Calc II Recitation D **Name: ____________________**

Recitation Eight: 7/23/2015

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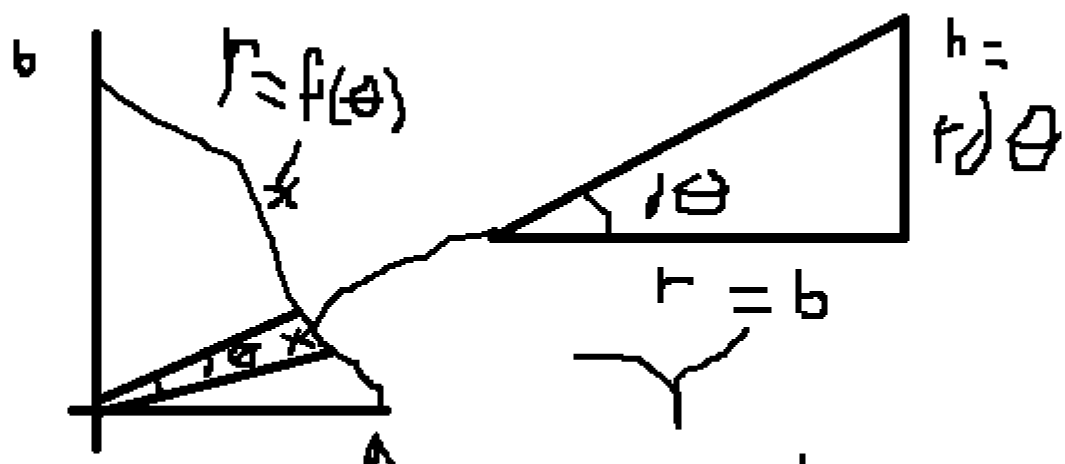
Objectives:

1. Area!
   - Polar
   - Regular (Skipping Unless Questions About It)
2. Volume!
   - By Cross Section
   - Disk & Washer Method
   - Shell Method (**Strong Focus**)
3. Practice!

Area

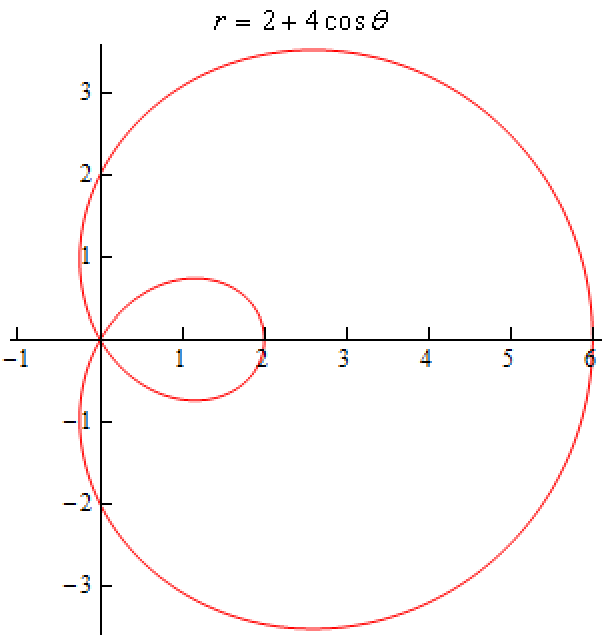
1. Polar Functions (Round Two for Recitation D)

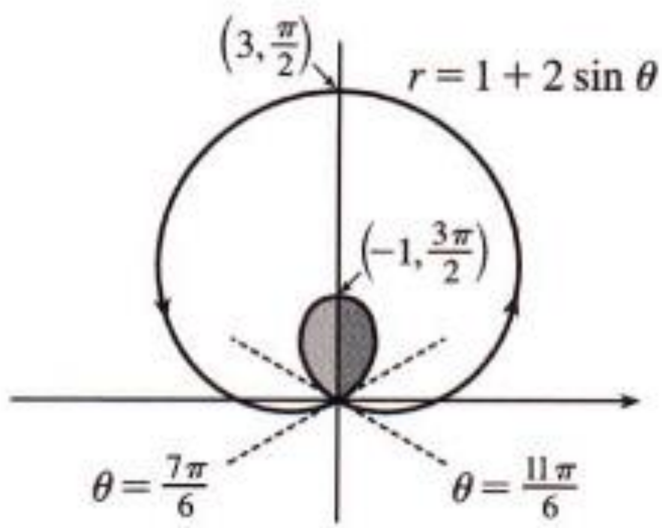


$$dA = \frac{1}{2} bh$$

$$A = \int_a^b \frac{1}{2} r^2 \, d\theta$$

***Absolutely Beautiful Drawing Matt \(^__^)/ (Thanks!)**
Procedure for Solving Polar Area Problems
1. Inner Loop Limacon





Procedure:
1. Set r = 0
2. Solve for $\theta_1, \theta_2, \theta_3$ (Note: $\theta_3 = 2\pi + \theta_1$) Why is this? (Think about # times interects origin)
3. Bounds for the inner loop are the two $\theta's\ with\ the\ least\ difference$
4. Bounds for the outer loop are the two $\theta's\ with\ the\ greatest\ difference$
5. Apply formulas

****We Probably Won't Have time to cover other types of Polar Area Graphs ☹**

Area of Polar Graphs Problems:

(Note: For the Following Problems, **find the Bounds of Integration and set up the integrals only)**

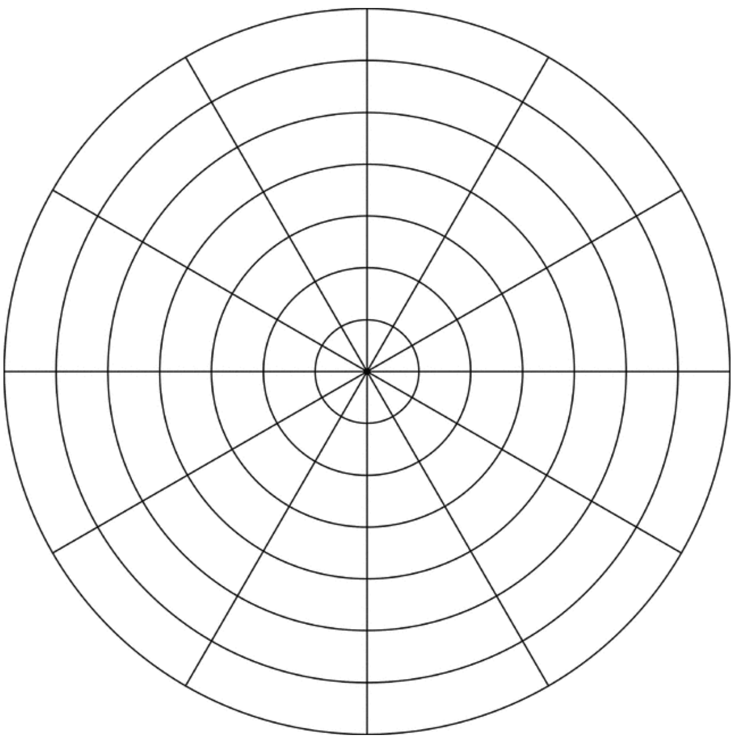
Bodyguard Huell, a four hundred pound man, doesn't enjoy long walks- he doesn't really enjoy exercising for that matter, but he does enjoy body slamming people who make him unhappy. (YOU! ☺ haha)

Unfortunately for you, you have displeased Huell. Huell now walks toward you - slowly. You know you can outrun him, but also know that eventually he will catch up (Huell is pretty relentless). Anyways you have the ability to make Huell happy again if you replace his food and then give him more food.

To pay for the food you need to solve several calculus problems (the café is kinda weird… which brings up the question how Huell got his food in the first place? but nevermind…) Given 5 minutes until Huell shows up, can you get him some food before he breaks your spine? (lol? ☺)

GOOD LUCK!

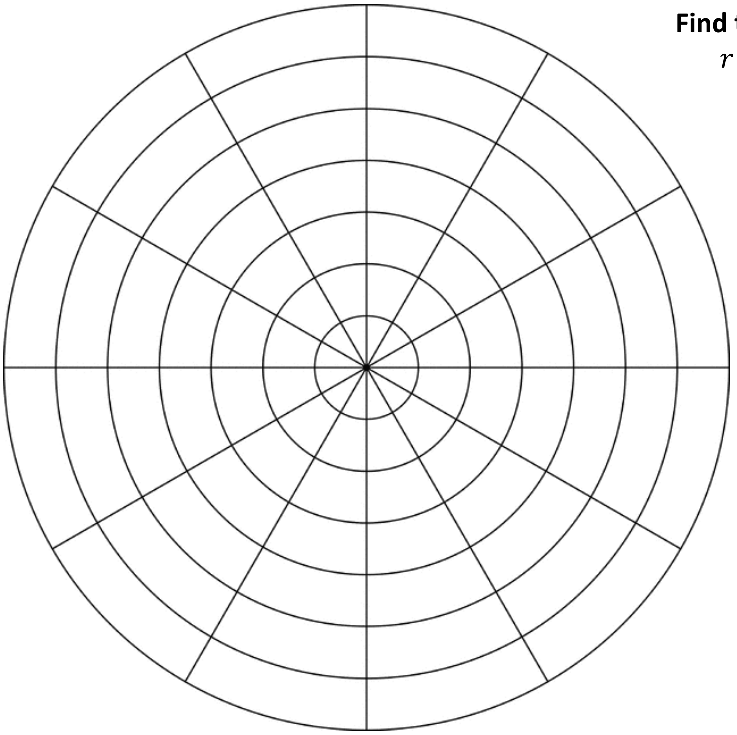
Problem One: Inner Loop (Setup Only)



Find the area of the inner loop (Big – little)

$r = 2 - 3\cos(\theta)$

Problem Two: Inner Loop (Setup Only)

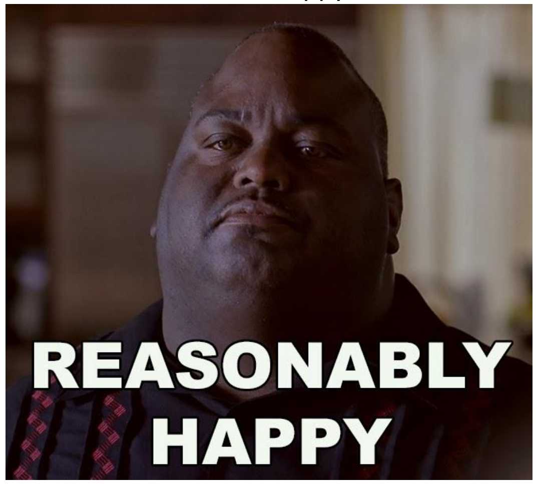


Find the area of the big loop & the little loop (Setup Only)

$r = 2 - 4\sin(\theta)$

Note: See Attached Sheet about Cardiod, Dimpled, Convex, and Rose Curve Area Graphs (We have limited time ☹)

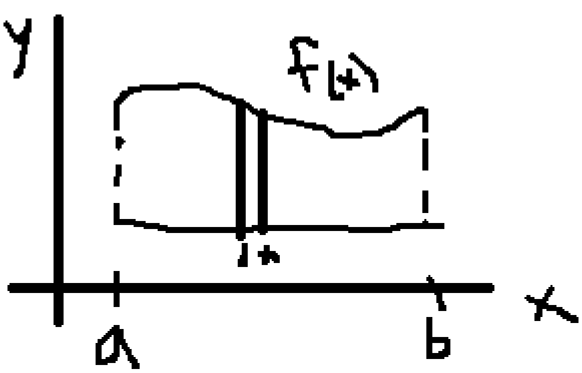
Is Huell Happy?



Area (Continued) – this will be skipped due to time constraints ☹

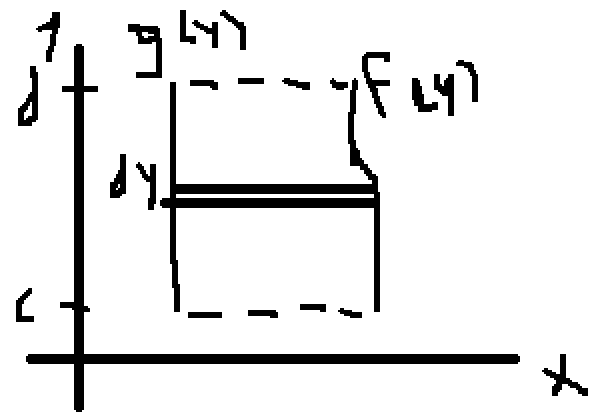
We're Back To Rectangles! ☺

$A = \int_a^b c \; dx$



$\boldsymbol{c = y_{outer} - y_{inner}}$

$A = \int_c^d c \; dy$



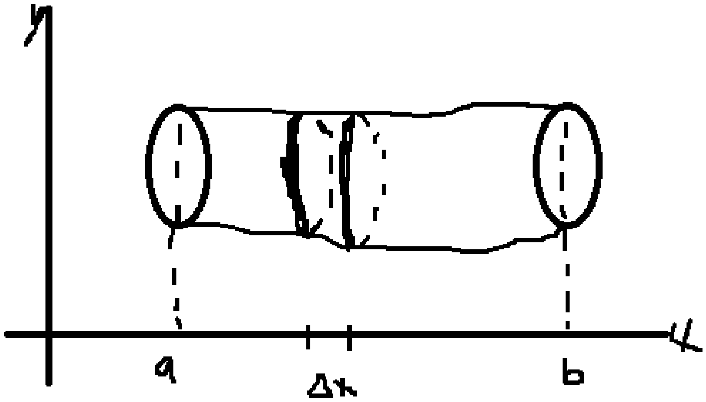
$\boldsymbol{c = x_{outer} - x_{inner}}$

***Note**: 1. inner means function closer to the axis perpendicular to the rectangle slices

2. Sometimes it is necessary to create multiple integrals to calculate the area of a shape.

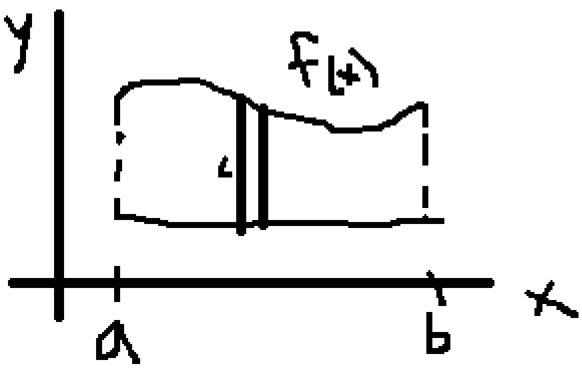
Volume: By Cross Sections! (Shapes)

Explanatory Drawing:



$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i)\Delta x = \underline{\hspace{4cm}}$$

Cross Section Perpendicular to x axis

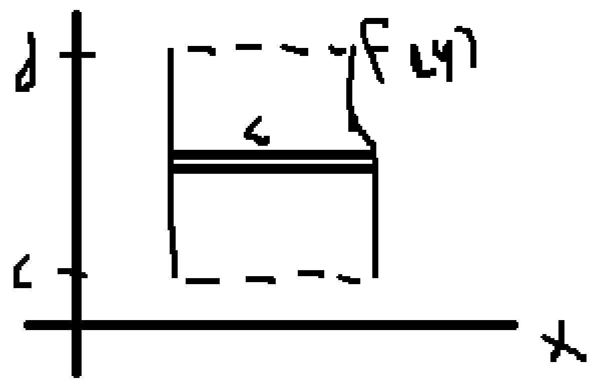


$V = \int_a^b A(x)dx$

$V = \int_a^b A(c)dx$

$c = y_{outer} - y_{inner}$

Cross Section Perpendicular to y axis



$V = \int_c^d A(x)dx$

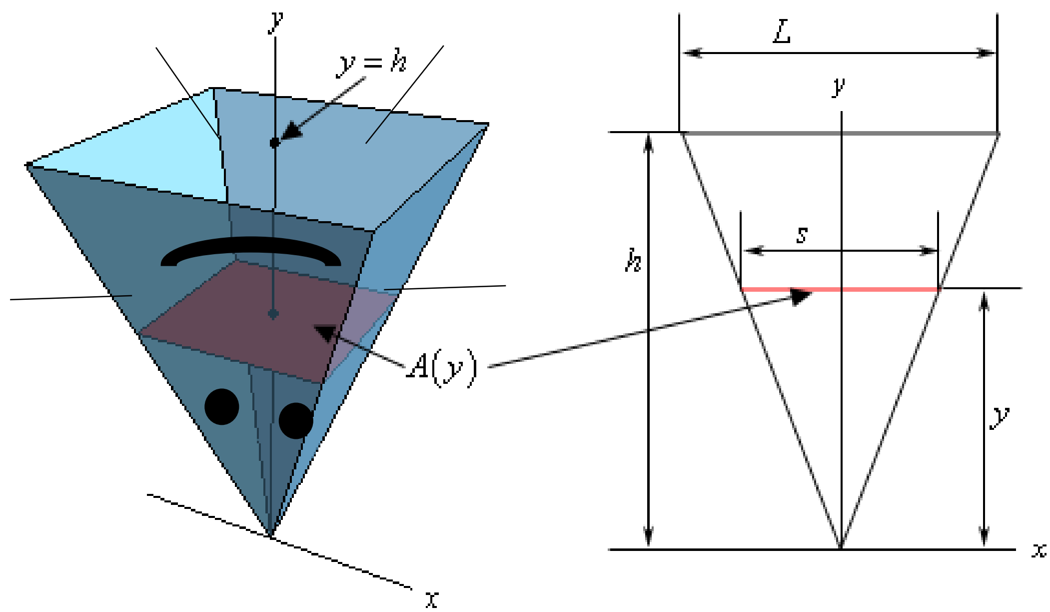
$V = \int_c^d A(c)dy$

$c = x_{outer} - x_{inner}$

Cross Section Types	**Area Formula**
Squares	$A = c^2$
Semi Circles	$A = \frac{\pi c^2}{8}$
Equilateral Triangles	$A = \frac{c^2\sqrt{3}}{4}$
Isosceles with a leg on the base	$A = \frac{c^2}{2}$
Isosceles with a hypotenuse on the base	$A = \frac{c^2}{4}$

Problem One: He's Back…
Triangle-chu is back and now in 3 dimensions! Hot Dang! Before he destroys all of your pokemon in battle (You only have a unless Magickarp ☹ and an Abra) can you find the volume of Triangle-chu given the following information?

Find the volume of a pyramid whose base is a square with sides of length *L* and whose height is *h*.



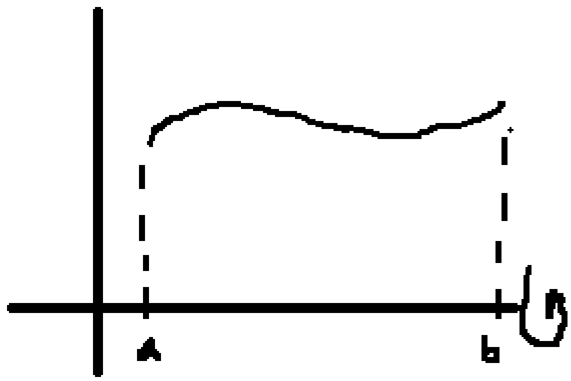
Note: Cross Sectional Problems will always tell you what type of cross section you are expected to take. Exception is the problem in the pset, which has you assume a circle cross section due to info give that you rotate around an axis.

Volume By Revolution: Disk & Washer Method
-This is really a **Cross Section of CIRCLES**

$$V = \pi r^2 h = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

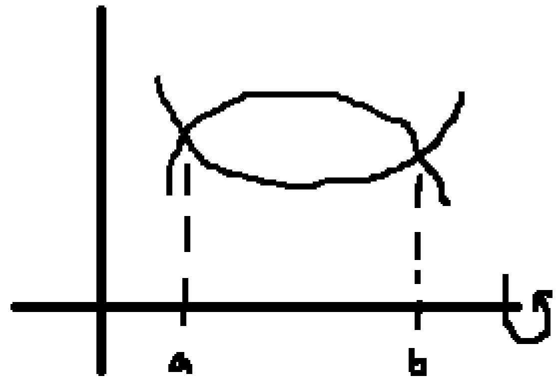
***Applies to both Disk and Washer Method**

Disk Method
Rotated about the x axis



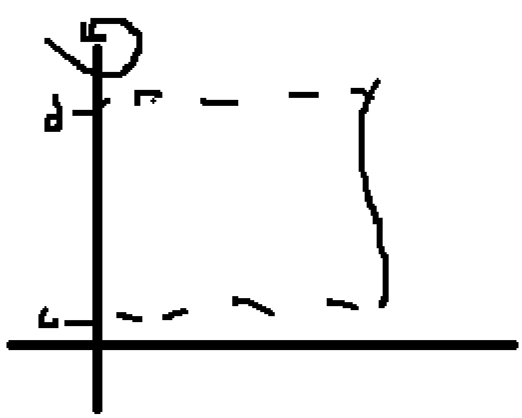
R = ___________________________
r = ___________________________
$V = \pi \int_a^b R^2 - 0^2 dx$
V = ___________________________

Washer Method
Rotated About the x axis



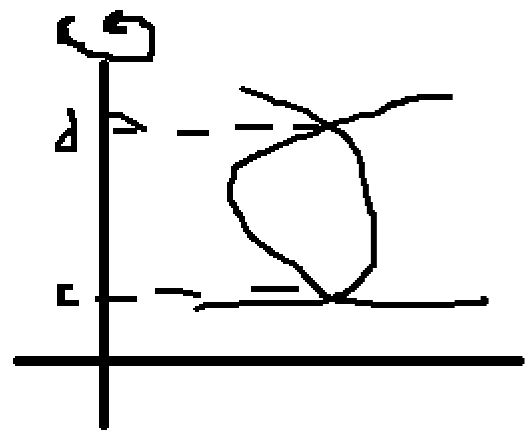
R = ___________________________
r = ___________________________
$V = \pi \int_a^b R^2 - r^2 dx$
V = ___________________________

Rotated about the y axis



R = ___________________________
r = ___________________________
$V = \pi \int_c^d R^2 - 0^2 dy$
V = ___________________________

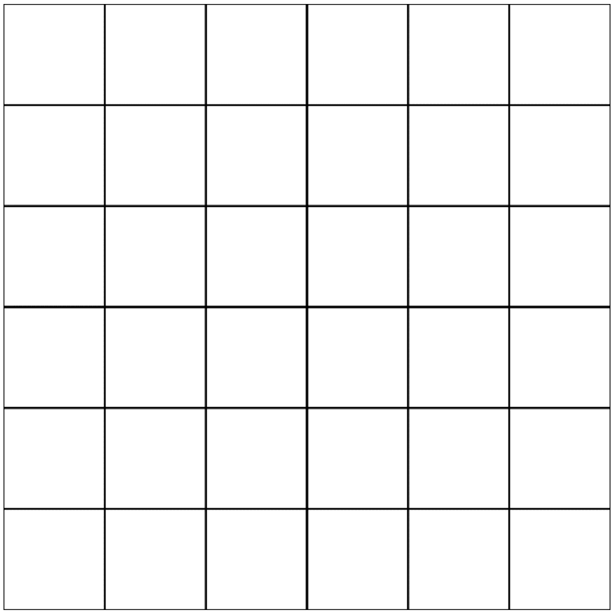
Rotated about the y axis



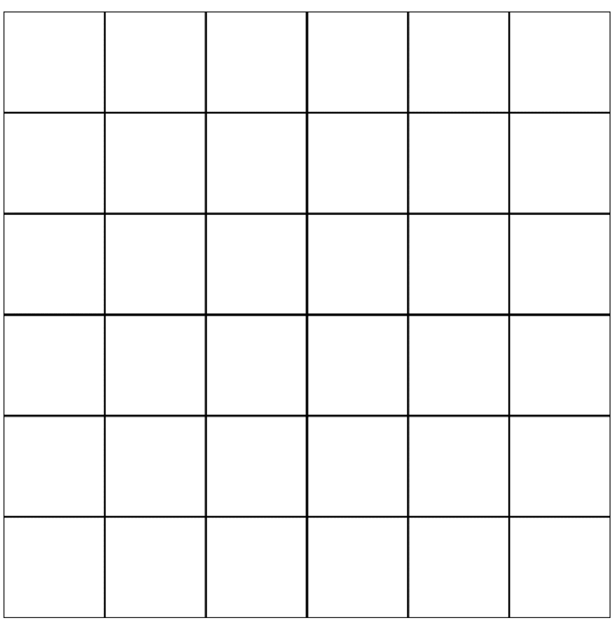
R = ___________________________
r = ___________________________
$V = \pi \int_c^d R^2 - r^2 dx$
V = ___________________________

*So Washer & Disk are essentially the same thing! ☺

Problem One: Apply Washer/Disk (Setup Only)
Find the volume of the solid generated by rotating about the line x = 3,the region bounded by the equations
$y = x^2, x = y^2$

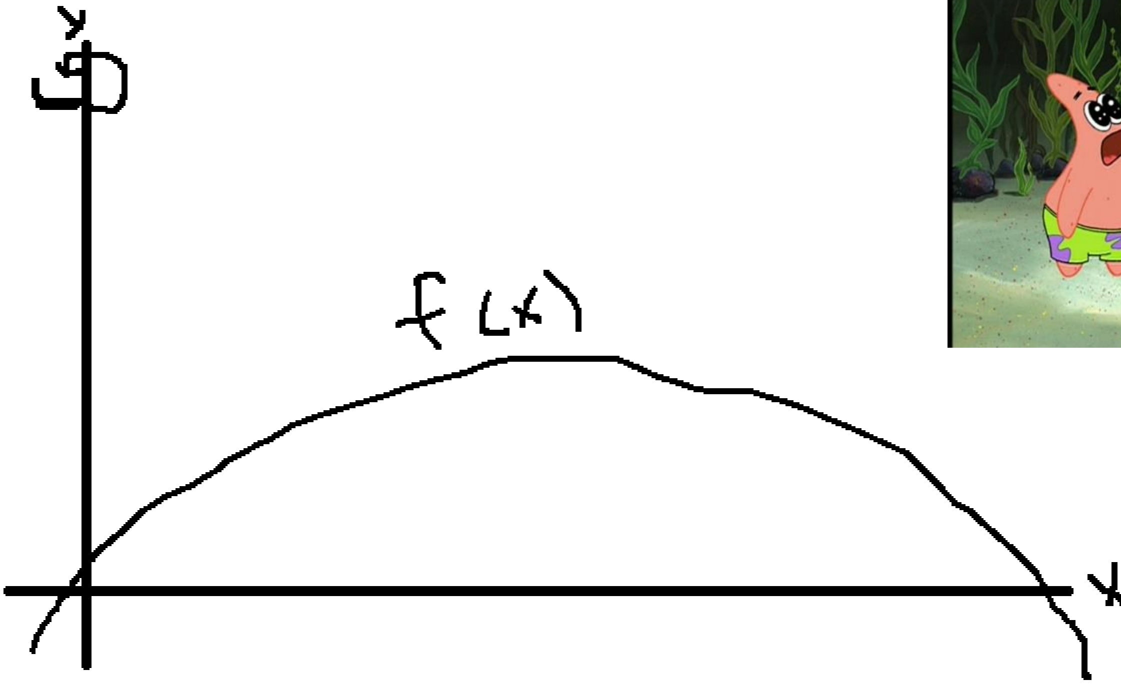


Problem Two: Apply Washer/Disk (Setup Only)
Find the volume of the solid generated by rotating about the line y = -1, the region bounded by the equations
$y = x - x^3$ & $y = 0$

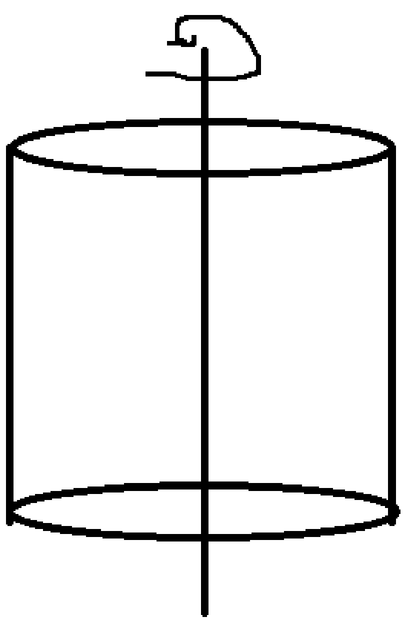


***Graphs Graciously provided by Matt to save time (Thank you Matt! ☺ You Welcome ☺)**

Volume by Revolution! Shell Method!!



- Washer and disk Method won't work here if problem is specifying you must rotate around the y axis
- Use Shell Method!
    - Using Shells to approximate volume means adding up infinitely many _____________.



*This is the shape we use

$$\boldsymbol{V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i)\Delta x =} \text{ \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

***The shell does NOT have a top or bottom & is of thickness Δx & Δy**
***The shell you use is ALWAYS ___________ to the axis of rotation!**

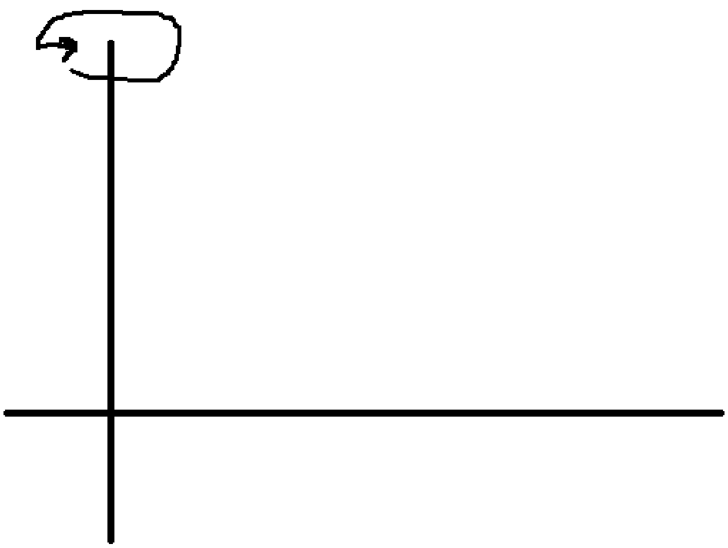
$$A = 2\pi r h$$

$$V = 2\pi \int_a^b rhdx = 2\pi \int_a^b radius * height\ dx\ (or\ dy)$$

*Our job is to find radius, height, and bounds of integration!

Steps:
1. Draw out graphs
2. If confused by what is height, radius then draw in a shell/cylinder. Remember that radius is from the axis of rotation to some point in the region & shells are always parallel to axis of rotation! ☺

Problem One: Find the volume using shells method! (Setup Only)
Find the volume of the solid rotated about the y axis of the region bounded by the following equations.
$y = 4x - x^2$ & $y = 0$



Problem Two: Shell's Method More! More! More!
Find the volume of the solid rotated about the y-axis of the region bounded by the following equations.
$y = f(x)$ & $y = g(x)$ within the bounds [a,b]