# Volatility: An Introductory Understanding

matthewzz

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## 1 Introduction

While we begin our discussion, we will look at different versions of volatility. Perhaps the most common measure of volatility is the measure of **variance**, and standard deviation.

### 1.1 A ***Very*** Brief Introduction To Variance

Take some set X = $\{x_1, x_2, x_3..., x_n\}$ in our context A can be thought of as some asset with prices $\{x_1, x_2, x_3..., x_n\}$ at time 1, 2, 3,..., n.

The variance of X is given by the formula: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ **, denoted as $s^2$. And the variance: $s = \sqrt{s^2}$.

*(Note: In our context, since we are always using sample data, we will almost always use going to use the MLE estimator of variance. For simplicity and familiarity we will use $\sigma^2$ for $s^2$, and $\sigma$ for $s$

### 1.2 A Brief Introduction To Options

<u>Take S to be a common asset.</u>
A call option (contract) gives the buyer the **right, but not the obligation to *buy*** the asset S at the **strike price (X)**, at or before the expiration date.

Similarly,
A put option (contract) gives the buyer the **right, but not the obligation to *sell*** the asset S at the **strike price (X)**, at or before the expiration date.

Immediately upon evaluation, we can see that both of these "options" must have some sort of value.

Let's look at an example to clarify,
Take S to be Apple (which we will denote by their stock ticker $AAPL).
Let us, for example, suppose AAPL to be at a current market price** of approximately **$100**.

<u>Say you own:</u>
An AAPL call option with a strike price of $90
An AAPL put option with a strike price of $110,
both of which expire next Friday.

<u>Let's consider what would happen if we "exercise" both of these contracts:</u>

**Call Option:** Since we have the right to buy the "underlying" stock (AAPL) for $90. Assuming we have the $90 to purchase the stock, we now can sell back the stock we just purchased at a sale price of $100 (since that is the current market price). This nets us a profit of $100 - $90 = $10.

Clearly, this call option has to have some value since it just allowed us to get $10 of profit, so the value should ***<u>at least</u>*** **$10**.

Similarly,

**Put Option:** Since we have the right to sell the "underlying" stock (AAPL) for $100. Assuming we can buy the stock at market price for $100, we now can sell back the stock we just purchased at a sale price of $110 (since this is the strike price of our option). This nets us a profit of $110 - $100 = $10.

Clearly, this put option has to have some value since it just allowed us to get $10 of profit, so the value should also be ***<u>at least</u>*** **$10**.

# 2 Pricing Options

## 2.1 Introduction: Looking at Intrinsic vs Extrinsic Value

As we saw in the previous section, given a call option whose strike price is below the current market price, or a put option whose strike price is above the market price, we can always generate some immediate amount of profit (aka cash flow). If this immediate cash flow, I, is greater than 0, we call the value of the cash flow that an option ($10 in the previous example) its **intrinsic value**.

Given that stock prices can change due to variance, it is fair to assume that there could also be some additional value to an option, called its extrinsic value, E, since there is a possibility of getting more cash flow from the option.

We can borrow from the last example, (AAPL to be at a current market price of approximately **$100**).

Say you now own only:
An AAPL call option with a strike price of $90

Suppose you wait until next Tuesday to exercise this option. On Tuesday, the current market price has risen to $110. Now you can see from only our call option we can get a cash flow of $20.

If we would have sold our option when the price was $100, we would have missed out on the extra $10 that we make on Tuesday.

So if we think of the price an option it is:

Option Price = E + I.

Where: E = The measure of immediate cash flows due to exercising the option **now** And, I = The measure of "option premium" due to the **potential of extra cash flows** being made by exercising it in the **future**.

**IMPORTANT: Still this does not tell us anything about how an option should be priced, given the underlying stock price. All this tells us is: given market prices how much of an option's value is intrinsic vs extrinsic**

## 2.2 What An Option Is Worth: Binomial Model Approach

To be continued...