

Multiple Linear Regression on Sleep Data

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Motivation

Sleep is an important contributor to our well-being and is deeply connected to our physical health, and cognitive function. However we often find that many individuals struggle with getting long hours sleep, often finding themselves tossing and turning throughout the night. Consequently, there is now a growing interest in understanding the lifestyle and bio metric factors that influence sleep patterns and duration. General knowledge and intuition suggest factors such as physical activity and occupational stress may influence sleep duration. Therefore, our goal is to use linear regression techniques to quantify exactly how much these factors reduce or extend sleep time.

Before, analyzing the data set, we expect a number of our indicators to contribute significantly to sleep duration. In general, we would assume that occupation will play an important role in analyzing our data as a more stressful work environment may hinder sleep patterns and disturb the body's natural sleep cycle. Additionally, we may expect that higher levels of physical activity will work in tandem with deeper and longer duration of sleep.

Also, we seek to filter through and find the most important variables in our analysis. We expect that variables like Daily Steps and Physical Activity Level will be a contribute to sleep duration in a similar manner. In our data set, we have numerous combinations of variables that are alike in this way and therefore we want to reduce our factors to give the most precise and simple model possible.

About the Data

Table 1: Variables From Our Data

Gender	Age	Occupation
Sleep Duration (hours)	Quality of Sleep (scale: 1–10)	Physical Activity Level (minutes/day)
Stress Level (scale: 1–10)	BMI Category	PP (Pulse Pressure)
Heart Rate (bpm)	Daily Steps	Sleep Disorder

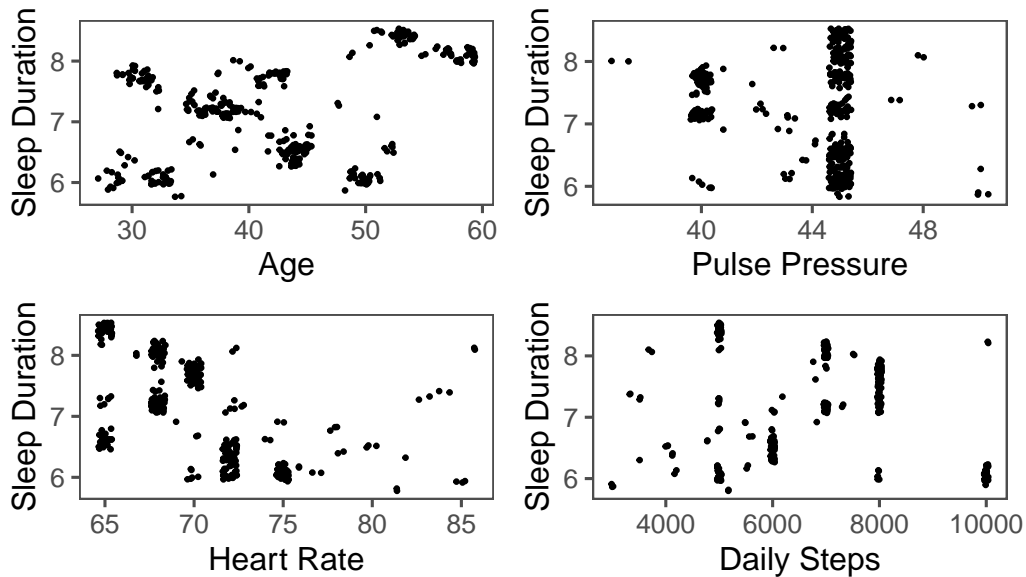
Sleep Duration is a continuous variable and will be our response variable. The rest will be our covariates. Gender, Occupation, Quality of Sleep, Stress Level, BMI Category and Sleep Disorder are categorical variables. Physical Activity, PP, agem and Daily steps are continuous variables. We have a total of 374 observations.

We had no missing data in the data set but we did have to make a mutation using `dplyr`. In the original data set there was a blood pressure variable that was encoded as a character but was a ratio, e.g. “ $\frac{120}{80}$ ”. We separated the blood pressure variable into two: Systolic and Diastolic. These two we suspected to be highly correlated but did not want to just drop one or the other. We had options of combining the two into something meaningful. One was Pulse Pressure (PP), which is calculated as $PP = \text{systolic} - \text{diastolic}$. PP is the force of the hearts contraction on the arteries. The other is called Mean Arterial Pressure (MAP), which is calculated as $MAP = \frac{\text{systolic} \cdot \text{diastolic}^2}{3}$. MAP is the average blood pressure throughout a cardiac cycle. We decided on PP because it was simpler to calculate and understand.

For transparency it should be noted that the data is synthetic. Even though it is synthetic we still treat the analysis and report as if it was real data.

Exploratory Data Analysis

Sleep Duration vs Continuous Variables



Visual relationship can be seen for Sleep Duration with Age and sleep_duration. Not so much for PP and daily_steps. The correlation between our numeric covariates is most notable between sleep_duration and heart_rate. age and hearts_pressure

Table 2: Correlation Matrix of Continuous Variables

	sleep_duration	age	PP	heart_rate	daily_steps
sleep_duration	1.00	0.34	-0.16	-0.52	-0.04
age	0.34	1.00	0.46	-0.23	0.06
PP	-0.16	0.46	1.00	0.27	-0.31
heart_rate	-0.52	-0.23	0.27	1.00	-0.03
daily_steps	-0.04	0.06	-0.31	-0.03	1.00

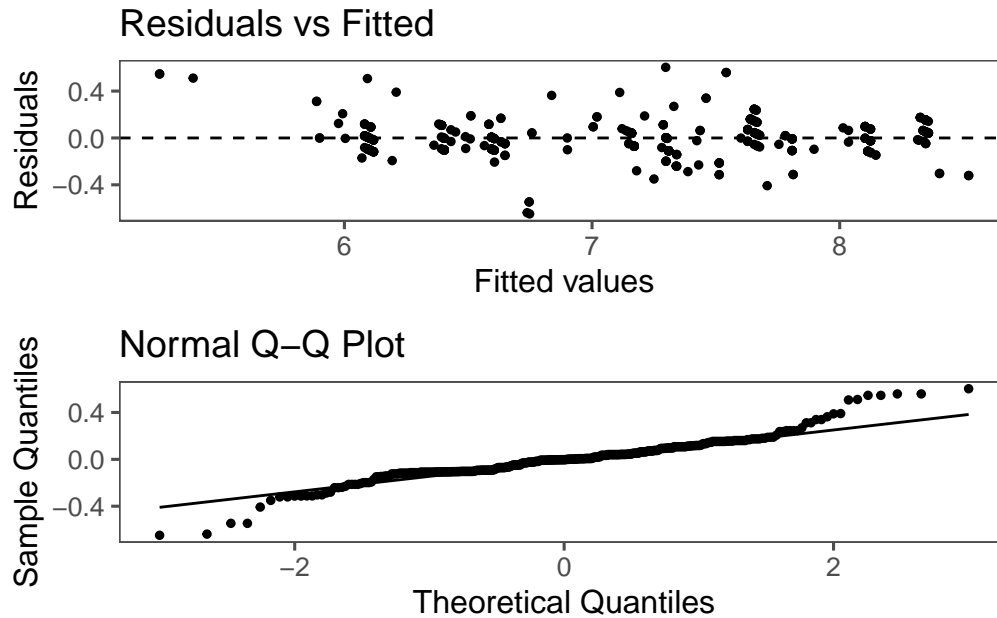
Full Model

Running the full model we can see that our omnibus hypothesis test has an F-stat of 426.6 which is greater than the 99th percentile for our F-distribution, $F_{31,342,.99} = 1.74$. So we reject the null that all coefficients are 0. We also see a high Adjusted R^2 and very low Residual Sum of Squares compared to the Total Sum of Squares.

Table 3: Full Linear Model Outcome

F-statistic	Adjusted R ²	Residual Sum of Squares	Total Sum of Squares
313.34	0.958	9.28	236.13

We also saw all coefficients have standard error less than one.



As far as assumptions go homoskedasticity seems to be true and normality might be slightly violated with heavy tails but there is no excessive deviation from normality. We will explore multicollinearity and outliers next.

Regression Diagnostics: Outliers and Multicollinearity

Looking now towards outliers, we selected observations that had one of following: leverage $> 2 \cdot \frac{p+1}{n}$, studentized residual > 2 , and a cooks threshold > 1 . We saw a total of 57 observations that exceeded at least one of these thresholds. Since we can't say that any of these observation are errors we will only remove row 264 because it has a leverage of 1 and NaN for the rest. This is clearly a problem observation.

Table 4: Row with Highest Leverage

	Leverage	Studentized	DFFITS	Cooks_Distance
264	1	NaN	NaN	NaN

Next, we wanted to take care of multicollinearity after removing observation 264 from the data set. We checked the VIF values for each input covariate and determined which features can be removed. We also set 5 as a threshold for a large VIF value and removed any features that exceed this value. Furthermore, we used the scaled GVIF value, $GVIF^{1/(2 \cdot Df)}$, as it allows categorical variables to be numerically compared to continuous variables. From our VIF table, we noticed that age and quality_of_sleep have $GVIF^{1/(2 \cdot Df)} > 5$.

	GVIF	Df	$GVIF^{1/(2 \cdot Df)}$
gender	15.83647	1	3.979507
age	26.02202	1	5.101178
occupation	8275.05049	9	1.650646
quality_of_sleep	30.58749	1	5.530596
physical_activity_level	10.66632	1	3.265933
stress_level	8680.08342	5	2.476580
bmi_category	125.37070	3	2.237172
heart_rate	14.83382	1	3.851470
daily_steps	10.02533	1	3.166281
sleep_disorder	12.50975	2	1.880668
PP	8.66875	1	2.944274

We removed variables age and quality_of_sleep and ran a MLR model. Our results showed that the reduced VIF model was very similar to the full model. Adjusted R^2 was essentially unchanged. Therefore, we concluded that age and quality_of_sleep did not contribute much to the regression analysis because they were linear combinations of other independent variables.

Table 6: Outlier + VIF Model

F-stat.	Adjusted R^2	Residual Sum of Squares	Total Sum of Squares
308.43	0.952	10.6	236.08

Variable Selection: Backward, Forward, and Stepwise

In order to avoid overfitting issues, we utilized a greedy algorithm process to select variables that provided the lowest AIC value. We ran 3 model selection process as discussed in class and recorded the variables selected for each method. We found that all 3 methods took all of our variables in our data set. We knew that AIC penalizes the fitted model for each additional variables taken. Despite this, we saw that all variables were significant. Note that the variable selection process was done after our high VIF variables were removed.

Table 7: Variable Selection

Variable	Backward	Forward	Stepwise
bmi_category	Yes	Yes	Yes
daily_steps	Yes	Yes	Yes
gender	Yes	Yes	Yes
heart_rate	Yes	Yes	Yes
occupation	Yes	Yes	Yes
physical_activity_level	Yes	Yes	Yes
PP	Yes	Yes	Yes
sleep_disorder	Yes	Yes	Yes
stress_level	Yes	Yes	Yes

Suggested Regression: LASSO

For our suggested regression, we used LASSO regression. We hoped to extract the most important input features to achieve a simple yet powerful model. To find the optimal λ or the regularization parameter, we used a cross-validation process by splitting our data set: train = 80%. We saw that gender, occupation, bmi_category, and sleep_disorder converged to zero.

Table 8: Non-Zero Coefficients from LASSO (Selected by Cross-Validation)

	Variable	Coefficient
(Intercept)	(Intercept)	10.7550
gender	gender	0.0000
occupation	occupation	0.0000
physical_activity_level	physical_activity_level	0.0128
stress_level	stress_level	-0.3145
bmi_category	bmi_category	0.0000
heart_rate	heart_rate	-0.0115
daily_steps	daily_steps	-0.0001
sleep_disorder	sleep_disorder	0.0000
PP	PP	-0.0282