$$H^\bullet(S) \xrightarrow[\mathcal{T}_S(\omega)=\pi^*(\omega)\wedge\Phi_W]{\sim} H^{\bullet+n-k}(W) \xrightarrow{j_*} H^{\bullet+n-k}(M)$$