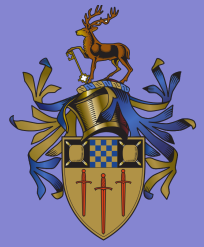

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The dynamics of the two process model for sleep/wake regulation

Matthew Bailey

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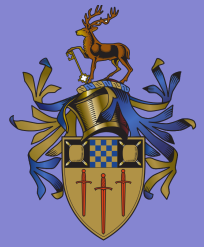

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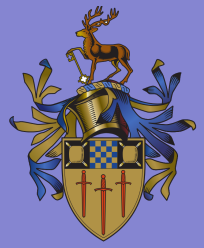

Why mathematically model sleep?

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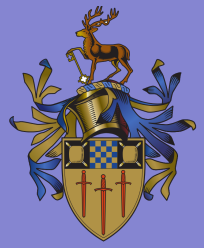
- Understood that poor sleep can lead to many diseases
- Models can help understand the underlying physiology
- Understanding sleep timing has many practical uses



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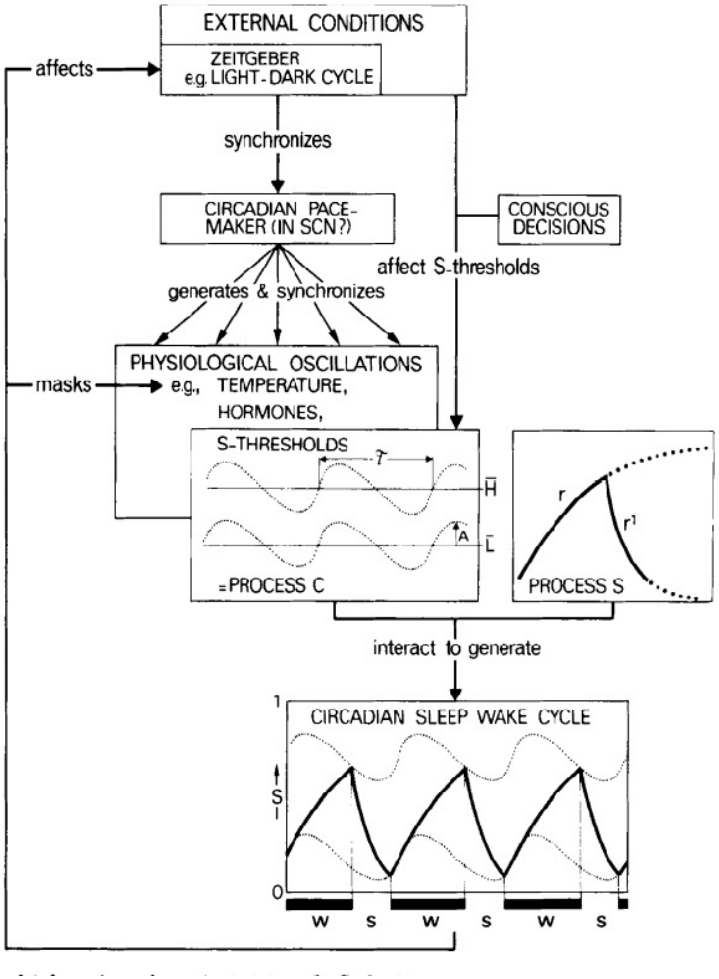
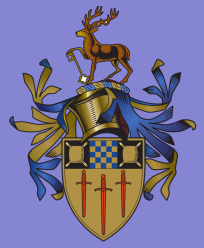


Daan, Borbély 1984

- **Circadian process:** oscillator entrained to 24 hours by external conditions

- **Homeostatic process:** denoting physiological oscillations

- Interaction of these two processes gives switching between sleep and wake



Model equations

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Homeostat:

$$H_s(t) = H^+(t_0)e^{\frac{t_0-t}{\chi_s}}.$$

$$H_w(t) = 1 + (H^-(t_0) - 1)e^{\frac{t_0-t}{\chi_w}}.$$

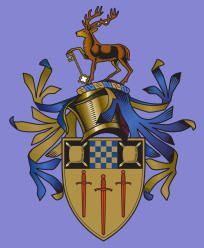
Switching:

$$H^+(t) = H_0^+ + a\sin(2\pi t),$$

$$H^-(t) = H_0^- + a\sin(2\pi t).$$

Assumptions:

- Homeostat increases on wake and decreases on sleep
- Switching between sleep and wake occurs



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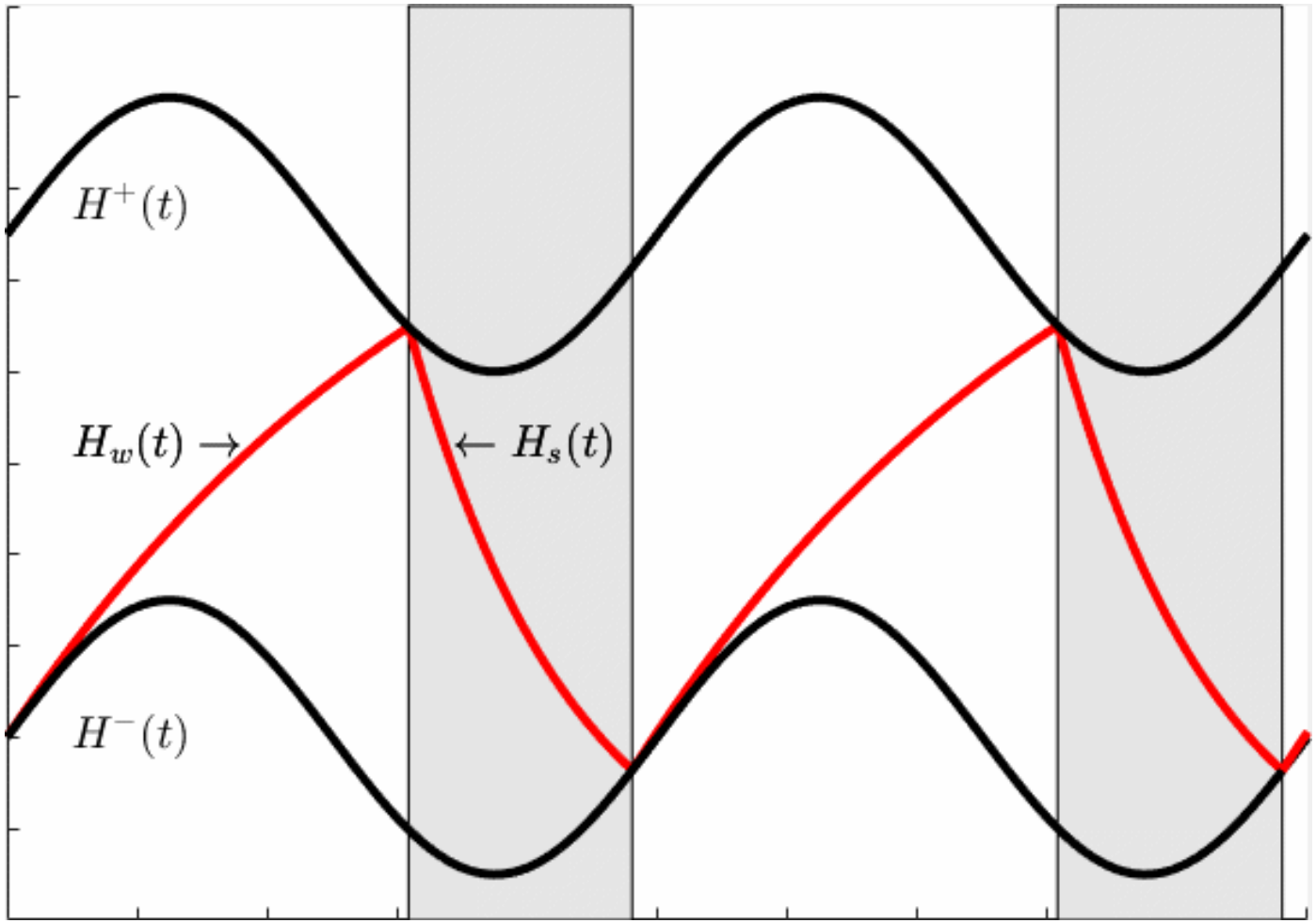

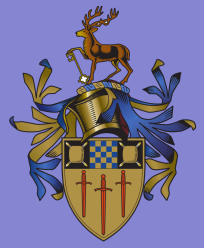

Sleep-wake dynamics

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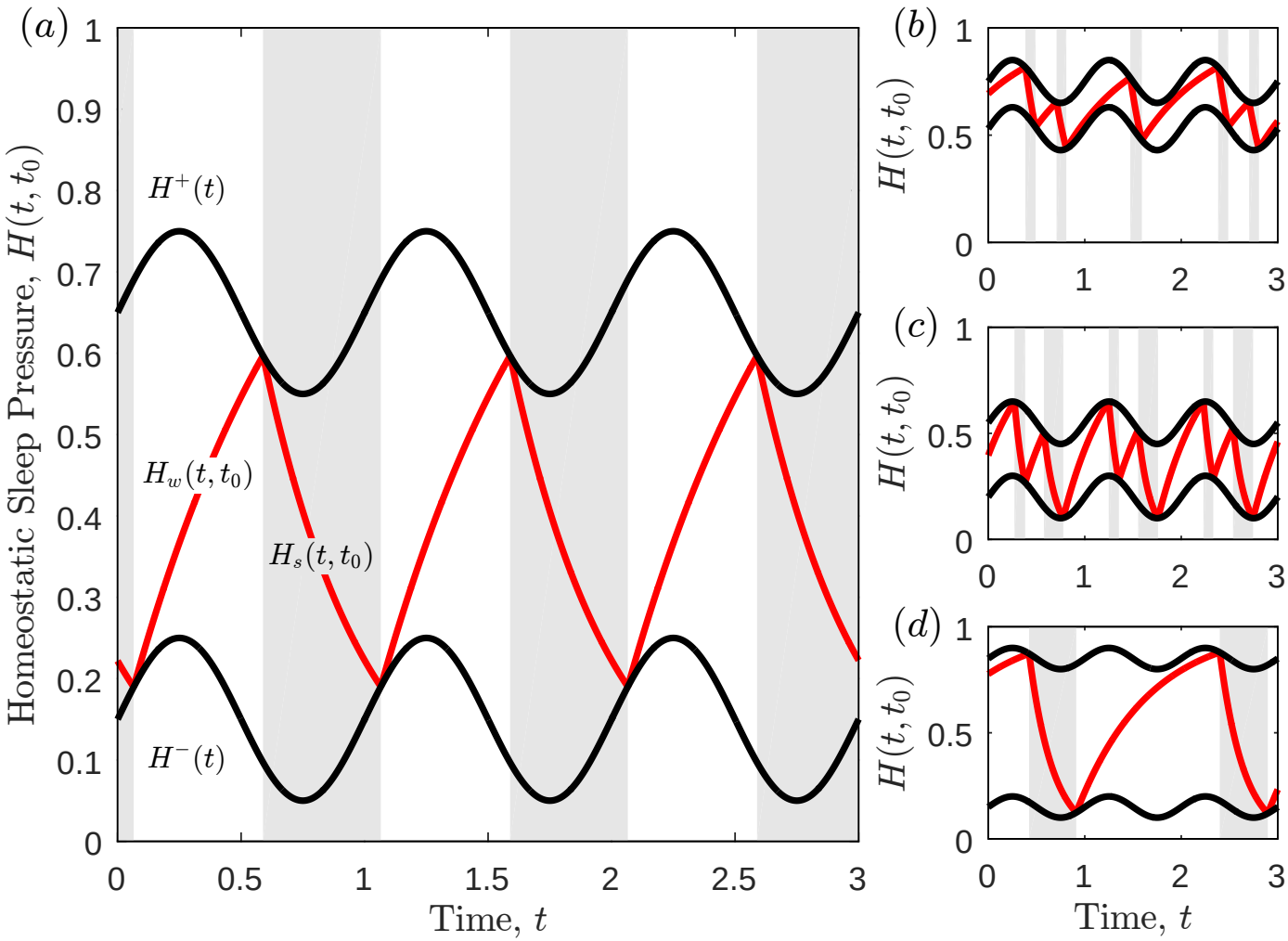

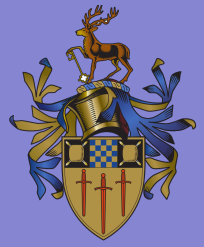

Human sleep

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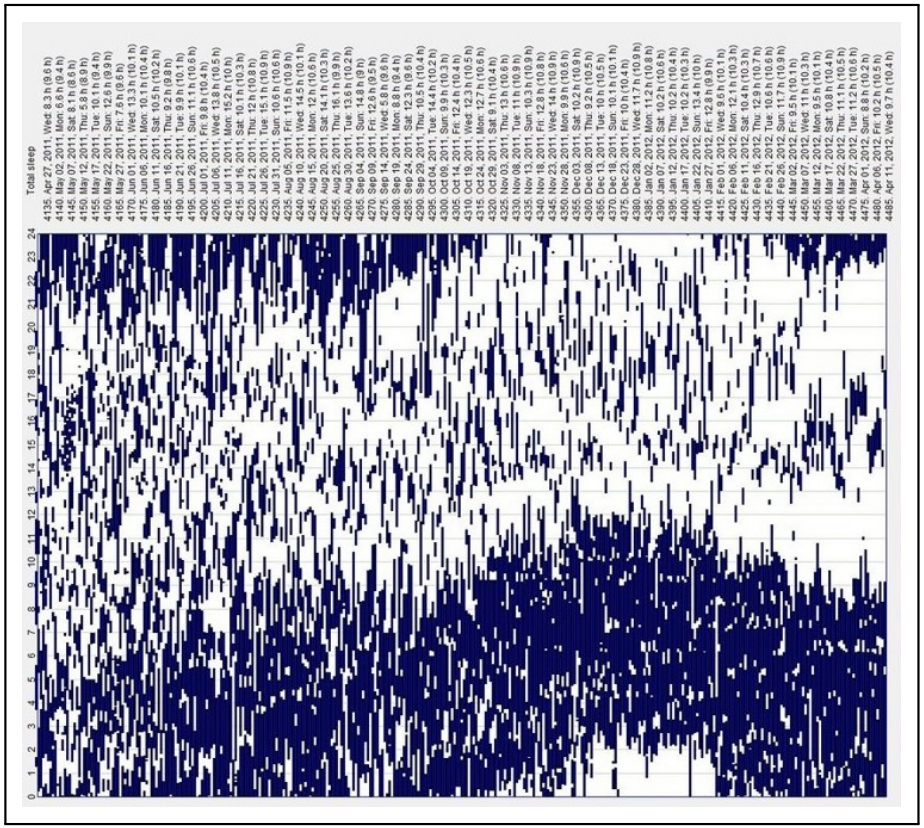
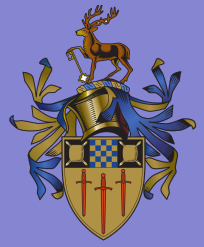
- Monophasic in adults (One sleep per day)
- Changes from birth to adulthood





One dimensional map

The up and down maps

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Down map

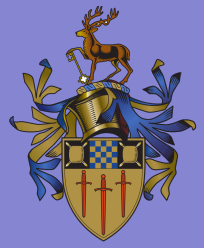
$T_d : \mathbb{R} \mapsto \mathbb{R}$ where $T_d(t_0)$ is the first time greater than t_0 such that

$$H_s(T_d(t_0), t_0) = H^-(T_d(t_0)).$$

Up map

$T_u : \mathbb{R} \mapsto \mathbb{R}$ where $T_u(t_0)$ is the first time greater than t_0 such that

$$H_w(T_u(t_0), t_0) = H^+(T_u(t_0)).$$



One dimensional maps

The composite map

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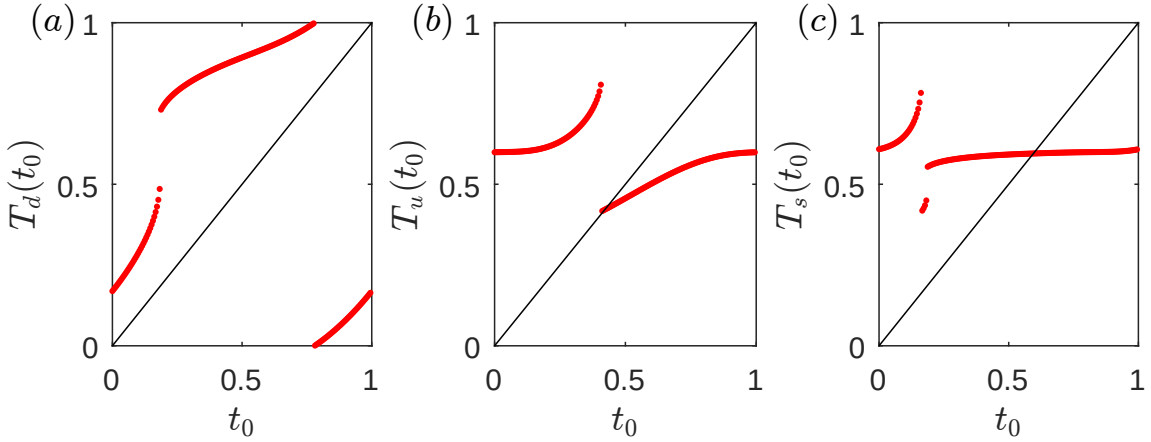
Composite map

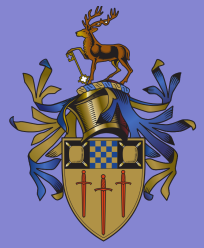
$$T_s : \mathbb{R} \mapsto \mathbb{R}, \quad T_s(t_0) = T_u(T_d(t_0)).$$



All three maps satisfy

$$T_i(t_0 + 1) = T_i(t_0) + 1, \quad i = d, u, s.$$

$T_i(t_0)$ can be seen as the lift of a monotonic circle map on the interval $[0, 1]$.



(p, q) periodic solutions

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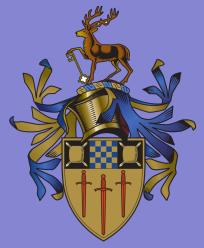
- Many different types of periodic orbits.
- p sleeps (and hence p wakes) in q days.

Hence we say that t_0 generates a (p, q) periodic orbit if

$$T_s^p(t_0) = t_0 + q.$$

Interested in the mechanisms which create/ annihilate periodic solutions



Saddle node bifurcations

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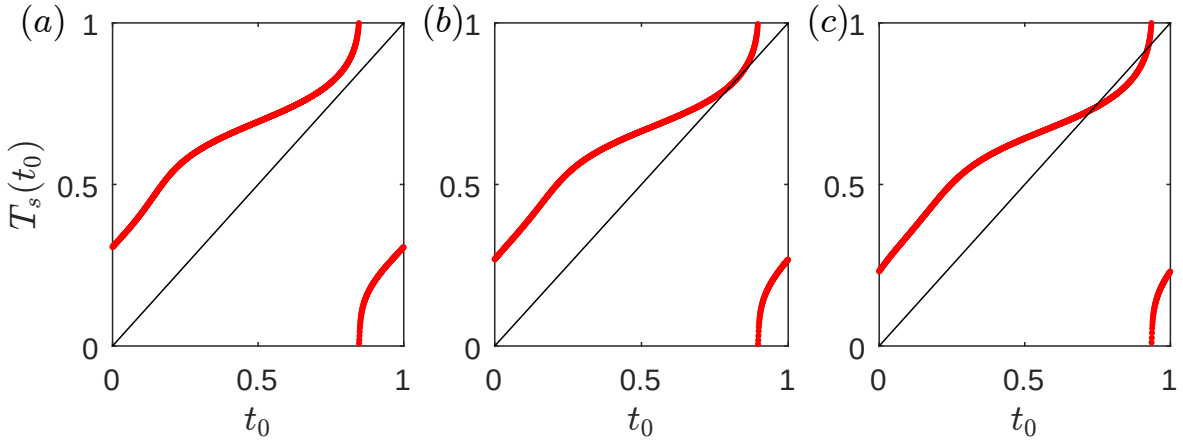
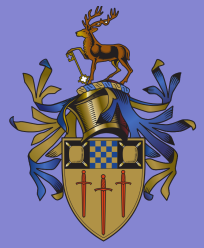
$$(T_s^p)'(t_0) = 1 \quad \text{and} \quad T_s^p(t_0) = t_0 + q$$



Creates and annihilates periodic solutions

$$0 \to 1 \to 2 \to 1 \to 0$$



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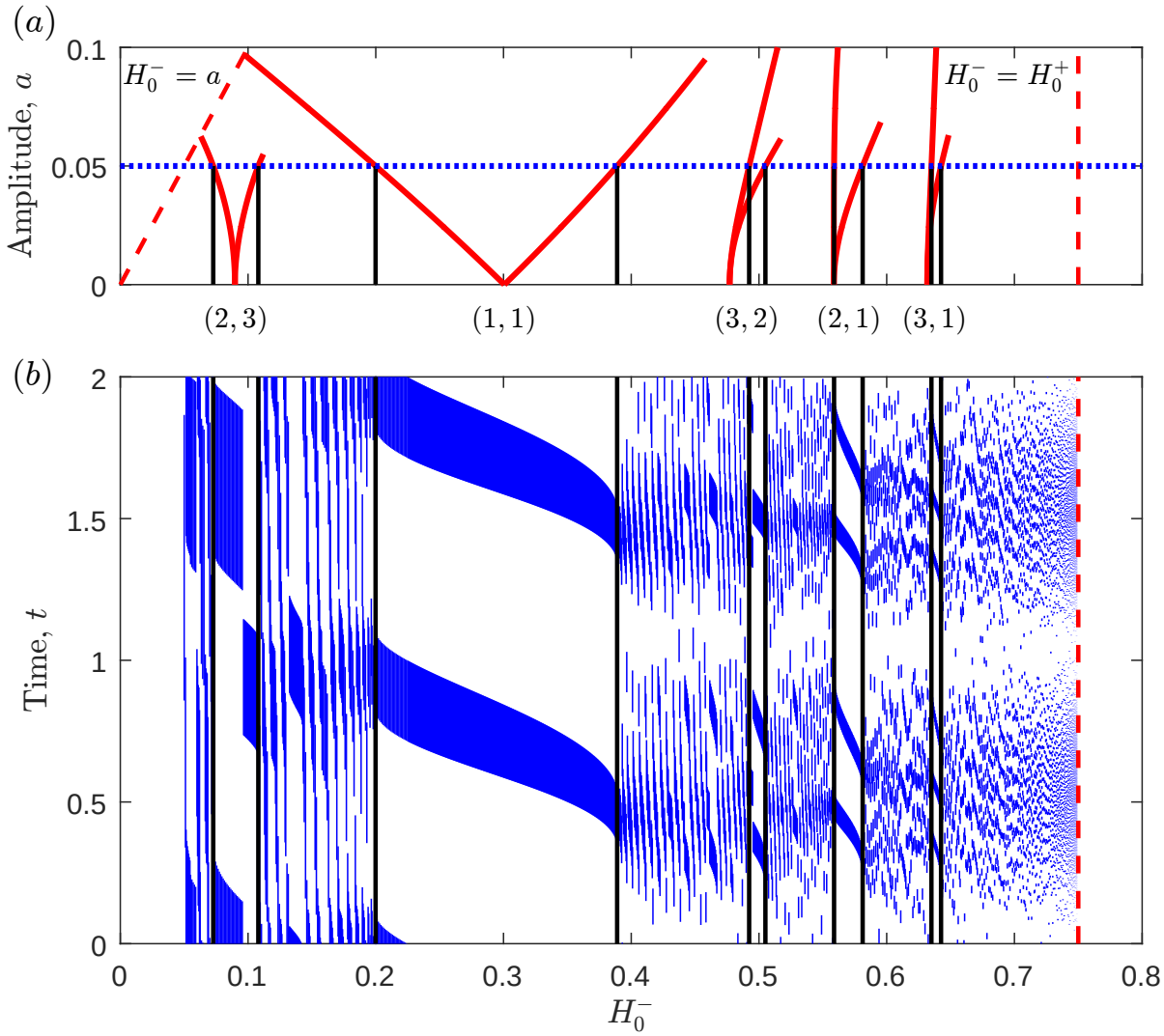

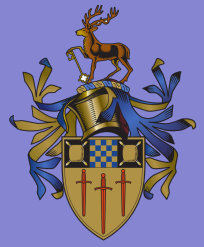

Border Collisions

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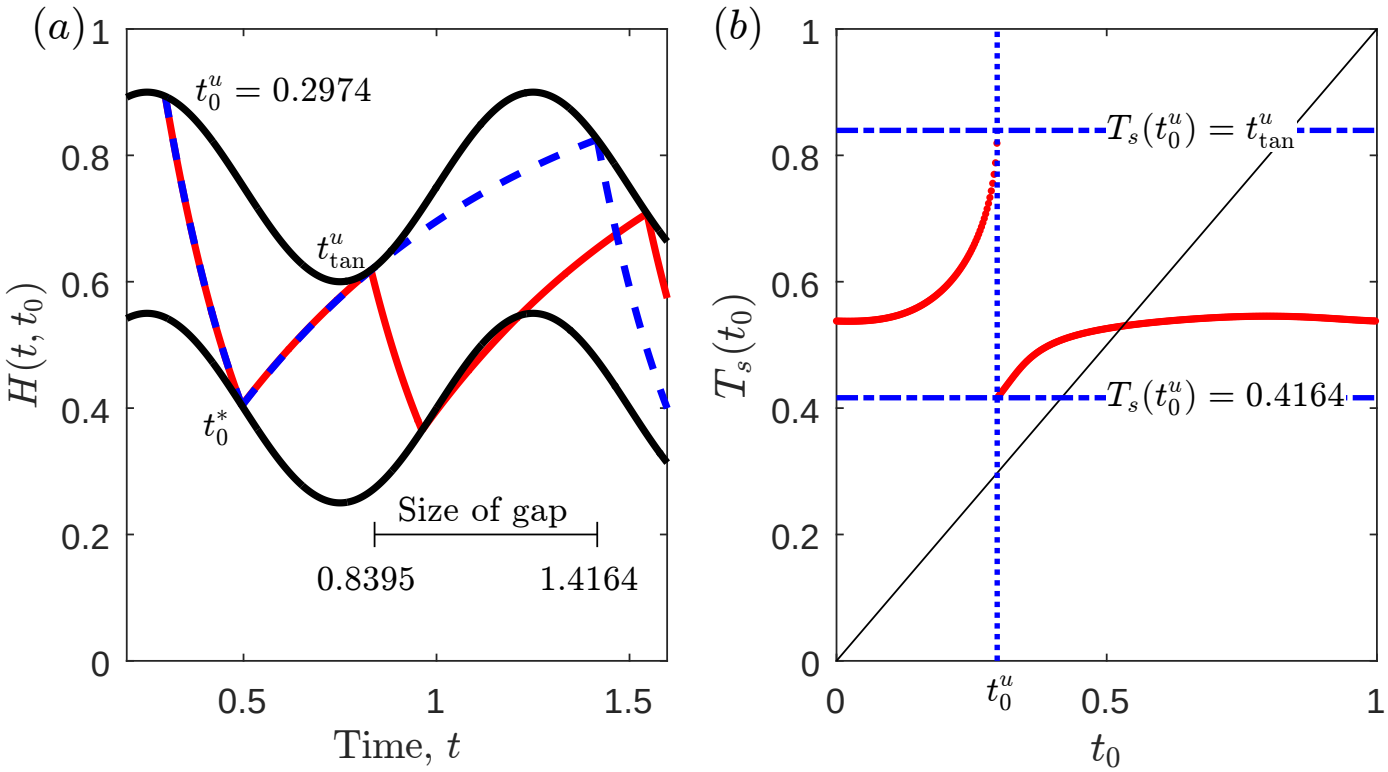
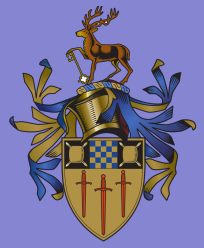


We can view the homeostat as having 2 potential paths at the tangency point



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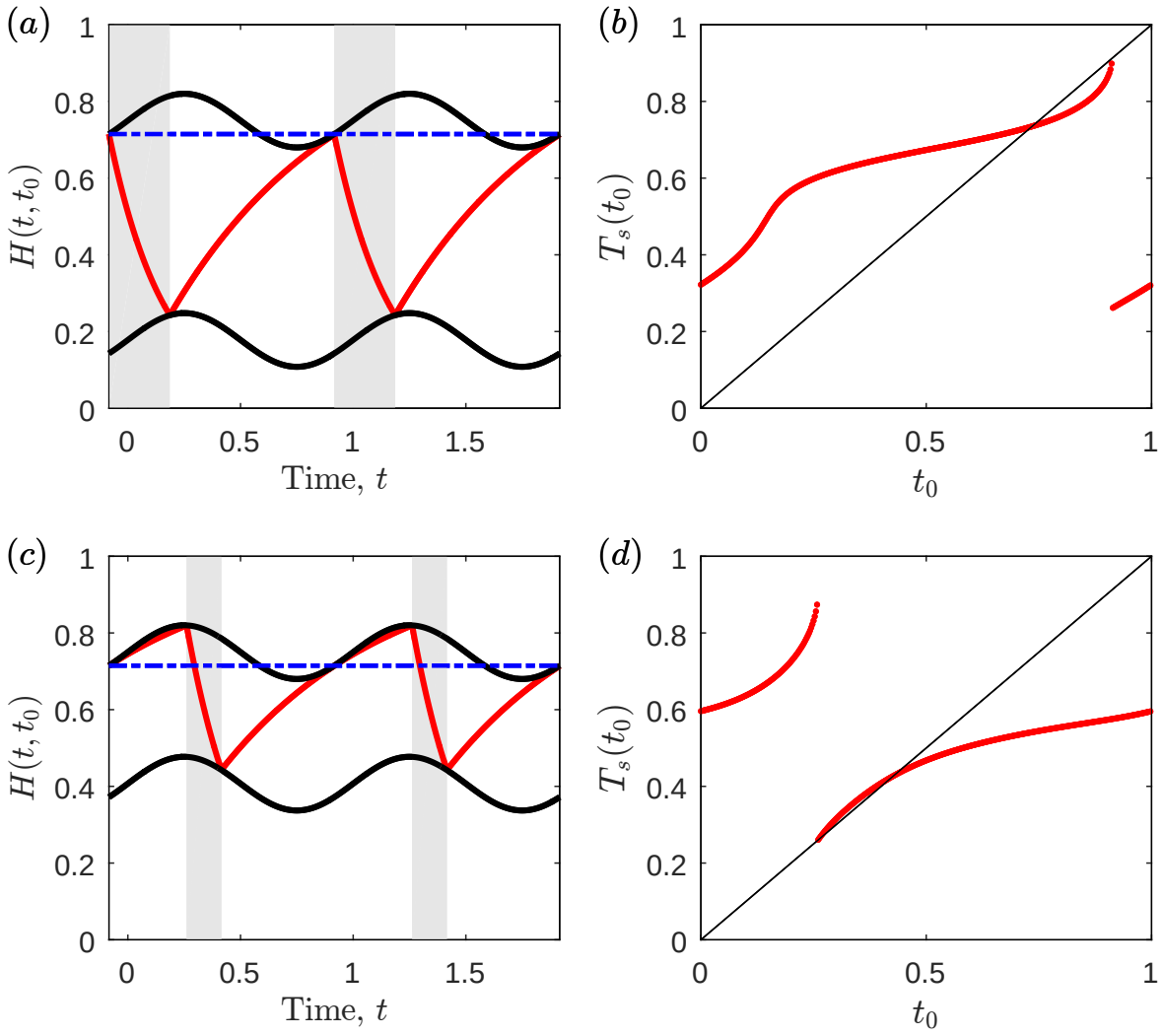

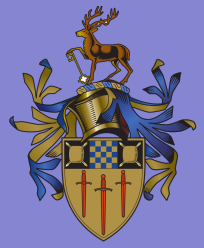

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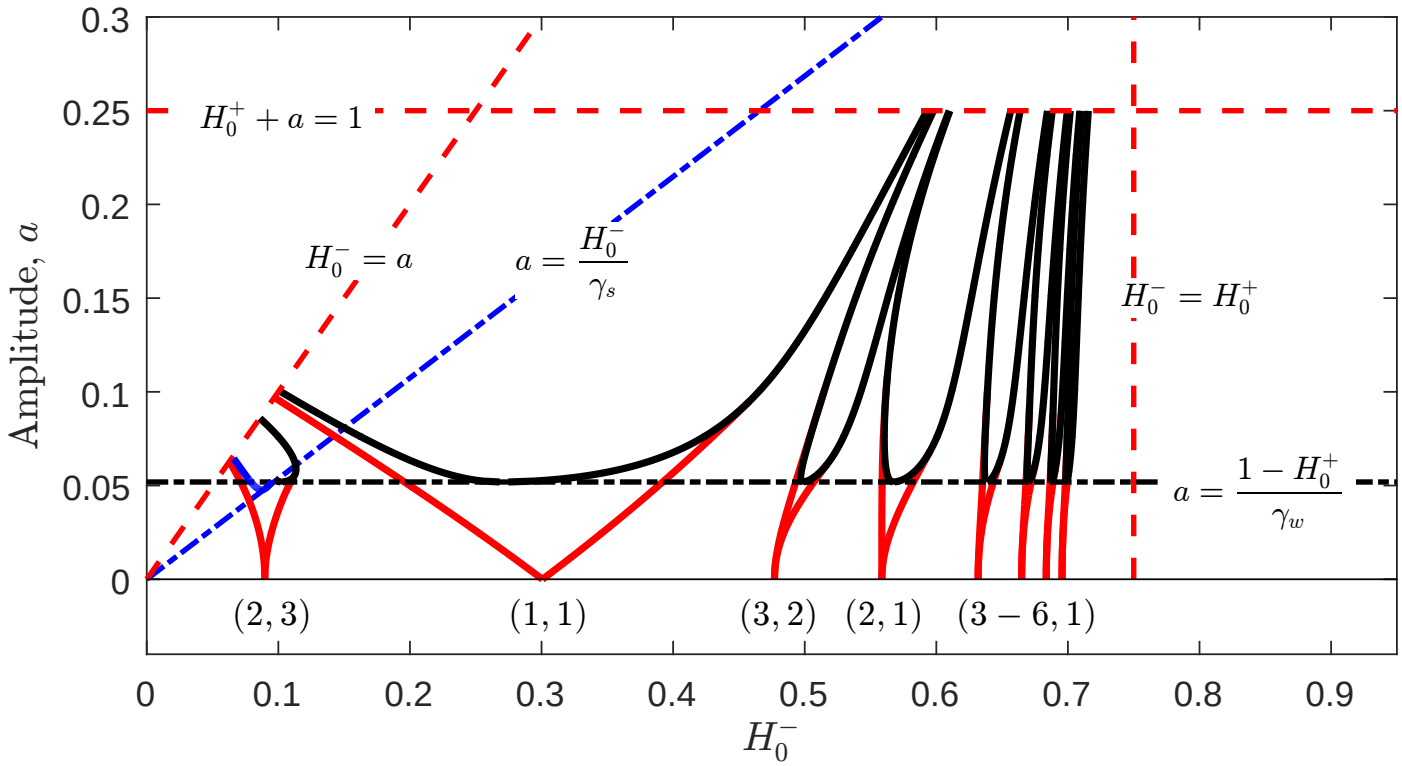
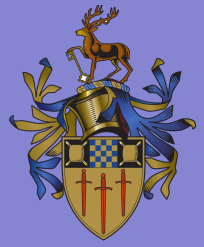


Border collisions take over the Arnold tongue boundaries for larger amplitudes.



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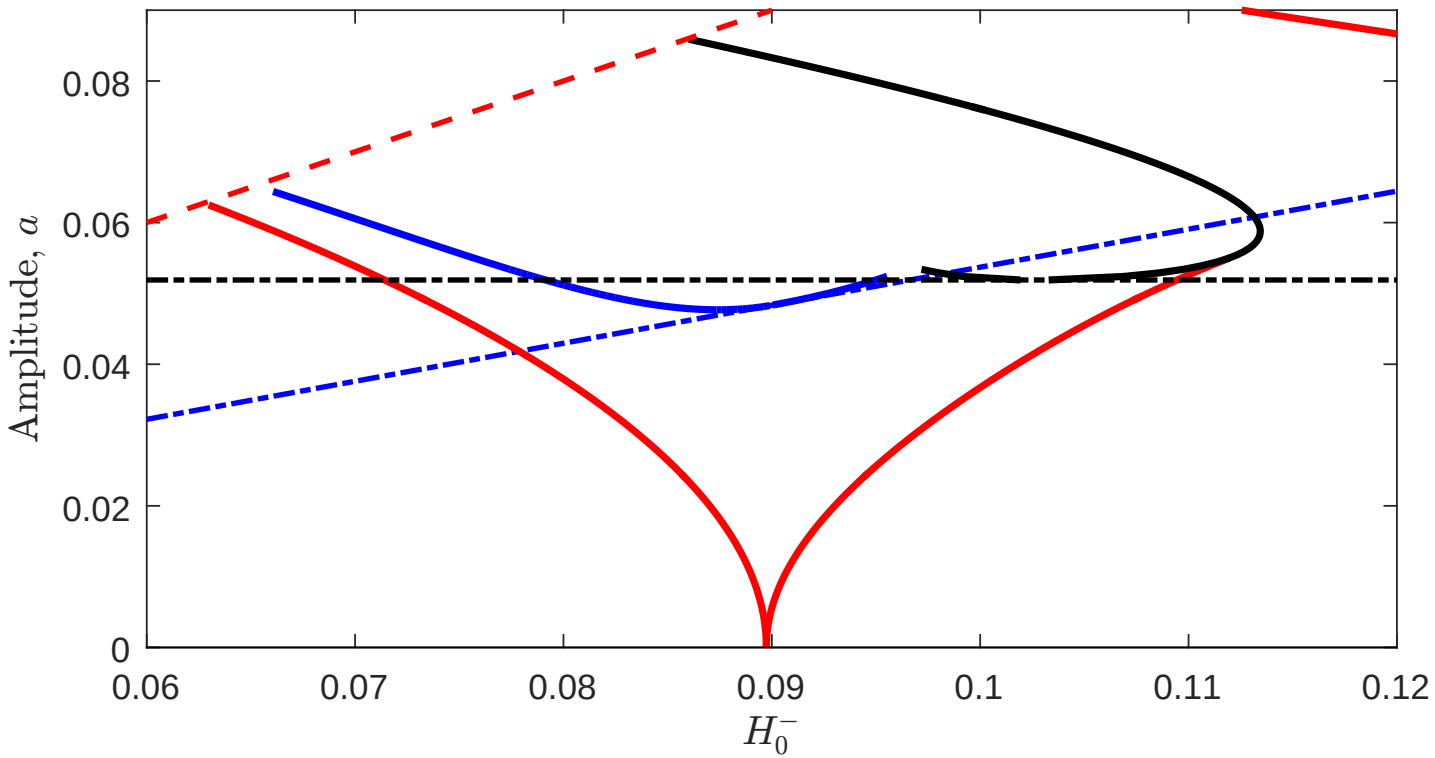
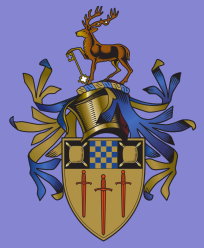
Zoomed into the $(2, 3)$ tongue



Tangencies on both the upper and lower thresholds are important in this region



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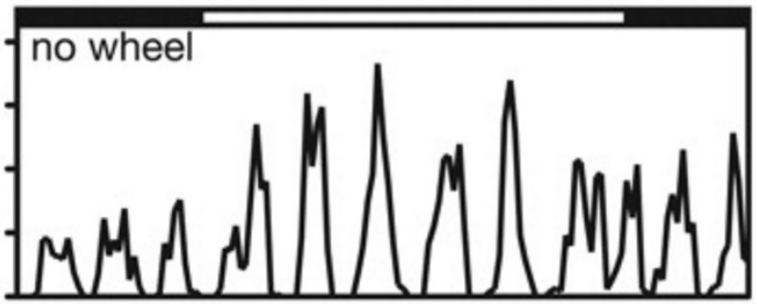
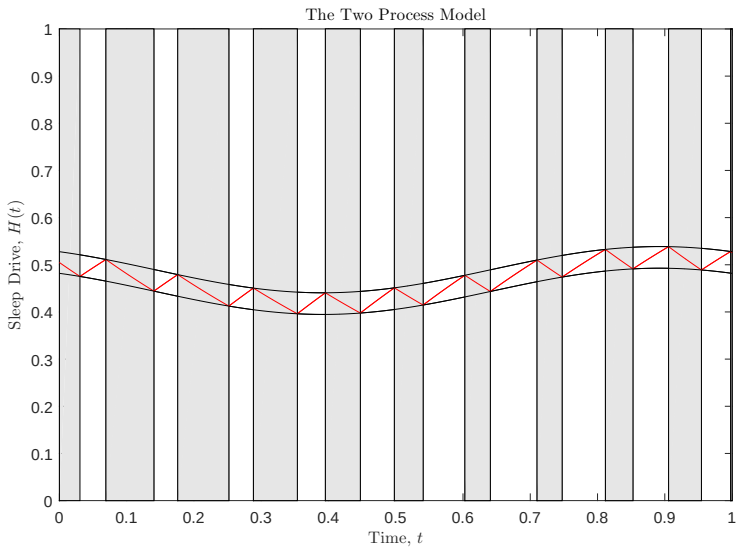
- Polyphasic (Many sleeps per day)
- Consolidated sleep and wake
- More sleep during the day



Van Der Veen, Minh 2006



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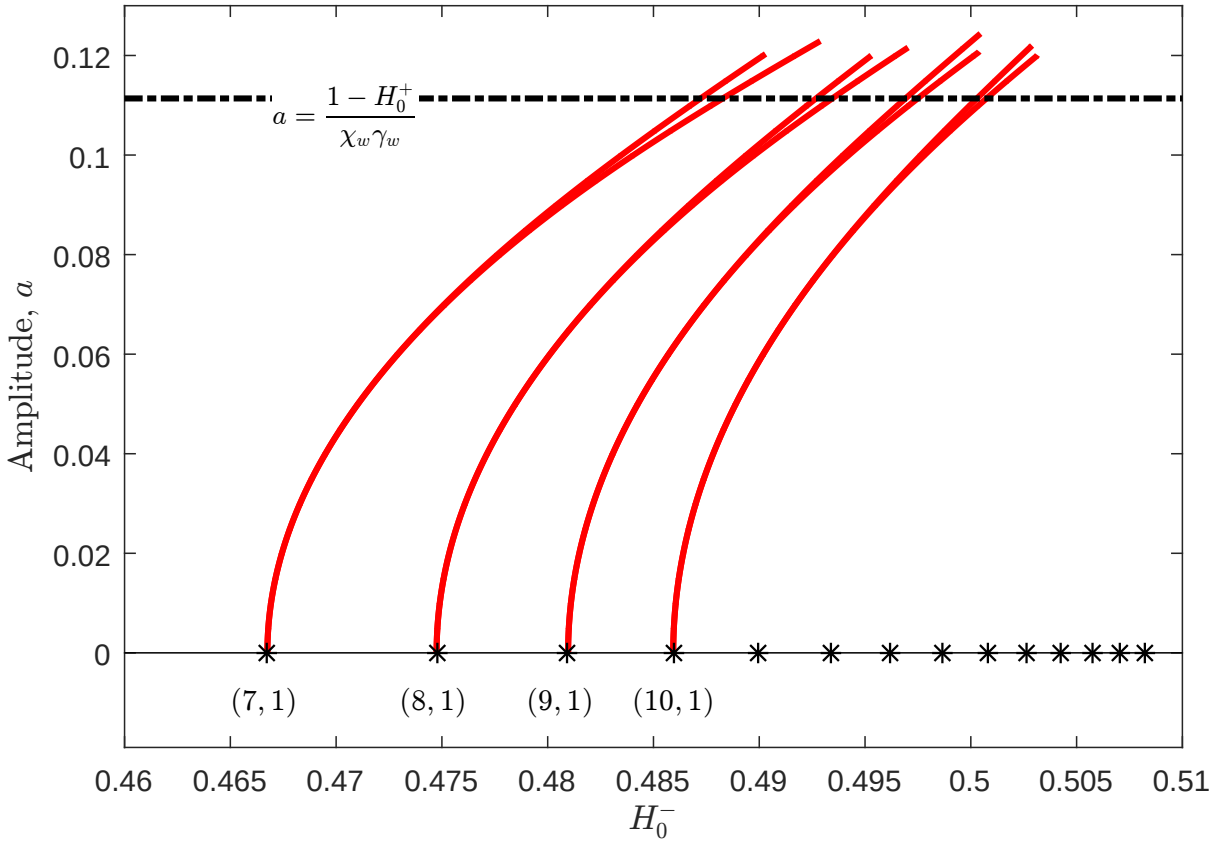
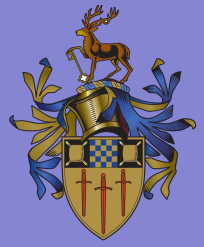


Expect that it is unlikely that the voles will be one-day periodic



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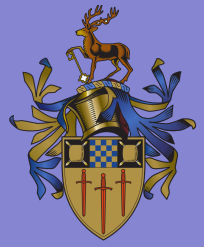
- The Two Process Model displays a wide range of dynamics (monophasic and polyphasic sleep - wake patterns)

- Transitions between periodic solutions occur as a result of saddle-node and border collision bifurcations

Future work

- Non-monotonicity

- A modified two process model including an external zeitgeber



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Thanks for listening!

Supervisors:
Dr Anne Skeldon
Prof Gianne Derks