Calc II Recitation D **Name: ____________________**

Recitation Seven: 7/21/2015

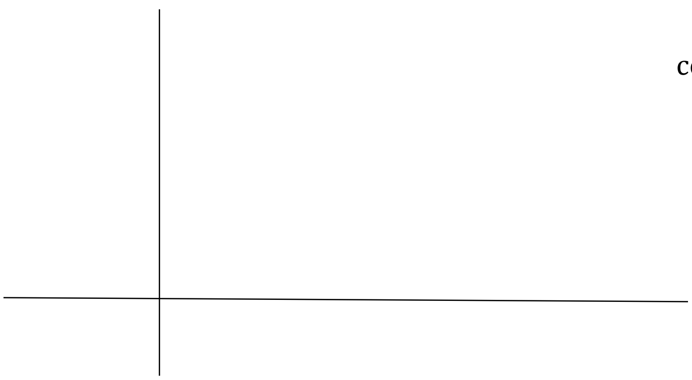
Alexis Cuellar
Ashley G Simon
Audrey B Ricks
Carissa R Gadson
Hector J Vazquez
Mael J Le Scouezac
Mario Contreras
Michael A Castillo
Paul A Herold
Thomas Varner
Clarissa Sorrells

Objectives:
1. Polar Coordinates
2. Polar Graphs
3. Parametric Equations
4. A Little Something Fun (Calculus & Instructional)

Polar Coordinates:

x = _______________ y = _______________

$\boldsymbol{x^2 + y^2 =}$ ____ $\boldsymbol{r = \sqrt{x^2 + y^2}}$



$\cos(\theta) = \frac{x}{r}$ $\sin(\theta) = \frac{y}{r}$

$\theta = \arctan(\frac{y}{x})$

Example One:
Write the given rectangular equation in polar form.
$xy = 1$

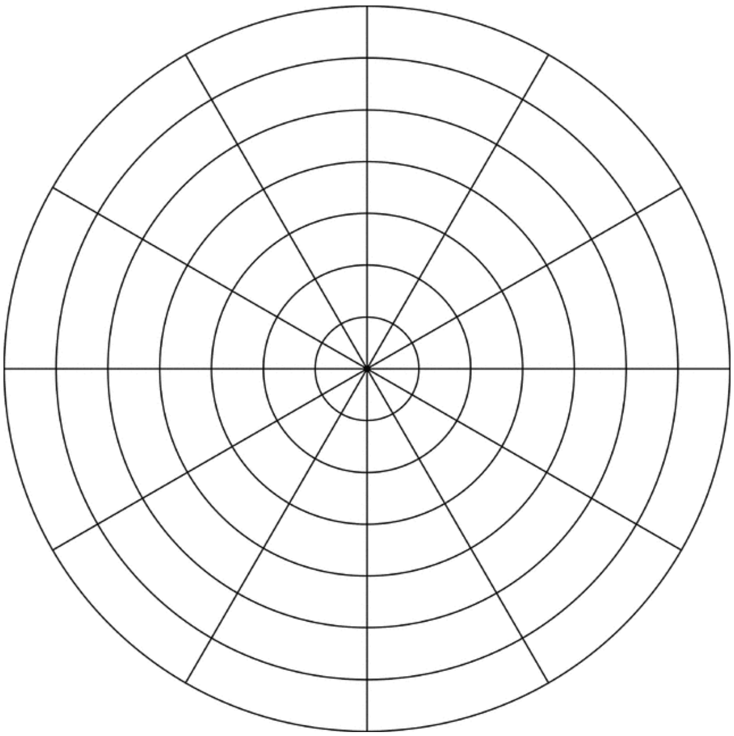
Example Two:
Write the given polar equation in rectangular form
$r = 1 + \cos(\theta)$ *Start by multiply both sides by r

Polar Graphs:

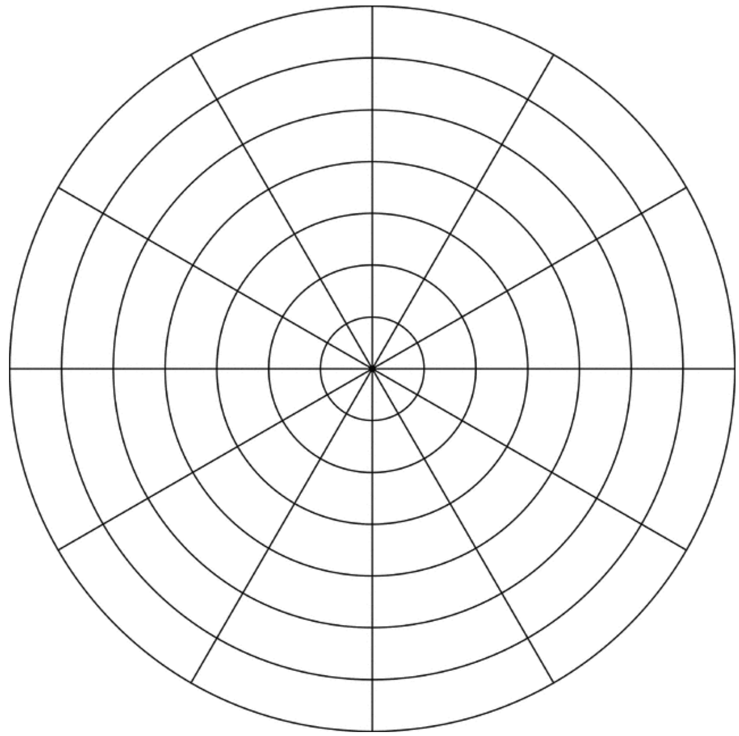
-Circles -Limacons (Please don’t say LIMA-CONE ☹)
-Cardiods -Rose Curves

Circles:

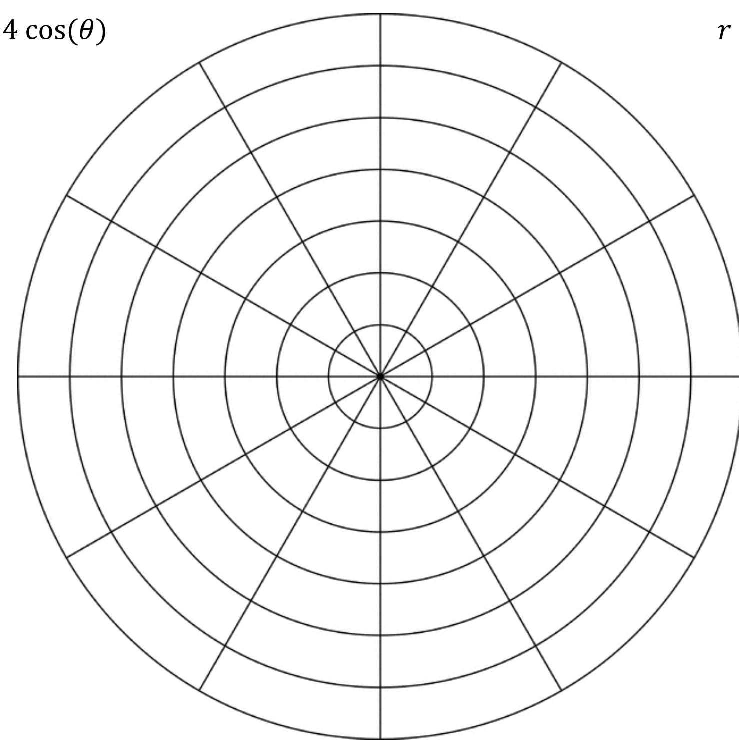
$\boldsymbol{r = asin(\theta)}$ * Circle will be centered along the ______ axis.
$\boldsymbol{r = acos(\theta)}$ *Circle will be centered along the ______ axis.
$\boldsymbol{r = a}$ *Circle will be centered along the _______. (a must be positive)



$r = 4\cos(\theta)$



$r = -4\sin(\theta)$

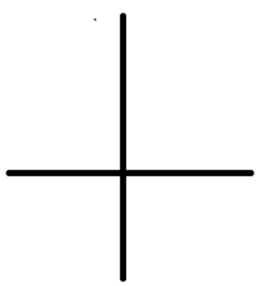
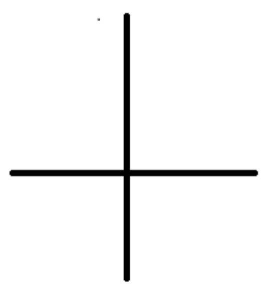
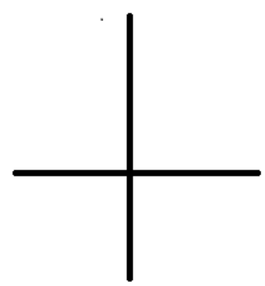
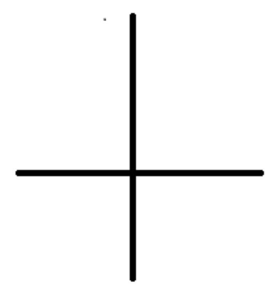


$r = 4$

LIMACONS! OUR FAVORITE! ☺ (According to Matt at least)

Graphing made easy! ☺ WE BREAK IT DOWN! ☺

$$r = a \pm b\cos(\theta) \qquad\qquad r = a \pm b\sin(\theta)$$

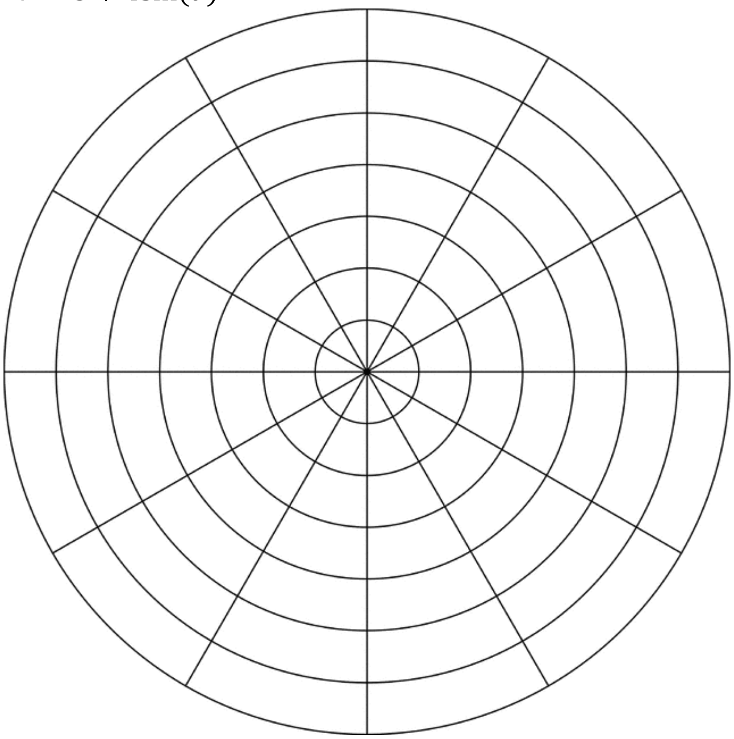
Ratio	$\left\lvert\frac{a}{b}\right\rvert < 1$	$\left\lvert\frac{a}{b}\right\rvert = 1$	$1 < \left\lvert\frac{a}{b}\right\rvert < 2$	$\left\lvert\frac{a}{b}\right\rvert \geq 2$
Shape (Name)	________________	________________	________________	________________
Diagram **(General Look)**				

Graphing Procedure:

*Note Following tables uses looking at the formula to determine features of the graph

$\cos(\theta)$ or $\sin(\theta)$	**Determines:**	**X axis or Y axis Respectively**
Ratio $\left\lvert\frac{a}{b}\right\rvert$	**Determines:**	
Value of $a + b$	**Determines:**	
Value of a	**Determines:**	
Value of $a - b$	**Determines:**	

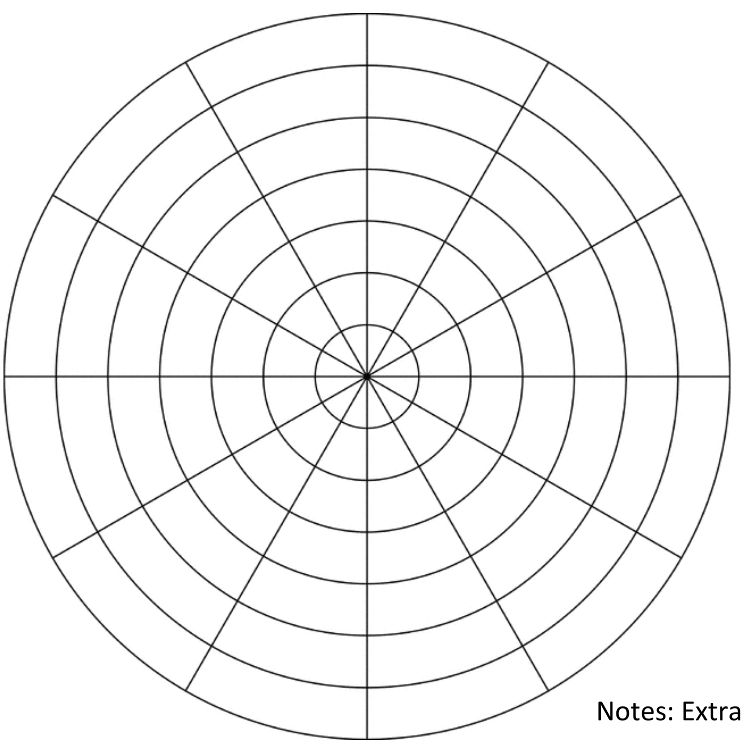
Example Three:
Graph the following Equation in the X/Y plane
$r = 3 + 4\sin(\theta)$



Cos/Sin	
Ratio: a/b	
a + b	
a	
a - b	

Example Four:
Graph the following equation in the X/Y plane (BUT MATTTTT… ISN'T This the $\frac{r}{\theta}$ $plane?$ – No) – Show an example if desired by students.

$r = 2 - \cos(\theta)$



Cos/Sin	
Ratio: a/b	
a + b	
a	
a - b	

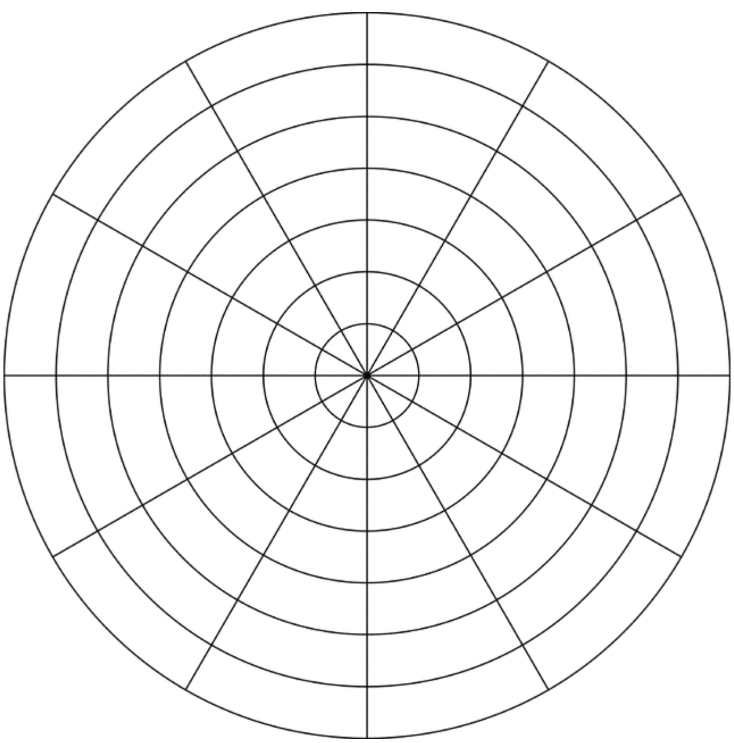
Notes: Extra Problems Will be Posted on the Back

Petal Curves:

$r = a\sin(n\theta)$ $\qquad\qquad$ $r = a\cos(n\theta)$

“a” determines:	**length of Petals**
# of petals: - **If n is odd** - **If n is even**	 **-** **-**
To find where the first petal starts	
To find where other petals start rotate by the following angle	

Example Five:
Graph the following Equation on the X/Y Axis
$r = 4\sin(2\theta)$



a	
n	
# of petals	
1^{st} Petal Begins	
Rotate ? Degrees for next petals	

Area of Polar Graphs

-One important equation

$$A = \frac{1}{2}\int_a^b r^2 d\theta$$

-*Ask if they want proof

Area of Polar Graphs Continued:

-Difficult part about these problems is finding the bounds of integration [a,b]

-Will do examples in order to show methodology

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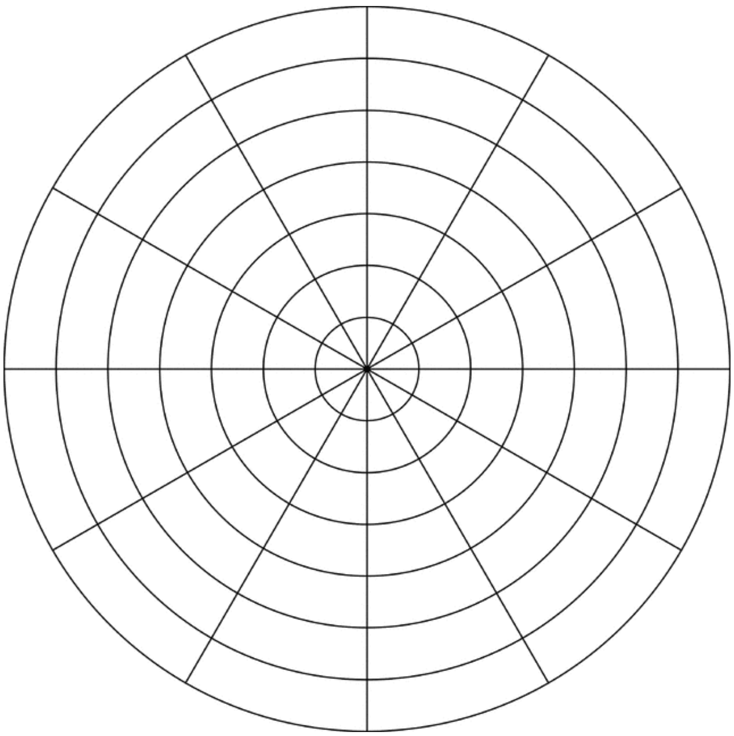
Example Six:

Find the area bounded by rose petal graph with the equation

$r = 4\sin(2\theta)$ #This is the equation from example Five



$$Area = \frac{1}{2}\int_a^b r^2 d\theta$$

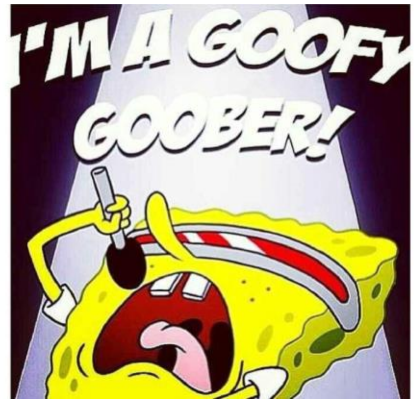
Steps:

-Rose petal graph: Solve for the area of one rose petal

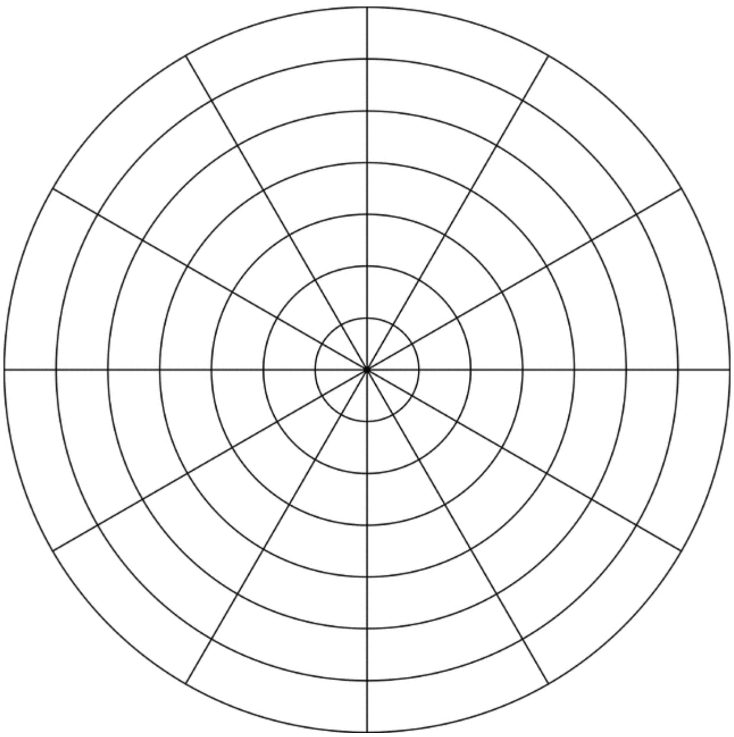
-Multiply Answer by # of Rose Petals

-To find the bounds integration set r = 0 & solve for θ

Example Seven: GOOFY GOOBER!!!
I'm a goofy goober!
What is a goofy goober! I am! Wow that's great! GOOFY GOOBER!!
Do you know what goofy goobers like best?
-Not doing calculus problems?
-HAHA No.
-They LOVE DOING CALCULUS PROBLEMS!
Specifically Area of Inner Loop Limacon!
Find the area of the each loop!



$\boldsymbol{r = 1 + 2cos(\theta)}$



Steps:
-Graph
-Look at Symmetry
-Set up equations (One for big hoop & One for Small)
-To find bounds of integration
    -first graph equation
    -then look at pts that would make symmetry
    Easy. Find θ there
    -Find θ there

Cos/Sin	
Ratio: a/b	
a + b	
A	
a – b	

Parametric Curves:

A Parameteric Curve in the plan is a pair of functions

______________________ ______________________ *(Two Dimensions)

that give x & y as continuous function of the real number, t, (the parameter)

Position:

$r(t) = \big(x(t), y(t)\big)$ = __________________

$$r(b) = [\, x(a) + \int_{t=a}^{t=b} x'(t)dt\,, \qquad y(a) + \int_{t=a}^{t=b} y'(t)dt]$$

Velocity:

$v(t) = \big(x'(t), y'(t)\big)$ = __________________

$$v(b) = [\, x'(a) + \int_{t=a}^{t=b} x''(t)dt\,, \qquad y'(a) + \int_{t=a}^{t=b} y''(t)dt]$$

Acceleration:

$a(t) = \big(x''(t), y''(t)\big)$ = _________________

Speed: (Magnitude of Velocity)

$$|v(t)| = \sqrt{\big(x'(t)\big)^2 + \big(y'(t)\big)^2}$$

Displacement: (How Far you are from start)

Displacement = $\sqrt{(\boldsymbol{x_2 - x_1})^2 + (\boldsymbol{y_2 - y_1})^2}$

Distance: (How far you have traveled)

$D = \int_{t=a}^{t=b} |v(t)|dt =$ ____________________

Derivative of Parametric Equations (IMPORTANT!)

1. First Derivative

$$\boldsymbol{\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}} \quad \textbf{OR} \quad \boldsymbol{\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}}$$

*Chain rule it yo! To make sense of it!

2. Second Derivative

$$\boldsymbol{\frac{d^2y}{dx^2} = \frac{\left(\frac{dy'}{dt}\right)}{\left(\frac{dx}{dt}\right)}} \quad \textbf{OR} \quad \boldsymbol{\frac{d^2y}{dx^2} = \frac{\left(\frac{dy'}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}}$$

PRACTICE!!

Example Eight:

Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$ $\qquad r = 1 + 3cos(\theta)$

Steps:

1. Find $x(\theta)$ & $y(\theta)$
2. Find Derivatives

Example Nine:

A. Find the position of the particle at time t = 5

B. Find the distance the particle moves from t = 1 to t = 5

$\boldsymbol{v(t) = < t^2 + 1\ , t >}$ $\qquad$ **given** $\boldsymbol{r(0) = (1, 2)}$