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# Assignment 3

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## Question 2: Extension of Assignment 2

In this question, a grid is set up, with the left sided boundary set to 1V. Two bottle-necks of conductivities different than the rest of the grid are introduced. Using the maxtrix form of the problem ( $GV=F$ ), the electrostatic potential within the region is found and plotted. Additionally, the conductivity, electric field and currents are plotted as well.

```
% Clear all previous variables, figures, etc, to ensure that the
workspace
% is clean.
clear all
clearvars
clearvars -GLOBAL
close all

for repeat = 1:1

    %Set the length and width of the grid.
    L=160;
    W=120;
    %b) - Vary the size of the grid
    if(repeat>1 && repeat <7)
        L = (repeat-1)*40;
        W = (repeat-1)*30;
    end
    Lb = L/5;
    Wb = W/5;

    %Initialize the G,B and conductivity matrices.
    G = sparse(L*W,L*W);
    B=zeros(L*W,1);
    B(1:W,1)=1;
    condMap = zeros(W,L);

    %Populate the conductivity matrix.
    for lCount = 1:L
        for wCount = 1:W
            if(lCount < L/2 + Lb/2 && lCount > L/2 - Lb/2 &&...
                (wCount > W-Wb || wCount < Wb))
                condMap( wCount,lCount) = 10^-2;
            else
                condMap(wCount,lCount) = 1;
            end
        end
    end
end
```

```

        end
    end
end

% Set the diagonal of the G matrix to 1. This value will be
overwritten
% later if it is not a boundary condition.
for count = 1:L*W
    G(count,count)=1;
end

```

Loop through rows and columns, if not a boundary case, set the gradient based on the sum of adjacent conductivities.

```

for col = 1:L
    if(col~=1 &&col~=L)
        for row = 1:W
            n = row + (col -1)*W;
            if(count~=1 && row ~=1 && row~=W)
                rxBefore = (condMap(row,col) +
condMap(row,col-1))/2.0;
                rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
                ryBefore = (condMap(row,col) +
condMap(row-1,col))/2.0;
                ryAfter = (condMap(row,col) + condMap(row
+1,col))/2.0;

                nyBefore = n-1;
                nyAfter = n+1;
                nxBefore = row+(col-2)*W;
                nxAfter = row+col*W;
                G(n,n) = -(rxBefore+rxAfter+ryBefore+ryAfter);
                G(n, nyBefore) =ryBefore;
                G(n, nyAfter)=ryAfter;
                G(n, nxBefore)=rxBefore;
                G(n, nxAfter) =rxAfter;

            elseif(row==1)
                %Special Case: Bottom of Grid
                rxBefore = (condMap(row,col) +
condMap(row,col-1))/2.0;
                rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
                ryAfter = (condMap(row,col) + condMap(row
+1,col))/2.0;

                nyAfter = n+1;
                nxBefore = row+(col-2)*W;
                nxAfter = row+col*W;
                G(n,n) = -(rxBefore+rxAfter+ryAfter);
                G(n, nyAfter)=ryAfter;
                G(n, nxBefore)=rxBefore;
                G(n, nxAfter) =rxAfter;
            end
        end
    end
end

```

```

elseif(row==W)
    %Special Case: Top of Grid
    rxBefore = (condMap(row,col) +
condMap(row,col-1))/2.0;
    rxAfter = (condMap(row,col) + condMap(row,col
+1))/2.0;
    ryBefore = (condMap(row,col) +
condMap(row-1,col))/2.0;

    nyBefore = n-1;
    nxBefore = row+(col-2)*W;
    nxAfter = row+col*W;
    G(n,n) = -(rxBefore+rxAfter+ryBefore);
    G(n, nyBefore) =ryBefore;
    G(n, nxBefore)=rxBefore;
    G(n, nxAfter) =rxAfter;
end
end
end
end
V=G\B;

%Map the voltage to original grid.
voltMap = zeros(W,L);
for cols = 1:L
    for rows = 1:W
        n= rows+(cols-1)*W;
        voltMap(rows,cols)=V(n);
    end
end
end

```

Find the Electric field knowing  $E = -\nabla V$

```

[Ex, Ey]=gradient(voltMap);
Ex=-Ex;
Ey=-Ey;

```

Find the current density knowing  $J = \sigma E$

```

Jx = condMap.*Ex;
Jy = condMap.*Ey;

```

```

%Sum the currents at both contacts (edges), take the average to
find
%the total.
current1 = sum(Jx(:,1));
current2 = sum(Jx(:,L));
totalCurrent = (current1+current2)/2;

```

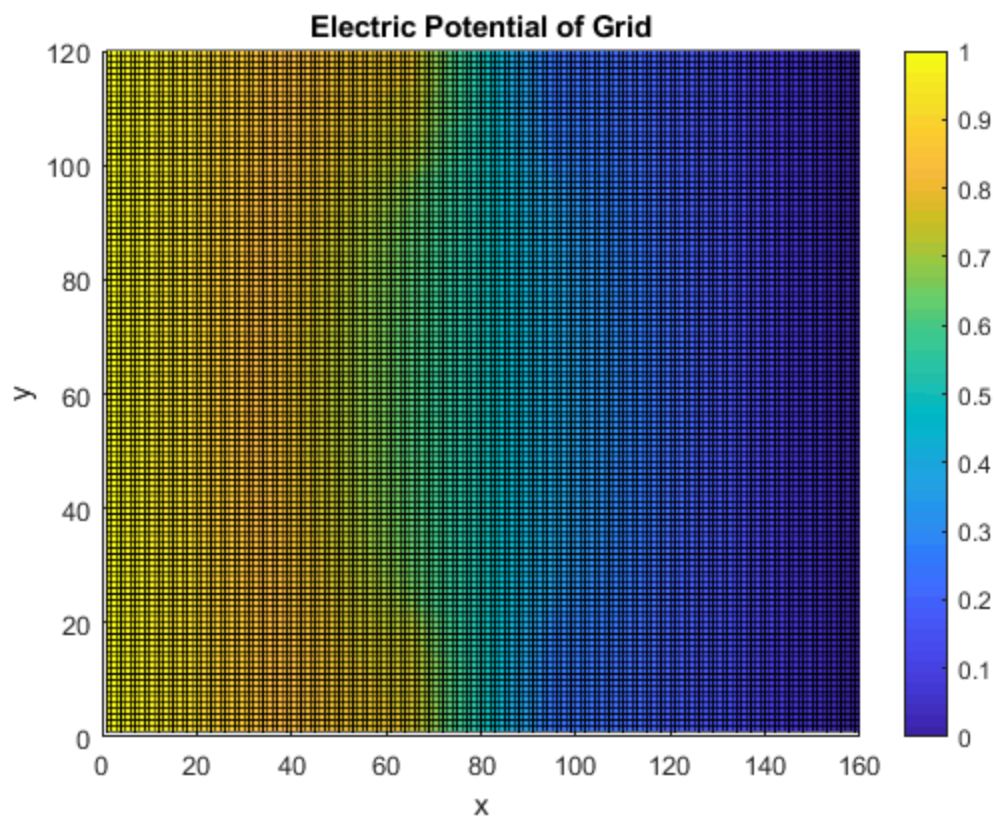
## Part A

As calculated above, the total current through the contacts for  $L=160$ ,  $W=120$ ,  $L_b = 32$  and  $W_b = 24$  is 0.6235. Additionally, there is no observed difference between the currents at each contact.

```
if(repeat==1)
```

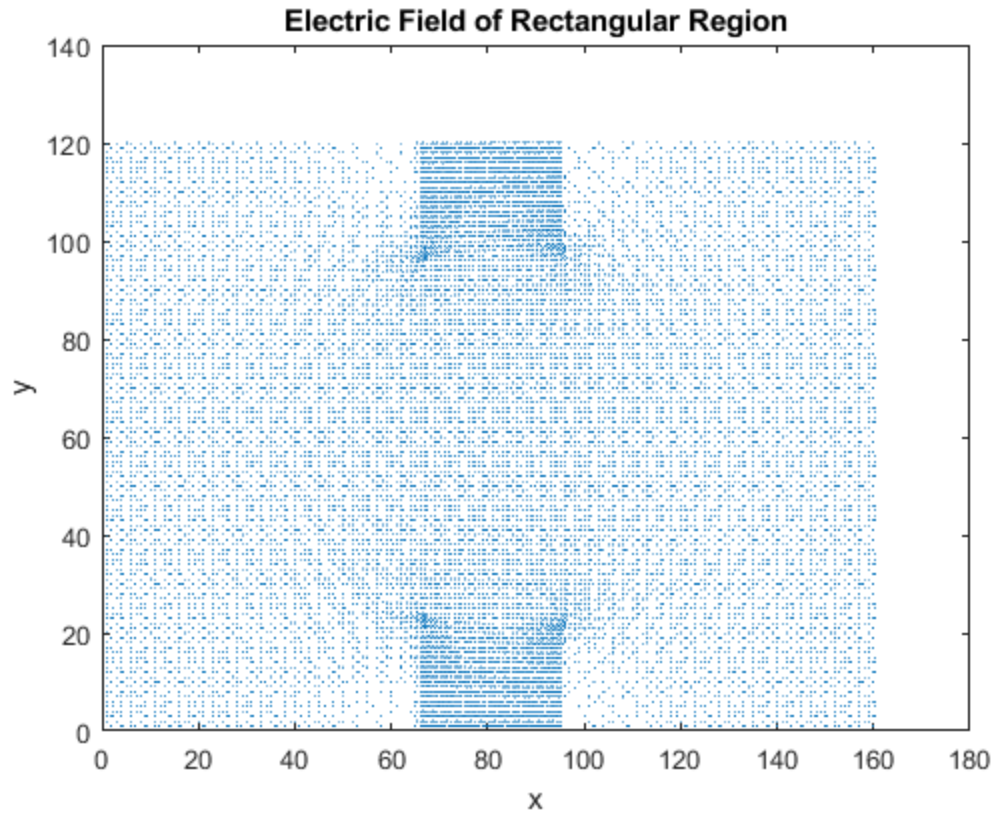
The plot of the electric potential can be seen in the figure below. The left contact is set to  $V=1$ , and there is an almost linear decrease up to the right contact, which is at  $V=0$ . This linearity is slightly disturbed due to the two bottle-neck regions.

```
figure
surf(1:L,1:W,voltMap)
xlabel('x')
ylabel('y')
zlabel('Voltage')
title('Electric Potential of Grid')
colorbar;
view(2)
```



The electric field can be seen in the figure below. The electric field is strongest in the bottle-neck regions.

```
figure;
x = 1:L;
y=1:W;
quiver(x,y,Ex,Ey);
xlabel('x')
ylabel('y')
title('Electric Field of Rectangular Region')
```



end

end

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