DYNAMIC HEDGING

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Abstract. This project explores the implementation of Delta and Delta-Gamma hedging strategies within the Black-Scholes framework. Using a simulated asset price process, we investigate the profit and loss distributions of both hedging approaches under varying drift (μ) assumptions and analyze their performance across 5,000 simulated paths. We also examine the hedging positions for sample paths, highlighting scenarios where the asset ends in and out of the money. We then assess the impact of model misspecification by comparing hedging outcomes under a range of real-world volatilities different from the assumed volatility. Transaction costs are included to make the analysis more realistic, providing insights into the strengths and weaknesses of Delta and Delta-Gamma hedging.

1. Introduction

Hedging strategies play an important role in managing the risk associated with derivative instruments. Delta and Delta-Gamma hedging are two approaches that leverage sensitivities of option prices to the underlying asset to mitigate risk. The Black-Scholes model provides a theoretical foundation for understanding these strategies, assuming the underlying asset price follows a geometric Brownian motion.

In this project, we examine the practical application of Delta and Delta-Gamma hedging for an at-the-money call option sold under the Black-Scholes framework. The hedging strategies are evaluated on their ability to manage risk and minimize profit and loss deviations across varying market conditions. By simulating 5,000 paths for the underlying asset price, we analyze the differences in outcomes between the two strategies, highlighting their respective strengths and limitations.

Furthermore, the project investigates the sensitivity of hedging outcomes to model assumptions, such as volatility (σ) and drift (μ), and incorporates realistic elements like transaction costs. These insights are crucial for understanding the robustness of hedging strategies under real-world market dynamics and for guiding practitioners in risk management decisions.

2. Methodolgy

Consider the following: Assume that an asset price process $S = (S_t)_{0\leq t}$ follows the Black-Scholes model, where the asset's current price is \$10. We immediately sell 10,000 units of an at-the-money, $\frac{1}{4}$ year call on this asset, option g, and we wish to hedge this short call option position. To execute this hedge, we shall trade: an at-the-money call with a maturity of 0.3 years, option h, the stock, and the bank account. We assume transaction costs such that we are charged \$0.005 per share on equity transactions and \$0.005 per option on option transactions. Further, we shall only trade integer value of stocks and options. Finally, we assume: The remaining model parameters are $\mu = 10\%, \sigma = 25\%, r = 5\%$, and we hedge daily.

To implement the hedging of the position in option g, we will utilize two different strategies: Delta hedging, and Delta-Gamma hedging.

2.1. **Delta Hedging.** Delta is defined as the change in the value of an option, relative to the change in movement in the price of an underlying asset. Delta can be derived from the price of a call option by taking its partial derivative with respect to the price of the underlying asset. Since

$$c(t,S_t) = S_t N\left(\frac{\log\left(\frac{S_t}{K}\right) + (r+\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right) - e^{-r(T-t)}K\left(\frac{\log\left(\frac{S_t}{K}N\right) + (r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\right)$$

it follows that

$$\begin{aligned}
\frac{d}{dS_t}c(t,S_t) &= N(d_+) + S_t N'(d_+) - e^{-r\tau}KN'(d_-) \\
&= N(d_+) + S_t\cdot\phi(d_+)\cdot\frac{d}{dS_t}d_+ - e^{-r\tau}K\cdot\phi(d_+-\sigma\sqrt{\tau})\cdot\frac{d}{dS_t}(d_+-\sigma\sqrt{\tau}) \\
&= N(d_+) + S_t\cdot\phi(d_+)\cdot\frac{d}{dS_t}d_+ - e^{-r\tau}K\cdot\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(d_+-\sigma\sqrt{\tau})^2}\cdot\frac{d}{dS_t}(d_+-\sigma\sqrt{\tau}) \\
&= N(d_+) + S_t\cdot\phi(d_+)\cdot\frac{d}{dS_t}d_+ - e^{-r\tau}K\cdot\phi(d_+)\frac{S_t}{K}e^{r\tau}\cdot\frac{d}{dS_t}(d_+-\sigma\sqrt{\tau}) \\
&= N(d_+) + S_t\cdot\phi(d_+)\cdot\frac{d}{dS_t}d_+ - S_t\cdot\phi(d_+)\cdot\frac{d}{dS_t}(d_+-\sigma\sqrt{\tau}) = N(d_+)
\end{aligned}$$

Where $N(\cdot)$ and $\phi(\cdot)$ are the standard normal cumulative distribution function and the probability density function, respectively, and $\tau = T-t$.

Delta hedging is a strategy used to hedge the directional risk associated with changes in the price of the underlying asset. Hence we want to construct a Delta-neutral portfolio to hedge the risk of shorting the option. As we are adjusting the hedge position on a daily basis, we will take initial positions in the underlying asset and bank account to neutralize Delta at time 0, and then adjust these positions over the life of option g. To implement Delta hedging, we follow the following procedure:

(1) Assume we start with nothing in our bank account, $B_0 = 0$, no position in the underlying asset, $\alpha_S = 0$, and no position in the option being hedged, $\alpha_g = 0$
(2) Determine the value of the bank account by computing the price of option g, and multiplying this value by 10,000, the magnitude of the short position.
(3) Compute the Delta of option g using the formula derived above.
(4) Determine the size position in the underlying asset required to neutralize Delta.

$$\alpha_{S_0} = \alpha_g \cdot \Delta_g$$

(5) Determine the number of shares that must be purchased/sold to so that our portfolio contains exactly α_{S_t} shares, and multiply $|\alpha_{S_t}|$ by $f_S = 0.005$, the cost to trade a share.
(6) Update the amount in the bank account,

$$B_t = B_{t-1} - (\alpha_{S_t} - \alpha_{S_{t-1}})\cdot S_t - |\alpha_{S_t} - \alpha_{S_{t-1}}|\cdot f_S$$

(7) Compute the future value of the bank account.
(8) At time T, we determine the payoff of the initial position in option g,

$$G(S_T) = -\alpha_g\cdot(S_T-K)^+$$

and update the bank account to include the proceeds from closing the position in the underlying asset,

$$B_T = B_{T-1} + \alpha_{S_T}\cdot S_T$$

(9) If the payoff is zero, then we incur final transaction costs from closing out our position in the underlying asset, and we have a final PnL of

$$\mathrm{PnL} = \mathrm{B}_T + G(S_T) - \alpha_{S_T} \cdot f_S$$

If the payoff is non-zero, then we have two sources of transaction costs at the terminal time T: 1. Costs incurred by closing out our position in the underlying asset, and 2. Costs incurred from option g being exercised. Hence we have a final PnL of

$$\mathrm{PnL} = \mathrm{B}_T + G(S_T) - |\alpha_{S_T}| \cdot f_S - |\alpha_g| \cdot f_S$$

We note that since we are the writer of option g, if the option were to be exercised at terminal time T, then we shall only incur transaction costs for the purchase of the shares that must be delivered to the option holder.

2.2. **Delta-Gamma Hedging.** An option's Gamma represents the rate of change in its Delta for a one-unit change in the underlying stock. Hence we can derive Gamma as the second derivative of the option's price with respect to the underlying asset price, or as the first derivative of the Delta with respect to the underlying asset price. From 2.1, we have that $\frac{d}{dS_t}c(t, S_t) = N(d_+)$, hence

$$\begin{aligned}
\frac{d^2}{dS_t^2}c(t, S_t) &= \frac{d}{dS_t}N(d_+) \\
&= \phi(d_+)\frac{d}{dS_t}\left(\frac{\log\frac{S_t}{K} + (r + \frac{1}{2}\sigma^2)(\tau)}{\sigma\sqrt{\tau}}\right) \\
&= \phi(d_+)\frac{d}{dS_t}\left(\frac{\log S_t}{\sigma\sqrt{\tau}} - \frac{\log K}{\sigma\sqrt{\tau}} + \frac{(r + \frac{1}{2}\sigma^2)(\tau)}{\sigma\sqrt{\tau}}\right) \\
&= \phi(d_+)\frac{1}{S_t\sigma\sqrt{\tau}}
\end{aligned}$$

Delta-Gamma hedging combines the Delta hedge described above with Gamma hedging, where the objective of the strategy is to achieve both a Delta-neutral and a Gamma-neutral portfolio. Hence we utilize two hedging instruments: the underlying asset for Delta neutrality, and a second hedging option for Gamma neutrality. Thus we implement the following hedging process:

(1) Assume we start with nothing in our bank account, $B_0 = 0$, no position in the underlying asset, $\alpha_S = 0$, no position in the hedging option, $\alpha_h = 0$, and finally, no position in the option being hedged, $\alpha_g = 0$.
(2) Determine the amount in the bank account by computing the price of option g, and multiplying it by 10,000, the magnitude of the short position.
(3) Compute the Delta and Gamma of both options g and h using the formula derived in 2.1, 2.2.
(4) Determine the size position in the hedging option required to neutralize Gamma.

$$\alpha_h = -\frac{\alpha_g \cdot \Gamma_g}{\Gamma_h}$$

(5) Determine the size position in the underlying asset required to neutralize Delta.

$$\alpha_{S_t} = \alpha_g \cdot \Delta_g + \alpha_h \cdot \Delta_h$$

(6) Determine the number of shares and the number of hedging options that must be purchased/sold to so that our portfolio contains α_{S_t} shares and α_h options, and multiply $|\alpha_{S_t}|$ and $|\alpha_h|$ by trading costs $f_S = 0.005$ and $f_O = 0.005$, respectively.
(7) Determine the price of the option $h(t, S_t)$.
(8) Update the amount in the bank account,

$$\begin{aligned} B_t = B_{t-1} &- (\alpha_{S_t} - \alpha_{S_{t-1}}) \cdot S_t - (\alpha_{h_t} - \alpha_{h_{t-1}}) \cdot h(t, S_t) \\ &- (\alpha_{S_t} - \alpha_{S_{t-1}}) \cdot f_S - (\alpha_{h_t} - \alpha_{h_{t-1}}) \cdot f_O \end{aligned}$$

(9) Compute the future value of the bank account.
(10) At time T, we determine the payoff of the initial position in option g,

$$G(S_T) = -\alpha_g \cdot (S_T - K)^+$$

and update the bank account to include the proceeds from closing the positions in the underlying asset and the hedging option,

$$B_T = B_{T-1} + \alpha_{S_T} \cdot S_T + \alpha_{h_T} \cdot h(T, S_T)$$

(11) If the payoff $G(S_T)$ is zero, then we have two sources of transaction costs at the terminal time T: 1. Costs incurred by closing out our position in the underlying asset, and 2. Costs incurred by closing out our position in option h. Hence we have a final PnL of

$$\text{PnL} = \text{B}_T + G(S_T) - \alpha_{S_T} \cdot f_S - \alpha_{h_T} \cdot f_O$$

If the payoff is non-zero, then we have three sources of transaction costs at the terminal time T: 1. Costs incurred by closing out our position in the underlying asset, 2. Costs incurred by closing out our position in option h, and 3. Costs incurred since option g will be exercised. Hence we have a final PnL of

$$\text{PnL} = \text{B}_T + G(S_T) - |\alpha_{S_T}| \cdot f_S - |\alpha_g| \cdot f_O - |\alpha_{h_T}| \cdot f_O$$

2.3. **Sample Path Simulation.** To determine the profit and loss distributions of the Delta and Delta-Gamma hedging strategies, we must first simulate sample paths of the asset price, which we assume follow a Geometric Brownian motion. We first consider the dynamic of GBM,

$$dS_t = \mu S_t dt + \sigma S_t dB_t$$

Consider $f(x) = \log X, f'(x) = \frac{1}{X}$, and $f''(X) = -\frac{1}{X^2}$. Applying Ito's, we have

$$\begin{aligned} d\log S_t &= f'(S_t)dS_t + \frac{1}{2}f''(S_t)(dS_t)^2 \\ &= \frac{1}{S_t}(\mu S_t dt + \sigma S_t dB_t) - \frac{1}{2S_t^2}(\mu S_t dt + \sigma S_t dB_t)^2 \\ &= \mu dt + \sigma dB_t - \frac{\sigma^2}{2}dt \\ &= (\mu - \frac{1}{2}\sigma^2)dt + \sigma dB_t \end{aligned}$$

Integrating, we get

$$\log S_t = \log S_0 + (\mu - \frac{1}{2}\sigma^2)t + \sigma B_t$$

where B_t follows a Brownian Motion. Finally, we have

$$S_t = S_0 \cdot e^{(\mu - \frac{1}{2}\sigma^2)t + \sigma B_t}$$

Once the asset price has been computed, we multiply through by the initial stock price and return the cumulative product of elements along a given simulation path.

3. Solutions

3.1. **Question 1.** We compare the profit and loss distributions of Delta and Delta-Gamma hedging strategies by simulating 5,000 sample paths using the methodology described in 2.3, where all paths follow independent Geometric Brownian Motion (GBM) random walks. On each path, we implement both the Delta hedging strategy outlined in 2.1 and the Delta-Gamma hedging strategy outlined in 2.2.

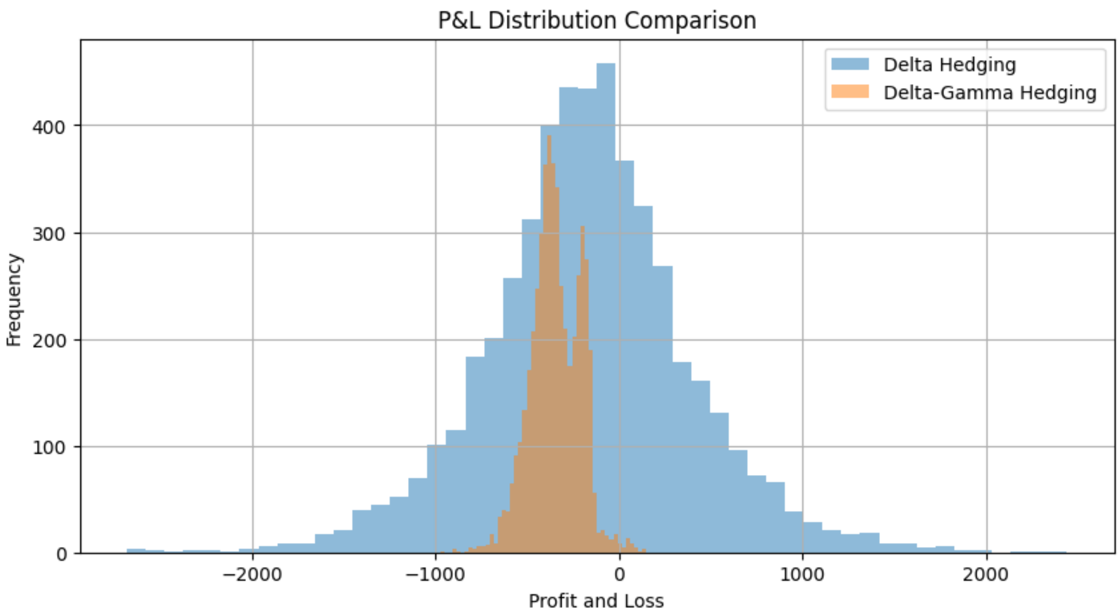

Figure 1. Delta and Delta-Gamma hedging strategies for 5,000 sample paths.

Qualitative Observations: In Figure 1, we see two very different PnL distributions for the two strategies. The Delta hedging strategy produces a distribution that appears fairly normal with somewhat symmetric tails and centered slightly negative. In contrast, the Delta-Gamma strategy produces a bimodal distribution with very short tails, again centered below zero. We expect the means of both distributions to be negative given the impact of trading costs.

Table 1. Hedging P&L Statistics

	Mean	**Std Dev**
Delta Hedging P&L	-171.739	560.2888
Delta-Gamma Hedging P&L	-346.1697	135.8541

Quantitative Observations: These aforementioned observations are consistent with the summary statistics found in Figure 1. First, the mean of the Delta-Gamma strategy is greater in magnitude than that of the Delta strategy indicating worse returns, however, the standard deviation is significantly smaller. This is shown in Figure 1 where the Delta-Gamma distribution is much more narrow, exhibiting very short tails. The long tails of the Delta distribution imply that while Delta hedging offers the potential for higher profits, it also carries a higher risk of significant losses. Conversely, the Delta-Gamma strategy provides more consistent but generally lower PnL outcomes with reduced risk.

Next, to determine the impact of μ on the PnL distributions of the strategies, we simulate sample paths for $\mu \in \{0.1, 0.2, ..., 0.9\}$. We then repeated the hedging strategies defined in 2.1, 2.2 on each of the new simulated paths to obtain the respective PnL.

Table 2. Delta and Delta-Gamma Hedging Results

	Delta		**Delta-Gamma**	
μ	**Mean**	**Std Dev**	**Mean**	**Std Dev**
0.1	-171.7390	560.2888	-346.1697	135.8541
0.2	-190.0028	550.0752	-356.7646	130.0090
0.3	-187.8742	540.3916	-369.1695	125.1276
0.4	-216.5648	513.5666	-371.4583	118.8866
0.5	-230.0154	494.7818	-376.9123	110.0328
0.6	-237.6413	478.2571	-377.1996	97.8482
0.7	-246.1595	458.3504	-371.8024	88.2584
0.8	-265.2032	431.8069	-369.2411	83.4370
0.9	-267.4748	387.5811	-362.4074	79.1769

Impact on Distribution Mean: As shown in Table 2, increasing μ generally leads to a more negative mean PnL for both strategies, indicating higher average losses. However, the relationship is not strictly linear, as seen with μ=0.3 for Delta hedging where the mean PnL slightly improves compared to μ=0.2. For the Delta-Gamma strategy, the mean PnL becomes more negative up to μ=0.6 and then starts to improve for higher μ values. Despite these variations, the overall impact of μ on the mean PnL appears limited.

Impact on Distribution Standard Deviation: In contrast to the impact on the distribution mean, changes in μ have a significant impact on the standard deviation, and consequently the shape, of the PnL distributions for both hedging strategies. Consider the Delta hedging distribution: as μ increases, the standard deviation of the distribution decreases significantly, indicating that higher drift values lead to reduced variability in PnL outcomes. Specifically, for the Delta strategy, the standard deviation decreases from 560.2888 when $\mu = 0.1$ to 387.5811 when $\mu = 0.9$. Similarly, for the Delta-Gamma strategy, the standard deviation decreases from 135.8541 at $\mu = 0.1$ to 79.1769 at $\mu = 0.9$. This trend suggests that as μ increases, the PnL distributions become narrower for both strategies, resulting in smaller potential profits and losses. The reduced standard deviation reflects a decrease in the risk associated with each strategy, as the outcomes are more tightly clustered around the mean. Notably, the Delta-Gamma strategy consistently exhibits a lower standard deviation compared to the Delta strategy across all μ values, highlighting its effectiveness in minimizing PnL variability. We observe that for each incremental increase in μ, the standard deviation of both hedging strategies decreases consistently. This consistent reduction underscores the influence of the drift parameter on the stability of PnL outcomes, making higher μ values desirable for strategies aiming to minimize risk.

Overall, while the Delta strategy offers higher potential returns, it does so at the expense of increased risk. The Delta-Gamma strategy provides a more stable PnL distribution with reduced risk but lower average returns. Figures 2-10 visualize these findings.

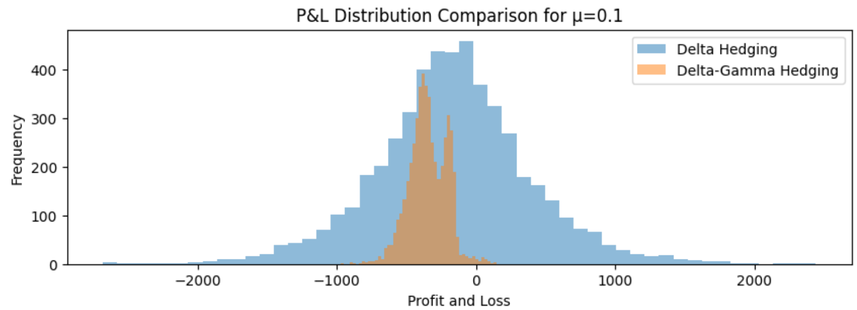

FIGURE 2. P&L Distribution Comparison for $\mu = 10\%$.

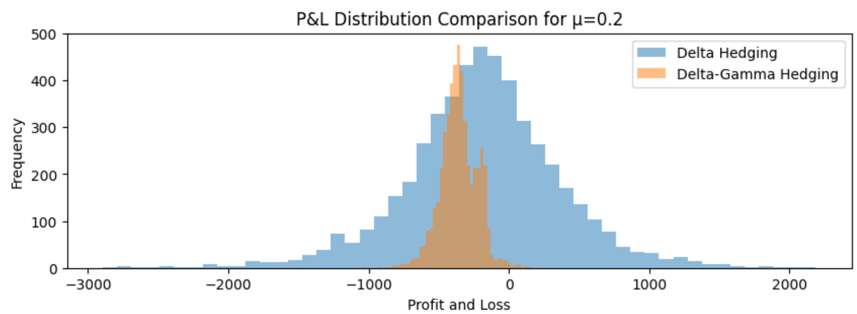

FIGURE 3. P&L Distribution Comparison for $\mu = 20\%$.

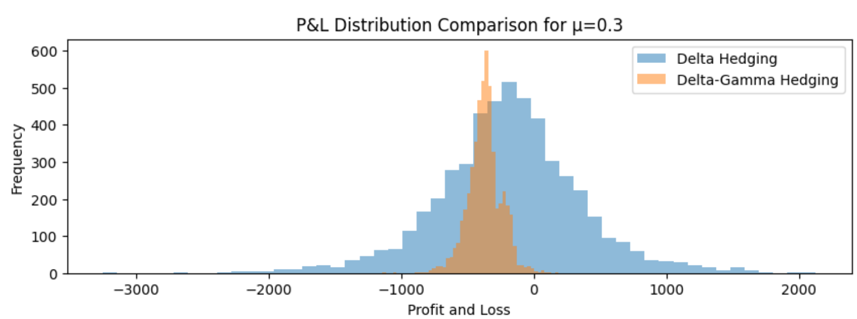

FIGURE 4. P&L Distribution Comparison for $\mu = 30\%$.

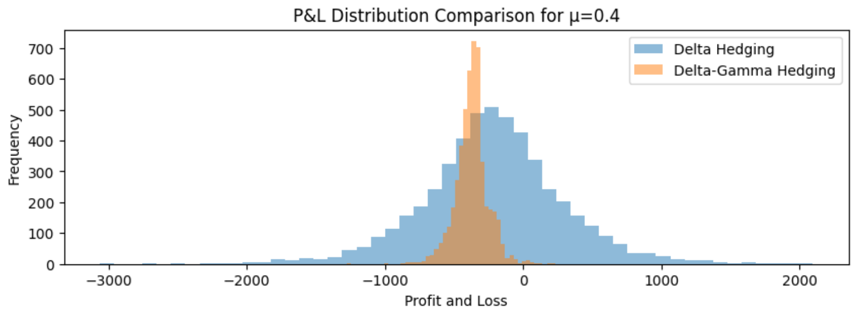

FIGURE 5. P&L Distribution Comparison for $\mu = 40\%$.

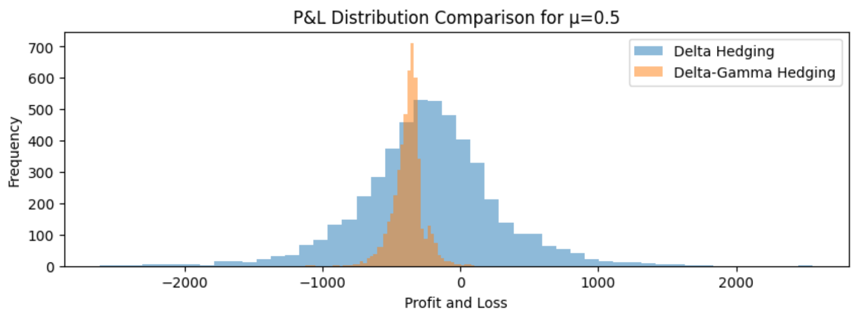

FIGURE 6. P&L Distribution Comparison for $\mu = 50\%$.

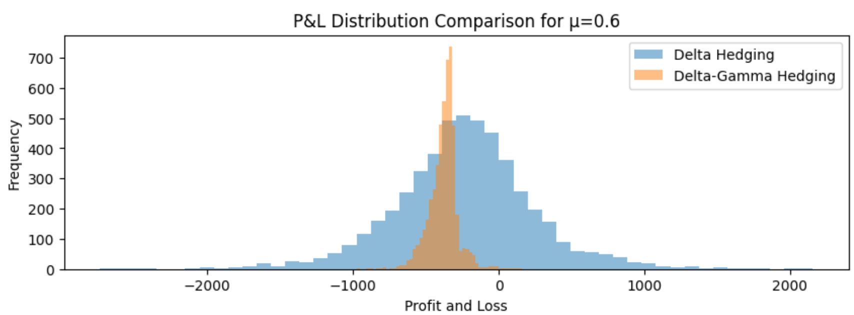

FIGURE 7. P&L Distribution Comparison for $\mu = 60\%$.

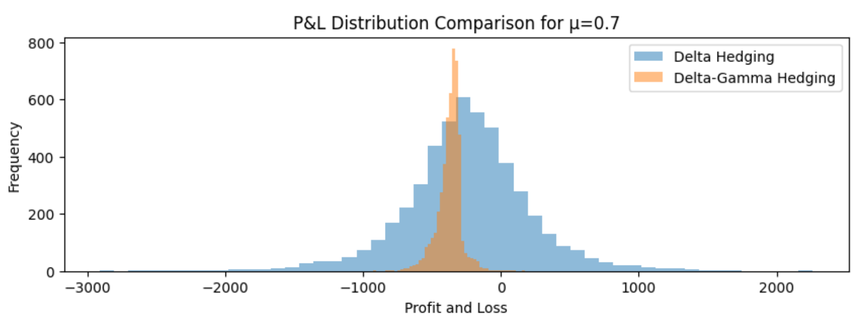

FIGURE 8. P&L Distribution Comparison for $\mu = 70\%$.

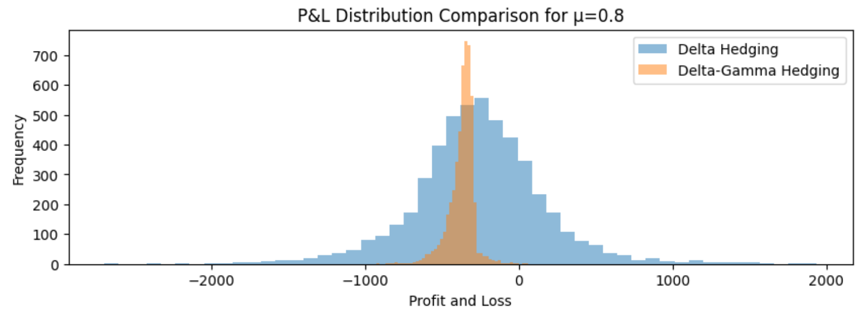

FIGURE 9. P&L Distribution Comparison for $\mu = 80\%$.

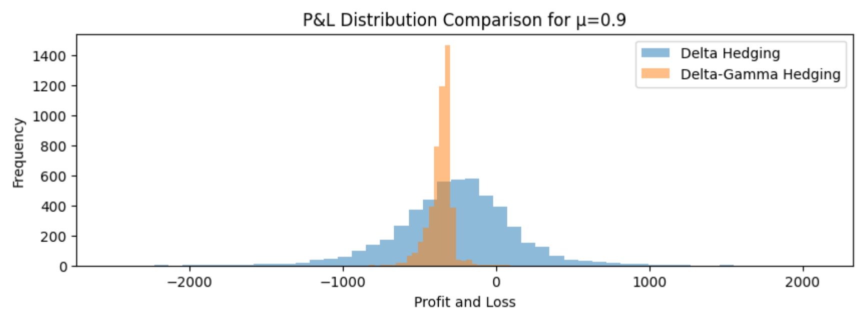

FIGURE 10. P&L Distribution Comparison for $\mu = 90\%$.

3.2. **Question 2.** This question examines the positions held in the asset and the hedging option under Delta-Gamma hedging for two distinct sample paths, simulated using Geometric Brownian Motion (GBM):

$$S_t = S_0 \cdot e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t}.$$

One path represents an in-the-money (ITM) scenario, where the asset price ends above the strike price, while the other represents an out-of-the-money (OTM) scenario, where the asset price ends below the strike price. A random seed of 47 was used to ensure consistent asset price trajectories for both Delta and Delta-Gamma hedging strategies. Figure 11 shows the sample paths and the corresponding positions in the asset and option for both scenarios.

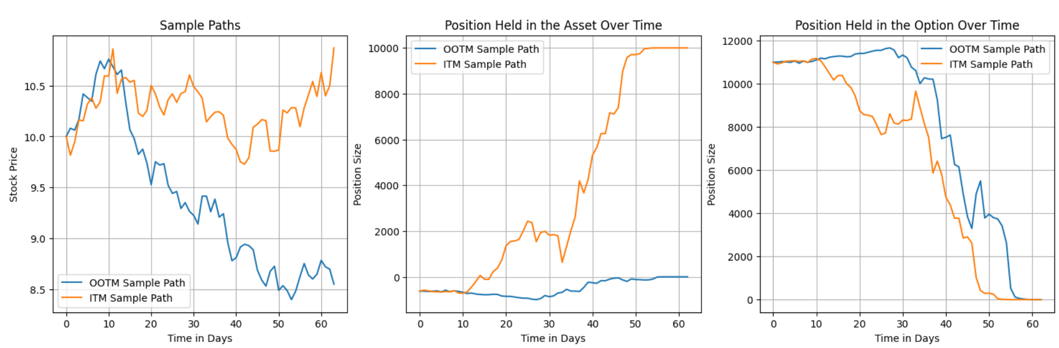

FIGURE 11. Sample Paths, Asset Positions, and Option Positions for ITM and OTM Scenarios.

Observations for In-the-Money (ITM) Path: For the ITM path, the asset price starts near the strike price and rises, ending above it. Initially, the option position is substantial due to higher Gamma sensitivity but decreases as the option moves deeper in-the-money and Delta approaches 1. This decline reflects reduced Gamma adjustments, with the underlying asset position steadily increasing to match the higher Delta. These dynamics are shown in Figure 12.

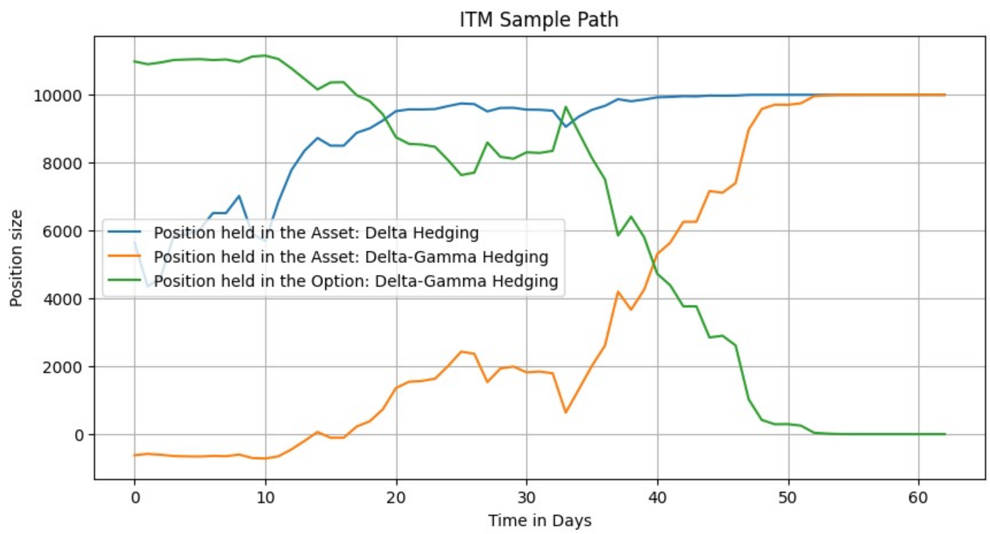

FIGURE 12. Delta-Gamma Hedging for In-the-Money Sample Path.

Observations for Out-of-the-Money (OTM) Path: For the OTM path, the asset price starts near the strike price and steadily declines, ending well below it. As the option moves further out-of-the-money, Delta and Gamma approach zero, leading to a sharp decline in the option position and minimal adjustments to the underlying asset position. These dynamics are illustrated in Figure 13.

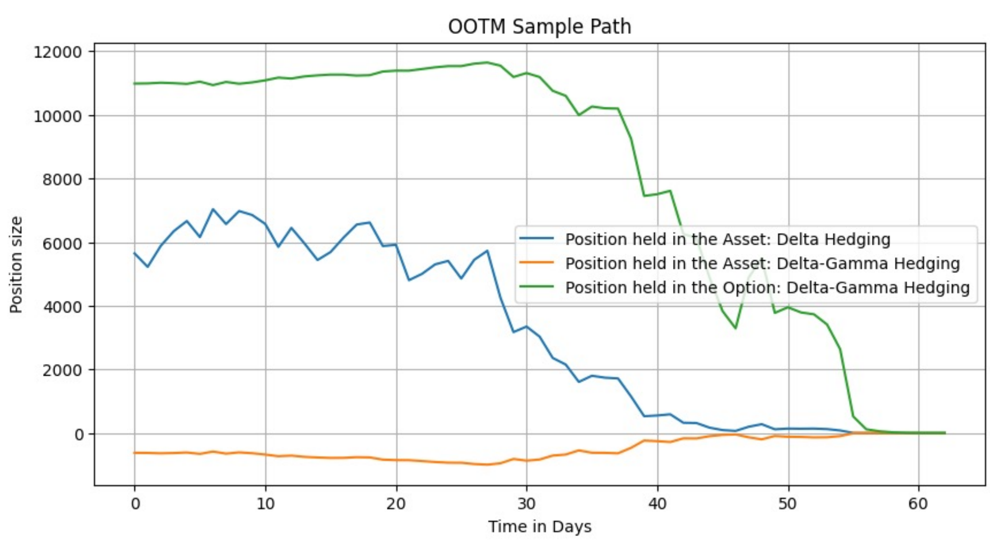

FIGURE 13. Delta-Gamma Hedging for Out-of-the Money Sample Path.

Comparison of ITM and OTM Hedging: The ITM and OTM paths demonstrate distinct behaviors in Delta-Gamma hedging due to the varying sensitivities of the option. For the ITM path, as the option becomes deeper in-the-money near expiration, Delta approaches 1, and Gamma decreases, reducing the need for frequent adjustments to the hedging position. This results in a steady increase in the position held in the underlying asset, while the position in the option decreases as the option's sensitivity stabilizes. In contrast, for the OTM path, the option's Delta and Gamma both approach zero as the option moves further out-of-the-money, reflecting diminished sensitivity to changes in the underlying asset price. This results in a consistent reduction in the option position over time, while the position in the underlying asset remains relatively stable due to the minimal impact of Delta and Gamma adjustments in this region.

These observations highlight how Delta-Gamma hedging adapts to different market scenarios, accounting for the changing sensitivities of the option as it moves in or out of the money.

3.3. **Question 3.** For this question, we analyze the performance of the Delta and Delta-Gamma hedging strategies where the real-world volatility (σ) deviates from the assumed value of $\sigma = 25\%$. Specifically, real-world volatilities are varied in the range $\{20\%, 22\%, 24\%, 26\%, 28\%, 30\%\}$. To reflect this, we modify the volatility parameter used for simulating asset price paths via Geometric Brownian Motion (GBM), while keeping the volatility for the Black-Scholes pricing model fixed at $\sigma = 25\%$. The results, including profit and loss (P&L) distributions and summary statistics, are presented in Figures 14–19 and Tables 3 and 4.

Summary Statistics: Tables 3 and 4 summarize the mean P&L and standard deviation for both strategies:

- **Delta Hedging:** The mean P&L decreases from positive to negative as real-world volatility increases, with the standard deviation rising, indicating higher risk and sensitivity to volatility misspecification:
  (1) At lower volatilities ($\sigma = 20\%$ and $\sigma = 22\%$), Delta hedging generates positive mean P&L due to over-hedging, but effectiveness declines as volatility rises.
  (2) At $\sigma = 25\%$, Delta hedging performs moderately well, with near-zero mean P&L and higher variance compared to Delta-Gamma hedging, reflecting greater risk exposure.
  (3) At higher volatilities ($\sigma = 28\%$ and $\sigma = 30\%$), Delta hedging incurs significant losses, with mean P&L dropping sharply and standard deviation increasing substantially.

- **Delta-Gamma Hedging:** This strategy shows smaller losses and significantly lower variance, mitigating extreme losses through second-order sensitivities:
  (1) At lower volatilities ($\sigma = 20\%$ and $\sigma = 22\%$), Delta-Gamma hedging has small negative mean P&L but maintains low variance.
  (2) At $\sigma = 25\%$, Delta-Gamma hedging yields lower mean P&L than Delta hedging but significantly reduces variance, limiting risk exposure.
  (3) At higher volatilities ($\sigma = 28\%$ and $\sigma = 30\%$), Delta-Gamma hedging outperforms Delta hedging with smaller losses and lower variance, effectively handling non-linear sensitivities.

Table 3. Delta Hedging

$\mathbb{P}$ Volatility	Mean (μ)	Std Dev (σ)
0.20	297.1090	388.019
0.22	94.6173	374.322
0.24	-148.6630	371.214
0.26	-405.5150	431.724
0.28	-699.9470	509.180
0.30	-1161.1300	782.052

Table 4. Delta-Gamma Hedging

$\mathbb{P}$ Volatility	Mean (μ)	Std Dev (σ)
0.20	-339.408	54.7174
0.22	-347.721	64.8019
0.24	-358.948	69.7094
0.26	-368.435	84.9804
0.28	-377.511	92.9713
0.30	-372.934	159.1160

Profit and Loss Distributions: The P&L distributions for Delta and Delta-Gamma hedging across varying real-world volatilities ($\sigma = 20\%, 22\%, \ldots, 30\%$) illustrate the impact of model misspecification. As the real-world volatility deviates further from the assumed volatility of 25%, the performance of Delta hedging worsens, characterized by significantly higher variance and increasingly extreme losses. This effect is really shown at higher volatilities ($\sigma = 28\%$ and $\sigma = 30\%$).

In contrast, Delta-Gamma hedging appears to perform better as volatility deviates from the assumed value, with reduced variance and more stable outcomes. However, this improvement is largely an illusion, as the stability of Delta-Gamma hedging is not due to its own performance but rather because Delta hedging becomes substantially worse under these conditions. The trade-off between mean P&L and stability remains, with Delta-Gamma hedging consistently producing worse mean P&L but managing risk by limiting extreme losses.

See Figures 14–19 below for a visualization of these trends.

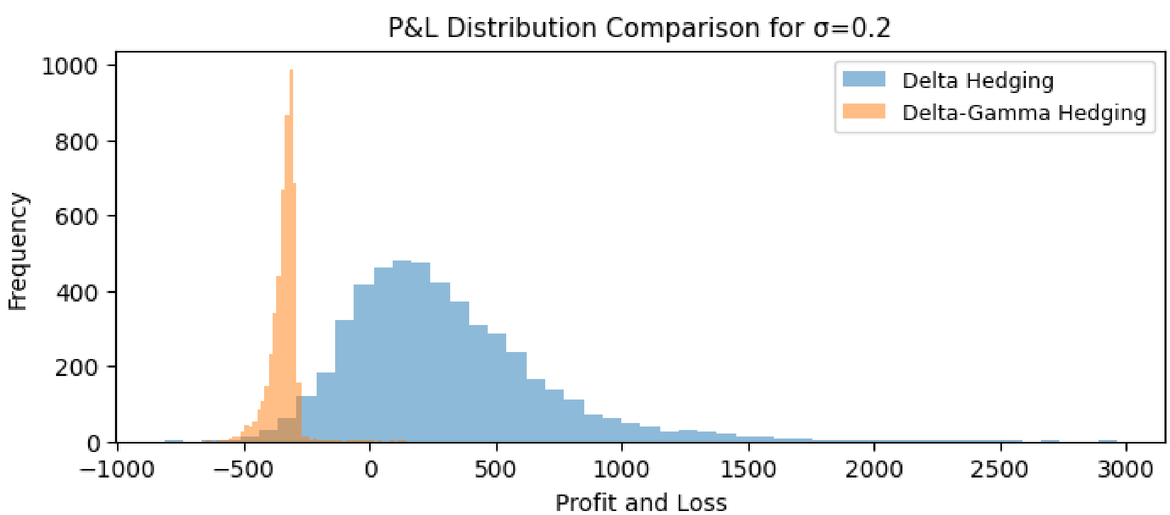

Figure 14. P&L Distribution Comparison for $\sigma = 20\%$.

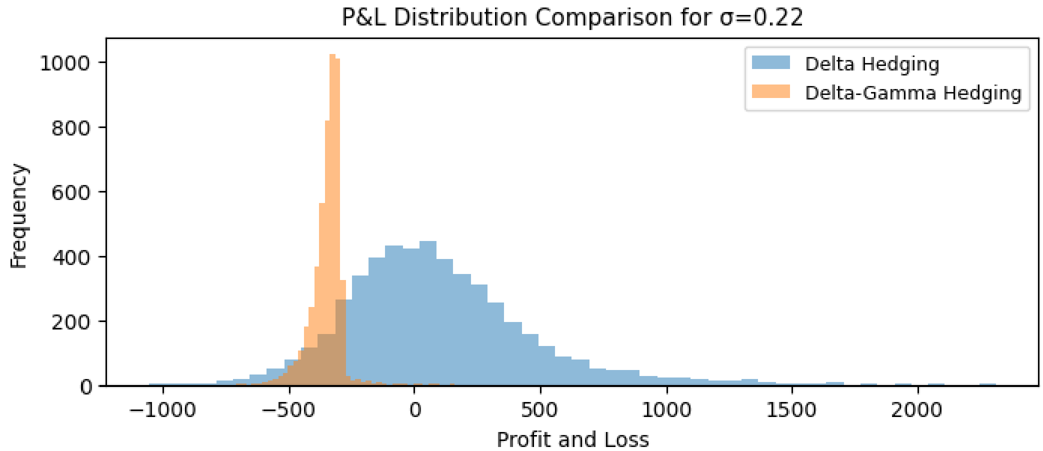

FIGURE 15. P&L Distribution Comparison for $\sigma = 22\%$.

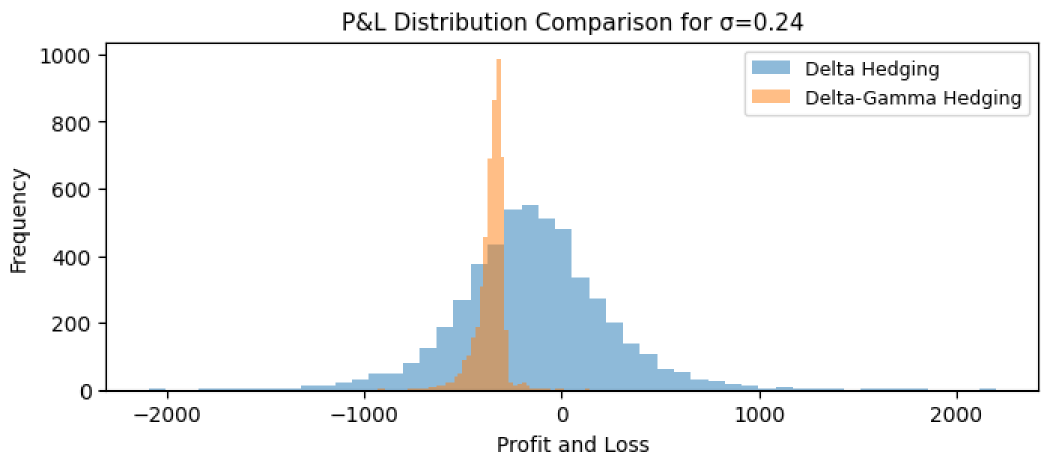

FIGURE 16. P&L Distribution Comparison for $\sigma = 24\%$.

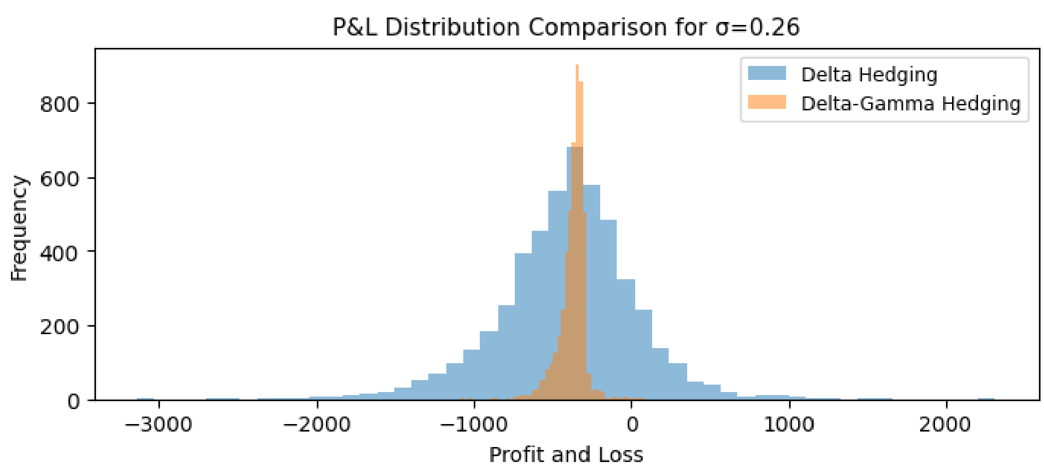

FIGURE 17. P&L Distribution Comparison for $\sigma = 26\%$.

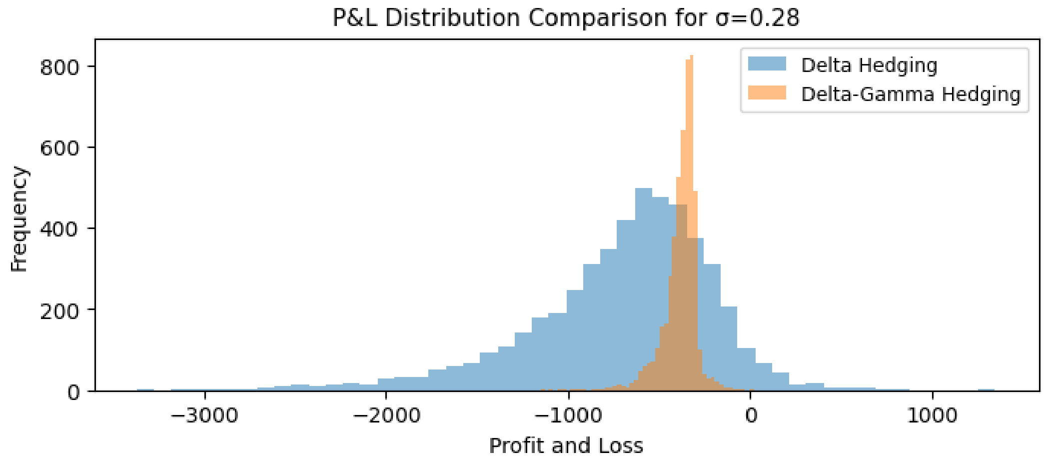

FIGURE 18. P&L Distribution Comparison for $\sigma = 28\%$.

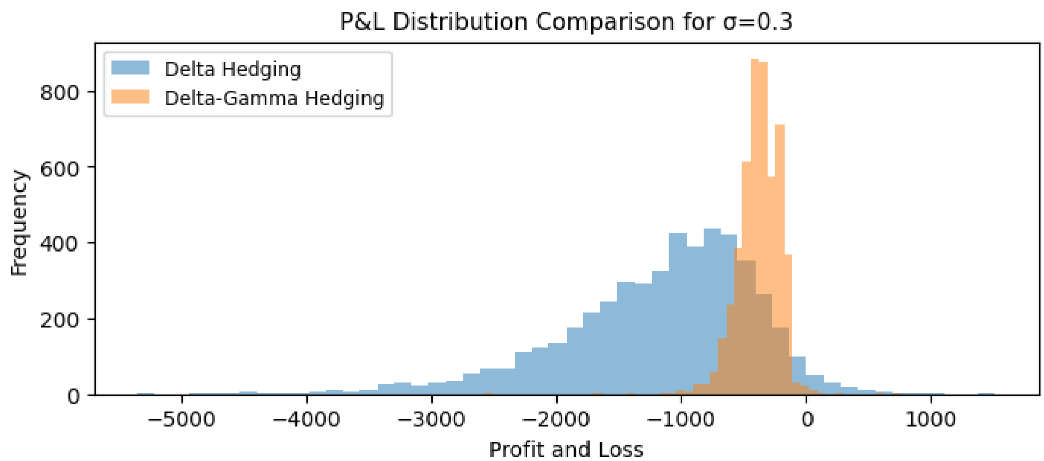

FIGURE 19. P&L Distribution Comparison for $\sigma = 30\%$.

4. CONCLUSION

This project explored the performance of Delta and Delta-Gamma hedging strategies under various conditions, including various drift assumptions, sample paths, and model misspecifications of real-world volatility. The analysis highlights the trade-offs between these strategies: Delta hedging offers higher potential returns but with greater risk, while Delta-Gamma hedging provides more stable outcomes with reduced variability, albeit at the cost of lower mean P&L. The results underscore the importance of incorporating second-order sensitivities for managing extreme losses and mitigating risk effectively.