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Economics 103
Introduction to Econometrics
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Mid-Term Practice (Second Example)
Tuesday, May 16, 2023

Instructions:

- This is a 1 hour and 10 minute exam. You are allowed to use the equations sheet provided in the site of the course and handwritten notes.
- You are not allowed to use any electronic device. No connection to the internet via WiFi or any other method is allowed. It is not permitted to use any kind of mobile phone.
- When you are finished with the exam, please **turn in the exam questions**.
- Cheating of any form will result in a score of 0 (zero) for the exam, in addition to the normal university disciplinary action.
- Please sign below that you have read, understood, and fulfilled all of the above instructions and conditions.

Please fill in the following personal information:

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Exam Version A	

Please start solving the examinations only when you are instructed to do so.
Please stop immediately when instructed to do so.

Good Luck!

Part I (Questions based on Regression Output):

Questions 1–4 are based on the following regression output

Consider the following linear model:

$$logwage = \beta_1 + \beta_2 \cdot logeduc + e_i, \text{ where}$$

1. *logwage* means the logarithm of daily wage, measured in dollars.
2. *logeduc* means the logarithm of education, measured in years of schooling.

The output for this linear regression is given below:

```
. reg logwage logeduc

      Source |       SS           df       MS      Number of obs   =       100
-------------+----------------------------------   F(1, 98)        =     96.47
       Model |  .25000000         1  .250000000   Prob > F        =    0.0000
    Residual |  .25000000        98  .002500000   R-squared       =    0.5000
-------------+----------------------------------   Adj R-squared   =    0.4950
       Total |  .50000000        99  .005000000   Root MSE        =    .05000

------------------------------------------------------------------------------
     logwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     logeduc |   1.000000   .100000     10.00   0.000     .7500000     1.15000
       _cons |   3.000000   .200000     15.00   0.000     2.750000     3.50000
------------------------------------------------------------------------------
```

Answer the following questions based on the regression output:

Question 1. What is the expected value of log of daily wage for a person whose value for log of education is 4?

(a) 13

(b) 19

(c) 5

(d) 43

(e) 7

Answer: e.

$E(logwage) = \beta_1 + \beta_2 logeduc$, so the estimate for log of daily wage for a person whose value for log of education is 4 is given by $\hat{y} = 3 + 1 \cdot 4 = 7$.

Question 2. Let the expected daily wage for a person that has 10 years of schooling be \$ 200. What is the average effect of an additional year of schooling?

(a) 2

(b) 3

(c) 10

(d) 20

(e) 30

Answer: c.
The marginal effect of the log-log regression is given by is given by $\beta_2 \cdot \frac{y}{x} = 1 \cdot \frac{200}{10} = 20$.

Question 3. What is the correlation between log of wage and log of education?

(a) $+0.5$

(b) $-\sqrt{0.5}$

(c) $+\sqrt{0.5}$

(d) -0.5

(e) 0.25

Answer: c.

We know that $R^2 = corr(logWage, logeduc)^2$ and that the R^2 of the regression is $R^2 = 0.5$. Thus, $corr(logWage, logeduc) = +\sqrt{0.5}$ or $corr(logWage, logeduc) = -\sqrt{0.5}$. Note that $\hat{(b)}_2 = 1$ which implies that $\hat{(b)}_2 = cov(logWage, logeduc)/var(logeduc) = 1$, therefore $cov(logWage, logeduc) > 0$ and thereby, $corr(logWage, logeduc) > 0$. In summary, it must be the case that $corr(logWage, logeduc) = +\sqrt{0.5}$.

Question 4. Let the expected daily wage for a person that has 10 years of schooling be \$ 200. Now consider a person with 11 years of schooling, that is, a 10% increase in schooling. What is the average wage you would expect for the person with 11 years of schooling?

(a) 210

(b) 220

(c) 230

(d) 240

(e) 260

Answer: b.

The elasticity is $\hat{b}_2 = 1$, so 1% increase in education implies, on average, on 1% increase in wage. Thus a 10% increase in education implies a 10% increase in wge, from 200 to 220.

Questions 5–8 are based on the following regression output

Consider the **Quadratic** linear model described below:

$$wage = \beta_1 + \beta_2 \cdot educ2 + e.$$

1. *wage* means daily wage measured in dollars.
2. *educ*2 means the squared of education variable $(educ)^2$.

The regression output associated with this regression is given by:

```
1 . reg wage educ2

      Source |       SS       df       MS              Number of obs =     100
-------------+------------------------------           F(  1,   98  ) =  100.00
       Model |   10000.00     1     10000.00           Prob > F      =  0.0000
    Residual |   10000.00    98       100.00           R-squared     =  0.5000
-------------+------------------------------           Adj R-squared =  0.4950
       Total |   20000.00    99       200.00           Root MSE      =   10.00

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ2|   1.00         0.100      10.00   0.000     0.7500     1.12500
       _cons |   100.00      10.000      10.00   0.000     90.000     125.000
------------------------------------------------------------------------------
```

The regression output for the covariance matrix of the estimated intercept $\hat{b}_1$ and slope $\hat{b}_2$ is given by:

```
Covariance matrix of coefficients of regress model

        e(V) |       educ       _cons
-------------+------------------------
        educ2|  0.010              .
       _cons | -0.500         100.000
```

Question 5. The best mean predictor for the average wage for a person with 10 years of education?

(a) 110

(b) 120

(c) 200

(d) 220

(e) 102

Answer: c.

Note that $E(Y|x) = \beta_1 + \beta_2 \cdot x^2$. Thus the estimate of $E(Y|x = 10)$ is $\hat{b}_1 + \hat{b}_2 \cdot x^2 = 100 + 1 \cdot 10^2 = 100 + 100 = 200$.

Question 6. What is the 95% confidence interval for the mean predictor of the wage of a person with 10 years of education (use critical value $t_c = 2$)?

(a) $[180, 220]$

(b) $[190, 210]$

(c) $[200, 220]$

(d) $[100, 140]$

(e) $[100, 120]$

Answer: a.

The linear predictor for the average wage for a person with 10 years of education is given by $\lambda = \beta_1 + \beta_2 \cdot 100$. The estimate for this mean predictor is: $\hat{\lambda} = \hat{b}_1 + \hat{b}_2 \cdot 10^2 = 200$. The estimated variance for this linear predictor is:

$$\begin{aligned} \widehat{var}(\lambda) &= \widehat{var}(b_1) + 100^2 \widehat{var}(\cdot b_2) + 2 \cdot 100 \cdot \widehat{cov}(b1, b_2) && (1) \\ &= 100 + 100^2 \cdot 0.01 - 2 \cdot 100 \cdot 0.5 && (2) \\ &= 100 + 100 - 2 \cdot 100 \cdot 0.5 && (3) \\ &= 100 && (4) \end{aligned}$$

The estimated standard erro for λ is

$$\widehat{se}(\lambda) = \sqrt{\widehat{var}(\lambda)} = \sqrt{100} = 10$$

The confidence interval is given by:

$$[\hat{\lambda} \pm t_c \cdot \widehat{se}(\lambda)] = [200 \pm 2 \cdot 10] = [180, 220]$$

Question 7. What is The best linear predictor for the marginal effect of schooling on wage for a person with 10 years of education?

(a) 10

(b) 110

(c) 20

(d) 102

(e) 120

Answer: c.

The best linear predictor for the marginal effect of schooling on wage for a person with 10 years of education is given by $\hat{\delta}_0 = 2 \cdot \hat{b}_2 \cdot 10 = 20 \cdot \hat{b}_2 = 20$, given that $\hat{b}_2 = 1$.

Question 8. What is the 95% confidence interval for the marginal effect of schooling on wage for a person with 10 years of education (use critical value $t_c = 2$) ?

(a) $[6, 14]$

(b) $[8, 12]$

(c) $[16, 24]$

(d) $[116, 124]$

(e) $[18, 22]$

Answer: c.

The marginal effect for a person with x years of education is $2 \cdot x \cdot \beta_2$. Thus marginal effect for a person with x years of education is $20 \cdot \beta_2$. The question is asking for the confidence interval for $20 \cdot \beta_2$, which is $[20 \cdot \hat{b}_2 \pm 20 \cdot t_c \cdot \widehat{se}(b_2)]$. Inserting the values from the regression output, we have that:

$$[20 \cdot 1 \pm 20 \cdot 2 \cdot 0.1] = [20 \pm 4] = [16, 24].$$

Part II (Questions that are not based on Regression Outputs):

Question 9. Consider the following regression model given by $y = \beta_1 + \beta_2 x_1 + \beta_3 x_2 + e$. Let $\hat{b}_1, \hat{b}_2, \hat{b}_3$ be the estimates of this regression whose variance-covariance matrix is given by:

$$\begin{pmatrix} 1 & 1 & -1 \\ 1 & 2 & -.5 \\ -1 & -.5 & 3 \end{pmatrix}.$$

The **standard error** of the estimator $b_2 + b_3$ is:

(a) 1

(b) 2

(c) 3

(d) 4

(e) 12

answer b

$$\begin{aligned} var(b_2 + b_3) &= var(b_2) + var(b_3) + 2cov(b_1, b_3) \\ &= 2 + 3 + 2 \cdot -.5 = 4 \\ se(b_2 + b_3) &= \sqrt{var(b_2 + b_3)} = \sqrt{4} = 2 \end{aligned}$$

Question 10. Let X, Y be two random variables and take values in $\{0, 1\}$ and whose joint distribution is given by:

Joint Distribution

	$Y = 0$	$Y = 1$	$f_X(x)$
$X = 0$	0	1	1
$X = 1$	0	0	0
$f_Y(y)$	0	1	1

Mark the choice that is **false**:

(a) X and Y are statistically **independent**.

(b) $E(X^2) = \big(E(X)\big)^2$

(c) $var(X) = var(Y)$

(d) $P(X = 1|Y = 1) = P(X = 1|Y = 0)$

(e) $E(X^2) = E(Y^2)$

Answer e

X takes value 0 with probability 1. Y takes value 1 with probability 1. These variables are indeed independent as $P(X = x, Y = y) = P(X = x)P(Y = y)$ for all $x \in \{0, 1\}$ and $y \in \{0, 1\}$. We have that $E(X) = 0$, $E(X^2) = 0$, $(E(X))^2 = 0$. In the same fashion, we also have that $E(Y) = 1$, $E(Y^2) = 1$, $(E(Y))^2 = 1$. The variance of both variables is equal to zero. Indeed there is no variation. It is true that $P(X = 1) = P(X = 1|Y = 1) = P(X = 1|Y = 0) = 0$. It is also true that $E(X^2) = 0 \neq 1 = E(Y^2)$.

Question 12. Mark the **FALSE** alternative regarding the precision of the least squares estimates.

(a) We can obtain more precise estimates by increasing the sample size of the regression.

(b) We can obtain more precise estimates by increasing the sample variance of x.

(c) Adding one to each of the values of $x_i; i = 1, .., N$ does not change the variance of slope estimator b_2.

(d) Adding one to each of the values of $x_i; i = 1, .., N$ does not change the variance of intercept estimator b_1.

(e) The correlation between the intercept estimator b_1 and the slope estimator b_2 is zero whenever the sample mean of x is zero.

Answer d

Adding one to x does not change its sample variance but changes its mean. The variance of the b_2 depends only on the sample variance or x while the variance of b_2 depends on the sample variance and the sample mean of x.

Question 13. The following statements describe properties of the Least square estimators. Mark the statement that is **TRUE**.

(a) The simple regression model is based on five essential assumptions. The most important assumption among these is that the Least square estimators are BLUE.

(b) The Jarque Bera statistic is used to test if the error terms are normally distributed. Therefore it cannot be used to inquire if a transformation of x is suitable to explain the dependent variable.

(c) The sample mean of the estimated residuals $\hat{e}$ is always zero, which implies that the covariance between x and $\hat{e}$ is also zero.

(d) The error terms of a regression are never observed. Thus, it is not useful to examine the residuals as they are not the actual error terms that generate the observed outcome.

(e) One consequence o the least square assumptions is that the least square estimators have the lowest variance among all linear estimators of β_1, β_2 that are unbiased.

Answer e

The BLUE property of the least squares come as a consequence of the least square assumptions. The Residual plot is useful the check if transformations of x are necessary to model the relation between x and y. The Jarque Bera test do inference on the distribution of residuals and it is useful to check if the model specification is sound.

Question 14. Consider the inference that tests the null hypothesis $H_0 : \beta = 0$. Which of the following statement is **FALSE**?

(a) The critical value of the test depends only on the alternative hypothesis, significance level and sample size.

(b) Depending on the alternative hypothesis, higher values of the test statistic can be evidence against or in favor of the null hypothesis.

(c) The test statistic is the same regardless of the alternative hypothesis.

(d) The p-value and the significance level are sufficient information to decide whether to reject the null hypothesis.

(e) The Error of Type I is determined by the p-value

Answer e

The Error of type 1 is determined by the significance level α

Question 15. Let $CI(\beta_2)$ be the estimated confidence interval for β_2 of a simple regression model at that 95% confidence level. Let $t_c(0.975, N-2)$ be the 97.5% quantile of the t-distribution with $N-2$ degrees of freedom, and $\hat{b}_2, \widehat{se}(b_2)$ be the estimates of β_2 and its standard error. Let $\hat{t}$ be your test statistic to test $H_0 : \beta_2 = 0$ against the alternative hypothesis $H_1 : \beta_2 \neq 0$ at significance level of 5%. . Which of the following statement is **FALSE**?

(a) $CI(\beta_2) = [\hat{b}_2 - t_c(0.975, N-2)\widehat{se}(b_2), \hat{b}_2 + t_c(0.975, N-2)\widehat{se}(b_2)]$.

(b) You reject the null hypothesis if $|\hat{t}| > t_c(0.975, N-2)$.

(c) $CI(\beta_2)$ alone provide sufficient information to decide whether to reject the null hypothesis or not.

(d) If $0 \in CI(\beta_2)$ then the p-value must be larger than 0.05.

(e) You reject H_0 if and only if $t_c(0.975, N-2)\widehat{se}(b_2) < \hat{b}_2$.

Answer e

You reject H_0 if $\hat{b}_2$ is far from zero, either to the negative tail or the positive tail. Namely, if $t_c(0.975, N-2)\widehat{se}(b_2) < \hat{b}_2$ of if $\hat{b}_2 < -t_c(0.975, N-2)\widehat{se}(b_2)$.

Question 16. Let the Simple Regression $Y = \beta_1 + \beta_2 x + \epsilon$ and consider the single hypothesis testing whose null hypothesis is $H_0 : \beta_2 = 0$ against $H_1 : \beta_2 \neq 0$. Let $\hat{t}_2$ be the appropriate test statistic. Then we are more likely to reject H_0 if (mark the **correct one**):

(a) The absolute value of test statistic $|\hat{t}_2|$ *decreases.*

(b) The p-value *decreases.*

(c) The significance level α *decreases*.

(d) The rejection region *decreases* (shrinks).

(e) The sample size *decreases*.

Answer b

If the p-value decreases, then it is more likely that p-value is smaller than α which would make you reject the null H_0. If the absolute value of test statistic $|\hat{t}_2|$ decreases, the p-value increases. The significance level α *decreases*, you are **less** likely to have that p-val $< \alpha$.
If the rejection region shrinks, there are fewer values of the test statistics that lead to a rejection of H_0.

Question 17. Consider the forecast error when predicting the outcome $Y_0 = \beta_1 + \beta_0 x_0 + e_0$ using the simple regression model $Y = \beta_1 + \beta_2 x + e$. Which of the following is **FALSE** regarding prediction:

(a) The prediction is more precise the closer x_0 is to the mean of the explanatory variable $\bar{x}$.

(b) The prediction is more precise the smaller the variance of the error term e.

(c) The prediction is more precise the larger the sample size N.

(d) The prediction is more precise the larger the variance of the exploratory variable $var(x)$.

(e) The prediction is more precise the larger the variance of the slope estimator $var(b_2)$.

Answer e
The question asks you to be able to interpret the equation of the variance of the forecast error:

$$\widehat{var(f)} = \sigma^2 \cdot \left(1 + \frac{1}{N} + \frac{(x_0 - \bar{x})^2}{\sum(x_i - \bar{x})^2}\right) \tag{5}$$

$$= \sigma^2 + \frac{\sigma^2}{N} + (x_0 - \bar{x})^2 var(b_2) \tag{6}$$

$$= \sigma^2 + \frac{\sigma^2}{N} + (x_0 - \bar{x})^2 \frac{\sigma^2}{N} \frac{1}{var(x)} \tag{7}$$

All the items are based on the interpretation of these equations.

Question 18. Consider data on (x, y) where $\bar{x}, \bar{y}$ are sample means, σ_x, σ_y are sample standard deviations and $corr(x, y)$ is the sample correlation. Let R^2 be the coefficient of determination of the simple regression $Y = \beta_1 + \beta_2 + \epsilon$. Which of the following statement is **FALSE**?

(a) The larger the absolute value of the covariance between y and x, the larger the R^2 (everything else constant).

(b) The correlation between x and the fitted values $\hat{y}$ is the same as the correlation between x and the actual dependent variable y.

(c) The R^2 can be computed as the ratio of the sum of squares of the regression $SSR = \sum(\hat{y} - \bar{y})^2$ and the total sum of the squares $SST = \sum(y - \bar{y})^2$.

(d) The regression of y on x generates the same R^2 of the regression of x on y.

(e) The R^2 measures the share of the variance of the dependent variable that is explained by the explanatory variable.

answer b

The correlation between x and the fitted values $\hat{y}$ is either 1 or -1 as:

$$corr(x, \hat{y}) = corr(x, \hat{b}_1 + \hat{b}_2 x) = corr(x, \hat{b}_2 x) = sign(\hat{b}_2)corr(x, x) = sign(\hat{b}_2).$$

The correlation between y and the and the fitted values $\hat{y}$ is:

$$corr(y, \hat{y}) = corr(y, \hat{b}_1 + \hat{b}_2 x) = corr(y, \hat{b}_2 x) = sign(\hat{b}_2)corr(y, x).$$

Question 19. Suppose that a simple regression using data x, y generates the estimates $\hat{b}_1 = \hat{b}_2 = \mathbf{1}$. Consider the transformed variables $x^* = x+1$, $y^* = y+1$. Let the new estimates be $\hat{b}_1^*$ and $\hat{b}_2^*$. Which of the following statement is **TRUE**?

(a) $\hat{b}_1^* = \hat{b}_1$ and $\hat{b}_2^* = \hat{b}_2$

(b) $\hat{b}_1^* \neq \hat{b}_1$

(c) $\hat{b}_2^* \neq \hat{b}_2$

(d) $\hat{b}_1^* = \hat{b}_2^* + 1$

(e) $\hat{b}_2^* = \hat{b}_1^* + 1$

Answer a

Let's migrate the original regression into the regression with the transformed variables:

$$Y = \beta_1 + \beta_2 x + \epsilon \tag{8}$$
$$\Rightarrow (Y+1) = \underbrace{(\beta_1 + 1 - \beta_2)}_{\beta_1^*} + \beta_2(x+1) + \epsilon \tag{9}$$
$$\tag{10}$$

We can assess that b_2^* still estimates β_2, while b_1^* estimates $\beta_1 - (1+\beta_2)$. Thus we have that $\hat{b}_2^* = \hat{b}_2$ and $\hat{b}_1^* = \hat{b}_1 + 1 - \hat{b}_2$. Now, given the estimates $\hat{b}_1 = \hat{b}_2 = 1$, we have that $\hat{b}_2^* = 1$ and $\hat{b}_1^* = 1$. Therefore $\hat{b}_1^* = \hat{b}_1$ and $\hat{b}_2^* = \hat{b}_2$.

Question 20. Consider data on (x, y) where $\bar{x}, \bar{y}$ are sample means,σ_x, σ_y are sample standard deviations and $corr(x, y)$ is the sample correlation. Consider the transformation that standardizes the variables, that is,

$$x^* = \frac{x - \bar{x}}{\sigma_x} \text{ and } y^* = \frac{y - \bar{y}}{\sigma_y}.$$

- Let the estimated coefficients and the R^2 for (x, y) be $\hat{b}_1, \hat{b}_2, R^2$.
- Let the estimate for the transformed data (x^*, y^*) be $\hat{b}_1^*, \hat{b}_2^*, R^{2*}$.

Which of the following statement is **FALSE**?

(a) $\hat{b}_1^* = 0$ and $\hat{b}_2^* = corr(x, y)$

(b) The coefficient of determination in each regression is the same

(c) The p-value for testing the slope $H_0 : \beta_2 = 0$ versus $H_0 : \beta_2 \neq 0$ in both regressions is the same

(d) $corr(x^*, y^*) = corr(x, y)$ and $R^{2*} = (\hat{b}_2^*)^2$

(e) The p-value for testing the intercept $H_0 : \beta_1 = 0$ versus $H_0 : \beta_1 \neq 0$ in both regressions is the same

Answer e

Note that the sample means of x^* and y^* are zero. Recall that the regression line passes trough the sample means. Thus it must pass by the point $(0, 0)$ and therefore the estimate of the intercept is zero. In this case, the p-value for testing the intercept $H_0 : \beta_1 = 0$ versus $H_0 : \beta_1 \neq 0$ is 1. Letter e is clearly false.

Note also that rescale x or y does not change the t-statistic of the slope estimator b_2. A shift of x or y does not change the t-statistic of the slope estimator b_2 either. The standardization employs a shift and a rescale of x and y and does not change the t statistic of β_2. Therefore its p-value remains the same. The slides of lecture 9 presents a useful table to examine transformation of variables: The table is presented below:

Original Model	New Slope Estimator $\tilde{b}_2$			New Intercept Estimator $\tilde{b}_1$			
Transformations	$\tilde{b}_2$	$\widehat{se}(\tilde{b}_2)$	$\hat{t}_2$	$\tilde{b}_1$	$\widehat{se}(\tilde{b}_1)$	t-stat	$\tilde{R}^2$
$x \rightarrow c_x \cdot x$	$\frac{\hat{b}_2}{c_x}$	$\frac{\widehat{se}(b_2)}{c_x}$	$\hat{t}_2$	$\hat{b}_1$	$\widehat{se}(b_1)$	$\hat{t}_1$	R^2
$y \rightarrow c_y \cdot y$	$c_y \cdot \hat{b}_2$	$c_y \cdot \widehat{se}(b_2)$	$\hat{t}_2$	$c_y \cdot \hat{b}_1$	$c_y \cdot \widehat{se}(b_1)$	$\hat{t}_1$	R^2
$y \rightarrow c_y \cdot y$ $x \rightarrow c_x \cdot x$	$\frac{c_y}{c_x} \cdot \hat{b}_2$	$\frac{c_y}{c_x} \cdot \widehat{se}(b_2)$	$\hat{t}_2$	$c_y \cdot \hat{b}_1$	$c_y \cdot \widehat{se}(b_1)$	$\hat{t}_1$	R^2
$y \rightarrow c \cdot y$ $x \rightarrow c \cdot x$	$\hat{b}_2$	$\widehat{se}(b_2)$	$\hat{t}_2$	$c \cdot \hat{b}_1$	$c \cdot \widehat{se}(b_1)$	$\hat{t}_1$	R^2
$y \rightarrow s_y + y$	$\hat{b}_2$	$\widehat{se}(b_2)$	$\hat{t}_2$	$\hat{b}_1 + s_y$	$\widehat{se}(b_1)$	$\frac{\hat{b}_1 + s_y}{\widehat{se}(b_1)}$	R^2
$x \rightarrow s_x + x$	$\hat{b}_2$	$\widehat{se}(b_2)$	$\hat{t}_2$	$\hat{b}_1 - s_x \hat{b}_2$	$\widehat{se}(b_1 - s_x b_2)$	$\frac{\hat{b}_1 - s_x \hat{b}_2}{\widehat{se}(b_1 - s_x b_2)}$	R^2

where $\widehat{se}(b_1 - s_x b_2) = \sqrt{\widehat{var}(b1) + s_x^2 \widehat{var}(b2) - 2 s_x \widehat{cov}(b_1, b_2)}$.