

$$\begin{array}{ccccccc}
 d^*\omega & \longmapsto & (d(\rho_V\omega), -d(\rho_U\omega)) & \longmapsto & d\omega = 0 \\
 & & \nearrow & & \nwarrow \\
 0 \longrightarrow \Omega^{k+1}(U \cup V) & \longrightarrow & \Omega^{k+1}(U) \oplus \Omega^{k+1}(V) & \longrightarrow & \Omega^{k+1}(U \cap V) & \longrightarrow & 0 \\
 & \uparrow d & \uparrow d & & \uparrow d & & \\
 0 \longrightarrow \Omega^k(U \cup V) & \longrightarrow & \Omega^k(U) \oplus \Omega^k(V) & \longrightarrow & \Omega^k(U \cap V) & \longrightarrow & 0 \\
 & & \searrow & & \swarrow \\
 & & (\rho_V\omega, -\rho_U\omega) & \longmapsto & \omega
 \end{array}$$