$$\begin{array}{ccc}
& & & \\
\downarrow & & & \\
H^{q}(S^{n}) & \longrightarrow & H^{q}(U) \oplus H^{q}(V) & \longrightarrow & H^{q}(S^{n-1} \times \mathbb{R}) \\
& & & \downarrow & & \\
& & & \downarrow & & \\
H^{q+1}(S^{n}) & \longrightarrow & H^{q+1}(U) \oplus H^{q+1}(V) & \longrightarrow & \dots
\end{array}$$