Using LuaLaTeX for Linear Algebra

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Example: elementary Gauss-Jordan

```
\directlua{
    A = RationalAlg.StringToMatrix(
        "{{4,12,4,0},{6,3,-6,9},{6,-7,-14,15},{-9,13,23,-24}}"
    )}

\[ A = \directlua{tex.print(RationalAlg.MatrixToTeX(A,true))}\]
\directlua{
    M, R = RationalAlg.GaussJordanRowReduce(A)
    tex.print(RationalAlg.RowOpListToTeX(R,2,true))
}
```

$$A = \begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix}$$

$$\begin{aligned}
& \begin{pmatrix} 4 & 12 & 4 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix}
\xrightarrow{R_1 \leftarrow 1/4R_1}
\begin{pmatrix} 1 & 3 & 1 & 0 \\ 6 & 3 & -6 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\
\xrightarrow{R_2 \leftarrow R_2 - 6R_1}
& \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 6 & -7 & -14 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix}
\xrightarrow{R_3 \leftarrow R_3 - 6R_1}
\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ -9 & 13 & 23 & -24 \end{pmatrix} \\
\xrightarrow{R_4 \leftarrow R_4 + 9R_1}
& \begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & -15 & -12 & 9 \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix}
\xrightarrow{R_2 \leftarrow -1/15R_2}
\begin{pmatrix} 1 & 3 & 1 & 0 \\ 0 & 1 & \sfrac{4}{5} & -\sfrac{3}{5} \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\
\xrightarrow{R_1 \leftarrow R_1 - 3R_2}
& \begin{pmatrix} 1 & 0 & -\sfrac{7}{5} & \sfrac{9}{5} \\ 0 & 1 & \sfrac{4}{5} & -\sfrac{3}{5} \\ 0 & -25 & -20 & 15 \\ 0 & 40 & 32 & -24 \end{pmatrix}
\xrightarrow{R_3 \leftarrow R_3 + 25R_2}
\begin{pmatrix} 1 & 0 & -\sfrac{7}{5} & \sfrac{9}{5} \\ 0 & 1 & \sfrac{4}{5} & -\sfrac{3}{5} \\ 0 & 0 & 0 & 0 \\ 0 & 40 & 32 & -24 \end{pmatrix} \\
\xrightarrow{R_4 \leftarrow R_4 - 40R_2}
& \begin{pmatrix} 1 & 0 & -\sfrac{7}{5} & \sfrac{9}{5} \\ 0 & 1 & \sfrac{4}{5} & -\sfrac{3}{5} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

Example: finding an inverse using Gauss-Jordan

To find the inverse of the matrix

```
\directlua{A = RationalAlg.StringToMatrix("{{2,3,-2},{1,0,4},{5,2,3}}")}
```

```
\[ A = \directlua{tex.print(RationalAlg.MatrixToTeX(A))}\]
```

$$A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 0 & 4 \\ 5 & 2 & 3 \end{pmatrix}$$

First we define the augmented matrix by including a 3x3 identity matrix,

```
\directlua{AAugmented = RationalAlg.Augment(A, RationalAlg.IdentityMatrix(3))}
\[\directlua{tex.print(RationalAlg.MatrixToTeX(AAugmented,true))}\]
```

$$\begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix}$$

We then row reduce this matrix using Gauss-Jordan

```
\directlua{
    _, R = RationalAlg.GaussJordanRowReduce(AAugmented)
    tex.print(RationalAlg.RowOpListToTeX(R,2,true))
}
```

$$\begin{pmatrix} 2 & 3 & -2 & 1 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow 1/2R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 1 & 0 & 4 & 0 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow R_2 - 1R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 5 & 2 & 3 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 - 5R_1} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & -3/2 & 5 & -1/2 & 1 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_2 \leftarrow -2/3R_2} \begin{pmatrix} 1 & 3/2 & -1 & 1/2 & 0 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - 3/2R_2} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & -11/2 & 8 & -5/2 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + 11/2R_2} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & -31/3 & -2/3 & -11/3 & 1 \end{pmatrix} \xrightarrow{R_3 \leftarrow -3/31R_3} \begin{pmatrix} 1 & 0 & 4 & 0 & 1 & 0 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - 4R_3} \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & -10/3 & 1/3 & -2/3 & 0 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 10/3R_3} \begin{pmatrix} 1 & 0 & 0 & -8/31 & -13/31 & 12/31 \\ 0 & 1 & 0 & 17/31 & 16/31 & -10/31 \\ 0 & 0 & 1 & 2/31 & 11/31 & -3/31 \end{pmatrix}$$

It follows that the inverse is given by

```
\directlua{l, r = RationalAlg.Split(A, 3)}
\[ A^{-1} = \directlua{tex.print(RationalAlg.MatrixToTeX(r,true))}\]
```

$$A^{-1} = \begin{pmatrix} -8/31 & -13/31 & 12/31 \\ 17/31 & 16/31 & -10/31 \\ 2/31 & 11/31 & -3/31 \end{pmatrix}$$

Random matrices

```
\directlua{
    M = RationalAlg.RandomMatrix(3,4,true)
    tex.print(
        "\\[ A = " .. RationalAlg.MatrixToTeX(M) .. "\\]"
    )
    _, R = RationalAlg.GaussJordanRowReduce(M)
    tex.print(RationalAlg.RowOpListToTeX(R,2,true))
}
```

$$A = \begin{pmatrix} 0 & 3 & 2 & 5 \\ 1 & 3 & 3 & 0 \\ 5 & -1 & 2 & 3 \end{pmatrix}$$

$$\begin{aligned}
&\begin{pmatrix} 0 & 3 & 2 & 5 \\ 1 & 3 & 3 & 0 \\ 5 & -1 & 2 & 3 \end{pmatrix} \xrightarrow{R_1 \longleftrightarrow R_2} \begin{pmatrix} 1 & 3 & 3 & 0 \\ 0 & 3 & 2 & 5 \\ 5 & -1 & 2 & 3 \end{pmatrix} \\
\xrightarrow{R_3 \leftarrow R_3 - 5R_1} &\begin{pmatrix} 1 & 3 & 3 & 0 \\ 0 & 3 & 2 & 5 \\ 0 & -16 & -13 & 3 \end{pmatrix} \xrightarrow{R_2 \leftarrow 1/3R_2} \begin{pmatrix} 1 & 3 & 3 & 0 \\ 0 & 1 & 2/3 & 5/3 \\ 0 & -16 & -13 & 3 \end{pmatrix} \\
\xrightarrow{R_1 \leftarrow R_1 - 3R_2} &\begin{pmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 2/3 & 5/3 \\ 0 & -16 & -13 & 3 \end{pmatrix} \xrightarrow{R_3 \leftarrow R_3 + 16R_2} \begin{pmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 2/3 & 5/3 \\ 0 & 0 & -7/3 & 89/3 \end{pmatrix} \\
\xrightarrow{R_3 \leftarrow -3/7R_3} &\begin{pmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 2/3 & 5/3 \\ 0 & 0 & 1 & -89/7 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - 1R_3} \begin{pmatrix} 1 & 0 & 0 & 54/7 \\ 0 & 1 & 2/3 & 5/3 \\ 0 & 0 & 1 & -89/7 \end{pmatrix} \\
\xrightarrow{R_2 \leftarrow R_2 - 2/3R_3} &\begin{pmatrix} 1 & 0 & 0 & 54/7 \\ 0 & 1 & 0 & 71/7 \\ 0 & 0 & 1 & -89/7 \end{pmatrix}
\end{aligned}$$

Surds

We can also use surds and complex numbers.

```
\directlua{
    require "Surd"
    z = Surd:new(nil, 2, 3, -1)
    w = Surd:new(nil, Rational:new(nil, 2, 3), Rational:new(nil, 1, 3), -1)
    tex.print("\\[ z = " .. z:totexstring(false) .. ", \\qquad w = " .. w:totexstring(false) ..
    "\\]")
}
```

$$z = 2 + 3i, \qquad w = \frac{2}{3} + \frac{1}{3}i$$

- $z + w = \frac{8}{3} + \frac{10}{3}i$
- $z - w = \frac{4}{3} + \frac{8}{3}i$
- $zw = \frac{1}{3} + \frac{8}{3}i$
- $z/w = \frac{21}{5} + \frac{12}{5}i$

The Surd library will automatically ensure the ‘base’ is square-free. For example the following code

```
\directlua{
    a = Surd:new(nil, 2, 5, -12)
    tex.print("\\[ a = " .. a:totexstring() .. "\\]")
}
```

produces the output:

$$a = 2 + 20i\sqrt{3}$$