Calc II Recitation D **Name: ____________________**

Recitation Ten: 7/30/2015

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Objectives:
0. Nth-term test
1. Infinite Geometric Series
2. Telescoping Series
3. P- Series
4. Integral Test
5. Comparison and Positive-Term Series
6. Alternating Series, Absolute Convergence

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0. Nth term test "Kick the bucket"

$$\lim_{n\to\infty} a_n = c$$

If "C":

1. $\neq 0$: **Series Diverges**
2. Equals 0 : No Conclusion can be drawn.

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1. Geometric Series

$$Example:\ 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

$$\sum_{n=1}^{\infty} 1*\left(\frac{1}{2}\right)^{n-1} = 1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$$

General Equation:

$$\boxed{\sum_{n=1}^{\infty} a_1*(r)^{n-1} = S}$$

When does it converge?

Apply Nth term test -> $\lim_{n\to\infty} a_n = a\lim_{n\to\infty} r^n = 0 \quad \boldsymbol{iff}\ \ |\boldsymbol{r}| < \mathbf{1}$

If it converges then

$$\boxed{\sum_{n=1}^{\infty} a_1*(r)^{n-1} = S = \frac{a_1}{(1-r)}}$$

We can find the sum!

2. Telescoping Series:

1. What are they?
   -A series whose partial sums have a limited number of terms due to cancelation.
2. It is one of two types (Geometric is the other) that can we can find the actual sum.

Note: How do I tell if it is a telescoping series?
It is a series that can be broken down into partial fractions
How do I tell if it converges?
First use nth term (to tell it doesn't converge)
Then use ratio test or other test (to tell if it converges)
How do I find the sum?
Break the sum into partial fractions and then write out the terms

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3. P Series:

General Form:

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots$$

Test for Convergence:

1. Converges if $p > 1$

2. Diverges if $p \le 1$

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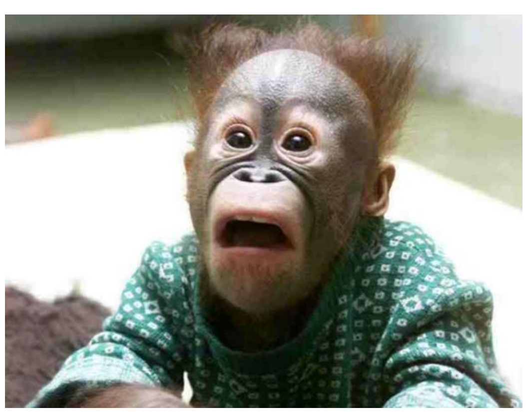
4. Integral Test

1. Can only be used if $a_n = f(n)$ & $f(n)$ *is* ***continous, decreasing, and positive*** *on* $[1, \infty]$
2. Tells us if

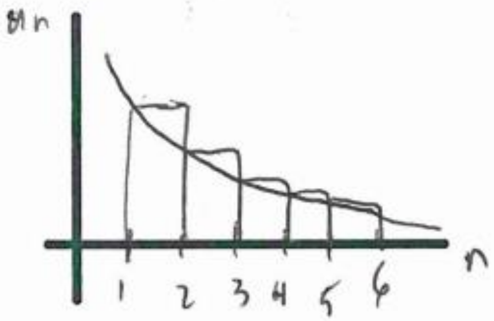
$$\sum_{n=1}^{\infty} a_n \qquad \& \qquad \int_1^{\infty} f(x)dx$$

<u>Both</u> Converge or Diverge



Proof:

Left Reimann Summ



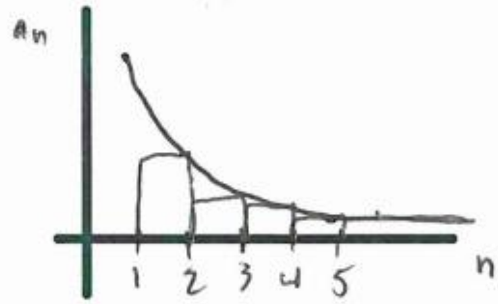
$S_n = a_1 + a_2 + a_3 + \cdots a_n$

$S_n \geq \int_1^{n+1} f(x)dx$

If Integral Diverges:

Σa_n diverges

Right Reimann Summ



$S_n - a_1 = a_2 + a_3 + \cdots a_n$

$S_n \leq a_1 + \int_1^{n} f(x)dx$

If Integral Converges:

Σa_n converges

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3. How to Use Integral Test:

Steps:

1. Check if $a_n = f(n)$ & $f(n)$ *is* ***continuous, decreasing, and positive*** *on* $[1, \infty]$
    - a. Continuous – **Are there no holes in denominator when you plug in n= 1, 2, 3,…?**
    - b. Decreasing – **Is $a_n > a_{n+1}$**
    - c. Positive – **Is $a_n > 0$**
2. Set up and Evaluate Integral
    - a. If Integral converges so does Series
    - b. If Integral diverges so does Series

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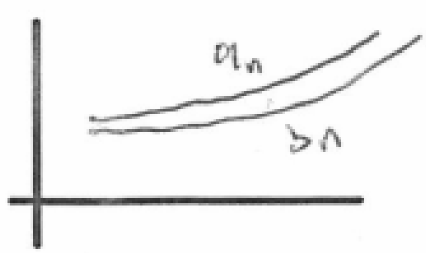
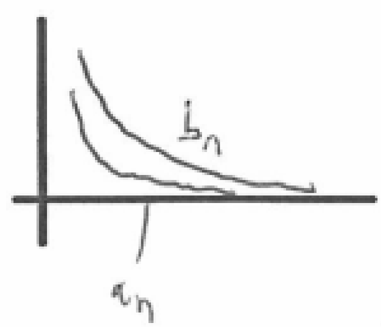
5. Direct Comparison Test

Given the following functions





If:

1. a_n *is smaller than* b_n & b_n *converges then* a_n *converges*
2. a_n *is larger than* b_n & b_n *diverges then* a_n *diverges*

General Procedure:

1. Chose a b_n
2. Prove that b_n is larger or smaller than a_n
3. Prove that b_n either diverges or converges
4. Draw Conclusions

Questions:

1. What if b_n is larger than a_n and it diverges?
   -Then we know nothing
2. What if b_n is smaller than a_n and it converges?
   -Then we know nothing

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6. Limit Comparison Test:

1. Conditions:

The series $\sum_{n=1}^{\infty} a_n$ & $\sum_{n=1}^{\infty} b_n$ must contain **only positive terms**

2. $\sum_{n=1}^{\infty} b_n$ is our comparison series. If $\lim_{n\to\infty} \frac{a_n}{b_n} = C$

*C must be a positive finite number than whatever behavior b_n exhibited (Convergence or Divergence) a_n will behave in the same way. Therefore, both will either converge or diverge.

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7. Alternating Series Test

-Does the series alternate? Aka first term positive and second term negative?

$\sum_{n=1}^{\infty}(-1)^{n+1}b_n$ OR $\sum_{n=1}^{\infty}(-1)^{n}b_n$

- b_n must be positive (Must check for this)
- If $b_{n+1} < b_n$ for all n then **the series converges**

Time for Jeopardy!!

****Rules will be announced**