$$i \geq 0 : \qquad \begin{array}{ccc} U_{L,i}/U_{L,i+1} & \xrightarrow{\;\sim\;} & \ell^+ \\[4pt] \scriptstyle N \big\downarrow & & \big\downarrow \scriptstyle \mathrm{Tr} \\[4pt] U_{K,i}/U_{K,i+1} & \xrightarrow{\;\sim\;} & \Bbbk^+ \end{array} \qquad\qquad i = 0 : \qquad \begin{array}{ccc} U_{L,0}/U_{L,1} & \xrightarrow{\;\sim\;} & \ell^* \\[4pt] \scriptstyle N \big\downarrow & & \big\downarrow \scriptstyle N \\[4pt] U_{K,0}/U_{K,1} & \xrightarrow{\;\sim\;} & \Bbbk^* \end{array}$$