先端データ解析論　第 8 回レポート

電子情報学専攻　48-176403 石毛真修

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$\boldsymbol{\mu}$ で偏微分すると，

$$\frac{\partial J}{\partial \boldsymbol{\mu}} = \frac{\partial}{\partial \boldsymbol{\mu}} \left(\max(0, 1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y)\right)^2 + \gamma \frac{\partial}{\partial \boldsymbol{\mu}} (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})^\top \tilde{\boldsymbol{\Sigma}}^{-1} (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})$$

右辺の第 1 項は，$1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y > 0$ のとき，

$$\begin{aligned}
\frac{\partial}{\partial \boldsymbol{\mu}} \left(\max(0, 1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y)\right)^2 &= \frac{\partial}{\partial \boldsymbol{\mu}} \left(1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y\right)^2 \\
&= \frac{\partial}{\partial \boldsymbol{\mu}} \left(1 - 2\boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y + y^2 \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x}) \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})\right) \\
&= \frac{\partial}{\partial \boldsymbol{\mu}} \left(1 - 2y\boldsymbol{\phi}(\boldsymbol{x})^\top \boldsymbol{\mu} + y^2 \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x}) \boldsymbol{\phi}(\boldsymbol{x})^\top \boldsymbol{\mu}\right) \\
&= -2y\boldsymbol{\phi}(\boldsymbol{x}) + 2y^2 \boldsymbol{\phi}(\boldsymbol{x}) \boldsymbol{\phi}(\boldsymbol{x})^\top \boldsymbol{\mu} \\
&= -2y\boldsymbol{\phi}(\boldsymbol{x}) \left(1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y\right)
\end{aligned}$$

$1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y < 0$ のとき，

$$\frac{\partial}{\partial \boldsymbol{\mu}} \left(\max(0, 1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y)\right)^2 = \boldsymbol{0}$$

これらをまとめて，

$$\frac{\partial}{\partial \boldsymbol{\mu}} \left(\max(0, 1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y)\right)^2 = -2y\boldsymbol{\phi}(\boldsymbol{x}) \max\left(0, 1 - \boldsymbol{\mu}^\top \boldsymbol{\phi}(\boldsymbol{x})y\right)$$

右辺の第 2 項は，

$$\gamma \frac{\partial}{\partial \boldsymbol{\mu}} (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})^\top \tilde{\boldsymbol{\Sigma}}^{-1} (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}}) \quad = \gamma \left(\tilde{\boldsymbol{\Sigma}}^{-1} + (\tilde{\boldsymbol{\Sigma}}^{-1})^\top\right) (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}}) = 2\gamma \tilde{\boldsymbol{\Sigma}}^{-1} (\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}})$$

第 1 項と第 2 項の式をまとめて，$\dfrac{\partial J}{\partial \boldsymbol{\mu}} = \mathbf{0}$ と置くと，

$$y\boldsymbol{\phi}(\boldsymbol{x}) \max\left(0, 1 - \hat{\boldsymbol{\mu}}^\top \boldsymbol{\phi}(\boldsymbol{x}) y\right) = \gamma \tilde{\boldsymbol{\Sigma}}^{-1} (\hat{\boldsymbol{\mu}} - \tilde{\boldsymbol{\mu}})$$

が成り立つ．

これを $\hat{\boldsymbol{\mu}}$ について解くと，$1 - \hat{\boldsymbol{\mu}}^\top \boldsymbol{\phi}(\boldsymbol{x}) y > 0$ のときは

$$\hat{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}} + \frac{y\left(1 - \tilde{\boldsymbol{\mu}}^\top \boldsymbol{\phi}(\boldsymbol{x}) y\right)}{y^2 \boldsymbol{\phi}(\boldsymbol{x})^\top \tilde{\boldsymbol{\Sigma}} \boldsymbol{\phi}(\boldsymbol{x}) + \gamma} \tilde{\boldsymbol{\Sigma}} \boldsymbol{\phi}(\boldsymbol{x})$$

$1 - \hat{\boldsymbol{\mu}}^\top \boldsymbol{\phi}(\boldsymbol{x}) y < 0$ のときは

$$\hat{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}}$$

となる．以上より，解 $\hat{\boldsymbol{\mu}}$ は

$$\hat{\boldsymbol{\mu}} = \tilde{\boldsymbol{\mu}} + \frac{y \max\left(0, 1 - \tilde{\boldsymbol{\mu}}^\top \boldsymbol{\phi}(\boldsymbol{x}) y\right)}{y^2 \boldsymbol{\phi}(\boldsymbol{x})^\top \tilde{\boldsymbol{\Sigma}} \boldsymbol{\phi}(\boldsymbol{x}) + \gamma} \tilde{\boldsymbol{\Sigma}} \boldsymbol{\phi}(\boldsymbol{x})$$

と表せる．