Time Series HW3

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March 2021

Problem 1

a. Correlation matrix

For this problem, I use the following properties:

$$corr(X_i, X_j) = \lambda_{i1} * \lambda_{j1} + \lambda_{i2} * \lambda_{j2} ... + \lambda_{im} * \lambda_{jm}$$

$$corr(X_i, X_i) = 1$$

$$corr(X_j, X_i) = corr(X_i, X_j)$$

Let R_{3x3} stand for the correlation matrix.

$$\begin{aligned}
R_{3x3} &= \begin{bmatrix} corr(X_1, X_1) & corr(X_1, X_2) & corr(X_1, X_3) \\ corr(X_2, X_1) & corr(X_2, X_2) & corr(X_2, X_3) \\ corr(X_3, X_1) & corr(X_3, X_2) & corr(X_3, X_3) \end{bmatrix} \\
corr(X_1, X_2) &= \lambda_{X_1 1} * \lambda_{X_2 1} + \lambda_{X_1 2} * \lambda_{X_2 2} \\
&= 0.8 * 0.6 + 0.4 * 0.6 \\
&= 0.72 \\
corr(X_1, X_3) &= 0.8 * 04 + 0.8 * 0.4 \\
&= 0.64 \\
corr(X_2, X_3) &= 0.6 * 0.4 + 0.6 * 0.8 \\
&= 0.72 \\
\boldsymbol{R}_{3x3} &= \begin{bmatrix} 1 & 0.72 & 0.64 \\ 0.72 & 1 & 0.72 \\ 0.64 & 0.72 & 1 \end{bmatrix}
\end{aligned}$$

b. Communalities

Communality is simply the λs squared. Let C_{X_i} stand for the communality of X_i

$$\begin{aligned}
\boldsymbol{C_{X_1}} &= 0.8^2 + 0.4^2 \\
&= 0.64 + 0.16 \\
&= \mathbf{0.80} \\
\boldsymbol{C_{X_2}} &= 0.6^2 + 0.6^2 \\
&= \mathbf{0.72} \\
\boldsymbol{C_{X_3}} &= 0.4^2 + 0.8^2 \\
&= \mathbf{0.80}
\end{aligned}$$

Problem 2

a.

For variance calculations, each factor's variance is simply λ_{ij}^2 where i signifies which manifest variable it belongs to and j represents which factor it belongs to. To make things clearer, Let F_j stand for each factor. The total variance is the sum of the communalities: 2.32.

$$Var(X_i) = \lambda_{i1}^2 + \lambda_{i2}^2 + ...\lambda_{ip}^2$$

The variance explained by the factors for X_1 taken proportionally are:

$$\begin{aligned}
\boldsymbol{F_1} &= \frac{0.80^2}{2.32} \\
&\approx \mathbf{27.59}\% \\
\boldsymbol{F_2} &= \frac{0.4^2}{2.32} \\
&\approx \mathbf{6.90}\%
\end{aligned}$$

For X_2:

$$\begin{aligned}
\boldsymbol{F_1} &= \frac{0.60^2}{2.32} \\
&\approx \mathbf{15.52}\% \\
\boldsymbol{F_2} &= \frac{0.6^2}{2.32} \\
&\approx \mathbf{15.52}\%
\end{aligned}$$

For X_3:

$$\begin{aligned}
\boldsymbol{F_1} &= \frac{040^2}{2.32} \\
&\approx \mathbf{6.90}\% \\
\boldsymbol{F_2} &= \frac{0.8^2}{2.32} \\
&\approx \mathbf{27.59}\%
\end{aligned}$$

I would just like to note that since these are approximations, the proportions do not perfectly add up to 100% but they are still how much each factor explains the total variance.

b.

This is not equivalent to PCA. (come back to this later)

Problem 3

a.

$$\begin{aligned}
\boldsymbol{F}* &= FM \\
&= \begin{bmatrix} 0.8 & 0.4 \\ 0.6 & 0.6 \\ 0.4 & 0.8 \end{bmatrix} \begin{bmatrix} 0.7071 & 0.7071 \\ 0.7071 & -0.7071 \end{bmatrix} \\
&= \begin{bmatrix} 0.8 * 0.7071 + 0.4 * 0.7071 & 0.8 * 0.7071 + 0.4 * -0.7071 \\ 0.6 * 0.7071 + 0.6 * 0.7071 & 0.6 * 0.7071 + 0.6 * -0.7071 \\ 0.8 * 0.7071 + 0.4 * 0.7071 & 0.4 * 0.7071 + 0.8 * -0.7071 \end{bmatrix} \\
&= \begin{bmatrix} 0.84852 & 0.28284 \\ 0.84852 & 0 \\ 0.84852 & -0.28284 \end{bmatrix}
\end{aligned}$$

When calculating the relevant scores/correlations:

$$\begin{aligned}
corr(X_1, X_2) &= 0.84852 * 0.84852 + 0.28284 * 0 \\
&\approx 0.72 \\
corr(X_1, X_3) &= 0.84852 * 0.84852 + 0.28284 * -0.28284 \\
&\approx 0.64 \\
corr(X_2, X_3) &= 0.84852 * 0.84852 + 0 * -0.28284 \\
&\approx 0.72
\end{aligned}$$

While there are some rounding errors, the correlation matrix for F* is equal to the correlation matrix for F.

b. Communalities

Again let C_{X_i} stand for the communality of X_i

$$\begin{aligned}
\boldsymbol{C_{X_1}} &= 0.84852^2 + 0.28284^2 \\
&\approx \mathbf{0.80} \\
\boldsymbol{C_{X_2}} &= 0.85^2 + 0 \\
&\approx \mathbf{0.72} \\
\boldsymbol{C_{X_3}} &= 0.84852^2 + (-0.28284)^2 \\
&\approx \mathbf{0.80}
\end{aligned}$$

Again, there are slight rounding errors that make this an $\approx$ but they are essentially equivalent. The reason as to why communality is preserved is because the transformation is orthonormal. The information of the factors is preserved in such a transformation. It's essentially a reflection that contains the same information as before the reflection.

Problem 4

a.

For these constants to be equivalent, the main 3 correlations that have to be equivalent are: $corr(X_1, X_2)$,$corr(X_1, X_3)$,$corr(X_2, X_3)$. THis gives the following system of equations:

$$\begin{aligned}
a * b &= 0.72 \\
a * c &= 0.64 \\
b * c &= 0.72
\end{aligned}$$

Solving this system goes as follows:

$$\begin{aligned}
a * b &= b * c \\
a &= c \\
c * c &= 0.64 \\
\boldsymbol{c} &= \mathbf{0.8} \\
b * 0.8 &= 0.72 \\
\boldsymbol{b} &= \mathbf{0.9} \\
\boldsymbol{a} &= \mathbf{0.8}
\end{aligned}$$

As seen here, it is possible to end up with the same correlation matrix in a one factor model as the two factor model.

b.

The main implication here is that the correlation matrix for a factor analysis is not unique so one should not just base their judgement solely off of correlations.

Problem 5

a. SAS code

```
* Create the data based on the text file;
data WORK.EvaluateSupervisors;
  input OVERALL BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
cards;
43 51 30 39 61 92 45
63 64 51 54 63 73 47
71 70 68 69 76 86 48
61 63 45 47 54 84 35
81 78 56 66 71 83 47
43 55 49 44 54 49 34
58 67 42 56 66 68 35
71 75 50 55 70 66 41
72 82 72 67 71 83 31
67 61 45 47 62 80 41
64 53 53 58 58 67 34
67 60 47 39 59 74 41
69 62 57 42 55 63 25
68 83 83 45 59 77 35
77 77 54 72 79 77 46
81 90 50 72 60 54 36
74 85 64 69 79 79 63
65 60 65 75 55 80 60
65 70 46 57 75 85 46
50 58 68 54 64 78 52
50 40 33 34 43 64 33
64 61 52 62 66 80 41
53 66 52 50 63 80 37
40 37 42 58 50 57 49
63 54 42 48 66 75 33
66 77 66 63 88 76 72
78 75 58 74 80 78 49
48 57 44 45 51 83 38
85 85 71 71 77 74 55
```

```
82 82 39 59 64 78 39
RUN;

*Problem 5;
PROC FACTOR DATA = WORK.EvaluateSupervisors METHOD = principal PRIORS = ONE NFACTORS = 6 MIN
VAR BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
TITLE "PC style Factor Analysis"
RUN;
```

b.

PC style Factor Analysis RUN

The FACTOR Procedure
Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 6 Average = 1				
	Eigenvalue	Difference	Proportion	Cumulative
1	3.16922321	2.16287646	0.5282	0.5282
2	1.00634675	0.24343802	0.1677	0.6959
3	0.76290873	0.21039227	0.1272	0.8231
4	0.55251646	0.23526997	0.0921	0.9152
5	0.31724648	0.12548811	0.0529	0.9680
6	0.19175838		0.0320	1.0000

6 factors will be retained by the NFACTOR criterion.

Factor Pattern						
	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6
BEEFS	0.78219	-0.31363	0.38883	-0.23490	-0.10787	0.26797
PRIVILEGE	0.70268	-0.30973	0.18990	0.60569	-0.02123	-0.08333
NEWLEARN	0.82140	-0.21777	-0.23756	-0.16709	0.43688	-0.05153
RAISES	0.87704	0.11590	0.00490	-0.27139	-0.25930	-0.27649
CRITICAL	0.40022	0.80479	0.39938	0.07429	0.16271	0.02533
ADVANCE	0.67791	0.32172	-0.59975	0.15293	-0.14347	0.18237

For retaining factors, I am going with the retention rule of the eigenvalue being greater than 1. As such, I would return **factors 1 and 2.**

c.

Looking at the factor pattern, it seems like factor 1 tends to retain high weights on everything except for critical. This implies that this factor is associated with supervisors who handle employee complaints well, play favorites, encourage learning, raise based on merit, and provide a pathway for employees to better jobs. However, the factor is not associated with critical supervisors. With the exception of favoritism, I would say that factor 1 is a good indicator of a good supervisor.

As for factor 2, it seems like the opposite is true. This factor appears to capture critical supervisors and discounts the other variables. As such, I would take factor 2 as an indicator of what makes a bad supervisor.

Problem 6

Using a linear regression model and the following SAS code:

```
*Problem 6;
PROC REG DATA=prinfactors5;
model OVERALL = Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 /ss1 ss2;
RUN;
```

The REG Procedure
Model: MODEL1
Dependent Variable: OVERALL

Number of Observations Read	30
Number of Observations Used	30

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	6	3147.96634	524.66106	10.50	<.0001
Error	23	1149.00032	49.95654		
Corrected Total	29	4296.96667			

Root MSE	7.06799	R-Square	0.7326
Dependent Mean	64.63333	Adj R-Sq	0.6628
Coeff Var	10.93552		

Parameter Estimates							
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Type I SS	Type II SS
Intercept	1	64.63333	1.29043	50.09	<.0001	125324	125324
Factor1	1	8.22970	1.31249	6.27	<.0001	1964.11131	1964.11131
Factor2	1	-3.41693	1.31249	-2.60	0.0159	338.58649	338.58649
Factor3	1	3.60689	1.31249	2.75	0.0115	377.27898	377.27898
Factor4	1	-3.63138	1.31249	-2.77	0.0110	382.41999	382.41999
Factor5	1	0.94266	1.31249	0.72	0.4799	25.76965	25.76965
Factor6	1	1.43599	1.31249	1.09	0.2852	59.79991	59.79991

From the linear regression model, we can see through the R^2 score that the factors collectively explain about 73.26% of the variation.

As for the individual factors, that would simply be their squared error over the total sum of square errors. Let F_i stand for factor i.

$$\begin{aligned} F_1 &= \frac{1964.11131}{4296.96667} \\ &\approx 45.71\% \\ F_2 &\approx 7.88\% \\ F_3 &\approx 8.78\% \\ F_4 &\approx 8.90\% \\ F_5 &\approx 0.60\% \\ F_6 &\approx 1.39\% \end{aligned}$$

One last thing to note here is that factors 5 and 6 are insignificant due to having rather high P-values.

Problem 7

a.

```
*Problem 7;
PROC FACTOR DATA = WORK.EvaluateSupervisors METHOD = principal PRIORS = SMC
NFACTORS = 6 MINEIGEN = 0 OUT = prinfactors7;
VAR BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
TITLE "PC style Factor Analysis SMC"
RUN;
```

b.

PC style Factor Analysis SMC RUN

The FACTOR Procedure
Initial Factor Method: Principal Factors

Prior Communality Estimates: SMC					
BEEFS	PRIVILEGE	NEWLEARN	RAISES	CRITICAL	ADVANCE
0.62505535	0.37534801	0.55967371	0.67513759	0.18574001	0.48759750

Eigenvalues of the Reduced Correlation Matrix: Total = 2.90855217 Average = 0.48475869				
	Eigenvalue	Difference	Proportion	Cumulative
1	2.71771612	2.31678131	0.9344	0.9344
2	0.40093481	0.22202808	0.1378	1.0722
3	0.17890672	0.18294128	0.0615	1.1337
4	-.00403456	0.16150739	-0.0014	1.1324
5	-.16554195	0.05388702	-0.0569	1.0754
6	-.21942897		-0.0754	1.0000

3 factors will be retained by the MINEIGEN criterion.

Factor Pattern			
	Factor1	Factor2	Factor3
BEEFS	0.74755	-0.36273	0.12483
PRIVILEGE	0.61091	-0.17725	-0.06404
NEWLEARN	0.76629	-0.05146	-0.21483
RAISES	0.84947	0.11042	0.13720
CRITICAL	0.32091	0.25308	0.26760
ADVANCE	0.61147	0.39882	-0.15045

Looking at the output, factor 1 appears to have almost the same interpretation as the previous factor 1 interpretation in problem 5. However, there exists a higher weighting towards raises and a lower weighting towards critical. This implies that the factor captures what employees consider to be easy to work for supervisors.

Factor 2 appears to retains a similar interpretation as last time as well. However, the critical rating is significantly lower and the advance is higher. It appears that this factor then captures supervisors who may be hard to deal with but are open towards rewarding merit based work and provide company advancement.

c.

In question 5, the factors could be interpreted as an indicator of a good or bad supervisor. For question 7, the factors are a bit more nuanced but generally suggest a good or bad supervisor. Additionally, question 7's factors explain a higher proportion of the variation in its correlation matrix.

Problem 8

a.

```
*Problem 8;
PROC FACTOR DATA = WORK.EvaluateSupervisors METHOD = principal PRIORS = SMC
NFACTORS = 2 ROTATE = varimax
OUT = WORK.EvaluateSupervisors_scores;
VAR BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
TITLE "PC style Factor Analysis Verimax"
RUN;
```

b.

PC style Factor Analysis Verimax RUN

The FACTOR Procedure
Initial Factor Method: Principal Factors

Prior Communality Estimates: SMC					
BEEFS	PRIVILEGE	NEWLEARN	RAISES	CRITICAL	ADVANCE
0.62505535	0.37534801	0.55967371	0.67513759	0.18574001	0.48759750

Eigenvalues of the Reduced Correlation Matrix: Total = 2.90855217 Average = 0.48475869				
	Eigenvalue	Difference	Proportion	Cumulative
1	2.71771612	2.31678131	0.9344	0.9344
2	0.40093481	0.22202808	0.1378	1.0722
3	0.17890672	0.18294128	0.0615	1.1337
4	-.00403456	0.16150739	-0.0014	1.1324
5	-.16554195	0.05388702	-0.0569	1.0754
6	-.21942897		-0.0754	1.0000

2 factors will be retained by the NFACTOR criterion.

Factor Pattern		
	Factor1	Factor2
BEEFS	0.74755	-0.36273
PRIVILEGE	0.61091	-0.17725
NEWLEARN	0.76629	-0.05146
RAISES	0.84947	0.11042
CRITICAL	0.32091	0.25308
ADVANCE	0.61147	0.39882

Yes, the two factors here are the same as in question 7.

c

For verifying orthonormality, the magnitude of each vector in a transformation matrix must be 1 which signifies normality and the dot product must be 0 which signifies orthogonality.

PC style Factor Analysis Verimax RUN

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix		
	1	2
1	0.79912	0.60117
2	-0.60117	0.79912

Let V_1 be the first vector and V_2 be the second vector. The magnitudes of the vectors are:

$$
\begin{aligned}
\|V_1\| &= (0.79912)^2 + (-0.60117)^2 \\
&= 1 \\
\|V_2\| &= (0.60117)^2 + (0.79912)^2 \\
&= 1
\end{aligned}
$$

While it should be self evident from just glancing at it the dot product is as follows:

$$V_1 \ldots V_2 = 0.79912 * 0.60117 - 0.60117 * 0.79912 = 0$$

Thus, both orthogonality and normality are satisfied for the transformation matrix which makes it orthonormal.

d

PC style Factor Analysis Verimax RUN

The FACTOR Procedure
Rotation Method: Varimax

Orthogonal Transformation Matrix		
	1	2
1	0.79912	0.60117
2	-0.60117	0.79912

Rotated Factor Pattern		
	Factor1	Factor2
BEEFS	0.81544	0.15954
PRIVILEGE	0.59475	0.22561
NEWLEARN	0.64329	0.41955
RAISES	0.61245	0.59891
CRITICAL	0.10431	0.39516
ADVANCE	0.24889	0.68630

Variance Explained by Each Factor	
Factor1	Factor2
1.8804192	1.2382317

Looking at the rotated factor pattern, it seems like factor yet again has high weights on everything except for being critical. However, this time handling employee complaints plays a bigger role in the meaning of the factor. Thus, I see factor 1 as a measurement of good supervisors who can handle complaints well and are not too critical on their employees.

For factor 2, it seems like it puts more emphasis on supervisors who provide new opportunities and raise based on merit. I would consider this factor to indicate supervisors that you can go to if you are looking to advance your career.

Problem 9

The SAS code and output are as follows:

```
*Problem 9;
PROC CORR DATA = Work.evaluatesupervisors_scores;
RUN;
```

The CORR Procedure

9 Variables:	OVERALL BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE Factor1 Factor2

Simple Statistics						
Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
OVERALL	30	64.63333	12.17256	1939	40.00000	85.00000
BEEFS	30	66.60000	13.31476	1998	37.00000	90.00000
PRIVILEGE	30	53.13333	12.23543	1594	30.00000	83.00000
NEWLEARN	30	56.36667	11.73701	1691	34.00000	75.00000
RAISES	30	64.63333	10.39723	1939	43.00000	88.00000
CRITICAL	30	74.76667	9.89491	2243	49.00000	92.00000
ADVANCE	30	42.93333	10.28871	1288	25.00000	72.00000
Factor1	30	0	0.85285	0	-1.86118	1.36525
Factor2	30	0	0.77645	0	-1.31002	2.01667

Pearson Correlation Coefficients, N = 30 Prob > |r| under H0: Rho=0									
	OVERALL	BEEFS	PRIVILEGE	NEWLEARN	RAISES	CRITICAL	ADVANCE	Factor1	Factor2
OVERALL	1.00000	0.82542 <.0001	0.42612 0.0189	0.62368 0.0002	0.59014 0.0006	0.15644 0.4091	0.15509 0.4132	0.82349 <.0001	0.19094 0.3122
BEEFS	0.82542 <.0001	1.00000	0.55829 0.0013	0.59674 0.0005	0.66920 <.0001	0.18771 0.3205	0.22458 0.2328	0.95614 <.0001	0.20547 0.2760
PRIVILEGE	0.42612 0.0189	0.55829 0.0013	1.00000	0.49333 0.0056	0.44548 0.0136	0.14723 0.4375	0.34329 0.0633	0.69736 <.0001	0.29057 0.1193
NEWLEARN	0.62368 0.0002	0.59674 0.0005	0.49333 0.0056	1.00000	0.64031 0.0001	0.11597 0.5417	0.53162 0.0025	0.75429 <.0001	0.54034 0.0021
RAISES	0.59014 0.0006	0.66920 <.0001	0.44548 0.0136	0.64031 0.0001	1.00000	0.37688 0.0401	0.57419 0.0009	0.71812 <.0001	0.77135 <.0001
CRITICAL	0.15644 0.4091	0.18771 0.3205	0.14723 0.4375	0.11597 0.5417	0.37688 0.0401	1.00000	0.28334 0.1292	0.12230 0.5197	0.50894 0.0041
ADVANCE	0.15509 0.4132	0.22458 0.2328	0.34329 0.0633	0.53162 0.0025	0.57419 0.0009	0.28334 0.1292	1.00000	0.29183 0.1176	0.88390 <.0001
Factor1	0.82349 <.0001	0.95614 <.0001	0.69736 <.0001	0.75429 <.0001	0.71812 <.0001	0.12230 0.5197	0.29183 0.1176	1.00000	0.29041 0.1195
Factor2	0.19094 0.3122	0.20547 0.2760	0.29057 0.1193	0.54034 0.0021	0.77135 <.0001	0.50894 0.0041	0.88390 <.0001	0.29041 0.1195	1.00000

a

Looking at the summary statistics, the means for the factors matches the standardized assumption in that they are both 0. However, the standard deviation for both factors are not 1 which violates our assumption of standardization. Additionally when looking at the Pearson correlation coefficient matrix, we can see that the two factors are correlated with each other which also violates another assumption.

b

From the SAS output, The first supervisor has $F_1 = -1.462881996$ and $F_2 = 0.4556968621$. Next, I had to standardize the data via:

```
*Problem 9 b.;
Proc STDIZE Data = Work.evaluatesupervisors_scores out = evaluatesupervisors_scores_STD;
Var BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
RUN;
```

Then referencing the standardized scores from the code in 8 and the standardized observations, I did the following calculations in excel. First, I elect to show the formulas. Then I show the results.

	A	B	C	D	E	F	G	H
1	Standardized Scoring Coefficients							
2		Factor1	Factor2		STD Observations			
3	BEEFS	0.55187	-0.32385		-1.171632332		=B3*E3	=E3*C3
4	PRIVILEGE	0.17401	-0.00598		-1.890684132		=B4*E4	=E4*C4
5	NEWLEARN	0.21916	0.07679		-1.479649622		=B5*E5	=E5*C5
6	RAISES	0.10168	0.48155		-0.3494522		=B6*E6	=E6*C6
7	CRITICAL	-0.06259	0.15298		1.7416366195		=B7*E7	=E7*C7
8	ADVANCE	-0.09195	0.40041		0.2008675013		=B8*E8	=E8*C8
9							=SUM(G3:G8)	=SUM(H3:H8)

	A	B	C	D	E	F	G	H
1	Standardized Scoring Coefficients							
2		Factor1	Factor2		STD Observations			
3	BEEFS	0.55187	-0.32385		-1.171632		-0.64659	0.379433
4	PRIVILEGE	0.17401	-0.00598		-1.890684		-0.329	0.011306
5	NEWLEARN	0.21916	0.07679		-1.47965		-0.32428	-0.11362
6	RAISES	0.10168	0.48155		-0.349452		-0.03553	-0.16828
7	CRITICAL	-0.06259	0.15298		1.7416366		-0.10901	0.266436
8	ADVANCE	-0.09195	0.40041		0.2008675		-0.01847	0.080429
9							-1.46288	0.455703

As seen from the excel calculation, the SAS calculated scores more or less match with the manually calculated ones with minute differences mostly due to rounding.

Problem 10

The SAS code is as follows

```
*Problem 10;
PROC FACTOR DATA = WORK.Evaluatesupervisors METHOD = ML PRIORS = SMC HEYWOOD;
VAR BEEFS PRIVILEGE NEWLEARN RAISES CRITICAL ADVANCE;
RUN;
```

The hypothesis test results are:

Significance Tests Based on 30 Observations			
Test	DF	Chi-Square	Pr > ChiSq
H0: No common factors	15	65.5127	<.0001
HA: At least one common factor			
H0: 2 Factors are sufficient	4	2.9161	0.5720
HA: More factors are needed			

a

For the first test, we can see that the p-value for the chi-square test is very low meaning that we can reject the null hypothesis of there being no common factors. Thus, it provides support that common factors exist.

b

For the second test, the chi-square p-value is rather large implying that we cannot reject the null hypothesis. This provides support that the default number of factors extracted by SAS was not sufficient.