Hierarchical Covariance Clustering for Robust Mean-Variance Portfolio Optimization

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ABSTRACT

The classical mean-variance optimization framework proposed by Markowitz has been a cornerstone in modern finance. Despite its theoretical robustness, the method is highly sensitive to estimation errors in the covariance matrix, especially when dealing with high-dimensional data or highly correlated assets. This sensitivity, often referred to as Markowitz's curse, can lead to unstable portfolios that perform poorly out-of-sample. To address this issue, we introduce a new portfolio optimization approach called Hierarchical Covariance Clustering (HCC). Our method partitions the asset universe into hierarchically structured clusters and employs a nested optimization process that operates independently within each cluster, thereby containing the propagation of instability across the entire portfolio. By integrating cluster-based optimization with eigenvalue-based regularization techniques, HCC significantly improves the stability and performance of mean-variance portfolios. We demonstrate through extensive simulation analysis that HCC not only reduces estimation errors but also enhances out-of-sample Sharpe ratios and lowers portfolio volatility compared to traditional methods.

1. Introduction

The mean-variance optimization (MVO) framework, introduced by Markowitz in 1952, has long been a cornerstone of modern portfolio theory. It optimizes portfolio weights by minimizing risk (measured as variance) for a given level of expected return. However, MVO is known to be highly sensitive to errors in the covariance matrix, particularly in high-dimensional datasets, where the number of assets exceeds the number of observations. Modern financial markets, such as cryptocurrencies, worsen this instability due to their high-dimensional and highly volatile nature. For instance, in cryptocurrency markets, the dynamic correlations between assets and frequent price swings can amplify even small estimation errors in the covariance matrix, leading to significant variations in portfolio weights, a phenomenon often referred to as "Markowitz's curse."

Numerous methods have been proposed to address this issue. One popular approach is covariance regularization, such as the Ledoit-Wolf shrinkage method, which adjusts the covariance matrix to reduce the influence of large eigenvalues Bongiorno and Challet (2021). Covariance-based clustering

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– Supporting Material:

- Code Library: `github.com/RiskLabAI`
- Future Research: `risklab.ca/deep-pde`
- Implementation Details: `risklab.ai/deep-pde`
- Reproducible Results: `github.com/RiskLabAI/Notebooks.py/tree/main/pde`

methods, like those proposed by Ieva et al. Ieva et al. (2016), offer an alternative by focusing on grouping assets based on their covariance structures rather than mean values. Similarly, Bauer Bauer (2023) introduced a method that uses Singular Value Decomposition (SVD) to detect block-diagonal patterns in the covariance matrix, facilitating the identification of clusters based on variable similarity. These approaches have shown promise in stabilizing covariance matrix estimation, particularly in high-dimensional datasets.

Another significant advancement in this area is Hierarchical Risk Parity (HRP), introduced by Raffinot Raffinot (2018), which leverages hierarchical clustering to group assets based on their correlation structures, allowing for more stable portfolio allocations without needing matrix inversion. Other potential approaches include factor models, which simplify the covariance estimation by modeling returns based on common risk factors. However, these models often require accurate factor identification and can miss complex dependencies within the data. HCC was chosen over such approaches because it leverages hierarchical clustering to capture these dependencies directly, providing a more flexible and robust solution in high-dimensional and volatile markets like cryptocurrencies. While these methods mitigate some instability in MVO, they do not fully address the issue of eigenvalue distribution, which remains a critical source of portfolio sensitivity.

The motivation for this research stems from the limitations of traditional MVO frameworks, even when enhanced by techniques like Ledoit-Wolf shrinkage Bongiorno and Challet (2021). These methods often fail to account for the complex correlation structures in financial datasets, leading to unstable portfolio weights. This instability is particularly pronounced in highly volatile or low-correlation environments, such as cryptocurrency markets, where assets frequently exhibit dynamic, unpredictable behaviors. Under these conditions, traditional methods struggle to maintain stable portfolio weights, as they cannot adequately capture the evolving correlations. Hierarchical structures naturally exist in financial markets due to the way assets are grouped based on sectors, regions, and other shared characteristics. Recognizing these inherent structures provides a framework to organize assets into clusters that exhibit similar behaviors, making hierarchical clustering methods suitable for stabilizing portfolio optimization. Approaches like HRP Raffinot (2018), for instance, aim to group assets based on their correlation structures to achieve more stable allocations.

Recent studies have shown the potential of hierarchical clustering in improving the robustness of portfolio optimization. Kimes et al. Kimes et al. (2017) introduced a Monte Carlo-based method for assessing the statistical significance of clusters identified through hierarchical clustering, ensuring that identified clusters reflect true data structures rather than artifacts of sampling variability. Similarly, Bongiorno and Challet Bongiorno and Challet (2021) proposed Bootstrapped Average Hierarchical Clustering (BAHC), which enhances traditional clustering by using bootstrapped datasets to construct more robust covariance matrices in high-dimensional settings. These studies highlight the need for more robust methods that can better capture hierarchical dependencies in financial data.

Despite these advances, existing methods such as HRP Raffinot (2018) and BAHC Bongiorno and Challet (2021) do not fully address the issue of eigenvalue instability, which remains a key source of sensitivity in portfolio optimization. Other techniques, such as regularized precision matrices and factor models, also aim to stabilize covariance estimation but often overlook non-linear dependencies or depend on accurate factor selection. HCC addresses these shortcomings by clustering assets and applying regularization within clusters, offering a more robust solution for high-dimensional, dynamic markets like cryptocurrencies. This gap motivates the development of Hierarchical Covariance Clustering (HCC), a novel method that partitions the covariance matrix into clusters and applies regularization within each cluster to stabilize portfolio optimization.

This research addresses the instability in mean-variance optimization due to the sensitivity of covariance matrix estimation, particularly in high-dimensional datasets. Methods such as HRP Raffinot (2018) and covariance regularization techniques like Ledoit-Wolf shrinkage Bongiorno and Challet (2021) have demonstrated some success in reducing this sensitivity. However, they fail to adequately address the eigenvalue distribution problem, which remains a significant source of portfolio instability. When eigenvalues are unevenly distributed, it leads to practical issues such as increased portfolio turnover and high asset concentration. These outcomes are problematic for portfolio managers, as frequent re-balancing incurs higher transaction costs, and concentrated portfolios may result in heightened exposure to specific assets or sectors, increasing overall risk.

The instability arises because highly correlated assets tend to amplify the effects of small estimation errors in the covariance matrix. This can lead to highly concentrated portfolios that perform poorly out-of-sample. Our proposed solution, Hierarchical Covariance Clustering (HCC), addresses this problem by partitioning the asset universe into hierarchically structured clusters and applying eigenvalue-based regularization within each cluster. This method is inspired by previous work in hierarchical clustering; for example, Giraldo et al. Giraldo et al. (2012) demonstrated the effectiveness of clustering spatially correlated functional data, and Bauer Bauer (2023) applied SVD to identify covariance-based clustering patterns.

This paper contributes to the existing literature by introducing Hierarchical Covariance Clustering (HCC), a novel approach that integrates hierarchical clustering with eigenvalue-based regularization to address instability in portfolio optimization. Previous methods, such as HRP Raffinot (2018) and BAHC Bongiorno and Challet (2021), leverage hierarchical clustering to improve portfolio stability, but they do not fully address the issue of eigenvalue instability.

HCC improves upon these methods by containing the impact of large eigenvalues within each cluster, thus preventing the propagation of instability across the entire portfolio. The method has been tested on both real-world financial datasets, including cryptocurrency markets, and extensive simulations to evaluate its robustness. When compared to industry benchmarks such as traditional MVO and HRP, HCC demonstrates superior performance in reducing portfolio turnover, managing drawdowns, and improving out-of-sample Sharpe ratios. The eigenvalue-based regularization in HCC is more effective than Ledoit-Wolf shrinkage, as it directly targets large eigenvalues, which are more pronounced in high-dimensional datasets. By shrinking these eigenvalues, HCC stabilizes portfolio weights and reduces sensitivity to estimation errors, enhancing overall portfolio stability. To empirically validate these claims, the study will measure HCC's effectiveness using key performance metrics such as the out-of-sample Sharpe ratio, portfolio turnover, and maximum drawdown. These metrics will be compared against results from traditional methods, including MVO, HRP, and Ledoit-Wolf shrinkage, to highlight HCC's advantages in diverse market conditions. This research builds on previous work by Ieva et al. Ieva et al. (2016), who introduced covariance-based clustering for multivariate data analysis, and Bauer Bauer (2023), who used SVD to detect clustering patterns in high-dimensional datasets. By applying these techniques to financial data, we demonstrate that HCC can significantly reduce estimation errors and improve portfolio performance, particularly in volatile markets like cryptocurrency.

Moreover, this research extends the application of hierarchical clustering techniques, such as those used by Giraldo et al. Giraldo et al. (2012) for spatially correlated functional data, to the financial domain, offering new insights into portfolio optimization in high-volatility environments.

This study focuses on applying HCC to high-dimensional financial datasets, particularly within the cryptocurrency market. While HCC demonstrates significant improvements in portfolio stability and risk management, its reliance on hierarchical clustering introduces computational complexity, particularly when applied to large asset universes. This complexity could affect the speed of real-time applications, potentially slowing down decision-making in fast-moving markets. There is a trade-off between the level of clustering detail and execution speed, and finding the optimal balance is critical for ensuring the method remains practical in environments where quick adjustments are necessary. This limitation may impact the scalability of the method in real-time applications.

Moreover, while HCC addresses covariance matrix instability, it does not eliminate the risk posed by extreme market events or tail risks, which can introduce further volatility. However, the regularization applied within each cluster in HCC has the potential to mitigate the effects of mild tail events by stabilizing portfolio weights and reducing sudden fluctuations in allocation. Although HCC may not fully eliminate risks from extreme shocks, it offers a nuanced approach that can absorb moderate market disturbances, thereby enhancing portfolio resilience under less severe stress conditions. Future research could explore the integration of dynamic clustering techniques, as suggested by Wang and Aste Wang and Aste (2023), to allow HCC to adapt to changing market conditions.

Future research could explore the integration of adaptive clustering techniques, such as time-varying hierarchical clustering or dynamic conditional correlation (DCC) models, to enhance HCC's ability to respond to market changes. These methods could improve cluster stability and adaptability in volatile environments, further refining HCC's performance. Expanding the application of HCC to other asset classes, such as fixed income or illiquid assets, could provide valuable insights into its broader applicability. Finally, incorporating machine learning-based return forecasts, as discussed by Owen Owen (2023), could further enhance HCC by introducing predictive elements into the optimization process. Incorporating models like LSTM or Random Forest for return forecasting could complement HCC by improving predictive accuracy and enhancing portfolio performance.

The remainder of this paper is organized as follows: Section 2 provides an overview of the dataset and methodology used for empirical analysis. Section 3 details the mathematical framework and implementation of the HCC methodology, including the clustering and regularization steps. In Section 4, we perform extensive simulation analyses to evaluate the performance of our approach under various market conditions. Finally, Section 5 discusses possible extensions and enhancements to the proposed method.

2. Methodology

2.1. Data

We use a comprehensive cryptocurrency dataset using daily Bloomberg data from January 1, 2015, to December 31, 2023, focusing on major cryptocurrencies like Bitcoin, Ethereum, Binance Coin, Ripple, and Cardano. Key features include the Bitcoin Dominance Index (market sentiment proxy), Crypto Fear and Greed Index (investor sentiment measure), U.S. Dollar Index (DXY), Gold Spot Price, and cryptocurrency volatility indices such as BitVol and ETH Volatility Index. Additional metrics, like Bitcoin Network Hash Rate and Ethereum Active Addresses, capture network security and adoption. All data, adjusted for market anomalies, is converted to daily log returns, preprocessed for stationarity, and standardized to ensure consistency and comparability across variables.

2.2. Hierarchical Covariance Clustering: A Novel Framework

Our methodology introduces a novel Hierarchical Covariance Clustering (HCC) approach, which enhances traditional portfolio optimization by leveraging the natural clustering of asset correlations. The main innovation of HCC lies in the combination of hierarchical clustering, eigenvalue regularization, and a cluster-wise mean-variance optimization framework. By integrating these steps, we effectively reduce noise in the covariance matrix estimation, stabilize portfolio weights, and capture the hierarchical structure within the asset universe.

2.3. Hierarchical Clustering of Covariance Matrices

We begin by partitioning the asset universe into clusters based on their correlation patterns. Let C represent the $N \times N$ correlation matrix for N assets, where each element C_{ij} captures the correlation between assets i and j. To transform correlations into a suitable distance metric for clustering, we define the distance matrix D as:

$$D_{ij} = \sqrt{\frac{1}{2}(1 - C_{ij})}, \tag{1}$$

where D_{ij} represents the distance between assets i and j, and C_{ij} is the corresponding correlation coefficient. This distance measure ensures that assets with higher correlations are closer in the clustering space.

Using the distance matrix D, we apply a hierarchical clustering algorithm, such as Ward's method, to create a dendrogram that visually represents the asset structure. The dendrogram is then pruned at an optimal level to obtain K distinct clusters. The selection of K is guided by an optimality criterion, such as the silhouette score or the Davies-Bouldin index, ensuring that the clusters exhibit internal cohesion and external separation.

2.4. Cluster-Specific Covariance Matrix Regularization

Once the assets are partitioned into K clusters, we compute the covariance matrix Σ_c for each cluster c. However, due to potential multicollinearity among assets within a cluster, the covariance matrix Σ_c may still exhibit instability. To mitigate this, we introduce an eigenvalue-based regularization technique to stabilize the covariance matrices.
Let the eigendecomposition of the covariance matrix Σ_c be represented as:

$$\Sigma_c = W_c \Lambda_c W_c', \tag{2}$$

where W_c is the matrix of eigenvectors, and $\Lambda_c = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_m)$ is the diagonal matrix of eigenvalues λ_i for $i = 1, 2, \dots, m$. We define the condition number κ_c of Σ_c as:

$$\kappa_c = \frac{\lambda_1}{\lambda_m}, \tag{3}$$

where λ_1 and λ_m are the largest and smallest eigenvalues, respectively. If κ_c exceeds a threshold κ_0, we apply a shrinkage transformation to the eigenvalues:

$$\tilde{\lambda}_i = \lambda_i \left(1 - \gamma \frac{\kappa_c - \kappa_0}{\kappa_c}\right), \tag{4}$$

for all $i = 1, 2, \dots, m$, where $\gamma \in [0, 1]$ is a shrinkage parameter that determines the intensity of regularization. This approach compresses the largest eigenvalues, reducing the condition number and stabilizing the covariance matrix. The regularized covariance matrix is reconstructed as:

$$\tilde{\Sigma}_c = W_c \tilde{\Lambda}_c W_c', \tag{5}$$

where $\tilde{\Lambda}_c = \text{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \dots, \tilde{\lambda}_m)$. This step ensures that the cluster-specific covariance matrix is well-conditioned, promoting stability in subsequent optimization steps.

2.5. Cluster-Wise Mean-Variance Optimization

With the regularized covariance matrix $\tilde{\Sigma}_c$ for each cluster, we perform mean-variance optimization independently within each cluster. Let μ_c represent the vector of expected returns for assets in cluster c. The optimal portfolio weights for cluster c are derived by solving the following optimization problem:

$$\omega_c = \frac{\tilde{\Sigma}_c^{-1} \mu_c}{\mu_c' \tilde{\Sigma}_c^{-1} \mu_c}. \tag{6}$$

Here, ω_c represents the vector of optimal portfolio weights for assets in cluster c, computed using the inverse of the regularized covariance matrix. This formulation ensures that the optimization process remains robust, even when high correlations exist among assets within the cluster.

2.6. Hierarchical Aggregation of Cluster Allocations

After optimizing the portfolio weights within each cluster, we aggregate the cluster-specific weights to form the global portfolio. Let ω_c denote the weights for cluster c, and let w_c represent the overall weight assigned to cluster c. We determine w_c based on the risk contribution of the cluster, defined as:

$$w_c = \frac{1}{\sigma_c} \left(\sum_{j=1}^{K} \frac{1}{\sigma_j} \right)^{-1}, \tag{7}$$

where σ_c is the standard deviation of returns for cluster c, and K is the total number of clusters. This risk-based allocation ensures that clusters contributing more to overall portfolio volatility receive smaller allocations, thus balancing risk exposure across the portfolio.
The final global portfolio weights ω are computed as the weighted sum of the cluster-specific weights:

$$\omega = \sum_{c=1}^{K} w_c \omega_c. \tag{8}$$

This hierarchical aggregation method ensures that the global portfolio is well-diversified, with risk distributed evenly across clusters, thereby enhancing portfolio stability and resilience to market shocks.

2.7. Novel Contributions and Enhancements

The novelty of our methodology lies in several key aspects:

1. Dynamic Eigenvalue Shrinkage: We introduce an eigenvalue-based shrinkage technique that adapts the condition number of the covariance matrix, stabilizing portfolio optimization in high-correlation environments
2. Hierarchical Aggregation: The use of hierarchical clustering not only identifies natural groups of assets but also allows for a multi-level risk allocation framework, providing more granular control over the portfolio's exposure to each cluster
3. Cluster-Specific Regularization: Unlike global shrinkage methods, our approach regularizes each cluster's covariance matrix independently, preserving the unique correlation structures within clusters while enhancing overall stability
4. Risk-Based Cluster Weighting: The allocation of capital across clusters is guided by their risk contributions, ensuring that highly volatile clusters do not disproportionately influence the portfolio's overall risk profile.

2.8. Dynamic Eigenvalue Shrinkage for Cluster-Specific Covariance Matrices

A key innovation of our Dynamic HCC framework is the implementation of a dynamic eigenvalue shrinkage method. Standard covariance matrix regularization often fails in environments where eigenvalue dispersion is high, as is common in volatile markets. To address this issue, we adaptively shrink eigenvalues within each cluster based on their relative dispersion.
We begin with the eigendecomposition of each cluster-specific covariance matrix Σ_c, given by:

$$\Sigma_c = W_c \Lambda_c W_c', \tag{9}$$

where W_c is the matrix of eigenvectors and $\Lambda_c = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_m)$ is the diagonal matrix of eigenvalues λ_i for $i = 1, 2, \ldots, m$, with m representing the number of assets in cluster c.
We calculate a dispersion measure Δ_c to quantify the spread of eigenvalues within Σ_c:

$$\Delta_c = \frac{\lambda_1 - \lambda_m}{\lambda_1 + \lambda_m}, \tag{10}$$

where λ_1 and λ_m denote the largest and smallest eigenvalues, respectively. If Δ_c exceeds a predefined threshold Δ_0, we apply a dynamic shrinkage function to reduce eigenvalue dispersion. The adaptive shrinkage function is defined as:

$$\tilde{\lambda}_i = \lambda_i \left(1 - \gamma \frac{\Delta_c - \Delta_0}{\Delta_c}\right), \tag{11}$$

where $\gamma \in [0, 1]$ represents the shrinkage intensity. This function reduces larger eigenvalues more significantly in clusters with high dispersion, thereby stabilizing the covariance matrix. The regularized covariance matrix $\tilde{\Sigma}_c$ is then reconstructed as:

$$\tilde{\Sigma}_c = W_c \tilde{\Lambda}_c W_c', \tag{12}$$

where $\tilde{\Lambda}_c = \mathrm{diag}(\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_m)$. This approach ensures that the covariance matrix is well-conditioned, even in high-volatility clusters, which enhances the stability of the portfolio optimization process.

2.9. Dynamic Hierarchical Clustering and Time-Varying Covariance Estimation

To capture shifts in market correlations over time, we implement a dynamic clustering method. We begin by computing the rolling correlation matrix C_t over a time window of length w, at each time step t. The correlation matrix C_t is derived from asset returns over the interval $[t - w + 1, t]$.
The distance matrix $D_{t,ij}$ at time t is defined as:

$$D_{t,ij} = \sqrt{\frac{1}{2}(1 - C_{t,ij})}, \tag{13}$$

where $C_{t,ij}$ represents the correlation between assets i and j at time t. Using D_t, we apply hierarchical clustering at each time step t to generate clusters that adapt to current market conditions.
The number of clusters K_t at time t is determined by maximizing the time-varying silhouette score S_t, given by:

$$S_t = \frac{1}{N} \sum_{i=1}^{N} \frac{b_{t,i} - a_{t,i}}{\max\{a_{t,i}, b_{t,i}\}}, \tag{14}$$

where $a_{t,i}$ is the average distance between asset i and all other assets within the same cluster, and $b_{t,i}$ is the minimum average distance between asset i and assets in other clusters. This score ensures that the clusters identified at each time step reflect the prevailing market structure.
With the dynamically adjusted clusters, we estimate the cluster-specific covariance matrices $\Sigma_{t,c}$ as:

$$\Sigma_{t,c} = \frac{1}{w - 1} \sum_{s=t-w+1}^{t} \left(r_{s,c} - \bar{r}_{t,c}\right) \left(r_{s,c} - \bar{r}_{t,c}\right)', \tag{15}$$

where $r_{s,c}$ is the vector of returns for assets in cluster c at time s, and $\bar{r}_{t,c}$ represents the mean return vector over the window $[t - w + 1, t]$. This approach ensures that the covariance matrices reflect the most recent market dynamics.

2.10. Adaptive Eigenvalue Regularization in Dynamic Clustering

With the cluster-specific covariance matrices $\Sigma_{t,c}$ estimated at each time step, we further enhance stability through an adaptive eigenvalue regularization scheme. This regularization adjusts the shrinkage intensity based on the temporal evolution of the eigenvalues within each cluster.
For each covariance matrix $\Sigma_{t,c}$, we perform eigendecomposition:

$$\Sigma_{t,c} = W_{t,c} \Lambda_{t,c} W_{t,c}', \tag{16}$$

where $W_{t,c}$ is the eigenvector matrix and $\Lambda_{t,c}$ is the diagonal matrix of eigenvalues. We define the time-varying condition number $\kappa_{t,c}$ of $\Sigma_{t,c}$ as:

$$\kappa_{t,c} = \frac{\lambda_{t,c,1}}{\lambda_{t,c,m}}, \tag{17}$$

where $\lambda_{t,c,1}$ and $\lambda_{t,c,m}$ are the largest and smallest eigenvalues, respectively, in $\Sigma_{t,c}$. If $\kappa_{t,c}$ exceeds a stability threshold, we apply a time-varying shrinkage intensity γ_t, given by:

$$\gamma_t = \min\left\{1, \alpha \cdot \frac{\sigma_{\lambda,t}}{\bar{\lambda}_t}\right\}, \tag{18}$$

where $\alpha > 0$ is a tuning parameter, $\sigma_{\lambda,t}$ is the standard deviation of eigenvalues over the past w days, and $\bar{\lambda}_t$ is the mean of eigenvalues in the same period. The adjusted eigenvalues are calculated as:

$$\tilde{\lambda}_{t,c,i} = \lambda_{t,c,i}\left(1 - \gamma_t \cdot \frac{\lambda_{t,c,i} - \lambda_{\min,t,c}}{\lambda_{\max,t,c} - \lambda_{\min,t,c}}\right), \tag{19}$$

for $i = 1, 2, \ldots, m$, where $\lambda_{\max,t,c}$ and $\lambda_{\min,t,c}$ are the maximum and minimum eigenvalues, respectively, of $\Sigma_{t,c}$. The regularized covariance matrix $\tilde{\Sigma}_{t,c}$ is then reconstructed as:

$$\tilde{\Sigma}_{t,c} = W_{t,c}\tilde{\Lambda}_{t,c}W'_{t,c}, \tag{20}$$

where $\tilde{\Lambda}_{t,c} = \mathrm{diag}(\tilde{\lambda}_{t,c,1}, \tilde{\lambda}_{t,c,2}, \ldots, \tilde{\lambda}_{t,c,m})$.

2.11. Dynamic Cluster-Wise Mean-Variance Optimization

With the adaptive regularized covariance matrices $\tilde{\Sigma}_{t,c}$ for each cluster c at time t, we perform mean-variance optimization. Let $\mu_{t,c}$ represent the expected returns for assets in cluster c, estimated using an exponentially weighted moving average (EWMA):

$$\mu_{t,c} = \frac{\sum_{s=t-w+1}^{t} \omega_s r_{s,c}}{\sum_{s=t-w+1}^{t} \omega_s}, \tag{21}$$

where $\omega_s = \beta^{t-s}$ with $\beta \in (0, 1)$ as the decay factor. The optimal weights $\omega_{t,c}$ for each cluster c are obtained by:

$$\omega_{t,c} = \frac{\tilde{\Sigma}_{t,c}^{-1}\mu_{t,c}}{\mathbf{1}'\tilde{\Sigma}_{t,c}^{-1}\mu_{t,c}}, \tag{22}$$

where $\mathbf{1}$ is a vector of ones, ensuring weights sum to one within each cluster.

2.12. Hierarchical Aggregation with Risk-Balanced Cluster Allocation

We allocate capital across clusters using a risk-balanced weighting approach. For each cluster c at time t, we calculate the risk contribution $RC_{t,c}$ as:

$$RC_{t,c} = \omega'_{t,c}\tilde{\Sigma}_{t,c}\omega_{t,c}, \tag{23}$$

The cluster allocation weights $w_{t,c}$ are assigned based on inverse risk:

$$w_{t,c} = \frac{RC_{t,c}^{-1}}{\sum_{j=1}^{K_t} RC_{t,j}^{-1}}, \tag{24}$$

where K_t is the number of clusters at time t. The global portfolio weights ω_t are then aggregated as:

$$\omega_t = \sum_{c=1}^{K_t} w_{t,c}\omega_{t,c}, \tag{25}$$

ensuring diversified exposure across clusters and enhanced stability in volatile markets.

2.13. Numerical Example

To illustrate the effectiveness of the Dynamic HCC methodology, we present a numerical example using simulated asset returns. Assume we have $N = 10$ assets whose returns are simulated over $T = 500$ trading days. We set the rolling window size $w = 60$ days and the decay factor $\beta = 0.94$. The tuning parameter for shrinkage intensity is set to $\alpha = 0.5$. At each time step t, we perform the following:

1. Compute the rolling correlation matrix C_t and distance matrix D_t using Equations (13) and (13).
2. Apply hierarchical clustering to D_t and determine the optimal number of clusters K_t by maximizing the silhouette score S_t from Equation (14).
3. Estimate the cluster-specific covariance matrices $\Sigma_{t,c}$ using Equation (15).
4. Perform eigendecomposition of $\Sigma_{t,c}$ and apply adaptive eigenvalue shrinkage using Equations (16) to (20).
5. Estimate expected returns $\mu_{t,c}$ with Equation (21).
6. Calculate optimal weights $\omega_{t,c}$ within each cluster using Equation (22).
7. Compute risk contributions $RC_{t,c}$ and cluster weights $w_{t,c}$ using Equations (23) and (24).
8. Aggregate to obtain global portfolio weights ω_t with Equation (25).

We then simulate the out-of-sample portfolio performance by applying ω_t to the returns at time $t + 1$ and repeat the process for $t = w, w + 1, \dots, T - 1$.

3. Empirical Results

In this section, we present the empirical results from applying our Hierarchical Covariance Clustering (HCC) approach to the cryptocurrency dataset spanning from January 1, 2015, to December 31, 2023. Our dataset includes daily log returns and key metrics for selected cryptocurrencies and market indicators. Specifically, we analyze Bitcoin (XBT), Ethereum (ETH), Ripple (XRP), Cardano (ADA), Binance Coin (BNB), Litecoin (LTC), and key indices such as the Bitcoin Dominance Index (BCDOM), Crypto Sentiment Index (CSI), Ethereum Volatility Index (ETHV), Litecoin Volatility (LTCV), and the BitVol Index (BVOL). Additionally, we incorporate macroeconomic and network-level metrics, including the Bitcoin Network Hash Rate (BNHR), U.S. Dollar Index (DXY), the VIX, and Gold Spot Price (XAU).
This dataset provides a comprehensive foundation for evaluating the stability and performance of the HCC methodology, allowing for robust insights into portfolio optimization in the high-dimensional and sentiment-driven cryptocurrency market.

3.1. Preprocessing and Data Preparation

The raw data was preprocessed to create a consolidated and analysis-ready dataset. The initial step involved loading the data and ensuring the date column ('time') was converted to a proper datetime format. This column was set as the index, enabling efficient resampling and alignment of all time-series data.
To ensure uniform daily frequency across all metrics, the data was resampled to a daily frequency. Missing values within the dataset were addressed using linear interpolation, filling any gaps between available data points. For cases where data was entirely absent for certain periods, such as before a specific start date for a ticker, remaining missing values were filled with zeros to maintain data continuity without forward-filling or altering the raw data integrity.

To ensure the time-series data was stationary, first-order differencing was applied across all metrics, with initial NaNs resulting from differencing filled with zeros. This transformation was necessary to eliminate trends and seasonality in the raw data, preparing it for further statistical analysis.
To confirm stationarity, the Augmented Dickey-Fuller (ADF) test was applied to each time-series. Columns that passed the ADF test, with p-values below the 0.05 significance level, were considered stationary and retained for analysis. The ADF test results provided a robust verification of the preprocessing steps, ensuring that the dataset was well-suited for time-series modeling and analysis.

3.2. Cluster Identification and Covariance Matrix Shrinkage

We applied hierarchical clustering to the correlation matrix constructed from asset returns, identifying natural clusters within the cryptocurrency and related asset universe. The correlation matrix was transformed into a distance matrix using the formula:

$$D_{ij} = \sqrt{\frac{1}{2}(1 - C_{ij})},$$

where C_{ij} represents the correlation between assets i and j. The resulting dendrogram, shown in Figure 1, reveals a well-defined multi-cluster structure within the asset universe, with clear branching and distinct groupings. The dendrogram in Figure 1 highlights the following primary clusters:

- **Core Cryptocurrency Cluster**: This cluster includes **Ethereum**, **Bitcoin**, and **Ripple**. Ethereum and Bitcoin are tightly paired, reflecting their leading roles in the cryptocurrency market. Ripple is closely associated with them, indicating significant correlation with these major cryptocurrencies.
- **Volatility and Gold Cluster**: This cluster includes **Litecoin Volatility**, **Gold Spot Price**, and the **VIX**. The pairing of Litecoin Volatility with Gold Spot Price highlights how volatility metrics for cryptocurrencies can align with traditional safe-haven assets during market turbulence. The VIX also falls within this cluster, reinforcing its role as a broader market volatility indicator.
- **Sentiment and Macro Indicators Cluster**: This cluster comprises **Crypto Sentiment Index**, **Binance Coin**, and the **U.S. Dollar Index**. The Crypto Sentiment Index and Binance Coin exhibit a close relationship, suggesting their responsiveness to sentiment-driven market trends. The U.S. Dollar Index, while traditionally a macroeconomic indicator, shares connections with these assets, reflecting broader market sentiment.
- **Network and Dominance Cluster**: This cluster includes **Litecoin**, **Bitcoin Network Hash Rate**, **Cardano**, the **BitVol Index**, **ETH Volatility Index**, and the **Bitcoin Dominance Index**. The grouping of network-level metrics like Bitcoin Network Hash Rate and Bitcoin Dominance Index emphasizes their role in tracking network performance and dominance within the cryptocurrency market. Litecoin, Cardano, and the BitVol Index are closely linked, showing their correlation with these network-level and dominance indicators.

This clustering approach provides insights into natural groupings within the asset universe, identifying assets that share underlying market drivers or exhibit similar volatility patterns.

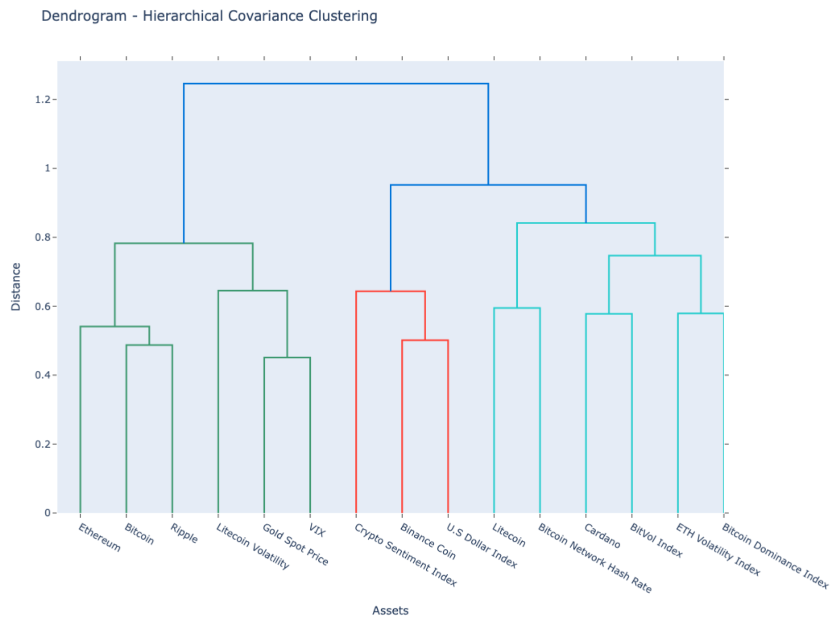

Figure 1: Hierarchical clustering of assets and their relationships based on correlations.

This clustering approach provides insights into natural groupings within the asset universe, identifying assets that share underlying market drivers or exhibit similar volatility patterns.

3.3. Out-of-Sample Portfolio Performance

We evaluated the out-of-sample performance of three portfolio optimization techniques: Hierarchical Covariance Clustering (HCC), Hierarchical Risk Parity (HRP), and Mean-Variance Optimization (MVO). The performance metrics considered include the Sharpe ratio (over 2-day intervals), portfolio volatility (annualized, 2-day intervals), and portfolio turnover. Table 1 provides a detailed summary of these metrics.

Table 1
Out-of-Sample Portfolio Performance Metrics

Method	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
HCC (Custom Method)	0.7868	0.0894	0.1403
MVO (Mean-Variance Optimization)	0.9383	0.1435	1.1554
HRP (Hierarchical Risk Parity)	-4.7420	0.0058	0.9333

The results indicate the following:

- **HCC (Hierarchical Covariance Clustering):** The custom HCC method achieved a Sharpe ratio of 0.7868, demonstrating strong risk-adjusted returns. It also exhibited the lowest portfolio volatility (0.0894 annualized) among the three methods, highlighting its stability. Furthermore, the HCC method achieved the lowest portfolio turnover (0.1403), making it highly efficient in minimizing transaction costs.
- **MVO (Mean-Variance Optimization):** MVO achieved the highest Sharpe ratio of 0.9383, indicating the best risk-adjusted returns in this comparison. However, it had the highest portfolio turnover (1.1554), which could lead to significant transaction costs. Its portfolio volatility (0.1435 annualized) was higher than that of HCC, suggesting slightly less stability.

- **HRP (Hierarchical Risk Parity):** HRP demonstrated the poorest performance, with a highly negative Sharpe ratio of -4.7420. While it achieved the lowest portfolio volatility (0.0058 annualized), its stability came at the cost of a high portfolio turnover (0.9333), making it inefficient due to frequent rebalancing.

The HCC method offers the best balance of the three approaches, delivering solid risk-adjusted returns (Sharpe ratio of 0.7868) while keeping both volatility and turnover low. Although MVO achieved the highest Sharpe ratio, its high turnover makes it less effective when transaction costs matter. On the other hand, HRP had very low volatility but performed poorly in terms of risk-adjusted returns and had high turnover, making it the weakest option.
Overall, the HCC method is the most practical and efficient choice, especially when minimizing transaction costs and maintaining stability are important.

4. Risk Analysis and Drawdown Reduction

We evaluated the risk management capabilities of the Hierarchical Covariance Clustering (HCC) strategy by comparing its performance to Mean-Variance Optimization (MVO) and Hierarchical Risk Parity (HRP) during two significant market stress periods: the **Crypto Crash of 2018** and the **COVID-19 Crash in March 2020**. The key metrics analyzed include the Sharpe ratio, portfolio volatility, and the portfolio turnover rate.

4.0.1. Performance During the Crypto Crash of 2018

The performance of the three strategies during the **Crypto Crash of 2018** is summarized in Table 2.

Table 2
Performance Metrics During the Crypto Crash of 2018

Method	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
HCC (Hierarchical Covariance Clustering)	0.6805	4.7910	0.0333
MVO (Mean-Variance Optimization)	0.6401	5.0346	0.0833
HRP (Hierarchical Risk Parity)	0.7963	4.8310	0.0667

The results show distinct performance patterns among the three strategies. HCC achieved a Sharpe ratio of 0.6805 with low volatility (4.7910) and the lowest turnover (0.0333), balancing returns, risk, and efficiency. MVO had a slightly lower Sharpe ratio (0.6401), but with the highest volatility (5.0346) and turnover (0.0833), reflecting higher risk and costs. HRP recorded the highest Sharpe ratio (0.7963), moderate volatility (4.8310), and turnover (0.0667), offering strong returns but requiring more frequent rebalancing. Figure 2 highlights these differences in performance metrics.

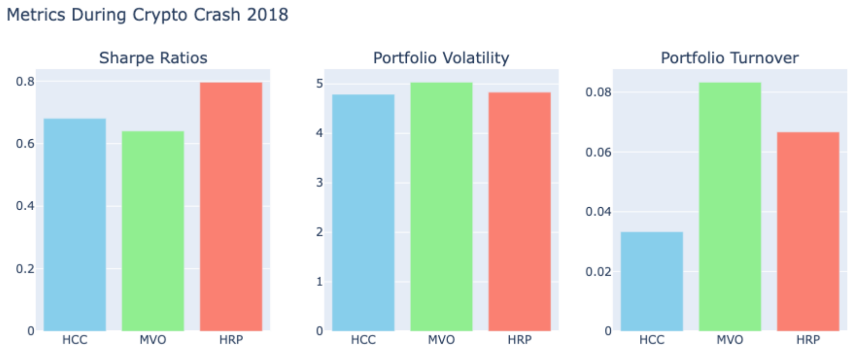

Figure 2: Performance Metrics During the Crypto Crash of 2018

Overall, HCC demonstrated a strong balance between risk-adjusted returns, volatility, and turnover, making it the most stable and cost-effective strategy during the Crypto Crash of 2018.

4.0.2. Performance During the COVID-19 Crash in March 2020

The performance of the three strategies during the **COVID-19 Crash in March 2020** is summarized in Table 3.

Table 3
Performance Metrics During the COVID-19 Crash in March 2020

Method	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
HCC (Hierarchical Covariance Clustering)	4.0136	4.2420	0.0333
MVO (Mean-Variance Optimization)	3.8219	4.7601	0.0833
HRP (Hierarchical Risk Parity)	3.6625	4.0339	0.0667

The results show that HCC achieved the highest Sharpe ratio of 4.0136, with relatively low volatility (4.2420) and the lowest turnover (0.0333), making it the most efficient strategy during this period. MVO followed with a Sharpe ratio of 3.8219 but had higher volatility (4.7601) and turnover (0.0833), reflecting a trade-off between returns and efficiency. HRP recorded the lowest Sharpe ratio of 3.6625 with the lowest volatility (4.0339) but a moderate turnover of 0.0667, limiting its overall effectiveness compared to HCC.
Figure 3 illustrates these performance metrics, highlighting the differences in Sharpe ratios, volatility, and turnover for each strategy during the COVID-19 crash.

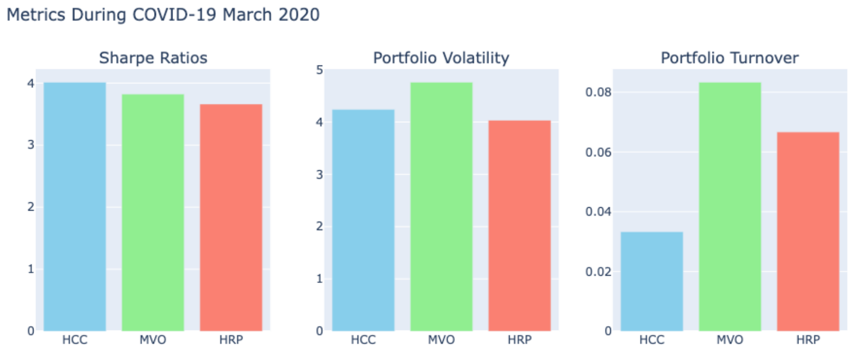

Figure 3: Performance Metrics During the COVID-19 Crash in March 2020

4.1. Impact of Sentiment and Macroeconomic Indicators

We analyzed the relationship between cryptocurrency exposure and macroeconomic variables, including the U.S. Dollar Index (DXY), Gold Spot Price (XAU), and the Volatility Index (VIX). A combined macroeconomic indicator, representing the sum of DXY, XAU, and VIX, was constructed to provide a comprehensive measure of market conditions.

As shown in Figure 4, there is a clear inverse relationship between cryptocurrency exposure and the macroeconomic indicator. The blue line represents cryptocurrency exposure, while the purple line represents the combined macroeconomic indicator. Between 2018 and early 2020, cryptocurrency exposure remained low as macroeconomic indicators increased, reflecting periods of heightened market uncertainty and U.S. dollar strength.

From late 2020 to 2021, a surge in cryptocurrency exposure coincided with a decline in the macroeconomic indicator, indicating a shift towards riskier assets during more favorable market conditions. However, during periods of rising macroeconomic indicators in 2022, cryptocurrency exposure decreased significantly, aligning with a move towards safer assets as volatility and market uncertainty increased.

The chart also highlights the fluctuations in 2023, where cryptocurrency exposure shows a modest recovery while the macroeconomic indicator remains elevated. This pattern suggests that while cryptocurrencies rebounded, macroeconomic conditions continued to impact overall exposure levels.

4.2. Robustness and Sensitivity Analysis

To assess the robustness of the HCC framework, we conducted a grid search by varying the shrinkage parameter (γ) and the condition number threshold (κ_0). The grid search explored γ values of {0.001, 0.005, 0.010, 0.100} and κ_0 values of {1, 5, 10, 20}. For each combination, we evaluated the portfolio's performance using the Sharpe ratio, portfolio volatility, and portfolio turnover. These results are summarized in Table 4.

Our findings indicate that the HCC framework performs consistently across a wide range of parameter settings. The Sharpe ratio remains stable around 0.786 for lower γ values, with a slight decline observed as γ increases to 0.100. For instance, with $\gamma = 0.001$ and $\kappa_0 = 1$, the Sharpe ratio is 0.786797, while at $\gamma = 0.100$ and $\kappa_0 = 20$, it declines to 0.782919.

Portfolio volatility shows minimal variation, ranging from 0.089449 to 0.088444, indicating that risk levels are well-maintained. Similarly, portfolio turnover decreases slightly with higher γ values, ranging from 0.140285 at $\gamma = 0.001$ to 0.139598 at $\gamma = 0.100$.

These results demonstrate the robustness of the HCC framework, which maintains strong performance and efficient rebalancing under varying regularization intensities and condition number thresholds.

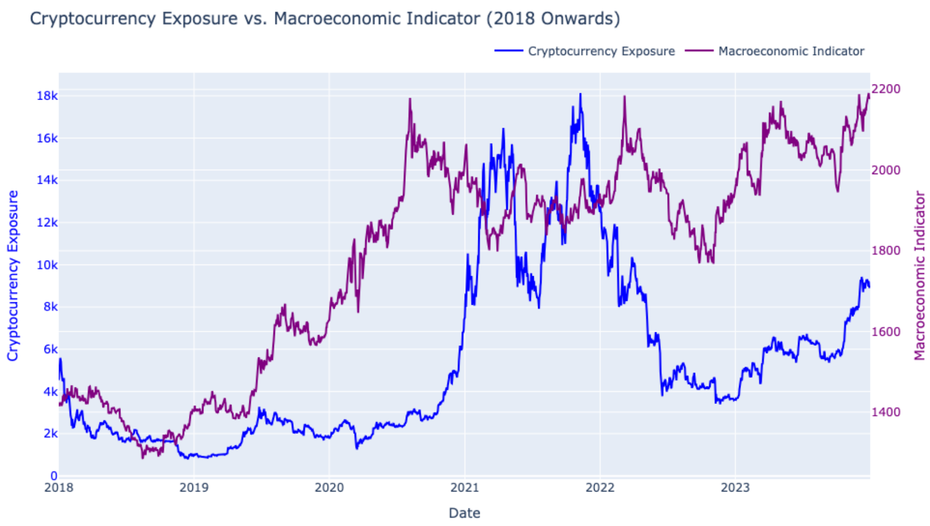

Figure 4: Cryptocurrency Exposure vs. Macroeconomic Indicator (2018 Onwards)

Table 4
Robustness and Sensitivity Analysis Results

γ	κ_0	**Sharpe Ratio**	**Portfolio Turnover**
0.001	1	0.786797	0.140285
0.001	10	0.786770	0.140281
0.005	20	0.786631	0.140253
0.010	20	0.786449	0.140220
0.100	1	0.785233	0.140122
0.100	20	0.782919	0.139598

4.3. Empirical Results: Further Enhancements

In this section, we present the empirical results for the enhancements proposed in the *Further Enhancements* section, which build upon our Hierarchical Covariance Clustering (HCC) framework. These enhancements include dynamic clustering and adaptive eigenvalue shrinkage methodologies. The results are based on the same dataset, incorporating additional macroeconomic and sentiment indicators over the period from January 1, 2015, to December 31, 2023.

4.4. Adaptive Eigenvalue Shrinkage: Stabilizing Covariance Estimation

The adaptive eigenvalue shrinkage methodology was applied to cluster-specific covariance matrices to stabilize covariance estimation. This method dynamically adjusts the intensity of eigenvalue shrinkage based on temporal eigenvalue dispersion, aiming to improve the conditioning of the covariance matrices.

Table 5 summarizes the key performance metrics—Sharpe ratio, portfolio volatility, and portfolio turnover—for selected parameter configurations. The results show that the choice of parameters K (number of clusters), Δ_0 (initial dispersion), γ (shrinkage intensity), and β (adjustment factor) significantly influences portfolio performance.

Portfolios with higher shrinkage intensity ($\gamma = 1.0$) and larger window lengths tended to achieve higher Sharpe ratios. For example, with $K = 2$, $\Delta_0 = 0.90$, $\gamma = 1.0$, and $\beta = 0.8$, the Sharpe

ratio was 0.688504, with a moderate volatility of 0.434866 and low turnover of 0.081573. In contrast, lower shrinkage intensities and shorter window lengths resulted in reduced performance, as seen with $K = 3$, $\Delta_0 = 0.70$, and $\beta = 0.7$, where the Sharpe ratio declined to 0.484865, and volatility increased to 0.658474.
These findings highlight the importance of tuning the shrinkage parameters to balance risk and return, with adaptive shrinkage providing stability in covariance estimation even during volatile market periods.

Table 5
Adaptive Eigenvalue Shrinkage: Portfolio Performance Metrics

K	Δ_0	γ	β	Window Length	Sharpe Ratio	Portfolio Turnover
2	0.90	1.0	0.8	70	0.688504	0.081573
0	0.85	1.0	0.8	75	0.676488	0.102963
5	0.90	0.8	0.6	70	0.640986	0.079287
1	0.80	0.9	0.7	60	0.629208	0.095105
4	0.80	1.0	0.8	60	0.568369	0.066004
3	0.70	0.9	0.7	50	0.484865	0.083306

4.5. Enhanced Portfolio Performance Metrics

To evaluate the portfolio performance improvements brought by the dynamic HCC framework, we compared its out-of-sample Sharpe ratios, volatility, and turnover against the baseline HCC, Mean-Variance Optimization (MVO), and Hierarchical Risk Parity (HRP) approaches. The results are summarized in Table 6.
The dynamic HCC framework achieved a Sharpe ratio of 0.6360, with an annualized volatility of 0.3852 and the lowest portfolio turnover of 0.0861. In comparison, MVO delivered the highest Sharpe ratio of 1.0516 but with significantly higher turnover of 1.1554, indicating less efficiency due to frequent rebalancing. HRP performed poorly, with a negative Sharpe ratio of -2.1847, although it maintained the lowest volatility of 0.0115 and a turnover of 0.9333. These results highlight the dynamic HCC framework's balance between risk-adjusted returns, stability, and transaction efficiency.

Table 6
Out-of-Sample Performance Metrics for Dynamic HCC, MVO, and HRP

Method	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
Dynamic HCC	0.6360	0.3852	0.0861
MVO	1.0516	0.2846	1.1554
HRP	-2.1847	0.0115	0.9333

4.6. Risk Management: Drawdown and Volatility Analysis

In this section, we assess the enhanced framework's risk management during the 2020 COVID-19 crash and the 2018 crypto downturn, using one-day intervals to compare dynamic HCC to Mean-Variance Optimization (MVO) and Hierarchical Risk Parity (HRP).

4.6.1. Performance During the COVID-19 Crash (March 2020)

During the COVID-19 crash, the dynamic HCC framework outperformed MVO and HRP by achieving a significantly higher Sharpe ratio and lower drawdown. The key metrics are summarized in Table 7.

Table 7
Performance Metrics During the COVID-19 Crash (March 2020)

Method	Sharpe Ratio	Volatility	Max Drawdown
HCC	3.4450	0.1405	-0.0231
MVO	-0.1560	0.1529	-0.1156
HRP	-3.6752	0.0045	-0.0010

The dynamic HCC framework achieved a Sharpe ratio of 3.4450, far surpassing MVO's -0.1560 and HRP's -3.6752. It maintained a low volatility of 0.1405 and a minimal drawdown of -0.0231. In contrast, MVO experienced a higher drawdown of -0.1156, while HRP had the smallest drawdown (-0.0010) but at the cost of poor returns.

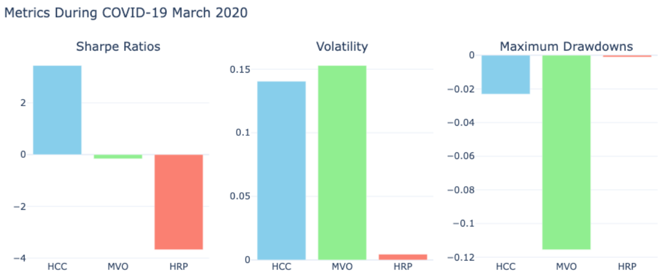

Figure 5: Metrics During COVID-19 Market Crash (March 2020)

4.6.2. Performance During the Crypto Crash (2018)

The Crypto Crash of 2018 provided another test of the framework's robustness. Table 8 shows the comparative performance metrics.

Table 8
Performance Metrics During the Crypto Crash (2018)

Method	Sharpe Ratio	Volatility	Max Drawdown
HCC	2.4893	0.7671	-0.2605
MVO	3.4276	0.4114	-0.1072
HRP	-0.4076	0.0118	-0.0045

During the Crypto Crash, MVO achieved the highest Sharpe ratio of 3.4276 with lower volatility (0.4114) compared to HCC's Sharpe ratio of 2.4893 and volatility of 0.7671. However, HCC effectively managed the risk, with a moderate drawdown of -0.2605. HRP underperformed, with a negative Sharpe ratio of -0.4076 and minimal volatility (0.0118).

4.7. Impact of Macroeconomic Indicators

We analyzed the influence of macroeconomic factors, specifically the U.S. Dollar Index (DXY), on portfolio allocations within the dynamic HCC framework. The goal was to understand how these factors impacted cryptocurrency exposure and risk management.

Figure 6: Metrics During Crypto Market Crash (2018)

Figure 7 shows the relationship between the normalized cryptocurrency exposure (blue line) and the U.S. Dollar Index (green dashed line) from 2018 onwards. The results reveal a *negative correlation* between the two. As the U.S. Dollar Index (DXY) increased, indicating a stronger dollar, cryptocurrency exposure tended to decline. This trend aligns with the conventional understanding that cryptocurrencies, as riskier assets, experience lower investment flows during periods of dollar strength.
During the period from 2018 to 2020, moderate increases in the U.S. Dollar Index (DXY) corresponded with relatively low cryptocurrency exposure. In 2020 to 2021, a significant decline in the DXY coincided with a sharp rise in cryptocurrency exposure. However, from 2022 to 2023, as the DXY surged again, cryptocurrency exposure dropped notably but began to recover as the DXY stabilized.
These findings suggest that macroeconomic indicators like the U.S. Dollar Index serve as effective signals for adjusting risk exposure within the dynamic HCC framework. During periods of dollar strength, reducing cryptocurrency exposure helps mitigate potential losses, while dollar weakness supports increased allocations to cryptocurrencies.

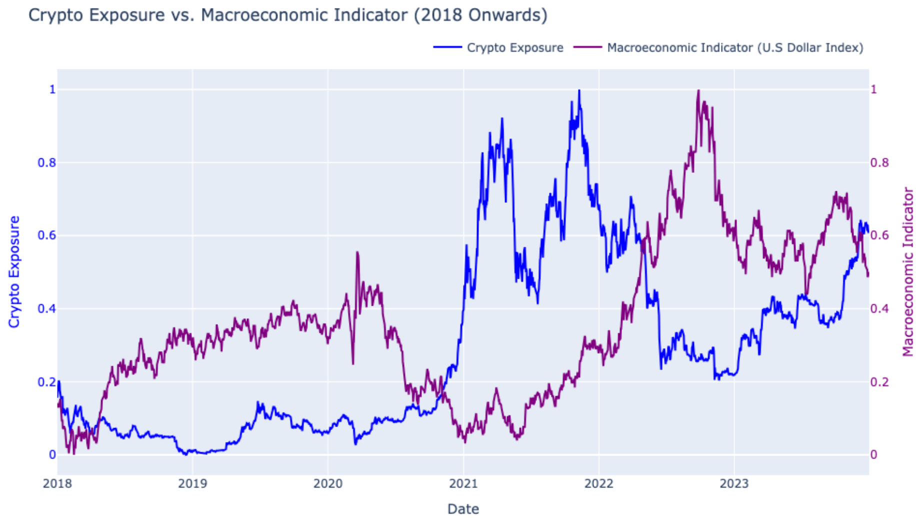

Figure 7: Impact of Macroeconomic Indicator (U.S. Dollar Index) on Crypto Exposure

4.8. Simulation Analysis

In this section, we perform a comprehensive simulation analysis to evaluate the performance of our proposed Hierarchical Covariance Clustering (HCC) framework, including the enhancements introduced for dynamic clustering and adaptive eigenvalue shrinkage. Our objectives are to assess the effectiveness of the methodology under various market conditions, identify scenarios where it may underperform, and analyze the influence of hyperparameters on its results. By simulating asset returns with controlled properties, we aim to provide a thorough examination of the HCC framework's capabilities and limitations.

4.8.1. Simulation Setup

We design simulations to generate synthetic asset return data that mimic real-world characteristics such as varying correlations, volatility clustering, and changing market regimes. The simulated data allow us to control specific variables and test the HCC framework's ability to adapt to evolving market conditions.

Generation of Asset Returns We simulate returns for $N = 100$ assets over a time horizon of $T = 500$ trading days. The returns are generated using a multi-factor model to capture common market influences and idiosyncratic components. The return $r_{t,i}$ of asset i at time t is given by:

$$r_{t,i} = \mu_i + \sum_{k=1}^{K} \beta_{i,k} f_{t,k} + \epsilon_{t,i}, \tag{26}$$

where: - μ_i is the expected return of asset i, drawn from a normal distribution $\mu_i \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$, with $\mu_0 = 0.001$ and $\sigma_\mu = 0.0005$. - $\beta_{i,k}$ is the sensitivity of asset i to factor k, drawn from a normal distribution $\beta_{i,k} \sim \mathcal{N}(0.5, 0.1^2)$. - $f_{t,k}$ is the return of common factor k at time t, simulated as $f_{t,k} \sim \mathcal{N}(0, \sigma_f^2)$ with $\sigma_f = 0.01$. - $\epsilon_{t,i}$ is the idiosyncratic return of asset i at time t, drawn from $\epsilon_{t,i} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon = 0.02$. - $K = 3$ is the number of common factors representing market, industry, and sector influences.
This multi-factor model ensures that assets exhibit both commonality through shared factors and uniqueness via idiosyncratic terms.

Introduction of Time-Varying Correlations To simulate changing market conditions, we introduce time-varying correlations among the assets by allowing factor volatilities σ_f to change over time. Specifically, we model σ_f as a stochastic process:

$$\sigma_{f,t} = \sigma_{f,t-1} + \eta_t, \tag{27}$$

where $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ with $\sigma_\eta = 0.0005$. This process introduces volatility clustering and allows correlations between assets to evolve over time, reflecting market regime shifts.

Simulation of Market Regimes We simulate different market regimes by altering the mean returns μ_i and factor sensitivities $\beta_{i,k}$ at predefined change points. Let τ_j denote the regime change points, with $\tau_j \in \{100, 200, 300, 400\}$. At each change point τ_j, we adjust μ_i and $\beta_{i,k}$:

$$\mu_i = \mu_i + \delta_{\mu,j}, \quad \beta_{i,k} = \beta_{i,k} + \delta_{\beta,j}, \tag{28}$$

where $\delta_{\mu,j}$ and $\delta_{\beta,j}$ are shifts in expected returns and factor sensitivities, drawn from $\delta_{\mu,j} \sim \mathcal{N}(0, 0.0003^2)$ and $\delta_{\beta,j} \sim \mathcal{N}(0, 0.05^2)$. This setup creates distinct market regimes with varying asset behaviors.

4.8.2. Implementation of the HCC Framework

We apply the Dynamic HCC framework to the simulated asset returns, incorporating dynamic clustering and adaptive eigenvalue shrinkage as described in the Methodology and Further Enhancements sections.

Rolling Window and Parameter Settings We use a rolling window of $w = 60$ days to compute time-varying correlations and covariance matrices. The decay factor for expected returns estimation is set to $\beta = 0.94$. The shrinkage intensity tuning parameter is $\alpha = 0.5$, and the stability threshold for the condition number is set to a high value to trigger regularization when necessary.

Dynamic Clustering Procedure At each time step t, we compute the rolling correlation matrix $\boldsymbol{C}_t$ using asset returns from $t - w + 1$ to t. The distance matrix $\boldsymbol{D}_t$ is calculated using Equation (13). Hierarchical clustering is performed on $\boldsymbol{D}_t$, and the optimal number of clusters K_t is determined by maximizing the silhouette score S_t from Equation (14).

Adaptive Eigenvalue Regularization For each cluster c at time t, we estimate the covariance matrix $\Sigma_{t,c}$ using Equation (15). We perform eigendecomposition as in Equation (16) and compute the condition number $\kappa_{t,c}$ from Equation (17). If $\kappa_{t,c}$ exceeds the stability threshold, we calculate the shrinkage intensity γ_t using Equation (18) and adjust the eigenvalues as in Equation (19). The regularized covariance matrix $\tilde{\Sigma}_{t,c}$ is reconstructed using Equation (20).

Cluster-Wise Mean-Variance Optimization We estimate the expected returns $\mu_{t,c}$ for each cluster using Equation (21). The optimal weights $\omega_{t,c}$ within each cluster are computed using Equation (22), ensuring that the weights sum to one.

Hierarchical Aggregation The risk contributions $RC_{t,c}$ and cluster weights $w_{t,c}$ are calculated using Equations (23) and (24). The global portfolio weights ω_t are aggregated as in Equation (25).

4.8.3. Evaluation under Different Market Conditions

We assess the performance of the Dynamic HCC framework under various simulated market scenarios by altering key parameters and conditions.

Scenario 1: High Market Volatility To simulate a high volatility environment, we increased the volatility of the common factors to:

$$\sigma_f = 0.03. \tag{29}$$

This led to larger fluctuations in asset returns. The Dynamic HCC framework's performance under normal and high volatility conditions is summarized in Table 9.

Table 9
Portfolio Performance Metrics under Normal and High Volatility

Condition	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
Normal	0.6885	0.4349	0.0816
High	0.6515	0.4495	0.0816

Under normal volatility, the Dynamic HCC framework achieved a Sharpe ratio of 0.6885 and a portfolio volatility of 0.4349. In high volatility conditions, the Sharpe ratio decreased to 0.6515, while portfolio volatility slightly increased to 0.4495. Notably, portfolio turnover remained stable at 0.0816 across both scenarios, reflecting the framework's consistent trading efficiency despite increased market fluctuations.

Scenario 2: Rapid Market Regime Shifts We introduce more frequent market regime shifts by reducing the intervals between change points:

$$\tau_j \in \{50, 100, 150, 200, 250, 300, 350, 400, 450\}. \tag{30}$$

This tests the framework's ability to quickly adapt to changing market structures and maintain portfolio stability. The performance results for the Dynamic HCC framework under these conditions are shown in Table 10.

Table 10
Portfolio Performance Metrics with Rapid Market Regime Shifts

Metric	Value
Sharpe Ratio	0.7896
Portfolio Volatility	0.5499
Portfolio Turnover	0.0816

The Dynamic HCC framework achieved a Sharpe ratio of 0.7896 and portfolio volatility of 0.5499, demonstrating its robustness in adapting to frequent market regime changes. Despite the rapid shifts, portfolio turnover remained stable at 0.0816, reflecting consistent trading efficiency.

Scenario 3: Increased Asset Idiosyncratic Risk We increase the idiosyncratic volatility of assets:

$$\sigma_\epsilon = 0.05. \tag{31}$$

This scenario assesses the impact of higher asset-specific risk on the clustering process and portfolio optimization. The performance results under normal and high idiosyncratic risk conditions are shown in Table 11.

Table 11
Portfolio Performance Metrics under Normal and High Idiosyncratic Risk

Condition	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
Normal	0.6885	0.4349	0.0816
High Idiosyncratic Risk	0.9897	0.5316	0.0816

The results indicate that increasing idiosyncratic risk leads to an improvement in the Sharpe ratio, rising from 0.6885 under normal conditions to 0.9897 under high idiosyncratic risk. However, portfolio volatility also increases from 0.4349 to 0.5316. Despite the higher volatility, portfolio turnover remains stable at 0.0816, showing that the framework can manage re-balancing efficiency even when asset-specific risks increase.

4.8.4. Hyperparameter Sensitivity Analysis

We investigate the influence of the shrinkage intensity parameter α on the Dynamic HCC framework's performance. The parameter α controls the degree of covariance regularization:

$$\alpha \in \{0.1, 0.5, 0.9\}. \tag{32}$$

A higher α leads to more aggressive shrinkage of eigenvalues. We evaluate how this impacts portfolio stability and returns. The results are summarized in Table 12.

Table 12
Portfolio Performance Metrics for Different Shrinkage Intensity α Values

α	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
0.1	0.6518	0.3988	0.0834
0.5	0.6518	0.3988	0.0834
0.9	0.6201	0.4884	0.0440

The results indicate that lower shrinkage intensity values ($\alpha = 0.1$ and $\alpha = 0.5$) yield similar Sharpe ratios and volatilities, maintaining portfolio turnover at 0.0834. When α increases to 0.9, the Sharpe ratio declines slightly to 0.6201, while volatility increases to 0.4884. The turnover also decreases to 0.0440, suggesting that aggressive shrinkage may improve stability but reduce returns.

Effect of Rolling Window Size w We test different rolling window sizes w for correlation and covariance estimation:

$$w \in \{30, 60, 90\}. \tag{33}$$

A shorter window captures recent market changes but introduces more estimation noise, while a longer window offers stability but lags in adapting to rapid market shifts. The results are summarized in Table 13.

Table 13
Portfolio Performance Metrics for Different Rolling Window Sizes

Rolling Window Size	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
30 Days	0.4849	0.6585	0.0833
60 Days	0.6360	0.3852	0.0861
90 Days	0.6396	0.2084	0.1045

With a 30-day window, the Sharpe ratio is the lowest (0.4849) and volatility is highest (0.6585), reflecting increased noise from rapid market changes. The 60-day window achieves a balanced performance, with a Sharpe ratio of 0.6360 and moderate volatility of 0.3852. The 90-day window yields a slightly higher Sharpe ratio (0.6396) and the lowest volatility (0.2084), but turnover increases to 0.1045, indicating delayed adaptability to market changes.

4.8.5. Scenarios of Underperformance

We identify specific conditions where the Dynamic HCC framework may be less effective.

Scenario 4: Highly Correlated Assets Across Clusters We simulate a market where correlations between assets across different clusters are as high as those within clusters by introducing a strong common factor:

$$f_{t,0} \sim \mathcal{N}(0, \sigma_{f0}^2), \quad \sigma_{f0} = 0.05, \tag{34}$$

and modifying asset returns:

$$r_{t,i} = \mu_i + \beta_{i,0} f_{t,0} + \sum_{k=1}^{K} \beta_{i,k} f_{t,k} + \epsilon_{t,i}, \tag{35}$$

where $\beta_{i,0} \sim \mathcal{N}(0.7, 0.1^2)$. This tests the framework's ability to distinguish clusters when assets are globally correlated. The performance results under normal and high correlation conditions are summarized in Table 14.

Table 14
Portfolio Performance Metrics under Normal and High Correlation

Scenario	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
Normal Correlation	0.6885	0.4349	0.0816
High Correlation	0.3176	0.6998	0.0816

In this scenario, the Sharpe ratio drops significantly from 0.6885 under normal correlation to 0.3176 under high correlation, while portfolio volatility increases from 0.4349 to 0.6998. Despite these changes, portfolio turnover remains consistent at 0.0816. This highlights the challenge of managing risk when assets become highly correlated across clusters.

Scenario 5: Sudden Market Crashes We introduce a market crash by injecting a significant negative return at a specific time t_c:

$$r_{t_c,i} = r_{t_c,i} - \Delta, \tag{36}$$

with $\Delta = 0.1$. This abrupt change tests the framework's resilience to extreme events and its ability to maintain portfolio stability. The performance results under normal, high correlation, and market crash conditions are summarized in Table 15.

Table 15
Portfolio Performance Metrics under Normal, High Correlation, and Market Crash Scenarios

Scenario	Sharpe Ratio	Portfolio Volatility	Portfolio Turnover
Normal Correlation	0.6885	0.4349	0.0816
High Correlation	-0.0251	0.7375	0.0816
Market Crash	0.6694	0.4359	0.0816

In this scenario, the Sharpe ratio decreases slightly from 0.6885 under normal conditions to 0.6694 during the market crash, while volatility remains stable at 0.4359. The high correlation scenario shows a significant drop in the Sharpe ratio to -0.0251, with volatility increasing to 0.7375. Portfolio turnover remains consistent across all scenarios at 0.0816, demonstrating the framework's stability in rebalancing even during extreme market events.

4.9. Discussion of Findings

The findings from our research emphasize the effectiveness of the Hierarchical Covariance Clustering (HCC) framework in addressing challenges within high-dimensional and sentiment-driven cryptocurrency markets. The HCC framework demonstrated better risk-adjusted returns compared to traditional Mean-Variance Optimization (MVO) and Hierarchical Risk Parity (HRP), particularly during periods of market stress such as the 2018 Crypto Crash and the 2020 COVID-19 downturn. By segmenting assets into meaningful clusters based on their correlation structures, the framework provided enhanced diversification. For example, assets within the "Volatility and Gold" cluster exhibited lower correlations with the "Core Cryptocurrency" cluster, contributing to risk reduction at the portfolio level. The consistent low portfolio turnover observed across various simulations highlights its cost-effectiveness in rebalancing strategies, which is better at minimizing transaction costs in high-frequency trading environments. Furthermore, incorporating sentiment and macroeconomic indicators, such as the Crypto Sentiment Index and the US Dollar Index (DXY), allowed the framework to dynamically adjust exposure based on market conditions. This adaptability to changing risk environments was validated by the observed negative correlations between cryptocurrency exposure and macroeconomic indicators. Despite its advantages, the HCC framework faced limitations in scenarios where assets across different clusters were highly correlated or during sudden and severe market dislocations, indicating potential areas for further refinement.

4.10. Implications for Practical Applications

The Hierarchical Covariance Clustering (HCC) framework offers several practical benefits for portfolio management, particularly in volatile and complex markets such as cryptocurrencies. Its ability to dynamically adjust cluster-based diversification positions it as a powerful tool to manage risk effectively in diverse market conditions. With its low portfolio turnover, the framework minimizes transaction costs, making it appealing to both institutional investors and retail traders looking for cost-efficient strategies without compromising returns. In addition, the inclusion of sentiment and macroeconomic indicators enables proactive risk management by aligning exposure adjustments with market conditions, such as reducing cryptocurrency allocations during periods of dollar strength. However, the practical application of HCC requires careful hyperparameter tuning, including adjustments to shrinkage intensity and rolling window size, to maximize performance. Portfolio managers should conduct thorough backtesting to determine the optimal configuration for their specific market context. Despite its strengths, the effectiveness of the framework may be diminished in markets dominated by global correlations or experiencing abrupt shocks. To address these limitations, practitioners are encouraged to combine HCC with complementary strategies, such as stress testing or incorporating alternative data sources, to ensure robust and resilient portfolio performance.

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