

**Mid-Term Practice (Second Example)**  
**Tuesday, May 16, 2023**

**Instructions:**

- This is a 1 hour and 10 minute exam. You are allowed to use the equations sheet provided in the site of the course and handwritten notes.
- You are not allowed to use any electronic device. No connection to the internet via WiFi or any other method is allowed. It is not permitted to use any kind of mobile phone.
- When you are finished with the exam, please **turn in the exam questions**.
- Cheating of any form will result in a score of 0 (zero) for the exam, in addition to the normal university disciplinary action.
- Please sign below that you have read, understood, and fulfilled all of the above instructions and conditions.

Please fill in the following personal information:

<b>First Name</b>	
<b>Last Name</b>	
<b>UCLA ID #</b>	
<b>Signature</b>	
<b>Exam Version A</b>	

Please start solving the examinations only when you are instructed to do so.  
Please stop immediately when instructed to do so.

Good Luck!

## Part I (Questions based on Regression Output):

Questions 1–4 are based on the following regression output

Consider the following linear model:

$$\logwage = \beta_1 + \beta_2 \cdot logeduc + e_i, \text{ where}$$

1. *logwage* means the logarithm of daily wage, measured in dollars.
2. *logeduc* means the logarithm of education, measured in years of schooling.

The output for this linear regression is given below:

```
. reg logwage logeduc
```

Source	SS	df	MS	Number of obs	=	100
Model	.25000000	1	.25000000	F(1, 98)	=	96.47
Residual	.25000000	98	.00250000	Prob > F	=	0.0000
				R-squared	=	0.5000
				Adj R-squared	=	0.4950
Total	.50000000	99	.00500000	Root MSE	=	.05000

  

logwage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
logeduc	1.000000	.100000	10.00	0.000	.750000 1.15000
_cons	3.000000	.200000	15.00	0.000	2.750000 3.50000

Answer the following questions based on the regression output:

**Question 1.** What is the expected value of log of daily wage for a person whose value for log of education is 4?

- (a) 13
- (b) 19
- (c) 5
- (d) 43
- (e) 7

**Question 2.** Let the expected daily wage for a person that has 10 years of schooling be \$ 200. What is the average effect of an additional year of schooling?

- (a) 2
- (b) 3
- (c) 10
- (d) 20

(e) 30

**Question 3.** What is the correlation between log of wage and log of education?

(a) +0.5

(b)  $-\sqrt{0.5}$

(c)  $+\sqrt{0.5}$

(d) -0.5

(e) 0.25

**Question 4.** Let the expected daily wage for a person that has 10 years of schooling be \$ 200. Now consider a person with 11 years of schooling, that is, a 10% increase in schooling. What is the average wage you would expect for the person with 11 years of schooling?

(a) 210

(b) 220

(c) 230

(d) 240

(e) 260

Questions 5–8 are based on the following regression output

Consider the **Quadratic** linear model described below:

$$wage = \beta_1 + \beta_2 \cdot educ2 + e.$$

1. *wage* means daily wage measured in dollars.
2. *educ2* means the squared of education variable (*educ*)<sup>2</sup>.

The regression output associated with this regression is given by:

```
1 . reg wage educ2
```

Source	SS	df	MS	Number of obs = 100		
Model	10000.00	1	10000.00	F( 1, 98 )	=	100.00
Residual	10000.00	98	100.00	Prob > F	=	0.0000
				R-squared	=	0.5000
				Adj R-squared	=	0.4950
Total	20000.00	99	200.00	Root MSE	=	10.00

  

wage	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
educ2	1.00	0.100	10.00	0.000	0.7500	1.12500
_cons	100.00	10.000	10.00	0.000	90.000	125.000

The regression output for the covariance matrix of the estimated intercept  $\hat{b}_1$  and slope  $\hat{b}_2$  is given by:

Covariance matrix of coefficients of regress model

e(V)	educ	_cons
educ2	0.010	.
_cons	-0.500	100.000

**Question 5.** The best mean predictor for the average wage for a person with 10 years of education?

- (a) 110
- (b) 120
- (c) 200
- (d) 220
- (e) 102

**Question 6.** What is the 95% confidence interval for the mean predictor of the wage of a person with 10 years of education (use critical value  $t_c = 2$ )?

- (a) [180, 220]

(b)  $[190, 210]$

(c)  $[200, 220]$

(d)  $[100, 140]$

(e)  $[100, 120]$

**Question 7.** What is The best linear predictor for the marginal effect of schooling on wage for a person with 10 years of education?

(a) 10

(b) 110

(c) 20

(d) 102

(e) 120

**Question 8.** What is the 95% confidence interval for the marginal effect of schooling on wage for a person with 10 years of education (use critical value  $t_c = 2$ ) ?

(a)  $[6, 14]$

(b)  $[8, 12]$

(c)  $[16, 24]$

(d)  $[116, 124]$

(e)  $[18, 22]$

**Part II (Questions that are not based on Regression Outputs):**

**Question 9.** Consider the following regression model given by  $y = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + e$ . Let  $\hat{b}_1, \hat{b}_2, \hat{b}_3$  be the estimates of this regression whose variance-covariance matrix is given by:

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & -.5 \\ -1 & -.5 & 3 \end{pmatrix}.$$

The **standard error** of the estimator  $b_2 + b_3$  is:

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 12

**Question 10.** Let  $X, Y$  be two random variables and take values in  $\{0, 1\}$  and whose joint distribution is given by:

Joint Distribution			
	$Y = 0$	$Y = 1$	$f_X(x)$
$X = 0$	0	1	1
$X = 1$	0	0	0
$f_Y(y)$	0	1	1

Mark the choice that is **false**:

- (a)  $X$  and  $Y$  are statistically **independent**.
- (b)  $E(X^2) = (E(X))^2$
- (c)  $var(X) = var(Y)$
- (d)  $P(X = 1|Y = 1) = P(X = 1|Y = 0)$
- (e)  $E(X^2) = E(Y^2)$

**Question 12.** Mark the **FALSE** alternative regarding the precision of the least squares estimates.

- (a) We can obtain more precise estimates by increasing the sample size of the regression.
- (b) We can obtain more precise estimates by increasing the sample variance of  $x$ .
- (c) Adding one to each of the values of  $x_i; i = 1, \dots, N$  does not change the variance of slope estimator  $b_2$ .

- (d) Adding one to each of the values of  $x_i; i = 1, \dots, N$  does not change the variance of intercept estimator  $b_1$ .
- (e) The correlation between the intercept estimator  $b_1$  and the slope estimator  $b_2$  is zero whenever the sample mean of  $x$  is zero.

**Question 13.** The following statements describe properties of the Least square estimators. Mark the statement that is **TRUE**.

- (a) The simple regression model is based on five essential assumptions. The most important assumption among these is that the Least square estimators are BLUE.
- (b) The Jarque Bera statistic is used to test if the error terms are normally distributed. Therefore it cannot be used to inquire if a transformation of  $x$  is suitable to explain the dependent variable.
- (c) The sample mean of the estimated residuals  $\hat{e}$  is always zero, which implies that the covariance between  $x$  and  $\hat{e}$  is also zero.
- (d) The error terms of a regression are never observed. Thus, it is not useful to examine the residuals as they are not the actual error terms that generate the observed outcome.
- (e) One consequence of the least square assumptions is that the least square estimators have the lowest variance among all linear estimators of  $\beta_1, \beta_2$  that are unbiased.

**Question 14.** Consider the inference that tests the null hypothesis  $H_0 : \beta = 0$ . Which of the following statement is **FALSE**?

- (a) The critical value of the test depends only on the alternative hypothesis, significance level and sample size.
- (b) Depending on the alternative hypothesis, higher values of the test statistic can be evidence against or in favor of the null hypothesis.
- (c) The test statistic is the same regardless of the alternative hypothesis.
- (d) The  $p$ -value and the significance level are sufficient information to decide whether to reject the null hypothesis.
- (e) The Error of Type I is determined by the  $p$ -value

**Question 15.** Let  $CI(\beta_2)$  be the estimated confidence interval for  $\beta_2$  of a simple regression model at that 95% confidence level. Let  $t_c(0.975, N - 2)$  be the 97.5% quantile of the  $t$ -distribution with  $N - 2$  degrees of freedom, and  $\hat{b}_2, \hat{se}(b_2)$  be the estimates of  $\beta_2$  and its standard error. Let  $\hat{t}$  be your test statistic to test  $H_0 : \beta_2 = 0$  against the alternative hypothesis  $H_1 : \beta_2 \neq 0$  at significance level of 5%. Which of the following statement is **FALSE**?

- (a)  $CI(\beta_2) = [\hat{b}_2 - t_c(0.975, N - 2)\hat{se}(b_2), \hat{b}_2 + t_c(0.975, N - 2)\hat{se}(b_2)]$ .
- (b) You reject the null hypothesis if  $|\hat{t}| > t_c(0.975, N - 2)$ .

- (c)  $CI(\beta_2)$  alone provide sufficient information to decide whether to reject the null hypothesis or not.
- (d) If  $0 \in CI(\beta_2)$  then the  $p$ -value must be larger than 0.05.
- (e) You reject  $H_0$  if and only if  $t_c(0.975, N - 2)\widehat{se}(b_2) < \hat{b}_2$ .

**Question 16.** Let the Simple Regression  $Y = \beta_1 + \beta_2 x + \epsilon$  and consider the single hypothesis testing whose null hypothesis is  $H_0 : \beta_2 = 0$  against  $H_1 : \beta_2 \neq 0$ . Let  $\hat{t}_2$  be the appropriate test statistic. Then we are more likely to reject  $H_0$  if (mark the **correct one**):

- (a) The absolute value of test statistic  $|\hat{t}_2|$  decreases.
- (b) The  $p$ -value decreases.
- (c) The significance level  $\alpha$  decreases.
- (d) The rejection region decreases (shrinks).
- (e) The sample size decreases.

**Question 17.** Consider the forecast error when predicting the outcome  $Y_0 = \beta_1 + \beta_0 x_0 + e_0$  using the simple regression model  $Y = \beta_1 + \beta_2 x + e$ . Which of the following is **FALSE** regarding prediction:

- (a) The prediction is more precise the closer  $x_0$  is to the mean of the explanatory variable  $\bar{x}$ .
- (b) The prediction is more precise the smaller the variance of the error term  $e$ .
- (c) The prediction is more precise the larger the sample size  $N$ .
- (d) The prediction is more precise the larger the variance of the exploratory variable  $var(x)$ .
- (e) The prediction is more precise the larger the variance of the slope estimator  $var(b_2)$ .

**Question 18.** Consider data on  $(x, y)$  where  $\bar{x}, \bar{y}$  are sample means,  $\sigma_x, \sigma_y$  are sample standard deviations and  $corr(x, y)$  is the sample correlation. Let  $R^2$  be the coefficient of determination of the simple regression  $Y = \beta_1 + \beta_2 x + \epsilon$ . Which of the following statement is **FALSE**?

- (a) The larger the absolute value of the covariance between  $y$  and  $x$ , the larger the  $R^2$  (everything else constant).
- (b) The correlation between  $x$  and the fitted values  $\hat{y}$  is the same as the correlation between  $x$  and the actual dependent variable  $y$ .
- (c) The  $R^2$  can be computed as the ratio of the sum of squares of the regression  $SSR = \sum(\hat{y} - \bar{y})^2$  and the total sum of the squares  $SST = \sum(y - \bar{y})^2$ .
- (d) The regression of  $y$  on  $x$  generates the same  $R^2$  of the regression of  $x$  on  $y$ .
- (e) The  $R^2$  measures the share of the variance of the dependent variable that is explained by the explanatory variable.



**Question 19.** Suppose that a simple regression using data  $x, y$  generates the estimates  $\hat{b}_1 = \hat{b}_2 = 1$ . Consider the transformed variables  $x^* = x + 1$ ,  $y^* = y + 1$ . Let the new estimates be  $\hat{b}_1^*$  and  $\hat{b}_2^*$ . Which of the following statement is **TRUE**?

- (a)  $\hat{b}_1^* = \hat{b}_1$  and  $\hat{b}_2^* = \hat{b}_2$
- (b)  $\hat{b}_1^* \neq \hat{b}_1$
- (c)  $\hat{b}_2^* \neq \hat{b}_2$
- (d)  $\hat{b}_1^* = \hat{b}_2^* + 1$
- (e)  $\hat{b}_2^* = \hat{b}_1^* + 1$

**Question 20.** Consider data on  $(x, y)$  where  $\bar{x}, \bar{y}$  are sample means,  $\sigma_x, \sigma_y$  are sample standard deviations and  $\text{corr}(x, y)$  is the sample correlation. Consider the transformation that standardizes the variables, that is,

$$x^* = \frac{x - \bar{x}}{\sigma_x} \text{ and } y^* = \frac{y - \bar{y}}{\sigma_y}.$$

- Let the estimated coefficients and the  $R^2$  for  $(x, y)$  be  $\hat{b}_1, \hat{b}_2, R^2$ .
- Let the estimate for the transformed data  $(x^*, y^*)$  be  $\hat{b}_1^*, \hat{b}_2^*, R^{2*}$ .

Which of the following statement is **FALSE**?

- (a)  $\hat{b}_1^* = 0$  and  $\hat{b}_2^* = \text{corr}(x, y)$
- (b) The coefficient of determination in each regression is the same
- (c) The  $p$ -value for testing the slope  $H_0 : \beta_2 = 0$  versus  $H_0 : \beta_2 \neq 0$  in both regressions is the same
- (d)  $\text{corr}(x^*, y^*) = \text{corr}(x, y)$  and  $R^{2*} = (\hat{b}_2^*)^2$
- (e) The  $p$ -value for testing the intercept  $H_0 : \beta_1 = 0$  versus  $H_0 : \beta_1 \neq 0$  in both regressions is the same