Calc II Recitation D **Name: ____________________**

Recitation Nine: 7/28/2015

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Objectives:
1. Review & Quiz
2. Taylor Polynomials & Taylor Series
3. Lagrange Remainder & Error
4. Infinite Series

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1. Review & Quiz!

Procedure to find Area of Polar Graphs

1. Check the ratio $\frac{a}{b}$ to determine type

Inner Loop	Cardiod	Dimple	Convex
$\left\lvert\frac{a}{b}\right\rvert < 1$	$\left\lvert\frac{a}{b}\right\rvert = 1$	$1 < \left\lvert\frac{a}{b}\right\rvert < 2$	$\left\lvert\frac{a}{b}\right\rvert \geq 2$
2.Write down general equation: $A = \frac{1}{2}\int_a^b r^2 d\theta$	2.Write down general equation: $A = \frac{1}{2}\int_a^b r^2 d\theta$	2.Write down general equation: $A = \frac{1}{2}\int_a^b r^2 d\theta$	2.Write down general equation: $A = \frac{1}{2}\int_a^b r^2 d\theta$
3.Find bounds of Integration: $r = 0 = a \pm b\, sin(\theta)$ Find $\theta_1, \theta_2, \theta_3$ -Big Loop has bounds $\theta_2 to\ \theta_3$ -Small Loop has bounds $\theta_1\ to\ \theta_{2,}$	3. Use symmetry & find bounds -Solve this for θ_1 $r = a + b = a \pm b\, sin(\theta)$ -Solve this for θ_2 $r = 0 = a \pm b\, sin(\theta)$ Integration from $\theta_1\ to\ \theta_{2,}$ -Muliply answer by two	3. Use symmetry & find bounds -Solve this for θ_1 $r = a + b = a \pm b\, sin(\theta)$ -Solve this for θ_2 $r = 0 = a \pm b\, sin(\theta)$ Integration from $\theta_1\ to\ \theta_{2,}$ -Muliply answer by two	3. Use symmetry & find bounds -Solve this for θ_1 $r = a + b = a \pm b\, sin(\theta)$ -Solve this for θ_2 $r = 0 = a \pm b\, sin(\theta)$ Integration from $\theta_1\ to\ \theta_{2,}$ -Muliply answer by two

Example One:
Find the area of the big loop and the inner loop. **Set Up Only**
$r = 1 + 2\cos(\theta)$

Arc Length

Integrating with Respect to X

$$S = \int_{x=a}^{x=b} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Integrating with Respect to Y

$$S = \int_{y=a}^{y=b} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Parametric Equations

$$S = \int_{t=a}^{t=b} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

Example Two:

Find the arc length of the following:

$y = x^2 \quad on\ interval\ \ [0,3]$

Surface Area

Rotating about the x axis

$$S = 2\pi \int_{x=a}^{x=b} f(x)\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Rotating about the y axis

$$S = 2\pi \int_{y=a}^{y=b} f(y)\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Parametric Equations

$$S = 2\pi \int_{t=a}^{t=b} y(t)\sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$S = 2\pi \int_{t=a}^{t=b} x(t)\sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

Example Three:

No Example three! Yay! Quiz Time!

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2. Taylor Polynomials & Series

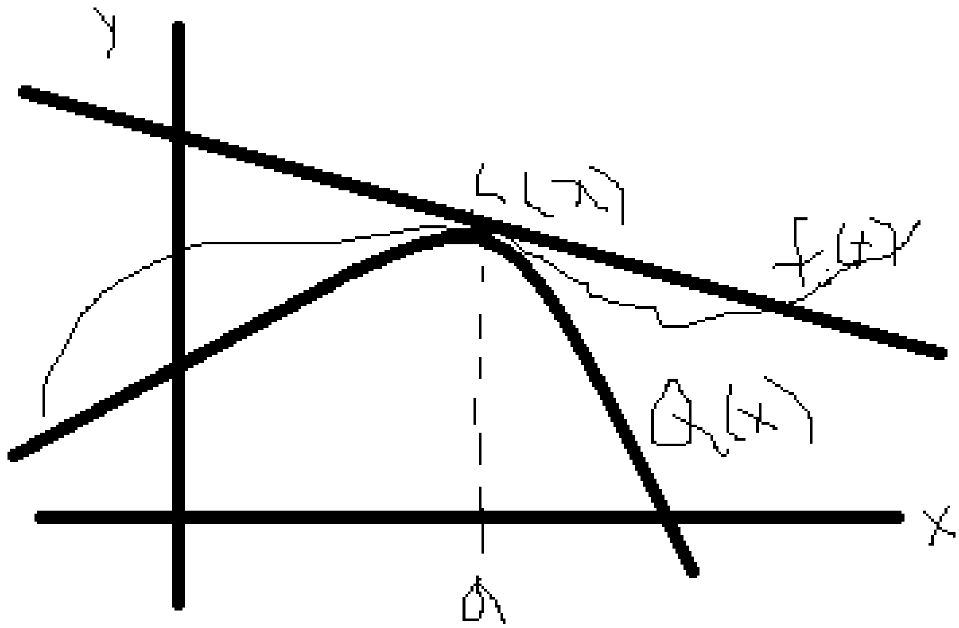
Recall!

Linear Approximations!

$L(x) = f(a) + f'(a)(x - a)$

Quadratic Approximations!

$Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)(x-a)^2)}{2}$



WOW! These are really Taylor Polynomials of Degrees:

__________ <-Degrees of Linear & Quad -> ___________

Taylor Polynomials are just adding more terms to the quadratic approximation to increase accuracy.

$$\boldsymbol{P_n(x)} = f(a) + f'(a)(x - a) + \frac{f''(a)(x-a)^2)}{2} + \ldots + \frac{f^n(a)(x-a)^n}{\boldsymbol{n}!}$$

Therefore the Taylor Series is:

$\boldsymbol{P_n(x)} =$ __________________ *Wow Matt that makes sense!

Procedure:

1. Notice how many degrees you are to set up your taylor polynomial with
2. Find the derivatives of the function you are approximating to that degree
3. Plug in your value for “a” (aka the x “guess” you are centering your polynomial around)
4. Plug into your P(x) to approximate any x value

Note:

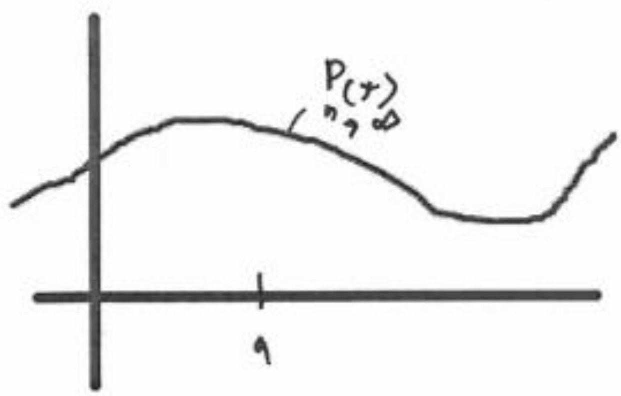
-Maclaurin Series are really Taylor Series with “a = 0”

3. Lagrange Error & Remainder

Actual Error:

Error = $|f(x) - P_n(x)|$

Lagrange Error:



$f(x) = P_n(x) + R_n(x)$
*Actual Function

a < Z < c

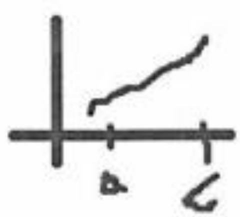
*Recall x = a is your starting value

$$R_n(x) = \frac{f^{(n+1)}(z)(x-a)^{(n+1)}}{(n+1)!}$$

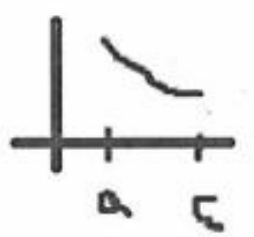
Max Lagrange Error:

$$R_{nmax}(x) = max\left|\frac{f^{(n+1)}(z)(c-a)^{(n+1)}}{(n+1)!}\right|$$

-Can have two different Values of Z if we are maximizing
-We must look at $f^{(n+1)}$



Z = c



Z = a

Example four:
Find the Max Lagrange Error of the following function:
$f(x) = sin(x) \quad on\ [0, 0.1]\ with\ n = 3$

Example Five:

(a)Find the Taylor polynomial degree 3 for the $y = \frac{1}{(x-4)^2}$ centered at x = 1

(b)Use the Taylor Polynomial to approximate the function at $x = 2$

(c)Find the Lagrange remainder

(d)Find the Lagrange error bound for the maximum error on the interval $[a,2]$, or $[2,a]$ (depending on which is bigger)

(e)Find the actual error in the Taylor approximation and show that it is less than or equal to the answer from part (d)

Example Six!: More Taylor!

(a)Find the Taylor polynomial degree 4 for the $y = \cos(x)$ centered at x = $\frac{\pi}{2}$

(b)Use the Taylor Polynomial to approximate the function at $x = 2$

(c)Find the Lagrange remainder

(d)Find the Lagrange error bound for the maximum error on the interval $[a,2]$, or $[2,a]$ (depending on which is bigger)

(e)Find the actual error in the Taylor approximation and show that it is less than or equal to the answer from part (d)

4. Infinite Sequences!

1. What is a sequence?

Example: 1,2,3,4,5,6,7,8,9,10 … $a_n = n$

Note: If you have a_n you could write out all the terms of a sequence

Alternating Sequences

Example:	1,	-2 ,	3,	-4,	5,	-6
Term:	____,	____,	____,	____,	____,	____,

$(-1)^{n+1} = 1st\ term\ postive$
$(-1)^{n} = 1st\ term\ negative$

$a_n =$ ______________

Convergence:

Test for Convergence:

Take the limit as $n \to \infty \quad of \ a_n$

$$\lim_{n \to \infty} a_n = c$$

1. If **c** = "a number" then sequence converges

2. If **c** DNE or if $c = \pm\infty$ then the sequence diverges

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Example Seven:

Write the following infinite sequence in terms of a_n

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

Practice! ☺ The team that gets the most correct will get a prize!

Work Sheet

For the following problems determine whether the following infinite sequences converge or diverge:

1. $a_n = 1 + \left(\frac{9}{10}\right)^n$

2. $a_n = 2 - \left(-\frac{1}{2}\right)^n$

3. $a_n = \frac{1+(-1)^n\sqrt{n}}{\left(\frac{3}{2}\right)^n}$

4. $a_n = \left(1 + \frac{1}{n}\right)^n$

5. $a_n = \frac{(1-n^2)}{2+3n^2}$

6. $a_n = \left(\frac{\ln(n)^2}{n}\right)$