end
```

Loop through rows and columns, if not a boundary case, set the gradient based on the sum of adjacent conductivies.

```
    for col = 1:L
        if(col~=1 &&col~=L)
            for row = 1:W
                n = row + (col -1)*W;
                if(count~=1 && row ~=1 && row~=W)
                    rxBefore = (condMap(row,col) +
 condMap(row,col-1))/2.0;
                    rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
                    ryBefore = (condMap(row,col) +
 condMap(row-1,col))/2.0;
                    ryAfter = (condMap(row,col) + condMap(row
+1,col))/2.0;

                    nyBefore = n-1;
                    nyAfter = n+1;
                    nxBefore = row+(col-2)*W;
                    nxAfter = row+col*W;
                    G(n,n) = -(rxBefore+rxAfter+ryBefore+ryAfter);
                    G(n, nyBefore) =ryBefore;
                    G(n, nyAfter)=ryAfter;
                    G(n, nxBefore)=rxBefore;
                    G(n, nxAfter) =rxAfter;

                elseif(row==1)
```

```
                    %Special Case: Bottom of Grid
                    rxBefore = (condMap(row,col) +
 condMap(row,col-1))/2.0;
                    rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
                    ryAfter = (condMap(row,col) + condMap(row
+1,col))/2.0;

                    nyAfter = n+1;
                    nxBefore = row+(col-2)*W;
                    nxAfter = row+col*W;
                    G(n,n) = -(rxBefore+rxAfter+ryAfter);
                    G(n, nyAfter)=ryAfter;
                    G(n, nxBefore)=rxBefore;
                    G(n, nxAfter) =rxAfter;

                elseif(row==W)
                    %Special Case: Top of Grid
                    rxBefore = (condMap(row,col) +
 condMap(row,col-1))/2.0;
                    rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
                    ryBefore = (condMap(row,col) +
 condMap(row-1,col))/2.0;

                    nyBefore = n-1;
                    nxBefore = row+(col-2)*W;
                    nxAfter = row+col*W;
                    G(n,n) = -(rxBefore+rxAfter+ryBefore);
                    G(n, nyBefore) =ryBefore;
                    G(n, nxBefore)=rxBefore;
                    G(n, nxAfter) =rxAfter;
                end
            end
        end
    end
    V=G\B;

    %Map the voltage to original grid.
    voltMap = zeros(W,L);
    for cols = 1:L
        for rows = 1:W
            n= rows+(cols-1)*W;
            voltMap(rows,cols)=V(n);
        end
    end
```

Find the Electric field knowing $E = -\nabla V$

```
    [Ex, Ey]=gradient(voltMap);
    Ex=-Ex;
    Ey=-Ey;
```

Find the current density knowing $J = \sigma E$

```
    Jx = condMap.*Ex;
    Jy = condMap.*Ey;

    %Sum the currents at both contacts (edges), take the average to
find
    %the total.
    current1 = sum(Jx(:,1));
    current2 = sum(Jx(:,L));
    totalCurrent = (current1+current2)/2;
```

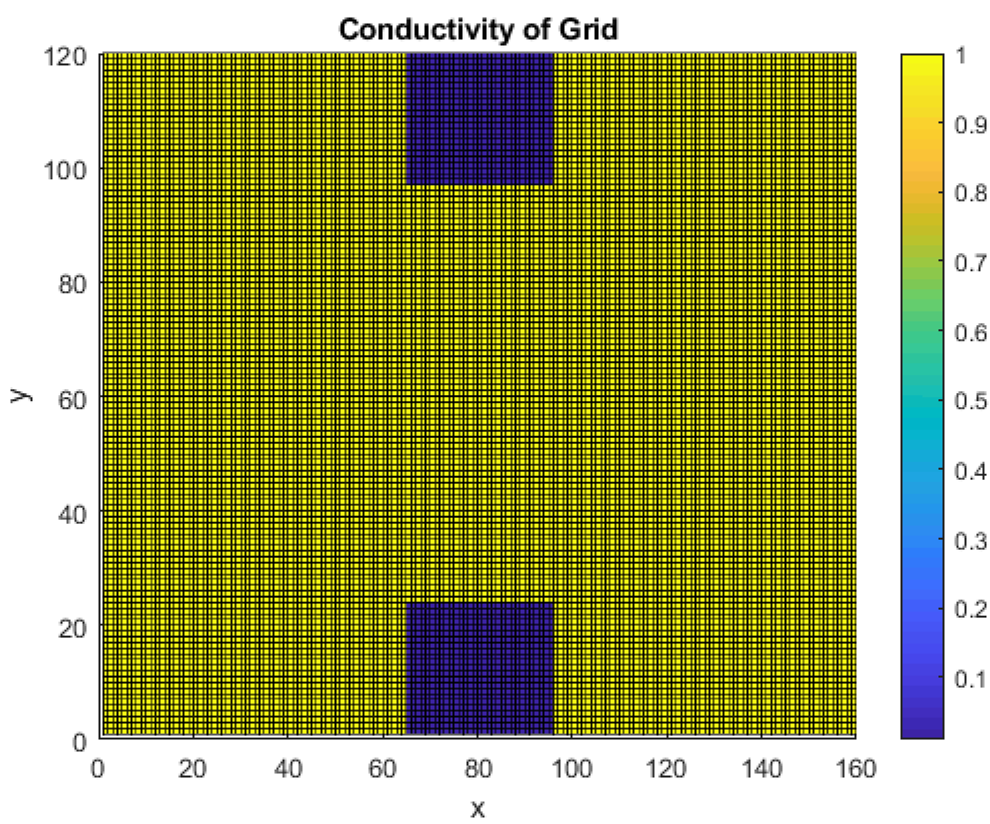
Part A

As calculated above, the total current through the contacts for L=160, W=120, Lb = 32 and Wb = 24 is 0.6235. Additionally, there is no observed difference between the currents at each contact.

```
    if(repeat==1)
```

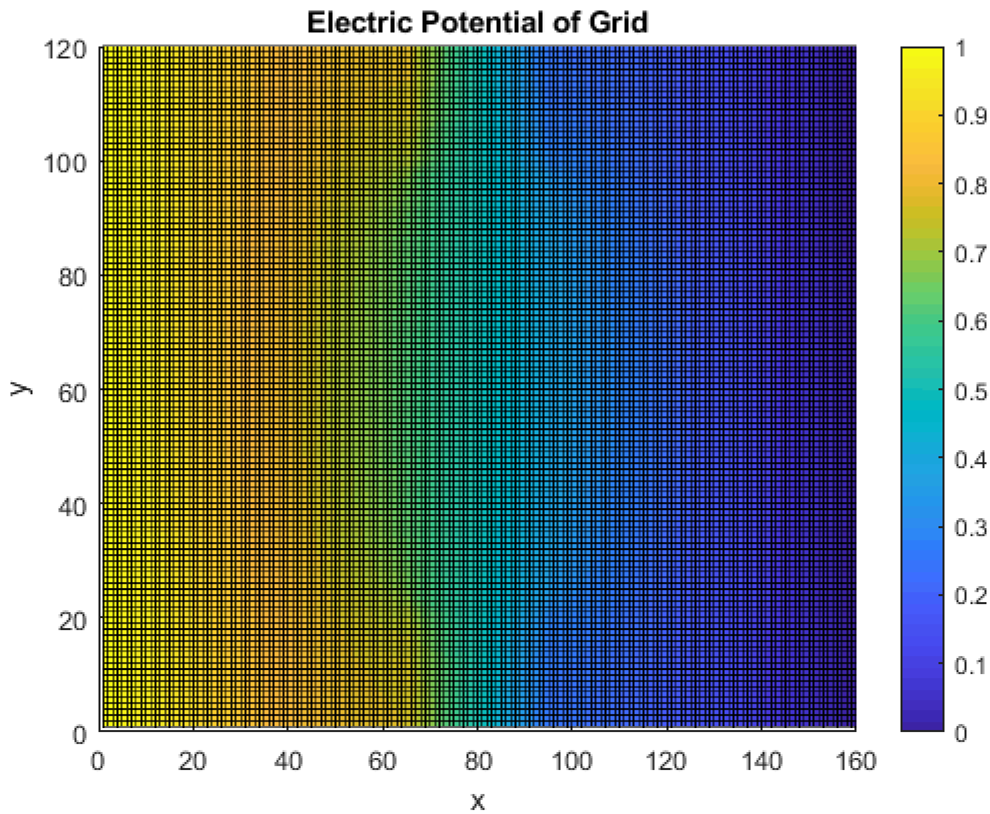
The conductivity plot can be seen in the figure below. The two bottle-neck regions can be seen clearly.

```
        figure
        surf(1:L,1:W,condMap)
        xlabel('x')
        ylabel('y')
        zlabel('Conductivity')
        title('Conductivity of Grid')
        colorbar;
        view(2)
```



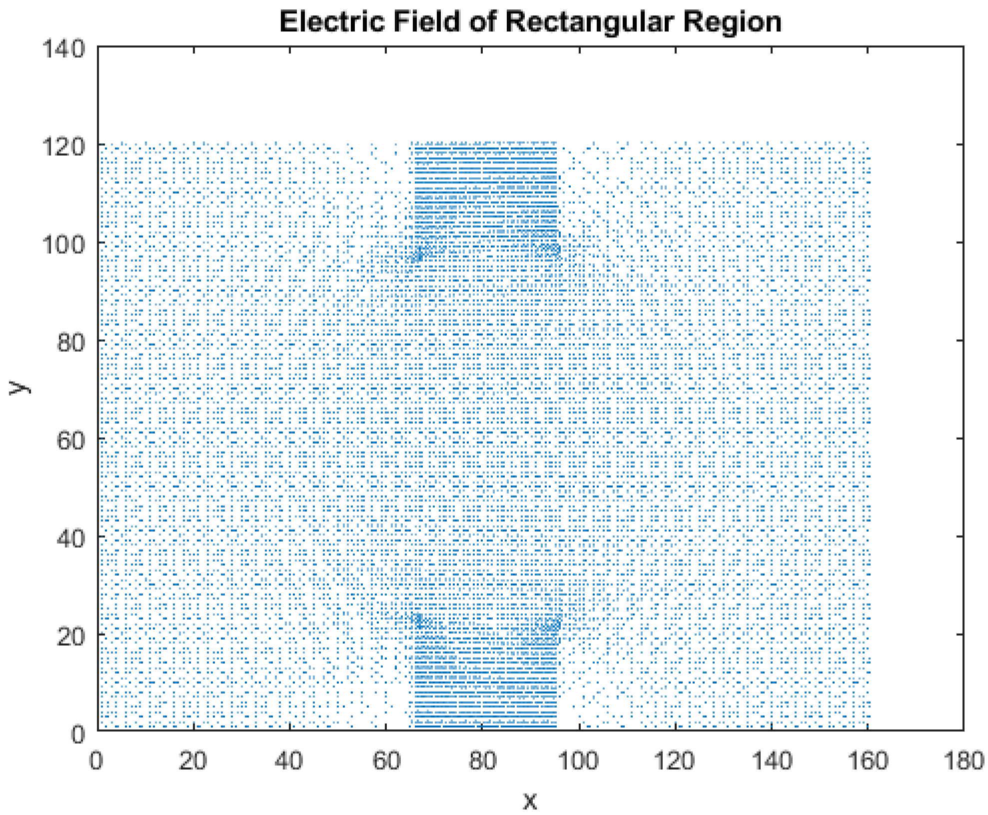
The plot of the electric potential can be seen in the figure below. The left contact is set to V=1, and there is an almost linear decrease up to the right contact, which is at V=0. This linearity is slightly disturbed due to the two bottle-neck regions.

```
figure
surf(1:L,1:W,voltMap)
xlabel('x')
ylabel('y')
zlabel('Voltage')
title('Electric Potential of Grid')
colorbar;
view(2)
```



The electric field can be seen in the figure below. The electric field is strongest in the bottle-neck regions.

```
figure;
x = 1:L;
y=1:W;
quiver(x,y,Ex,Ey);
xlabel('x')
ylabel('y')
title('Electric Field of Rectangular Region')
```



Seen in the figure below is the current flow. It can be seen that the current flows around the bottle-neck regions.

```
figure;
quiver(x,y,Jx,Jy);
xlabel('x')
ylabel('y')
title('Current Flow of Rectangular Region')
```

Part B

The current flow through the grid was calculated for various mesh densities (i.e. mesh sizes). As can be seen in the figure below ("Effect of Mesh Size on Current"), as the size of the mesh increases, the current decreases.

```
elseif(repeat>1 && repeat<7)

    meshSize(repeat-1) = L*W;
    currents_Mesh(repeat-1) = totalCurrent;

    if(repeat==6)
        figure
        plot(meshSize, currents_Mesh)
        xlabel("Mesh Size (L*W)")
        ylabel("Current")
        title("Effect of Mesh Size on Current")
```

```
        end
```

Part C

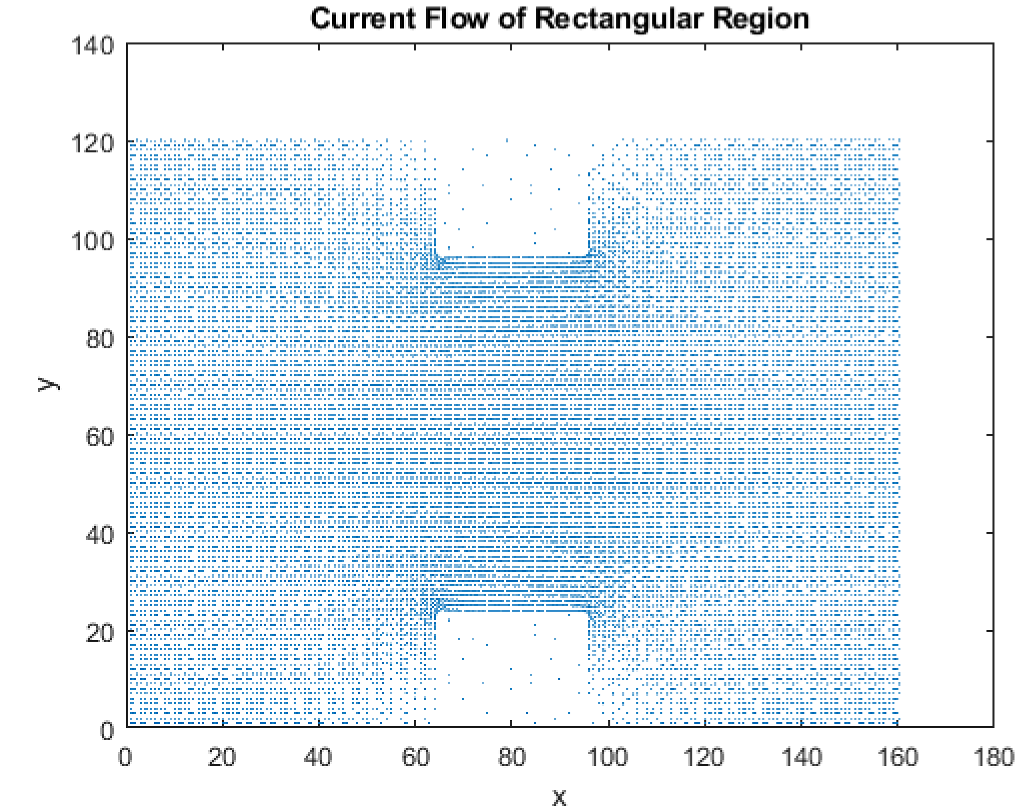
The current flow through the grid was calculated for multiple different bottle-neck sizes. As displayed in the figure below, ("Effect of Bottle-Neck Size on Current") the larger the bottle-neck, the smaller the current.

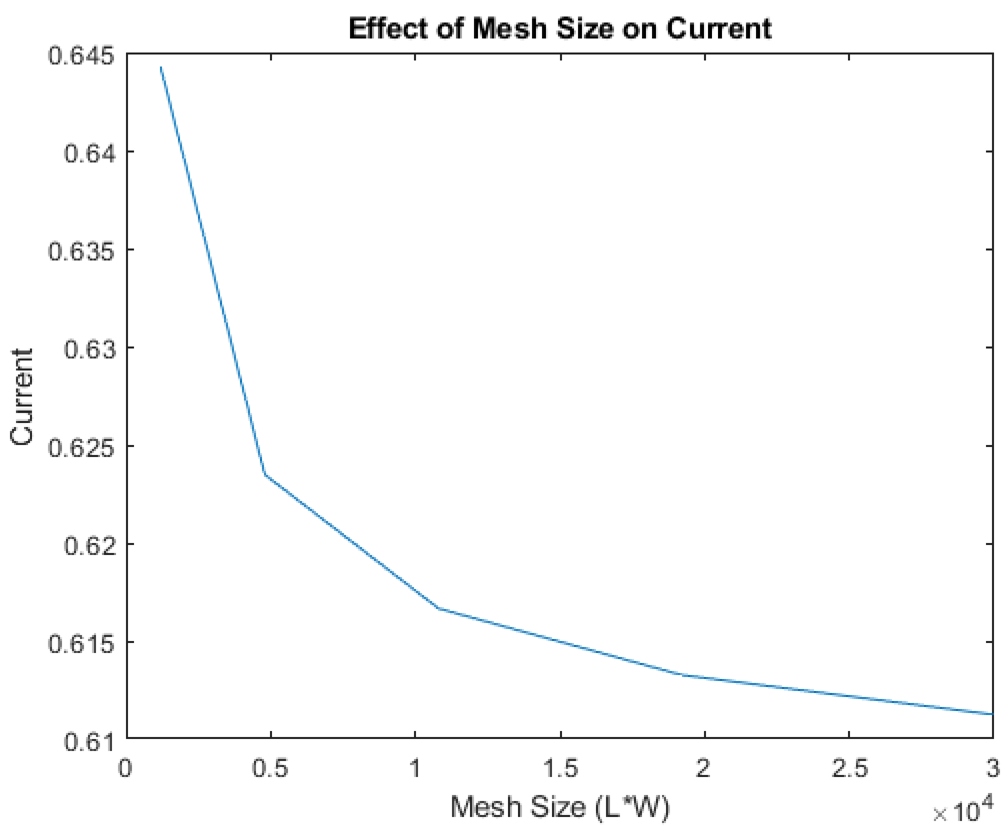
```
    elseif(repeat>6&&repeat<12)

        currents_BN(repeat-6)=totalCurrent;
        bnSize(repeat-6)=Lb*Wb;
        if(repeat==11)
            figure
            plot(bnSize, currents_BN)
            xlabel("Bottle-Neck Size (Lb*Wb)")
            ylabel("Current")
            title("Effect of Bottle-Neck Size on Current")
        end
```

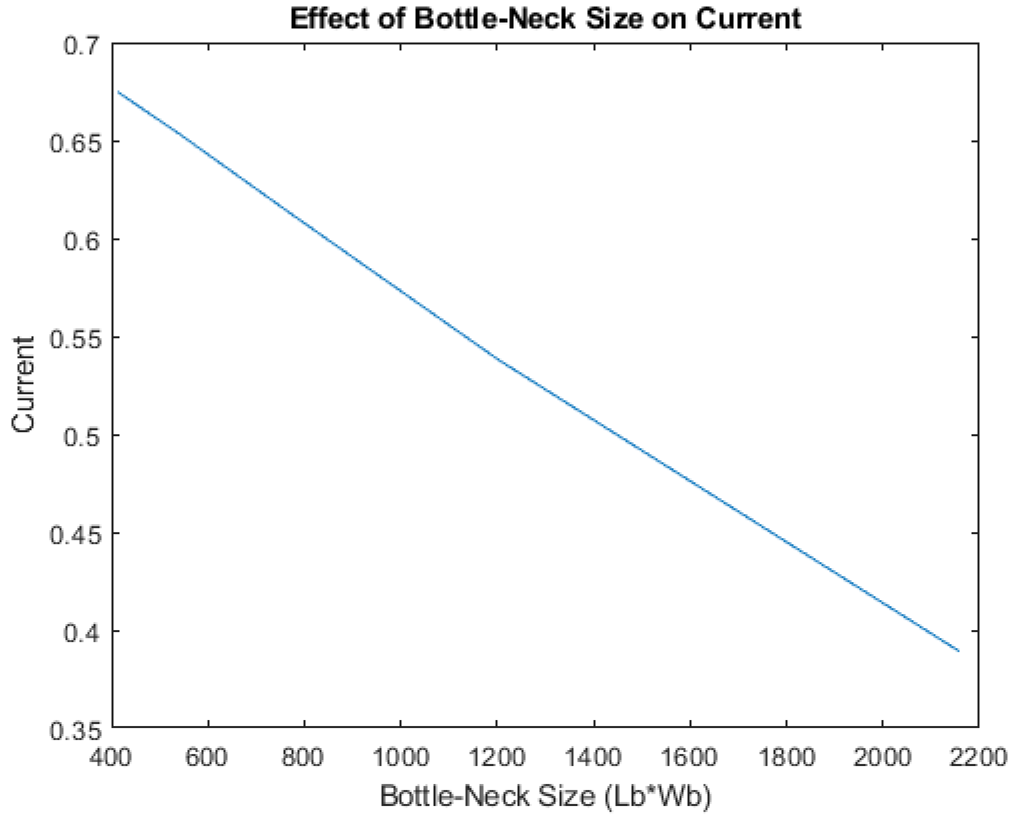
Part D

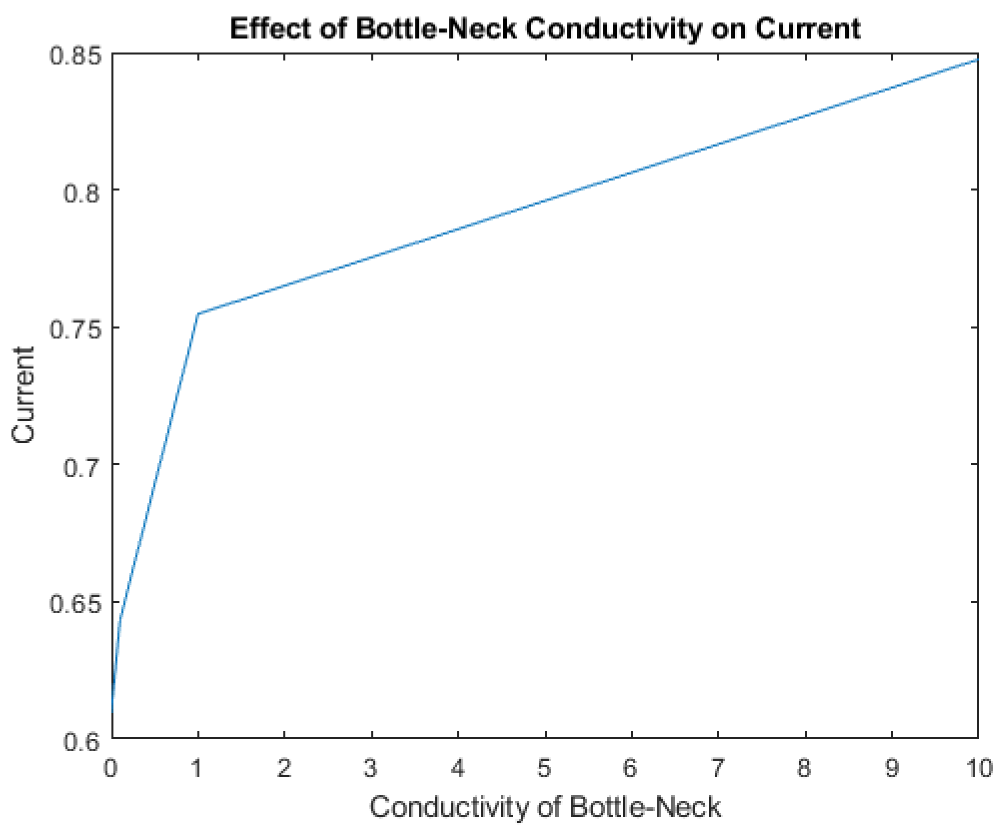
The current flowing through the region was calculated for a number of different bottle-neck conductivities. As shown in the figure below ("Effect of Bottle-Neck Conductivity on Current"), the greater the conductance of the bottle-neck regions, the greater the current.

```
    else
        currents_Cond(repeat-11)=totalCurrent;
        cond(repeat-11)=10^(repeat-15);
        if(repeat==16)
            figure
            plot(cond,currents_Cond);
            xlabel("Conductivity of Bottle-Neck")
            ylabel("Current")
            title("Effect of Bottle-Neck Conductivity on Current")
        end
    end
```


Current Flow of Rectangular Region
y
x
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
180


Effect of Mesh Size on Current
Current
Mesh Size (L*W)
0.645
0.64
0.635
0.63
0.625
0.62
0.615
0.61
0
0.5
1
1.5
2
2.5
3
×10^4


Effect of Bottle-Neck Size on Current
Current
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
400
600
800
1000
1200
1400
1600
1800
2000
2200
Bottle-Neck Size (Lb*Wb)


Effect of Bottle-Neck Conductivity on Current
Current
0.85
0.8
0.75
0.7
0.65
0.6
0
1
2
3
4
5
6
7
8
9
10
Conductivity of Bottle-Neck

```
end
```

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