$$(f_1, f_2, f_3) = V \xrightarrow{\hspace{8cm}} f_1 \mathrm{d}x + f_2 \mathrm{d}y + f_3 \mathrm{d}z \in \Omega^1(\mathbb{R}^3)$$

$$\Big\downarrow \text{rot} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \Big\downarrow \text{d}$$

$$\operatorname{rot}V = \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}, \frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}, \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) \xleftarrow{\hspace{2cm}} \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) \mathrm{d}y \wedge \mathrm{d}z + \left(\frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}\right) \mathrm{d}z \wedge \mathrm{d}x + \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) \mathrm{d}x \wedge \mathrm{d}y$$