Confidence Weighted Mean Reversion for Online Portfolio Selection

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1 Introduction

Online portfolio selection is a type of sequential decision-making problem with the goal of maximizing the cumulative wealth in the whole investment period. At the beginning of each period, a new investment strategy is determined based on the newly received market information. At the beginning of period t, the investor adjusts his/her strategy based on the historical investment strategies and historical assets' return. Then the investor adjusts his/her strategy at each period based on the historical information.

2 Problem Formulation

The online portfolio selection problem is clearly a sequential decision-making problem over a given set of assets. The investor needs to rebalance the portfolio allocation at each period to improve the total investment return. In this section, we introduce the decision-making process of online portfolio selection with transaction costs in n consecutive trading days. We assume that the market contains m risky assets. And we would like to invest in n consecutive trading days. The last price of asset i at day t is $p_{t,i}$. And we adopt $\mathbf{p}_t \in \mathbb{R}_+^m$ to denote the price vector. The price change is captured by a price relative vector $\mathbf{x}_t = \frac{\mathbf{p}_t}{\mathbf{p}_{t-1}}$, with elementwise vector division.

At the beginning of day t, the portfolio manager makes an allocation based on his/her expectation of asset changes and historical allocations. The new portfolio allocation at day t is represented by a portfolio vector $\mathbf{b}_t \in \Delta_m$, where

$$\Delta_m = \left\{ \mathbf{b} \in \mathbb{R}^m \mid \mathbf{b} \geq \mathbf{0}, \mathbf{b}\mathbf{1} = 1 \right\}.$$

where $\mathbf{1}$ is a row vector of all ones. In particular, let $b_{t,i}$ be the proportion of wealth we invest in asset i at day t. The simplex set Δ_m is used to ensure self-finance and no short selling is allowed. Typically, the initial portfolio is set to be $\mathbf{b}_1 = (\frac{1}{m}, \dots, \frac{1}{m})$. After implementing the portfolio $\mathbf{b}_t$ at day t, the wealth of the portfolio will change in accordance with market fluctuations until the end of the period. Owing to the price changes, the end-of-period portfolio is given by

$$\tilde{\mathbf{b}}_t = \frac{\mathbf{b_t} \cdot \mathbf{x_t}}{\mathbf{b}_t \mathbf{x}_t^\top},$$

where "$\cdot$" denotes the element-wise multiplication of two vectors.

Let $\gamma \in [0, 1]$ be the transaction cost rates, and w_{t-1} represents the transaction remainder factor, which is the net proportion after transaction costs are deducted. The sales occur when the proportion before rebalancing is greater than the proportion after rebalancing, i.e. $\tilde{b}_{t-1,i} - b_{t,i} w_{t-1} > 0$, while the purchases occur when $b_{t,i} w_{t-1} - \tilde{b}_{t-1,i} \geq 0$. We have

$$1 = w_{t-1} + \gamma \sum_{i=1}^{m} |b_{t,i} w_{t-1} - \tilde{b}_{t-1,i}|, \tag{1}$$

which means that the summation of the transaction remainder factor and transaction costs always equals 1. We thus treat the net proportion after transaction costs incurred as a function of two consecutive portfolios and the last price relative vector, i.e. $w_{t-1} = w\left(\mathbf{b}_t, \hat{\mathbf{b}}_{t-1}, \mathbf{x}_{t-1}\right)$. Note that w_{t-1} can be efficiently solved via the bisection method. As a result of rebalancing, the remaining capital becomes $S_{t-1} \times w_{t-1}$.

During the trading day t, the allocation $\mathbf{b}_t$ changes the wealth by a factor of

$$\mathbf{b}_t \mathbf{x}_t^\top = \sum_{i=0}^{m} b_{t,i} x_{t,i}.$$

To summarize, the portfolio wealth changes from S_{t-1} to $S_{t-1} \times w_{t-1} \times (\mathbf{b}_t \mathbf{x}_t^\top)$ in day t. Since we reinvest and use relative prices, the wealth grows in a multiplicative manner. The cumulative wealth of the portfolio at the end of period n can be expressed as follows

$$S_n = S_0 \prod_{t=1}^{n} w_{t-1} \times (\mathbf{b}_t \mathbf{x}_{t-1}^\top), \tag{2}$$

where the initial wealth S_0 is set to 1 for convenience.

For each trading day t, the manager accesses the sequence of previous portfolio vectors $(\mathbf{b}_1, \mathbf{b}_2, \cdots, \mathbf{b}_{t-1})$ and price relative vectors $(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_{t-1})$. Based on the historical information, the portfolio manager determines a new portfolio vector $\mathbf{b}_t$ for the next price relative vector $\mathbf{x}_t$. The resulting portfolio $\mathbf{b}_t$ is scored based on return $(\mathbf{b}_t \mathbf{x}_t^\top)$. This procedure is repeated until the final period, at which point the portfolio strategy is scored based on its portfolio cumulative wealth S_n.

It is worth noting that several general assumptions are made. Firstly, we assume proportional transaction costs on risky assets purchases and sales. Secondly, we assume that each asset

Algorithm 1: Online Portfolio Selections

Input: An OLPS algorithm; Historical price relatives $\mathbf{x}_1^{t-1}$; Transaction cost rate γ.

Output: Final cumulative wealth S_n.

Begin

Initialize $\mathbf{b}_1 = (\frac{1}{m}, \ldots, \frac{1}{m}), S_0 = 1$.

for t=2, ..., n **do**

1. Record current allocation $\hat{\mathbf{b}}_{t-1}$;
2. Calculate next portfolio vector $\mathbf{b}_t$ via Algorithm 2;
3. Derive the transaction reminder factor w_{t-1};
4. Update the wealth after transaction cost is deducted: $S' = S_{t-1} \times w_{t-1}$;
5. Receive market price relatives $\mathbf{x}_t$;
6. Update the cumulative wealth S_t:

$$S_t = S_{t-1} \times w_{t-1} \times (\mathbf{b}_t \mathbf{x}_t^\top);$$

end for

End

share is arbitrarily divisible and that any required quantities of shares, even fractional, can be bought and sold at the last closing price in any trading period. Thirdly, we assume that market behavior and stock prices are not affected by any trading strategy.

3 Confidence Weighted Mean Reversion

3.1 Formulation

Let us model the portfolio vector for the t^{th} trading day as a Gaussian distribution with mean $\boldsymbol{\mu} \in \mathbb{R}^m$ and the diagonal covariance matrix $\boldsymbol{\Sigma} \in \mathbb{R}^{m \times m}$ with nonzero diagonal elements σ_i^2 and zero for off-diagonal elements. The value μ_i represents the knowledge of asset i in the portfolio. The diagonal covariance matrix term σ_i^2 stands for the confidence we have in the portfolio mean value μ_i. At the beginning of $t-1^{th}$ trading day, we construct a portfolio $\mathbf{b}_{t-1}$ based on the distribution $\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Then after the price relative $\mathbf{x}_{t-1}$ is revealed, the portfolio increases its wealth by a factor of $\mathbf{b}_{t-1}\mathbf{x}_{t-1}^\top$. It is straightforward that the portfolio daily return can be viewed as a random variable of a univariate Gaussian distribution, D $\sim \mathcal{N}\left(\boldsymbol{\mu}\mathbf{x}_{t-1}^\top, \mathbf{x}_{t-1}\boldsymbol{\Sigma}\mathbf{x}_{t-1}^\top\right)$.

According to the mean reversion trading idea, the probability of a profitable portfolio for the next trading day $\mathbf{b}$ with respect to a mean reversion threshold ϵ is defined as,

$$\Pr_{\mathbf{b}\sim\mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma})}[\mathrm{D} \leq \epsilon] = \Pr_{\mathbf{b}\sim\mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma})}\left[\mathbf{b}\mathbf{x}_{t-1}^{\top} \leq \epsilon\right].$$

For simplicity, we write $\Pr\left[\mathbf{b}\mathbf{x}_{t-1}^{\top} \leq \epsilon\right]$ instead. The manager adjusts the distribution to ensure the probability of a profitable portfolio is higher than a confidence level $\theta \in [0,1]$,

$$\Pr\left[\mathbf{b}\mathbf{x}_{t-1}^{\top} \leq \epsilon\right] \geq \theta.$$

If the expected return using the $t-1^{th}$ price relative is less than a threshold with high probability, the actual return for the t^{th} trading day tends to be high with correspondingly high probability since the price relative tends to reverse.

Then, our algorithm chooses the distribution closest (in the KL divergence sense) to the current distribution $\mathcal{N}\left(\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}\right)$. As a result, on the t^{th} trading day, the algorithm sets the parameters of the distribution by solving the following optimization problem:

$$\begin{aligned}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t) = \arg\min_{\boldsymbol{\mu},\boldsymbol{\Sigma}} & \mathrm{D}_{\mathrm{KL}}\left(\mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma}) \| \mathcal{N}\left(\boldsymbol{\mu}_{t-1},\boldsymbol{\Sigma}_{t-1}\right)\right) \\ \text{s.t. } & \Pr\left[\boldsymbol{\mu}\mathbf{x}_{t-1}^{\top} \leq \epsilon\right] \geq \theta \\ & \boldsymbol{\mu} \in \Delta_m.\end{aligned}$$

Under the distribution of $\mathcal{N}(\boldsymbol{\mu},\boldsymbol{\Sigma})$, the return for the $t-1^{th}$ trading day has a Gaussian distribution with mean $\mu_D = \boldsymbol{\mu}\mathbf{x}_{t-1}^{\top}$ and covariance $\boldsymbol{\Sigma}_D = \mathbf{x}_{t-1}\boldsymbol{\Sigma}\mathbf{x}_{t-1}^{\top}$ of diagonal elements σ_D^2. Thus, the probability of a profitable portfolio, $\Pr[\mathrm{D} \leq \epsilon] = \Pr\left[\frac{\mathrm{D}-\mu_D}{\sigma_D} \leq \frac{\epsilon-\mu_D}{\sigma_D}\right]$. In this formula, $\frac{\mathrm{D}-\mu_D}{\sigma_D}$ is a normally distributed random variable, the above probability equals $\Phi\left(\frac{\epsilon-\mu_D}{\sigma_D}\right)$, where Φ is the cumulative distribution function of the Gaussian distribution. As a result, we can rewrite the constraint as $\frac{\epsilon-\mu_D}{\sigma_D} \geq \Phi^{-1}(\theta)$. Substituting μ_D and σ_D by their definitions and rearranging terms, and replacing the portfolio return term $\boldsymbol{\mu}\mathbf{x}_{t-1}^{\top}$ by its logarithmic $\log\left(\boldsymbol{\mu}\mathbf{x}_{t-1}^{\top}\right)$, we obtain the constraint,

$$\epsilon - \log\left(\boldsymbol{\mu}\mathbf{x}_{t-1}^{\top}\right) \geq \phi\sqrt{\mathbf{x}_{t-1}\boldsymbol{\Sigma}\mathbf{x}_{t-1}^{\top}},$$

where $\phi = \Phi^{-1}(\theta)$.

To this end, we rewrite the above optimization problem as the Revised Optimization Problem:

$$\begin{aligned}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t) = \arg\min_{\boldsymbol{\mu},\boldsymbol{\Sigma}} & \frac{1}{2}\left(\log\left(\frac{\det \boldsymbol{\Sigma}_{t-1}}{\det \boldsymbol{\Sigma}}\right) + \mathrm{Tr}\left(\boldsymbol{\Sigma}_{t-1}^{-1}\boldsymbol{\Sigma}\right)\right) \\ & + \frac{1}{2}\left(\left(\boldsymbol{\mu}_{t-1} - \boldsymbol{\mu}\right)\boldsymbol{\Sigma}_{t-1}^{-1}\left(\boldsymbol{\mu}_{t-1} - \boldsymbol{\mu}\right)^{\top}\right) \\ \text{s.t. } & \epsilon - \log\left(\boldsymbol{\mu}\mathbf{x}_{t-1}^{\top}\right) \geq \phi\sqrt{\mathbf{x}_{t-1}\boldsymbol{\Sigma}\mathbf{x}_{t-1}^{\top}} \\ & \boldsymbol{\mu}\mathbf{1} = 1, \boldsymbol{\mu} \succeq \mathbf{0}.\end{aligned}$$

For the above-revised optimization problem, the constraint is not convex in $\mathbf{\Sigma}$. we decompose $\mathbf{\Sigma}$ since it is positive semidefinite (PSD), i.e., $\mathbf{\Sigma} = \mathbf{\Upsilon}^2$ with $\mathbf{\Upsilon} = \mathbf{Q}\text{diag}\left(\eta_1^{1/2}, \ldots, \eta_m^{1/2}\right)\mathbf{Q}^\top$, where $\mathbf{Q}$ is orthonormal and $\eta_1, \ldots, \eta_m$ are the eigenvalues of $\mathbf{\Sigma}$ and thus $\mathbf{\Upsilon}$ is also PSD.

This reformulation yields the Final Optimization Problem:

$$
\begin{aligned}
(\boldsymbol{\mu}_t, \mathbf{\Upsilon}_t) = \arg\min_{\boldsymbol{\mu}, \mathbf{\Upsilon}} & \frac{1}{2}\left(\log\left(\frac{\det \mathbf{\Upsilon}_{t-1}^2}{\det \mathbf{\Upsilon}^2}\right) + \operatorname{Tr}\left(\mathbf{\Upsilon}_{t-1}^{-2}\mathbf{\Upsilon}^2\right)\right) \\
& + \frac{1}{2}\left(\left(\boldsymbol{\mu}_{t-1} - \boldsymbol{\mu}\right)\mathbf{\Upsilon}_{t-1}^{-2}\left(\boldsymbol{\mu}_{t-1} - \boldsymbol{\mu}\right)^\top\right) \\
\text{s.t. } & \epsilon - \log\left(\boldsymbol{\mu}\mathbf{x}_{t-1}^\top\right) \geq \phi\|\mathbf{\Upsilon}\mathbf{x}_{t-1}^\top\|, \mathbf{\Upsilon} \text{ is PSD} \\
& \boldsymbol{\mu}\mathbf{1} = 1, \boldsymbol{\mu} \succeq \mathbf{0}.
\end{aligned}
$$

3.2 Algorithm

Now let us develop the proposed algorithm based on the solution using the typical techniques from convex optimization. The solution to the Final Optimization Problem is expressed as:

$$
\boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} - \lambda_t \mathbf{\Sigma}_{t-1}\frac{\mathbf{x}_{t-1} - \overline{\mathbf{x}}_{t-1}\mathbf{1}}{\boldsymbol{\mu}_{t-1}\mathbf{x}_{t-1}^\top}, \mathbf{\Sigma}_t^{-1} = \mathbf{\Sigma}_{t-1}^{-1} + \lambda_t\phi\frac{\mathbf{x}_{t-1}\mathbf{x}_{t-1}^\top}{\sqrt{U_{t-1}}},
$$

where $V_{t-1} = \mathbf{x}_{t-1}\mathbf{\Sigma}_{t-1}\mathbf{x}_{t-1}^\top$ and $\sqrt{U_{t-1}} = \frac{-\lambda_t V_{t-1}\phi + \sqrt{\lambda_t^2 V_{t-1}^2\phi^2 + 4V_{t-1}}}{2}$, and $\overline{\mathbf{x}}_{t-1} = \frac{\mathbf{1}^\top\mathbf{\Sigma}_{t-1}\mathbf{x}_{t-1}^\top}{\mathbf{1}^\top\mathbf{\Sigma}_{t-1}\mathbf{1}}$ represents the confidence weighted average of the $t-1^{th}$ price relative.

We calculate the Lagrangian multiplier $\lambda_t = \max\left\{\gamma_{t-1,1}, \gamma_{t-1,2}, 0\right\}$, where

$$
\gamma_{t-1,1} = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \gamma_{t-1,2} = \frac{-b - \sqrt{b^2 - 4ac}}{2a},
$$

with $a = \left(\frac{V_{t-1} - \overline{\mathbf{x}}_{t-1}\mathbf{x}_{t-1}^\top\mathbf{\Sigma}_{t-1}\mathbf{1}}{M_{t-1}^2} + \frac{V_{t-1}\phi^2}{2}\right)^2 - \frac{V_{t-1}^2\phi^4}{4}$, $b = 2\left(\epsilon - \log M_{t-1}\right)\left(\frac{V_{t-1} - \overline{\mathbf{x}}_{t-1}\mathbf{x}_{t-1}^\top\mathbf{\Sigma}_{t-1}\mathbf{1}}{M_{t-1}^2} + \frac{V_{t-1}\phi^2}{2}\right)$, , $c = \left(\epsilon - \log M_{t-1}\right)^2 - V_{t-1}\phi^2$ and $M_{t-1} = \boldsymbol{\mu}_{t-1}\mathbf{x}_{t-1}^\top$.

Initially, we simply set $\boldsymbol{\mu}_1 = \frac{1}{m}\mathbf{1}, \quad \mathbf{\Sigma}_1 = \frac{1}{m^2}\mathbf{I}$. We also project the resulting $\boldsymbol{\mu}$ to the simplex domain to ensure the simplex constraint. To be consistent with the projection of the $\boldsymbol{\mu}$, we try to rescale $\mathbf{\Sigma}$ by normalizing its maximum value to $\frac{1}{\mathrm{m}^2}$. The final CWMR algorithm is presented below.

Algorithm 2: Confidence Weighted Mean Reversion

Input: $\phi = \Phi^{-1}(\theta)$: Confidence parameter; $\epsilon < 0$: Mean reversion parameter

Output: $\mathbf{b}_t$: Next portfolio;

Begin

1. Draw a portfolio $\mathbf{b}_{t-1}$ from $\mathcal{N}\left(\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}\right)$
2. Receive stock price relatives: $\mathbf{x}_{t-1}$
3. Calculate the following variables:

$$M_{t-1} = \boldsymbol{\mu}_{t-1}\mathbf{x}_{t-1}^{\top}, \quad V_{t-1} = \mathbf{x}_{t-1}\boldsymbol{\Sigma}_{t-1}\mathbf{x}_{t-1}^{\top}, \quad \overline{\mathbf{x}}_{t-1} = \frac{\mathbf{1}^{\top}\boldsymbol{\Sigma}_{t-1}\mathbf{x}_{t-1}^{\top}}{\mathbf{1}^{\top}\boldsymbol{\Sigma}_{t-1}\mathbf{1}}$$

5. Update the portfolio distribution:

$$\text{CWMR}\begin{cases} \lambda_t \text{ as calculated above} \\ \sqrt{U_{t-1}} = \frac{-\lambda_t\phi V_{t-1} + \sqrt{\lambda_t^2\phi^2 V_{t-1}^2 + 4V_{t-1}}}{2} \\ \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t-1} - \lambda_t\boldsymbol{\Sigma}_{t-1}\frac{\mathbf{x}_{t-1} - \overline{\mathbf{x}}_{t-1}\mathbf{1}}{M_{t-1}} \\ \boldsymbol{\Sigma}_t = \left(\boldsymbol{\Sigma}_{t-1}^{-1} + \lambda_t\phi\frac{\mathbf{x}_{t-1}\mathbf{x}_{t-1}^{\top}}{\sqrt{U_{t-1}}}\right)^{-1} \end{cases}$$

6. Normalize $\boldsymbol{\mu}_t$ and $\boldsymbol{\Sigma}_t$:

$$\boldsymbol{\mu}_t = \arg\min_{\boldsymbol{\mu}\in\Delta_m} \left\|\boldsymbol{\mu} - \boldsymbol{\mu}_t\right\|^2, \boldsymbol{\Sigma}_t = \frac{\boldsymbol{\Sigma}_t}{m^2\,\mathrm{Tr}\left(\boldsymbol{\Sigma}_t\right)}$$

7. $\mathbf{b}_t = \boldsymbol{\mu}_t$

End

In all cases, we empirically set the parameters, that is, confidence parameter $\phi = 2$ or equivalently confidence level $\theta = 95\%$, and mean reversion parameter $\epsilon = -0.5$.

4 Experiments

In the numerical experiment, we only use the daily last price in the given dataset. First, we need to filter the dataset. Based on the data in the 'ticker' column, we perform frequency counts. It is found that each element does not appear with exactly the same frequency, meaning that the data time length is not the same for each stock. The longest data duration is 2005 trading days from 2013/1/4 to 2021/3/19. We identify the individual elements with the highest frequency of occurrence, i.e., the stocks that satisfy the above time lengths. And we get the stock tickers are:

['9501 JT', '2914 JT', '5401 JT', '2801 JT', '1721 JT', '6857 JT', '9062 JT', '8253 JT', '7731

JT', '5631 JT', '8035 JT', '6701 JT', '9433 JT', '5108 JT', '7911 JT', '7912 JT', '9502 JT', '5201 JT', '8308 JT', '7270 JT', '9432 JT', '9021 JT', '5713 JT', '7751 JT', '5801 JT', '8750 JT', '5707 JT', '3382 JT', '5020 JT', '7201 JT', '8058 JT', '1605 JT', '9104 JT', '6702 JT', '6113 JT', '6301 JT', '6103 JT', '2503 JT', '6472 JT', '5202 JT', '3103 JT', '7012 JT', '5332 JT', '9022 JT', '3402 JT', '6473 JT', '9983 JT', '2502 JT', '6758 JT', '4452 JT', '9007 JT', '5101 JT', '7003 JT', '4523 JT', '9020 JT', '8233 JT', '4502 JT', '5333 JT', '9005 JT', '7261 JT', '5233 JT', '2531 JT', '7205 JT', '4901 JT', '5703 JT', '6674 JT', '6971 JT', '1925 JT', '6752 JT', '4063 JT', '9009 JT', '7211 JT', '5301 JT', '1801 JT', '6762 JT', '6503 JT', '9735 JT', '9503 JT', '9107 JT', '3401 JT', '9064 JT', '6703 JT', '8604 JT', '7011 JT', '9984 JT', '7202 JT', '7762 JT', '1963 JT', '4324 JT', '3861 JT', '5541 JT', '8031 JT', '4503 JT', '7267 JT', '9531 JT', '8354 JT', '8725 JT', '6302 JT', '7951 JT', '5214 JT', '4004 JT', '4005 JT', '4151 JT', '4183 JT', '1928 JT', '2501 JT', '9412 JT', '8795 JT', '4689 JT', '9602 JT', '4043 JT', '4061 JT', '9001 JT', '4911 JT', '4519 JT', '4543 JT', '7004 JT', '1802 JT', '6326 JT', '8601 JT', '2269 JT', '4507 JT', '4506 JT', '9202 JT', '8801 JT', '8316 JT', '5232 JT', '6361 JT', '8802 JT', '8628 JT', '8053 JT', '8303 JT', '5406 JT', '3407 JT', '6501 JT', '2282 JT', '7752 JT', '8766 JT', '9008 JT', '4208 JT', '3101 JT', '8355 JT', '8804 JT', '7013 JT', '5411 JT', '5714 JT', '1812 JT', '8001 JT', '4188 JT', '9532 JT', '7269 JT', '3436 JT', '7203 JT', '8331 JT', '1803 JT', '8304 JT', '8267 JT', '8411 JT', '6841 JT', '2871 JT', '9613 JT', '8252 JT', '6976 JT', '6471 JT', '5802 JT', '7733 JT', '9101 JT', '5711 JT', '8630 JT', '5803 JT', '8002 JT', '9301 JT', '4568 JT', '8306 JT', '4042 JT', '6770 JT', '4021 JT', '2002 JT', '8015 JT', '6305 JT', '3405 JT', '6952 JT', '5901 JT', '1332 JT', '9766 JT', '7735 JT', '8309 JT', '6902 JT', '8830 JT', '6504 JT', '5706 JT', '3086 JT', '2768 JT', '6954 JT', '3105 JT', '6506 JT', '4704 JT', '4902 JT', '2802 JT', '6479 JT', '3099 JT', '6367 JT'].

From this, we selected a total of 202 stocks, each with a total of 2005 trading days from 2013/1/4 to 2021/3/19. After that, we choose the daily last price of the above stocks and derive a new dataset. Applying the above trading strategy to the current data, we have total trading days $n = 2005$ and the number of risky assets $m = 202$. For convenience, we set the initial capital to 1 and the initial target position to be equally allocated. The default transaction costs rate $\gamma = 0.03\%$. At the beginning of each new trading day, we obtain a new portfolio by solving the optimization problem, adjusting the weights of each stock, and thus generating a new position.

We measure the performance of the strategy by the final cumulative wealth, along with some other metrics: the Sharpe ratio, Volatility, Maximum drawdown, and Winning ratio. Sharpe ratio is a measure of the risk-adjusted excess return of a portfolio. It compares the excess return to the volatility of the portfolio. In general, the higher the Sharpe ratio, the better the risk-adjusted return of the portfolio. Volatility is a measure of portfolio risk. It reflects the degree of volatility of a portfolio's return. The higher the volatility, the higher the risk of the portfolio. Maximum drawdown is a measure of portfolio risk. It indicates the maximum drawdown in the value of a portfolio over a given time period. The higher the maximum drawdown, the higher the risk of the portfolio. The winning ratio measures the percentage of all trading periods in which the portfolio has a positive return; the larger the Winning ratio, the more often the portfolio has a positive return.

```
Final cumulative wealth: 1.5515
Summary:
    Sharpe ratio: 0.0214
    Volatility: 25.5586%
    Max drawdown:  59.6699%
    Winning ratio:  51.9701%
```

Figure 1: The trading result of CWMR

The specific strategy performance and the experimental results can be referred to in the code. The "OLPS-code" folder contains all the code for the entire trading framework. In order to make the code runnable, you can read the "README.md" file first. After opening the "mainrun.py" file and setting the corresponding parameters, you can run the "mainrun.py" file for backtesting. Also, you can find the code corresponding to the strategy "CWMR" in the "algos" folder.