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Economics 103
Introduction to Econometrics
Spring, 2022

Mid-Term Practice (First Example)
Tuesday, May 16, 2023

Instructions:

- This is a 1 hour and 10 minute exam. You are allowed to use the equations sheet provided in the site of the course and handwritten notes.
- You are not allowed to use any electronic device. No connection to the internet via WiFi or any other method is allowed. It is not permitted to use any kind of mobile phone.
- When you are finished with the exam, please **turn in the exam questions**.
- Cheating of any form will result in a score of 0 (zero) for the exam, in addition to the normal university disciplinary action.
- Please sign below that you have read, understood, and fulfilled all of the above instructions and conditions.

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Exam Version A	

Please start solving the examinations only when you are instructed to do so.
Please stop immediately when instructed to do so.

Good Luck!

Part I (Questions based on Regression Output):

Questions 1–3 are based on the following regression output
Consider the following linear model based on the **Cubic** function:

$$wage = \beta_1 + \beta_2 \cdot educ3 + e, \text{ where}$$

1. *wage* means daily wage measured in dollars.
2. *educ* means years of education, measured in years of schooling.
3. *educ*3 means the cubic of education, that is $(educ)^3$.

The R Output for this linear regression is given below:

```
reg wage educ3

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,  98  ) =  100.00
       Model |   10000.00      1    10000.00           Prob > F      =  0.0000
    Residual |   10000.00     98      100.00           R-squared     =  0.5000
-------------+------------------------------           Adj R-squared =  0.4950
       Total |   20000.00     99      200.00           Root MSE      =   10.00

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       educ3|   0.05          0.0050      10.00   0.000      0.0400     0.0600
       _cons |   100.00      10.0000      10.00   0.000     80.000     120.000
------------------------------------------------------------------------------
```

Answer the following questions based on the regression output:

Question 1. What is the expected wage for a person with 10 years of education?

(a) 15

(b) 50

(c) 100

(d) 150

(e) 500

Answer: d.

$$\begin{aligned} E(Y|x_0 = 10) &= \hat{b}_1 + \hat{b}_2 \cdot x_0^3 \\ &= \hat{b}_1 + \hat{b}_2 \cdot 10^3 \\ &= 100 + 0.05 \cdot 1000 \\ &= 100 + 50 = 150 \end{aligned}$$

Question 2. What is the marginal effect of another year of education for a person with 10 years of education?

(a) 1.5

(b) 3

(c) 5

(d) 10

(e) 15

Answer: e.

$$\begin{aligned}\delta = \frac{\Delta E(Y)}{\Delta x} &= 3 \cdot \hat{b}_2 \cdot x_0^2 \\ &= 3 \cdot 0.05 \cdot 10^2 \\ &= 3 \cdot 0.05 \cdot 100 \\ &= 3 \cdot 5 = 15\end{aligned}$$

Question 3. What is the estimated elasticity for a person with 10 years of education at his expected wage?

(a) 0.1

(b) 0.5

(c) 1

(d) 1.5

(e) 2

Answer:c.

$$\begin{aligned}\eta = \underbrace{\frac{\Delta E(Y)}{\Delta x}}_{\delta} \cdot \frac{x_0}{\hat{y}} &= \delta \cdot \frac{x_0}{\hat{y}} \\ &= 15 \cdot \frac{10}{150} \\ &= 1\end{aligned}$$

Questions 4–12 are based on the following regression output
Consider the quadratic model:

$$wage = \beta_1 + \beta_2 \cdot educ + \beta_3 \cdot educ2 + e.$$

1. *wage* means daily wage measured in dollars.
2. *educ* means the education variable measured in schooling years.
3. *educ*2 means the squared of education variable $(educ)^2$.

The regression output for this linear regression is given below:

```
. reg wage educ educ2

      Source |       SS           df       MS      Number of obs   =       100
-------------+----------------------------------   F(2, 97)        =     50.00
       Model |  10000.00         2     5000.00     Prob > F        =    0.0000
    Residual |  10000.00        97      100.00     R-squared       =    0.5000
-------------+----------------------------------   Adj R-squared   =    0.4900
       Total |  20000.00        99      200.00     Root MSE        =    10.000

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |    -20.000    50.0000    -0.40   0.686    -120.0000    80.00000
       educ2 |      2.000     2.0000     1.00   0.448      -2.0000     6.00000
       _cons |    200.000   200.0000     1.00   0.422    -200.0000    600.0000
------------------------------------------------------------------------------
```

Question 4. What is the expected wage for a person with 10 years of education?

(a) 10

(b) 20

(c) 100

(d) 200

(e) 2000

Answer: d.

$$\begin{aligned} E(Y|x_0 = 10) &= \hat{b}_1 - \hat{b}_2 \cdot x_0 + \hat{b}_3 \cdot x_0^2 \\ &= \hat{b}_1 - \hat{b}_2 \cdot 10 + \hat{b}_3 \cdot 10^2 \\ &= 200 - 20 \cdot 10 + 2 \cdot 100 \\ &= 200 \end{aligned}$$

Question 5. What is the marginal effect of another year of education for a person with 10 years of education?

(a) 200

(b) 100

(c) 2

(d) 10

(e) 20

Answer: e.

$$\begin{aligned}
\delta = \frac{\Delta E(Y)}{\Delta x} &= \hat{b}_2 + 2 \cdot \hat{b}_3 \cdot x_0 \\
&= -20 + 2 \cdot 2 \cdot 10 \\
&= -20 + 40 = 20
\end{aligned}$$

Question 6. What is the estimated elasticity for a person with 10 years of education at his expected wage?

(a) 0.1

(b) 0.5

(c) 1

(d) 1.5

(e) 2

Answer: c.

$$\begin{aligned}
\eta = \underbrace{\frac{\Delta E(Y)}{\Delta x}}_{\delta} \cdot \frac{x_0}{\hat{y}} &= \delta \cdot \frac{x_0}{\hat{y}} \\
&= 20 \cdot \frac{10}{200} \\
&= 1
\end{aligned}$$

Question 7. Parameter λ_0 is defined as a linear combination of the model coefficients:

$$\lambda_0 = \frac{\beta_1}{200} - \frac{\beta_3}{2}.$$

What is the value of the Best Linear Unbiased Predictor (BLUP) for λ_0?

(a) 20

(b) 1

(c) 10

(d) 2

(e) 0

Answer: e.

$$\begin{aligned}\hat{\lambda} &= \frac{\hat{b}_1}{200} - \frac{\hat{b}_3}{2} \\ &= \frac{200}{200} - \frac{2}{2} \\ &= 1 - 1 \\ &= 0\end{aligned}$$

Question 8. The estimated covariance between estimators b_1, b_3 is $\widehat{cov}(b_1, b_3) = 200$. What is the estimated standard error for λ?

(a) -1

(b) 0

(c) 1

(d) 2

(e) 4

Answer: c.

$$\begin{aligned}v\hat{a}r(\lambda) &= \left(\frac{1}{200}\right)^2 v\hat{a}r(\hat{b}_1) + \left(\frac{1}{2}\right)^2 v\hat{a}r(\hat{b}_3) + 2 \cdot \left(\frac{1}{200}\right) \cdot \left(-\frac{1}{2}\right) \cdot c\hat{o}v(b_1, b_3) \\ &= \left(\frac{1}{200}\right)^2 \cdot 200^2 + \left(\frac{1}{2}\right)^2 \cdot 2^2 + 2 \cdot \left(\frac{1}{200}\right) \cdot \left(-\frac{1}{2}\right) \cdot 200 \\ &= 1 + 1 - 1 \\ &= 1\end{aligned}$$

Thus we have that $\hat{se}(\lambda) = \sqrt{v\hat{a}r(\lambda)} = \sqrt{1} = 1$

Question 9. What is the test statistic for the null hypothesis $H_0 : \lambda_0 = 0$ against the alternative $H_1 : \lambda_0 \neq 0$.

(a) -1

(b) 0

(c) 1

(d) 2

(e) 10

Answer: b.
Test statistic: $\hat{t} = \frac{\hat{\lambda}-0}{\hat{se}(\lambda)} = \frac{0-0}{1} = 0$

Question 10. Using the critical value of $t_c = 2$, the confidence interval for λ is given by:

(a) $[-1, 0]$

(b) $[0, 1]$

(c) $[0, 2]$

(d) $[-2, 2]$

(e) $[-1, 1]$

Answer: d.

$$\begin{aligned} \text{Confidence Interval: } CI &= [\hat{\lambda} \pm t_c \cdot \hat{se}(\lambda)] \\ &= [0 \pm 2 \cdot 1] \\ &= [-2, 2] \end{aligned}$$

Question 11. A econometrician spotted a typo in the p-values of the regression output. Let $p_1 = 0.422$ be the p-value associated with intercept β_1.
Let $p_2 = 0.686$ be the p-value associated with intercept β_2 (for *educ*).
Let $p_3 = 0.448$ be the p-value associated with intercept β_3 (for *educ*2)
Given the estimated coefficients, and its respective standard errors, we should have that:

(a) We should have that $p_2 < p_3$

(b) We should have that $p_1 = p_2$

(c) We should have that $p_2 = p_3$

(d) We should have that $p_2 < p_1$

(e) We should have that $p_1 = p_3$

Answer: e.
Note that $\hat{t}_1 = \hat{t}_3 \Rightarrow \hat{p}_1 = \hat{p}_3$
Moreover the absolute value of the t-statistic $\hat{t}_2$ is smaller than the value for the t-statistic $\hat{t}_2, \hat{t}_3$, that is: $|\hat{t}_2| = 0.4 < 1 = |\hat{t}_1| = |\hat{t}_3|$.
Therefore we must have that: $\hat{p}_2 > \hat{p}_1 = \hat{p}_3$.

Part II (Multiple Choice Questions that do not use Regression Output):

Question 12. Let X_1, X_2 be two random variables and take values in $\{0,1\}$ and whose joint distribution is given by:

Joint Distribution

	$X_1 = 0$	$X_1 = 1$	$f_{X_2}(x)$
$X_2 = 0$	0.2	0.2	0.4
$X_2 = 1$	0.3	0.3	0.6
$f_{X_1}(x)$	0.5	0.5	1

Mark the choice that is correct:

(a) X_1 and X_2 are statistically **independent**.

(b) $E(X_1) = E(X_2)$

(c) $P(X_2 = 1|X_1 = 1) = P(X_2 = 0|X_1 = 0)$

(d) $var(X_1) = var(X_2)$

(e) $Cov(X_1, X_2) \neq 0$

Answer: a.
X_1, X_2 are statistically independent because $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1)P(X_2 = x_2)$ for all $x_1, x_2 \in \{0,1\}$.
Independence implies that $Cov(X_1, X_2) = 0$. Indeed, we have that:

$$\begin{aligned} E(X_1) &= P(X_1 = 1) \cdot 1 = 0.5 \cdot 1 = 0.5 \\ E(X_2) &= P(X_2 = 1) \cdot 1 = 0.6 \cdot 1 = 0.6 \\ E(X_1 \cdot X_2) &= P(X_1 = 1, X_2 = 1) \cdot 1 = 0.3 \\ Cov(X_1, X_2) &= E(X_1 \cdot X_2) - E(X_1)E(X_2) = 0.5 \cdot 0.6 - 0.3 = 0 \end{aligned}$$

Also:

$$P(X_2 = 1|X_1 = 1) = \frac{P(X_2 = 1, X_1 = 1)}{P(X_1 = 1)} = \frac{0.3}{0.5} = 0.6 = P(X_2 = 1),$$

and

$$P(X_2 = 0|X_1 = 0) = \frac{P(X_2 = 0, X_1 = 0)}{P(X_1 = 0)} = \frac{0.2}{0.5} = 0.4 = P(X_2 = 0).$$

Question 13. Let $X_1 \sim N(1,1)$ and $X_2 \sim N(2,4)$ be two normally distributed random variables. Let the correlation between X_1 and X_2 be ρ. Mark the choice that is **correct**:

(a) $Var(c_1 \cdot X_1 + c_2 \cdot X_2) \neq Var(c_0 + c_1 \cdot X_1 + c_2 \cdot X_2)$ whenever $c_0 \neq 0$.

(b) $P((X_1 - 1) > 3) = P((X_2 - 2) > 3)$ *regardless* if X_1, X_2 correlate.

(c) Let $Z_1 = \left(\frac{X_1-1}{1}\right)$ and $Z_2 = \left(\frac{X_2-2}{2}\right)$, then $E(Z_1^2 + Z_2^2) = 2$ *regardless* if X_1, X_2 correlate.

(d) If X_1 and X_2 correlate, then their covariance may take any value in the real line $[-\infty, \infty]$.

(e) We have that $E\big((X_1 - 1) \cdot (X_2 - 2)\big) = 0$ *regardless* if X_1, X_2 correlate.

Answer: c.
Observe that Z_1, Z_2 are standard normal random variables that might be potentially correlated. Regardless of its correlation, we still have that $E(Z_1^2 + Z_2^2) = E(Z_1^2) + E(Z_2^2)$.
Now observe that Z_1, Z_2 are the standardized variables of X_1, X_2. Thus they have means zero and variance one. Now when a variable X has mean zero, it is always the case that $E(X^2) = Var(X)$. Therefore we have that:

$$E(Z_1^2 + Z_2^2) = E(Z_1^2) + E(Z_2^2) = Var(Z_1) + Var(Z_2) = 1 + 1 = 2.$$

Let's compute the mean and variance of these variables for sake of completeness.
Variables Z_1, Z_2 have mean zero because:

$$\begin{aligned} E(Z_1) = E\left(\frac{X_1 - 1}{1}\right) &= \left(\frac{E(X_1) - 1}{1}\right) = \left(\frac{1-1}{1}\right) = 0, \\ E(Z_2) = E\left(\frac{X_2 - 2}{2}\right) &= \left(\frac{E(X_2) - 2}{2}\right) = \left(\frac{2-2}{2}\right) = 0. \end{aligned}$$

Variables Z_1, Z_2 have variance one because:

$$\begin{aligned} Var(Z_1) = Var\left(\frac{X_1 - 1}{1}\right) &= \left(\frac{Var(X_1)}{1^2}\right) = \left(\frac{1}{1}\right) = 1, \\ Var(Z_2) = Var\left(\frac{X_2 - 2}{2}\right) &= \left(\frac{Var(X_2)}{2^2}\right) = \left(\frac{4}{4}\right) = 1. \end{aligned}$$

Thus the expectation $E(Z_1^2 + Z_2^2)$ is equal to:

$$\begin{aligned} E(Z_1^2 + Z_2^2) &= E(Z_1^2) + E(Z_2^2) \\ &= E((Z_1 - 0)^2) + E((Z_2 - 0)^2) \\ &= E((Z_1 - E(Z_1))^2) + E((Z_1 - E(Z_2))^2) \\ &= Var(Z_1) + Var(Z_2) \\ &= 1 + 1 = 2 \end{aligned}$$

In letter (a), the addition of a constant term does not modify the variance.

In letter (b), the probabilities are different because the random variables defined as $(X_1 - 1)$ and $(X_2 - 2)$ have mean zero but different variances.

In letter (d), consider two random variables X_1, X_2 such that $var(X_1) > Var(X_2)$, that is, X_1 has the bigger variance. Then we have that:

$$\begin{aligned}
-1 &\leq Corr(X_1, X_2) \leq 1 \\
-1 &\leq \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1)Var(X_2)}} \leq 1 \\
-1 &\leq \frac{Cov(X_1, X_2)}{se(X_1)se(X_2)} \leq 1 \\
-se(X_1)se(X_2) &\leq Cov(X_1, X_2) \leq se(X_1)se(X_2)
\end{aligned}$$

Note that it is clear that the covariance between two variables is bounded by $\pm se(X_1)se(X_2)$, which falsifies letter d. Now if $Var(X_1) > Var(X_2)$ then $se(X_1)se(X_1) > se(X_1)se(X_2)$, then it is also true that:

$$\begin{aligned}
-se(X_1)se(X_1) &\leq Cov(X_1, X_2) \leq se(X_1)se(X_1) \\
-Var(X_1) &\leq Cov(X_1, X_2) \leq Var(X_1)
\end{aligned}$$

In summary, it is always the case that the covariance of two variables is bounded by positive and the negative value of the largest variance.

In letter e, by the definition of the covariance, we have that:

$$E\big((X_1 - 1) \cdot (X_2 - 2)\big) = E\big((X_1 - E(X_1)) \cdot (X_2 - E(X_2))\big) = Cov(X_1, X_2).$$

The covariance can also be written in terms of the correlation ρ :

$$\rho = \frac{Cov(X_1, X_2)}{\sqrt{Var(X_1)Var(X_2)}} \Rightarrow Cov(X_1, X_2) = \rho \cdot \sqrt{Var(X_1)Var(X_2)}.$$

Thus, if $\rho \neq 0$ then $Cov(X_1, X_2) \neq 0$, which falsifies letter e.

Question 14. Consider the following regression model:

$$Y_i = \beta_1 + \beta_2 x_i + \epsilon_i,$$

for $i = 1, ..., N$. Let $e_i \sim \left(0, \sigma_i^2\right)$. That is, e_i has a distribution whose mean is 0 and its variance is σ^2. Let

$$s_x^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \overline{x})^2, \quad \text{where } \overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{1}$$

A data analyst ran a regression of y on x and obtained the following estimates for β_1 and β_2: $\hat{b}_1 = 4$, $\hat{b}_2 = .5$. Define $x_i^* = 10 \times x_i$. If one were to run a regression of y on x^* the estimates $\hat{b}_1^*$ and $\hat{b}_2^*$ would be

(a) $\hat{b}_1^* = 4$, $\hat{b}_2^* = 5$

(b) $\hat{b}_1^* = 4$, $\hat{b}_2^* = .5$

(c) $\hat{b}_1^* = 4$, $\hat{b}_2^* = .05$

(d) $\hat{b}_1^* = .4$, $\hat{b}_2^* = .5$

(e) $\hat{b}_1^* = .4$, $\hat{b}_2^* = 5$

Answer: c.
x-transformation $x^* = c \cdot x$ only changes the estimate for β_2, that is, $\hat{b}_2^* = \frac{\hat{b}_2}{c}$. the values of the t-statistics, the inference and the R^2 remain the same.

Question 15. Regarding the Simple Regression Model $y = \beta_1 + \beta_2 \cdot x + e$, which of the following is **FALSE**?

(a) $\bar{y} - \hat{b}_1 - \hat{b}_2\bar{x} = 0$, where $\bar{x}, \bar{y}$ denote sample means.

(b) The LS estimates for the quadratic regression $Y = \beta_1 + \beta_2 \cdot x^2 + e$ is **not** BLUE because the relation between Y and x is not linear, so the linearity assumption is violated.

(c) In the Simple Regression Model, $y = \beta_1 + \beta_2 \cdot x + e$, $\hat{b}_2 = \frac{cov(x,y)}{var(x)}$, where $cov(x, y)$ is the sample covariance and $var(x)$ is the sample variance of x.

(d) Let $\hat{b}_1^*, \hat{b}_2^*$ be estimates for β_1, β_2 other than the least squares estimates $\hat{b}_1, \hat{b}_2$, then it must be that:

$$\sum_{i=1}^{N}(\hat{b}_1^* + \hat{b}_2^* x_i - y_i)^2 \geq \sum_{i=1}^{N}(\hat{b}_1 + \hat{b}_2 x_i - y_i)^2.$$

(e) The sign of the covariance between $\hat{b}_1, \hat{b}_1$ depends only on the sample mean of x.

Answer: b.
Letters a, c display the standard equations that are used to estimate parameters in the simple regression model.
Letter d is correct because the least squares estimators minimize the sum of the square of the

residuals.
Letter e is correct because the formula for the covariance between estimators b_1, b_2 depends on $-\bar{x}$ and other terms that are always positive.
Letter b is false because the equation is quadratic in x but linear in x^2.

Question 16. Let the simple regression model $Y = \beta_1 + \beta_2 \cdot X + e$. Consider the inference that tests the null hypothesis $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 \neq 0$ at significance level α. Which of the statements is **false**?

(a) If the standard error of $\hat{b}_2$ decreases, then it is more likely to reject H_0, (everything else constant).

(b) The higher the absolute value of $\hat{b}_2$, the more likely it is to reject H_0 (everything else constant).

(c) The higher the significance level α, the more likely it is to reject H_0 (everything else constant).

(d) Hypothesis $H_0 : \beta_2 = 0$ is not rejected whenever the value 0 belongs to its confidence interval (with confidence level of $1 - \alpha$).

(e) The larger the sample size, the more likely it is to reject H_0, (everything else constant).

(f) The higher the p-value, the more likely you are to reject H_0.

Answer f.

The question requires you to interpret the equations for the estimator standard errors in the simple regression model:

$$
\begin{aligned}
Var(b_2) &= \sigma^2 \cdot \frac{1}{N} \cdot \frac{1}{var(x)} \\
Var(b_1) &= \sigma^2 \cdot \frac{1}{N} \cdot \frac{var(x) + \bar{x}^2}{var(x)} \\
Cov(b_1, b_2) &= \sigma^2 \cdot \frac{1}{N} \cdot \frac{-\bar{x}}{var(x)}
\end{aligned}
$$

According to the equations above, we have that:

- The larger the error variance σ^2, less precise (greater variance) of estimators b_1, b_2.
- The larger the sample size N, more precise (smaller variance) of estimators b_1, b_2.
- The larger the sample variance of the explanatory variable $var(x)$ more precise (smaller variance) of estimators b_1, b_2.
- $Cov(b_1, b_2)$ has always the opposite sign of $\bar{x}$.

The question also requires you to understand the mechanism of single hypothesis testing.
The table below presents a summary of the components of the single hypothesis testing using p-values:

Hypothesis Testing Using p-Values				
Null Hypothesis	Alternative Hypothesis	p-value Calculation	Rejection Rule	Which Tail?
$H_0 : \beta = c$	$H_1 : \beta \neq c$	$p = P(\|t(d.f)\| > \|\hat{t}\|)$	$p < \alpha$	Two-tail test
$H_0 : \beta = c$	$H_1 : \beta > c$	$p = P(t(d.f) > \hat{t})$	$p < \alpha$	Right-tail test
$H_0 : \beta = c$	$H_1 : \beta < c$	$p = P(t(d.f) < \hat{t})$	$p < \alpha$	Left-tail test
Test statistic depends on c $\hat{t} = \frac{\hat{b}-c}{\hat{S}e(b)}$		Depends on H_1 $(\neq, >, <)$	Depends on p and α	

For a given p-value, the larger the significance level α, the more likely you are to reject the null hypothesis.

Question 17. Let the simple regression model $Y = \beta_1 + \beta_2 \cdot X + e$. Consider the inference that tests the null hypothesis $H_0 : \beta_k = 0$ against $H_1 : \beta_k \neq 0$ at significance level α for $k = 1, 2$. Which of the statements is **correct**?

(a) It is less likely to reject the null hypothesis $H_0 : \beta_1 = 0$ if we do the transformation $y_{new} = y \cdot c$.

(b) It is less likely to reject the null hypothesis $H_0 : \beta_2 = 0$ if we do the transformation $x_{new} = x \cdot c$.

(c) It is less likely to reject the null hypothesis $H_0 : \beta_2 = 0$ if we do the transformation $y_{new} = y + c$.

(d) It is less likely to reject the null hypothesis $H_0 : \beta_1 = 0$ if we *standardize* both y and x.

(e) It is less likely to reject the null hypothesis $H_0 : \beta_2 = 0$ if we *standardize* both y and x.

Answer d.

The question explores the properties of linear transformations of the dependent variable y and the explanatory variable x. A summary of these properties is listed below:

- The transformation $x_{new} = x \cdot c$ modifies the estimates $\hat{b}_2$ and $\hat{se}(b_2)$. The estimates $\hat{b}_1, \hat{se}(b)_1$, the t-statistics for testing coefficients β_1, β_2 and R^2 remain the same.

- The transformation $y_{new} = y \cdot c$ modifies the estimates $\hat{b}_1, \hat{se}(b_1), \hat{b}_2, \hat{se}(b_2), \hat{\sigma}^2$. The inference, t-statistics for testing coefficients β_1, β_2 and R^2 remain the same.

- The transformation $x_{new} = x + c$ modifies the estimates $\hat{b}_1, \hat{se}(b_1)$ and its t-statistic $\hat{t}_1$. The estimates $\hat{b}_2, \hat{se}(b)_2, \hat{t}_2$ and R^2 remain the same.

- The transformation $y_{new} = y + c$ modifies the estimates $\hat{b}_1$ but does not modifies $\hat{se}(b_1)$, so it affects its t-statistic. The transformation does not change the estimates $\hat{b}_2, \hat{se}(b)_2, \hat{t}_2$ and R^2.

- If $\bar{x} = \bar{y} = 0$ then the estimate $\hat{b}_1 = 0, \hat{t}_1 = 0$ and we never reject the null hypothesis $H_0 : \beta_1 = 0$.

- If we *standardize* both y and x, then the estimate $\hat{b}_1$ is zero and the estimate $\hat{b}_2$ is equal to the sample correlation of x, y.

In more general terms, you must be able to read the following table:

Original Model	Slope			Intercept			
Transformations	Est. $\hat{b}_2$	Std.Err. σ_2	t-stat $\hat{t}_2$	Est. $\hat{b}_1$	Std.Err. σ_1	t-stat $\hat{t}_1$	R^2
$x \to c_x \cdot x$	$\hat{b}_2/c_x$	$\hat{\sigma}_2/c_x$	$\hat{t}_2$	$\hat{b}_1$	$\hat{\sigma}_1$	$\hat{t}_1$	R^2
$y \to c_y \cdot y$	$c_y \cdot \hat{b}_2$	$c_y \cdot \hat{\sigma}_2$	$\hat{t}_2$	$c_y \cdot \hat{b}_1$	$c_y \cdot \hat{\sigma}_1$	$\hat{t}_1$	R^2
$y \to c_y \cdot y$ $x \to c_x \cdot x$	$\frac{c_y \cdot \hat{b}_2}{c_x}$	$\frac{c_y \cdot \hat{\sigma}_2}{c_x}$	$\hat{t}_2$	$c_y \cdot \hat{b}_1$	$c_y \cdot \hat{\sigma}_1$	$\hat{t}_1$	R^2
$y \to c \cdot y$ $x \to c \cdot x$	$\hat{b}_2$	$\hat{\sigma}_2$	$\hat{t}_2$	$c \cdot \hat{b}_1$	$c \cdot \hat{\sigma}_1$	$\hat{t}_1$	R^2
$y \to s_y + y$	$\hat{b}_2$	$\hat{\sigma}_2$	$\hat{t}_2$	$\hat{b}_1 + s_y$	$\hat{\sigma}_1$	$\hat{t}_1 + s_y/\hat{\sigma}_1$	R^2
$x \to s_x + x$	$\hat{b}_2$	$\hat{\sigma}_2$	$\hat{t}_2$	$\hat{b}_1 - s_x \cdot \hat{b}_2$	$\sqrt{\hat{\sigma}_1{}^2 + s_x^2 \hat{\sigma}_2{}^2}$	$\frac{(\hat{b}_1 - s_x \cdot \hat{b}_2)}{\sqrt{\hat{\sigma}_1{}^2 + s_x^2 \hat{\sigma}_2{}^2}}$	R^2

(a) Note that the sample means of $\tilde{y}$ and $\tilde{x}$ are zero, that is $\bar{\tilde{y}} = 0$ and $\bar{\tilde{x}} = 0$. Thus the estimated value of $\hat{\tilde{b}}_1$ is given by:

$$\hat{\tilde{b}}_1 = \bar{\tilde{y}} - \hat{\tilde{b}}_2 \bar{\tilde{x}} = 0.$$

(b) Moreover the estimated t-statistic associated with $\hat{\tilde{b}}_1$ must be zero as $\hat{\tilde{t}}_1 = \hat{\tilde{b}}_1 / \widehat{se}(\tilde{b}) = 0$.
(c) Note also that a linear transformation of two random variables X, Y does not change its sample correlation. But the goodness of fit is simply the square of the correlation between variables. Thereby the goodness of fit of the original regression and the one that uses the transformed variables must be the same.
(d) Lastly, note that the sample variance of each of the transformed variables is one. Thus the covariance of the transformed variables is equal to its correlation.

Question 18. Consider two regressions: Y on X and Y on X. Notationally, let $Y = \beta_1^y + \beta_2^y X + \epsilon^y$, where $\hat{b}_1^y, \hat{b}_2^y$ denotes the estimates for β_1^y, β_2^y, $\hat{t}_1^y, \hat{t}_2^y$ are its t-statistics and R_y^2 denotes its goodness of fit. Similarly, let $X = \beta_1^x + \beta_2^x Y + \epsilon^x$, where $\hat{b}_1^x, \hat{b}_2^x, \hat{t}_1^x, \hat{t}_2^x, R_x^2$ denote its respective estimates. Mark the **correct** statement regarding these two regressions:

(a) It is always the case that $\hat{b}_2^y = \hat{b}_2^x$.

(b) It is always the case that $\hat{b}_2^y = 1/\hat{b}_2^x$.

(c) It is always the case that $\hat{b}_1^y = \hat{b}_1^x$.

(d) It is always the case that $\hat{t}_1^y = \hat{t}_1^x$.

(e) It is always the case that $R_y^2 = R_x^2$.

Answer e.

Letter e is true because the goodness of fit, R^2 can be expressed as $R^2 = (corr(x,y))^2$. The correlation between x and y are the same if we regress Y on X or vice-versa.

Letter a is false because $\hat{b}_2^y = \frac{cov(x,y)}{var(x)}$ while $\hat{b}_2^x = \frac{cov(x,y)}{var(y)}$. For $\hat{b}_2^x = \hat{b}_2^y$ to be true, we would need that $var(x) = var(y)$.

Letter b is false because $\hat{b}_2^y = \frac{cov(x,y)}{var(x)}$ while $1/\hat{b}_2^x = \frac{var(y)}{cov(x,y)}$. Note that one could mistakenly think that the letter is correct by some erroneous rationale. For instance, one could think on isolating X in the equation: $Y = \beta_1 + \beta_2 X + \epsilon$. This would generated the following equation $X = \frac{\beta_1}{\beta_2} + \frac{1}{\beta_2}X - \frac{1}{\beta_2}\epsilon$, which suggests the wrong statement that $\hat{b}_2^y = 1/\hat{b}_2^x$.

A simple way to see that letter c is wrong think of a variable x such that $\bar{x} = 0$. In this case, $\hat{b}_1^y = \bar{y} - \hat{b}_2^y\bar{x} = \bar{y}$.
On the other hand, $\hat{b}_1^x = \bar{x} - \hat{b}_2^x\bar{y} = -\bar{y}\cdot\frac{cov(x,y)}{var(y)}$, which falsifies the letter. In more general terms, for $\hat{b}_1^y = \hat{b}_1^x$ we need that:

$$
\begin{aligned}
&\bar{y} - \hat{b}_2^y\bar{x} = \bar{x} - \hat{b}_2^x\bar{y}\\
\Rightarrow &\bar{y} + \hat{b}_2^x\bar{y} = \bar{x} + \hat{b}_2^y\bar{x}\\
\Rightarrow &\bar{y}(1+\hat{b}_2^x) = \bar{x}(1+\hat{b}_2^y)\\
\Rightarrow &\underbrace{\frac{\bar{y}}{\bar{x}}}_{\text{Depends on Sample Means}} = \underbrace{\frac{1+\hat{b}_2^x}{1+\hat{b}_2^y}}_{\text{Depends on Sample Covariance}}
\end{aligned}
$$

The statement is wrong because the left-hand side depends only on sample means while the right-hand side of the equation depends on the sample covariances and we can change sample means without changing the sample covariances by simple adding constants to x or y.

A simple way to check that Letter d is wrong consists in investigating the case where $\bar{x} = 0$, $\bar{y} > 0$ and $cov(x,y) > 0$. Recall that, if $\bar{x} = 0$, then $\hat{b}_1^y = \bar{y}$ and $\hat{b}_1^x = -\bar{y}\cdot\frac{cov(x,y)}{var(y)}$. In this case, $\hat{b}_1^y > 0$ while $\hat{b}_1^x < 0$ and thereby it cannot be the case that $\hat{t}_1^y = \hat{t}_1^x$.

Question 19. Consider the model $Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon$. Mark the **correct** statement regarding the sample correlation between x_2 and x_3 :

(a) The *lower* the absolute value of the sample correlation, the *larger* the standard errors for the estimators b_2 and b_3.

(b) Any linear transformation of x_2 changes the correlation between x_2 and x_3 and thereby affects the standard error of estimator b_3.

(c) Re-scaling x_2 say $x_2^* = 5\cdot x_2$ changes the standard error of estimator b_2 but does not change its t-statistic $\hat{t}_2 = \hat{b}_2/\widehat{se}(b_2) = \hat{b}_2^*/\widehat{se}(b_2^*) = \hat{t}_2^*$.

(d) The estimators b_2, b_3 would be most precise if the explanatory variables were equal, that is, $x_2 = x_3$.

(e) If the sample means $\bar{x}_2$ and $\bar{x}_1$ were zero, that is, $\bar{x}_2 = \bar{x}_1 = 0$, then the estimate of the intercept is also zero, $\hat{b}_1 = 0$.

Answer c.

Letter c states a correct consequence of the linear transformation (see question 17)

Letter e is false because if $\bar{x}_2 = \bar{x}_1 = 0$, then $\hat{b}_1 = \bar{y}$.

The remaining of the items of the question explore the fact that, in multiple regression models, the higher the correlation between explanatory variables, the higher the estimated standard error of the leas squares estimates for β_2, β_3.

In is useful to investigate the formula for the variance of estimators b_2 and b_3. The formula for variance of estimators b_2 is given by:

$$Var(b_2) = \sigma^2 \cdot \frac{1}{N} \cdot \frac{1}{var(x_2)} \cdot \frac{1}{(1 - r_{2,3}^2)}$$

The interpretation of this formula is the following :

- The first term (σ^2) means that the **larger** the variance of the error term, the **larger** the variance of error term $Var(e_i) = E(e_i^2) = \sigma^2$, the **larger** the variance of $Var(b_2)$.
- The second term ($1/N$) means that the larger the sample size N, the **smaller** the variance of $Var(b_2)$.
- The Third term ($1/var(x_2)$) means that the **larger** the sample variance of exploratory variable x_2, the **smaller** the variance of $Var(b_2)$.
- The fourth term ($1/(1 - r_{2,3}^2)$) means that the **larger** the sample correlation of exploratory variables x_2, x_3, the **larger** the variance of $Var(b_2)$.

We can examine the analogous formula for b_3 :

$$Var(b_3) = \sigma^2 \cdot \frac{1}{N} \cdot \frac{1}{var(x_3)} \cdot \frac{1}{(1 - r_{2,3}^2)}$$

- The first term (σ^2) means that the **larger** the variance of the error term, the **larger** the variance of error term $Var(e_i) = E(e_i^2) = \sigma^2$, the **larger** the variance of $Var(b_2)$.
- The second term ($1/N$) means that the larger the sample size N, the **smaller** the variance of $Var(b_2)$.
- The Third term ($1/var(x_3)$) means that the **larger** the sample variance of exploratory variable x_3, the **smaller** the variance of $Var(b_2)$.
- The fourth term ($1/(1 - r_{2,3}^2)$) means that the **larger** the sample correlation of exploratory variables x_2, x_3, the **larger** the variance of $Var(b_2)$.

Note that the last term is identical for both $Var(b_2)$ and $Var(b_3)$. The correct answer is (a). The most important cases when the correlation is mention in mentioned in the course are:

- The increase in the sample correlation among **explanatory variables** (x_2, x_3) in the **multiple regression** increases the variance of the estimators b_2, b_3.

- The square of the sample correlation between the **dependent** variable y and **exploratory** variable x in the **simple** regression is the goodness of fit R^2. Thus an increase correlation between the **dependent** variable y and **exploratory** variable x in the **simple** regression increases the R^2.

Question 20. Consider the linear Regression model

$$\ln(Y_i) = \beta_1 + \beta_2 x_i + e_i$$

where x denotes annual household income (in thousands) and y denotes annual expenses on consumption goods. Let $\hat{y}$ be the estimated value of Y given the value of the explanatory variable x_0. Consider the following estimates:

$$\begin{aligned} \hat{\gamma}_x &= \hat{b}_2 x_0 \\ \hat{\gamma}_y &= \hat{b}_2 \hat{y} \\ \hat{\gamma}_{x,y} &= \hat{b}_2 \frac{x_0}{\hat{y}} \end{aligned}$$

Which of the following statements is **TRUE**?

(a) $\hat{\gamma}_y$ estimates the percentage increase in consumption expenses associated with an additional \$1,000 in income.

(b) $\hat{\gamma}_x$ estimates the income elasticity of consumption for the average family.

(c) $\hat{\gamma}_y$ estimates the increase in consumption expenses for a 1% increase in income.

(d) $\hat{\gamma}_x$ estimates the average increase in consumption for an addition thousand dollars in income for the average household.

(e) $\hat{\gamma}_{x,y}$ estimates the average % change in food consumption for a 1% increase in income.

Answer b.
The question explores the properties of the Log-Linear Model.
Slope is given by $\hat{\gamma}_y = \hat{b}_2 \bar{y}$
Thus a thousand dollars increase leads to $\hat{\gamma}_y$ increase in consumption expenditure.
Elasticity is given by $\hat{\gamma}_x = \hat{b}_2 \bar{x}$.
Thus, 1% increase in income leads to $\hat{\gamma}_x$ percentage increase in consumption expenditure.

A general question on this topic requires you to use the information of the following non-linear transformations of both the explanatory variable x and the dependent variable y :

The first column present a range of models.
The second column presents the function form of the model.

Table 4.1 **Some Useful Functions, their Derivatives, Elasticities and Other Interpretation**

Name	Function	Slope $= dy/dx$	Elasticity
Linear	$y = \beta_1 + \beta_2 x$	β_2	$\beta_2 \frac{x}{y}$
Quadratic	$y = \beta_1 + \beta_2 x^2$	$2\beta_2 x$	$(2\beta_2 x)\frac{x}{y}$
Cubic	$y = \beta_1 + \beta_2 x^3$	$3\beta_2 x^2$	$(3\beta_2 x^2)\frac{x}{y}$
Log-Log	$\ln(y) = \beta_1 + \beta_2 \ln(x)$	$\beta_2 \frac{y}{x}$	β_2
Log-Linear	$\ln(y) = \beta_1 + \beta_2 x$	$\beta_2 y$	$\beta_2 x$
	or, a 1 unit change in x leads to (approximately) a 100 β_2% change in y		
Linear-Log	$y = \beta_1 + \beta_2 \ln(x)$	$\beta_2 \frac{1}{x}$	$\beta_2 \frac{1}{y}$
	or, a 1% change in x leads to (approximately) a $\beta_2/100$ unit change in y		

The third column presents the the equation for the slope.
The fourth column presents column presents the the equation for the elasticity.
The interpretation of the slope and elasticity are: The two most important estimates are the slope and the elasticity.

- **Slope** $= \Delta Y/\Delta X$ If the estimate value of the slope is $\hat{\delta}$ then:
  - One **unity** change in x leads to (approximately) $\hat{\delta}$ **units** change in y
  - $\hat{b}_2$ in the linear regression $Y = \beta_1 + \beta_2 \cdot x + e$, estimates the slope of x on y.
- **Elasticity** $= \frac{\Delta Y}{\Delta X} \cdot \frac{X}{Y}$**:** If the estimate value of the slope is $\hat{\eta}$ then:
  - One **percent** change in x leads to (approximately) $\hat{\eta}$ **percent** change in y
  - $\hat{b}_2$ in the Log-log regression $\ln(Y) = \beta_1 + \beta_2 \cdot \ln(x) + e$, estimates the elasticity of x on y.