

# Using Lua<sup>A</sup>T<sub>E</sub>X for Linear Algebra

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## Example: elementary Gauss-Jordan

```
1 \directlua{
2   require "RationalAlg"
3
4   M = {
5     {Rational:new(nil, 4), Rational:new(nil,12),Rational:new(nil,4),Rational:new(nil,0)},
6     {Rational:new(nil, 6), Rational:new(nil,3),Rational:new(nil,-6),Rational:new(nil,9)},
7     {Rational:new(nil,6),Rational:new(nil,-7),Rational:new(nil,-14),Rational:new(nil,15)},
8     {Rational:new(nil,-9),Rational:new(nil,13),Rational:new(nil,23),Rational:new(nil,-24)}
9   }
10
11   tex.print("\\[A = "..RationalAlg.MatrixToTeX(M,true).."\\]")
12
13   A, R = RationalAlg.GaussJordanRowReduce(M)
14
15   tex.print(RationalAlg.RowOpListToTeX(R,2,true))
16 }
```

$$A = \begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix}$$

$$\begin{array}{ccc} \begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} & \xrightarrow{R_1 \leftarrow 1/4 R_1} & \begin{pmatrix} 1 & 3 & 1 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\ \xrightarrow{R_2 \leftarrow R_2 - 6R_1} & \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} & \xrightarrow{R_3 \leftarrow R_3 - 6R_1} \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\ \xrightarrow{R_4 \leftarrow R_4 + 9R_1} & \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} & \xrightarrow{R_2 \leftarrow -1/15 R_2} \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\ \xrightarrow{R_1 \leftarrow R_1 - 3R_2} & \begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} & \xrightarrow{R_3 \leftarrow R_3 + 25R_2} \begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & 0 & 0 & 0 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\ \xrightarrow{R_4 \leftarrow R_4 - 40R_2} & \begin{pmatrix} 1 & 0 & -7/5 & 9/5 \\ 0 & 1 & 4/5 & -3/5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \end{array}$$

## Example: finding an inverse using Gauss-Jordan

```

1 \directlua{
2   M = {
3     {Rational:new(nil,2), Rational:new(nil,3), Rational:new(nil,-2)},
4     {Rational:new(nil,1), Rational:new(nil,0), Rational:new(nil,4)},
5     {Rational:new(nil,5), Rational:new(nil,2), Rational:new(nil,3)}
6   }
7 }
8 To find the inverse of the matrix
9 \directlua{
10   tex.print("\[A = "..RationalAlg.MatrixToTeX(M,true).."\\]")
11 }
12 First we define the augmented matrix by including a 3x3 identity matrix,
13 \directlua{
14   MA = RationalAlg.Augment(M, RationalAlg.IdentityMatrix(3))
15   tex.print("\["..RationalAlg.MatrixToTeX(MA,true).."\\]")
16 }
17 We then row reduce this matrix using Gauss-Jordan
18 \directlua{
19   A, R = RationalAlg.GaussJordanRowReduce(MA)
20   tex.print(RationalAlg.RowOpListToTeX(R,2,true))
21 }
22
23 It follows that the inverse is given by,
24 \directlua{
25   l, r = RationalAlg.Split(A, 3)
26   tex.print("\[ A^{-1} = " .. RationalAlg.MatrixToTeX(r,true) .. "\\]")
27 }

```

To find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 0 & 4 \\ 5 & 2 & 3 \end{pmatrix}$$

First we define the augmented matrix by including a 3x3 identity matrix,

$$\begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix}$$

We then row reduce this matrix using Gauss-Jordan

$$\begin{array}{ccc}
 \begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} & \xrightarrow{R_1 \leftarrow 1/2 R_1} & \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} \\
 \xrightarrow{R_2 \leftarrow R_2 - 1R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} & \xrightarrow{R_3 \leftarrow R_3 - 5R_1} & \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix} \\
 \xrightarrow{R_2 \leftarrow -2/3 R_2} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix} & \xrightarrow{R_1 \leftarrow R_1 - 3/2 R_2} & \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix} \\
 \xrightarrow{R_3 \leftarrow R_3 + 11/2 R_2} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & -31/3 & -2/3 & -11/3 & 1 \end{pmatrix} & \xrightarrow{R_3 \leftarrow -3/31 R_3} & \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix} \\
 \xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix} & \xrightarrow{R_2 \leftarrow R_2 + 10/3 R_3} & \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & 0 & 17/31 & 16/31 & -10/31 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix}
 \end{array}$$

It follows that the inverse is given by

$$A^{-1} = \begin{pmatrix} -8/31 & -13/31 & 12/31 \\ 17/31 & 16/31 & -10/31 \\ 2/31 & 11/31 & -3/31 \end{pmatrix}$$

## Random matrices

```

1 \directlua{
2   M = RationalAlg.RandomMatrix(3,4,true)
3   tex.print(
4     "\\[ A = " .. RationalAlg.MatrixToTeX(M) .. "\\]"
5   )
6   R = RationalAlg.GaussJordanRowReduce(M)
7   tex.print(RationalAlg.RowOpListToTeX(R,2,true))
8 }

```

$$A = \begin{pmatrix} -5 & -2 & -4 & -5 \\ 3 & 3 & 0 & -2 \\ 1 & 1 & 0 & -5 \end{pmatrix}$$

$$\begin{array}{ccc}
 \begin{pmatrix} -5 & -2 & -4 & -5 \\ 3 & 3 & 0 & -2 \\ 1 & 1 & 0 & -5 \end{pmatrix} & \xrightarrow{R_1 \leftarrow -1/5 R_1} & \begin{pmatrix} 1 & 2/5 & 4/5 & 1 \\ 3 & 3 & 0 & -2 \\ 1 & 1 & 0 & -5 \end{pmatrix} \\
 \xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & 2/5 & 4/5 & 1 \\ 0 & 9/5 & -12/5 & -5 \\ 1 & 1 & 0 & -5 \end{pmatrix} & \xrightarrow{R_3 \leftarrow R_3 - 1R_1} & \begin{pmatrix} 1 & 2/5 & 4/5 & 1 \\ 0 & 9/5 & -12/5 & -5 \\ 0 & 3/5 & -4/5 & -6 \end{pmatrix} \\
 \xrightarrow{R_2 \leftarrow 5/9 R_2} \begin{pmatrix} 1 & 2/5 & 4/5 & 1 \\ 0 & 1 & -4/3 & -25/9 \\ 0 & 3/5 & -4/5 & -6 \end{pmatrix} & \xrightarrow{R_1 \leftarrow R_1 - 2/5 R_2} & \begin{pmatrix} 1 & 0 & 4/3 & 19/9 \\ 0 & 1 & -4/3 & -25/9 \\ 0 & 3/5 & -4/5 & -6 \end{pmatrix} \\
 \xrightarrow{R_3 \leftarrow R_3 - 3/5 R_2} \begin{pmatrix} 1 & 0 & 4/3 & 19/9 \\ 0 & 1 & -4/3 & -25/9 \\ 0 & 0 & 0 & -13/3 \end{pmatrix} & \xrightarrow{R_3 \leftarrow -3/13 R_3} & \begin{pmatrix} 1 & 0 & 4/3 & 19/9 \\ 0 & 1 & -4/3 & -25/9 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
 \xrightarrow{R_1 \leftarrow R_1 - 19/9 R_3} \begin{pmatrix} 1 & 0 & 4/3 & 0 \\ 0 & 1 & -4/3 & -25/9 \\ 0 & 0 & 0 & 1 \end{pmatrix} & \xrightarrow{R_2 \leftarrow R_2 + 25/9 R_3} & \begin{pmatrix} 1 & 0 & 4/3 & 0 \\ 0 & 1 & -4/3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
 \end{array}$$