Lua Test

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Example: elementary Gauss-Jordan

$$A = \begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix}$$

$$\begin{aligned}
&\begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \xrightarrow{R_1 \leftarrow 1/4R_1} \begin{pmatrix} 1 & 3 & 1 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\
\xrightarrow{R_2 \leftarrow R_2 - 6R_1} &\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - 6R_1} \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\
\xrightarrow{R_4 \leftarrow R_4 + 9R_1} &\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} \xrightarrow{R_2 \leftarrow -1/15R_2} \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\
\xrightarrow{R_1 \leftarrow R_1 - 3R_2} &\begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 + 25R_2} \begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & 0 & 0 & 0 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\
\xrightarrow{R_4 \leftarrow R_4 - 40R_2} &\begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

Example: finding an inverse using Gauss-Jordan

To find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 0 & 4 \\ 5 & 2 & 3 \end{pmatrix}$$

First we define the augmented matrix by including a 3x3 identity matrix,

$$\begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix}$$

We then row reduce this matrix using Gauss-Jordan

$$\begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow 1/2R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - 1R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - 5R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow -2/3R_2} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - 3/2R_2} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + 11/2R_2} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & -31/3 & -2/3 & -11/3 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow -3/31R_3} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 10/3R_3} \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & 0 & 17/31 & 16/31 & -10/31 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix}$$

It follows that the inverse is given by

$$A^{-1} = \frac{1}{31}\begin{pmatrix} -8 & -13 & 12 \\ 17 & 16 & -10 \\ 2 & 11 & -3 \end{pmatrix}$$