



The Hidden
Rhythm

Matthew Bailey

The Two Process
Model

Mammalian Sleep

Sleep Deprivation

Ultradian
Rhythms

Conclusion and
Future Work

The Hidden Rhythm

Sleep Patterns of the Common Vole

Matthew Bailey



matthew.bailey@surrey.ac.uk



Overview

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

1 The Two Process Model

2 Mammalian Sleep

3 Sleep Deprivation

4 Ultradian Rhythms

5 Conclusion and Future Work



Why Mathematically Model Sleep?

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



- Understood that poor sleep can lead to many diseases
- Models can help understand the underlying physiology
- Understanding sleep timing has many practical uses



The Two Process Model

The Hidden Rhythm

Matthew Bailey

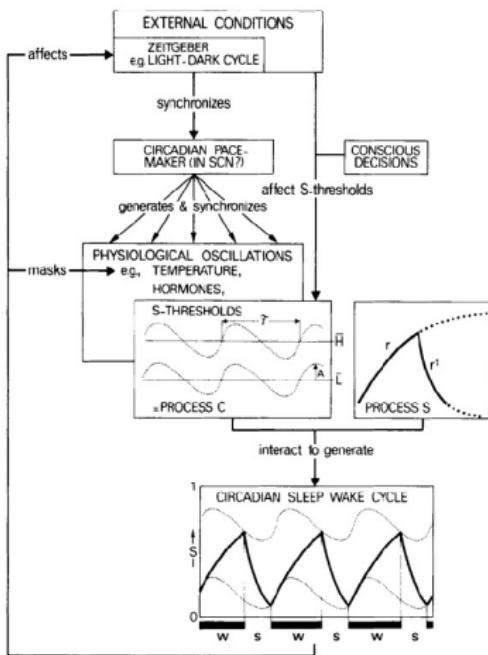
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



Daan, Borbély 1984

- **Circadian process:** oscillator entrained to 24 hours by external conditions

- **Homeostatic process:** denoting physiological oscillations

- Interaction of these two processes gives switching between sleep and wake



Model Equations

The Hidden
Rhythm

Matthew Bailey

The Two Process
Model

Mammalian Sleep

Sleep Deprivation

Ultradian
Rhythms

Conclusion and
Future Work

Homeostat:

$$H_s(t) = H^+(t_0)e^{\frac{t_0-t}{\chi_s}}.$$

$$H_w(t) = 1 + (H^-(t_0) - 1)e^{\frac{t_0-t}{\chi_w}}.$$

Switching:

$$H^+(t) = H_0^+ + a \sin(2\pi t),$$

$$H^-(t) = H_0^- + a \sin(2\pi t).$$

Assumptions:

- Homeostat increases on wake and decreases on sleep
- Switching between sleep and wake occurs



The Two Process Model

The Hidden
Rhythm

Matthew Bailey

The Two Process
Model

Mammalian Sleep

Sleep Deprivation

Ultradian
Rhythms

Conclusion and
Future Work



Sleep-Wake Dynamics

The Hidden Rhythm

Matthew Bailey

The Two Process Model

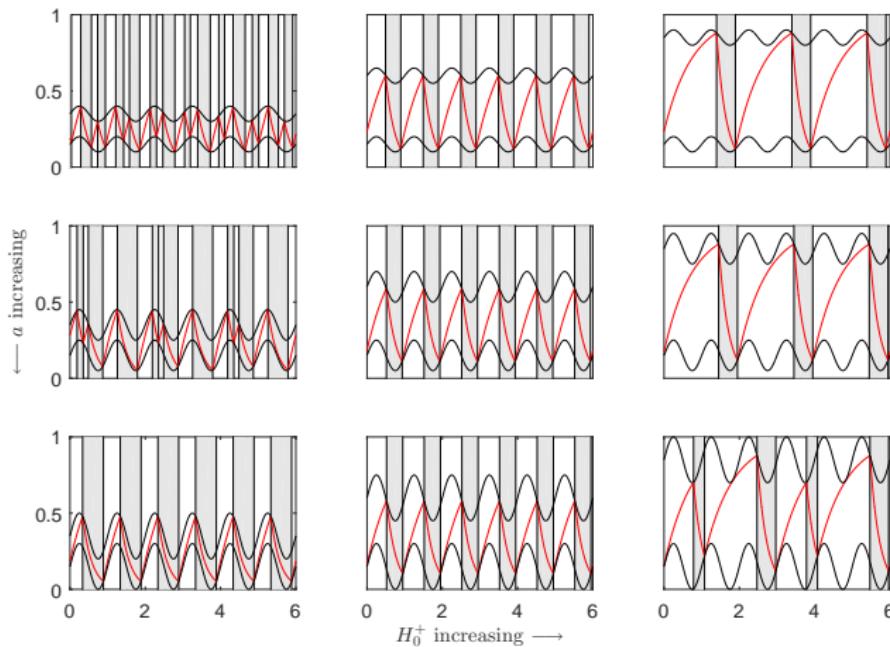
Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

Model Behaviour as Parameters Vary





Human Sleep

The Hidden Rhythm

Matthew Bailey

The Two Process Model

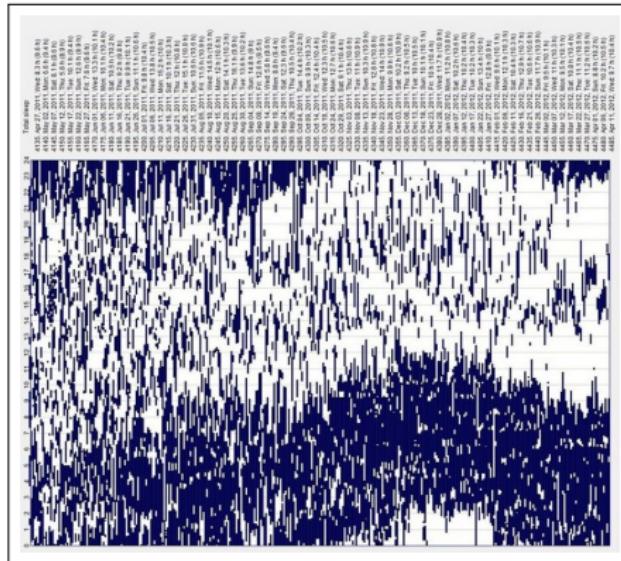
Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

- Monophasic in adults (One sleep per day)
- Changes from birth to adulthood





The Common Vole

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

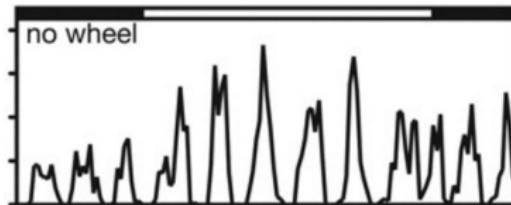
Sleep Deprivation

Ultradian Rhythms

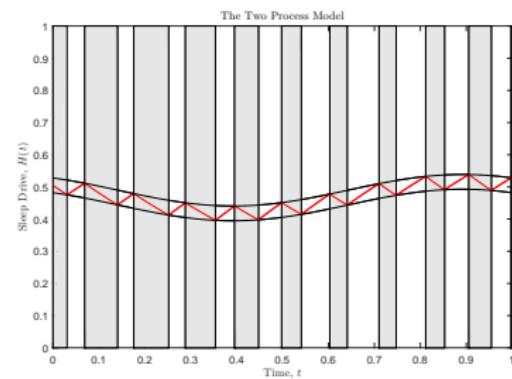
Conclusion and Future Work



- Polyphasic (Many sleeps per day)
- Consolidated sleep and wake
- More sleep during the day



Van Der Veen, Minh 2006





Sleep Deprivation

The Hidden Rhythm

Matthew Bailey

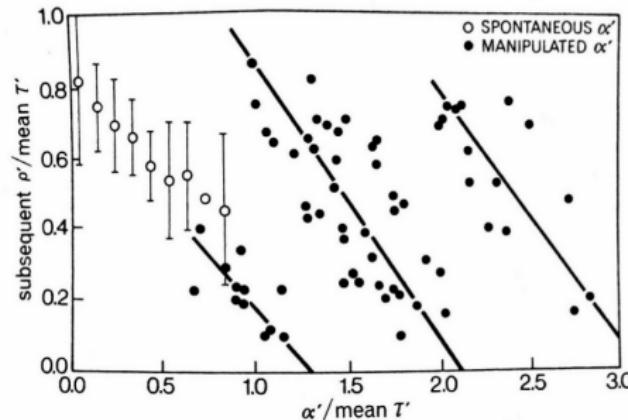
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



α' - Length of Wake

ρ' - Length of Sleep

T' - Average Sleep-Wake Period Length $(\frac{1}{8}, \frac{1}{9}, \frac{1}{10})$

Gerkhema 1991



Sleep Deprivation - Two Process Model

The Hidden Rhythm

Matthew Bailey

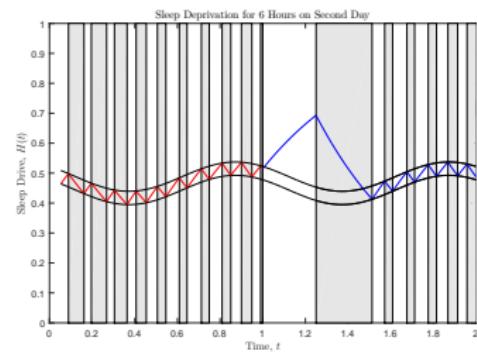
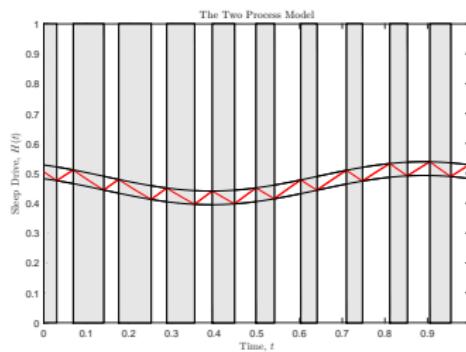
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



How does the model predict extended deprivation?



Sleep Deprivation

The Hidden Rhythm

Matthew Bailey

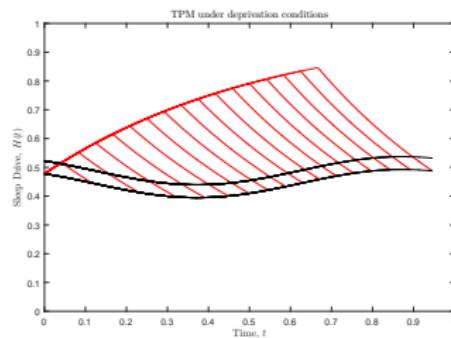
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work





Sleep Deprivation

The Hidden Rhythm

Matthew Bailey

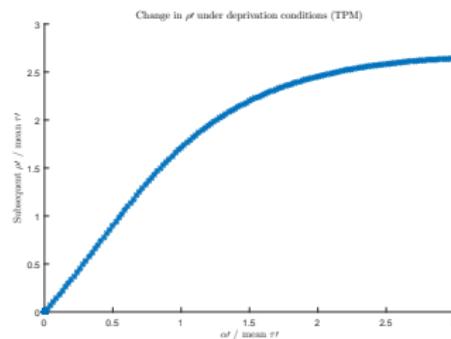
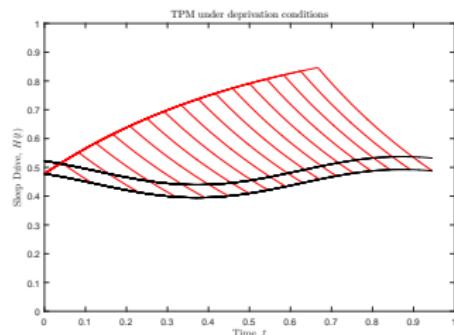
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work





Sleep Deprivation

The Hidden Rhythm

Matthew Bailey

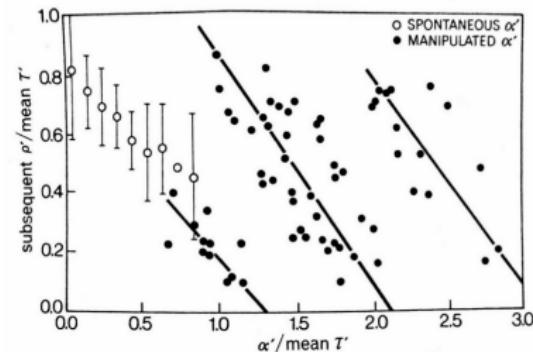
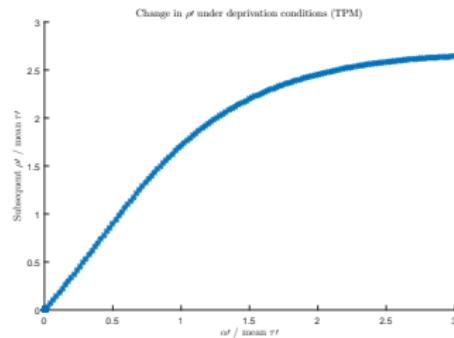
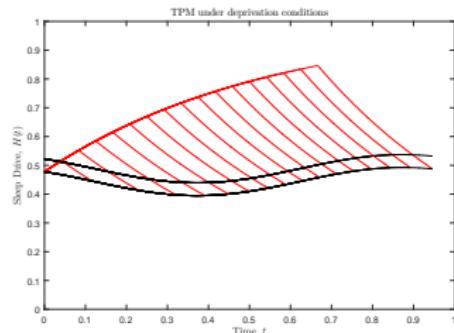
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



- Jump at each τ_t period
- Decreasing sleep length
- Max one τ_t sleep length



Ultradian Rhythmicity

The Hidden
Rhythm

Matthew Bailey

The Two Process
Model

Mammalian Sleep

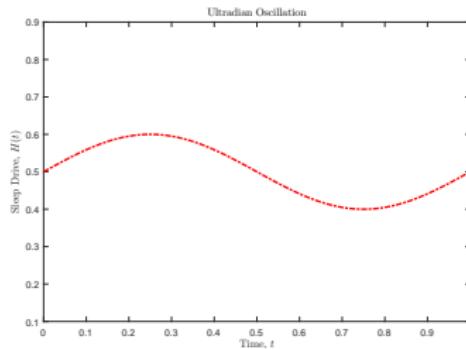
Sleep Deprivation

Ultradian
Rhythms

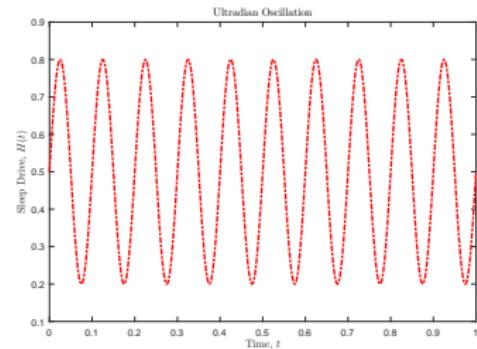
Conclusion and
Future Work

A recurrent period or cycle repeated throughout a 24-hour circadian day.

$$H^\pm(t) = H_0^\pm + a \sin(2\pi t) + a_u \sin\left(\frac{2\pi}{\tau'} t\right)$$



+





Ultradian Rhythmicity

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

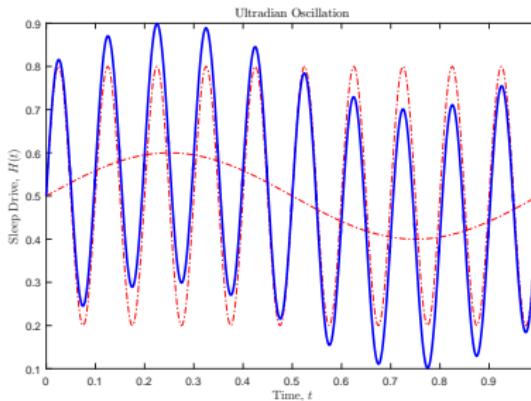
Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

A recurrent period or cycle repeated throughout a 24-hour circadian day.

$$H^\pm(t) = H_0^\pm + a \sin(2\pi t) + a_u \sin\left(\frac{2\pi}{\tau'} t\right)$$





An Ultradian Solution

The Hidden Rhythm

Matthew Bailey

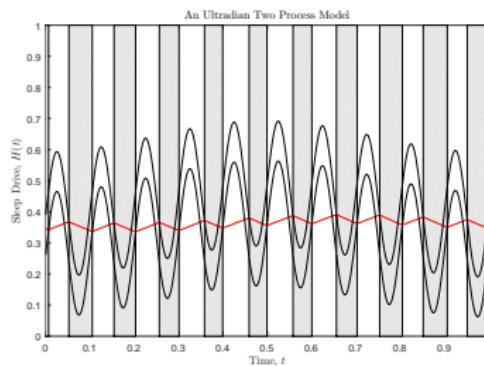
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



- Periodic
- Hidden Ultradian



An Ultradian Solution

The Hidden Rhythm

Matthew Bailey

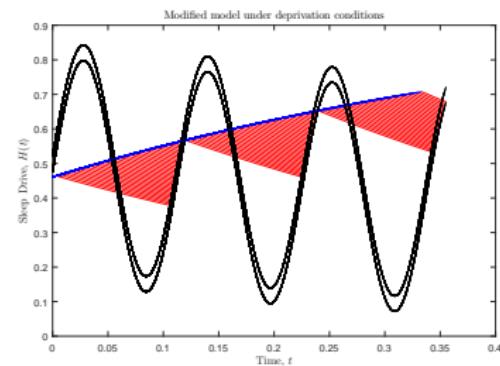
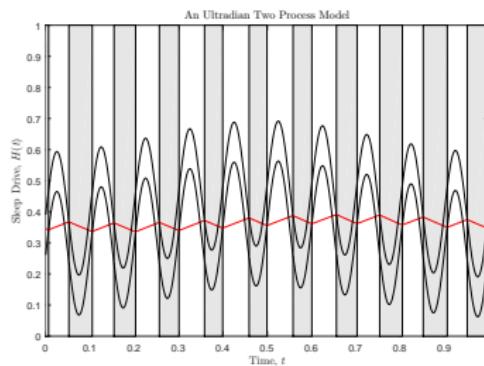
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work



- Periodic
- Hidden Ultradian
- a_u creates a barrier
- Jumps at $\sim \tau'$



An Ultradian Solution

The Hidden Rhythm

Matthew Bailey

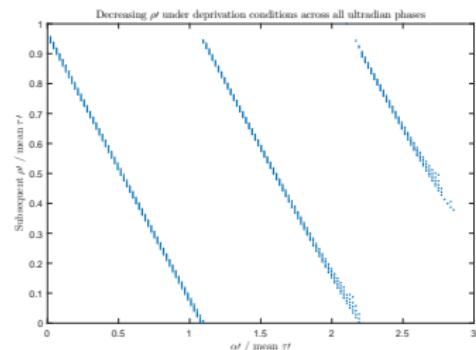
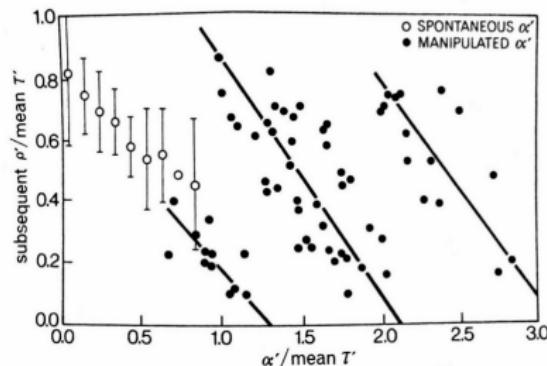
The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work





Conclusions

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

- The Two Process Model displays a wide range of dynamics
- Can describe both monophasic and polyphasic sleep - wake patterns
- Usual method of deprivation doesn't hold
- Only through desynchrony do we see the 'hidden' ultradian oscillation



Future Work

The Hidden Rhythm

Matthew Bailey

The Two Process Model

Mammalian Sleep

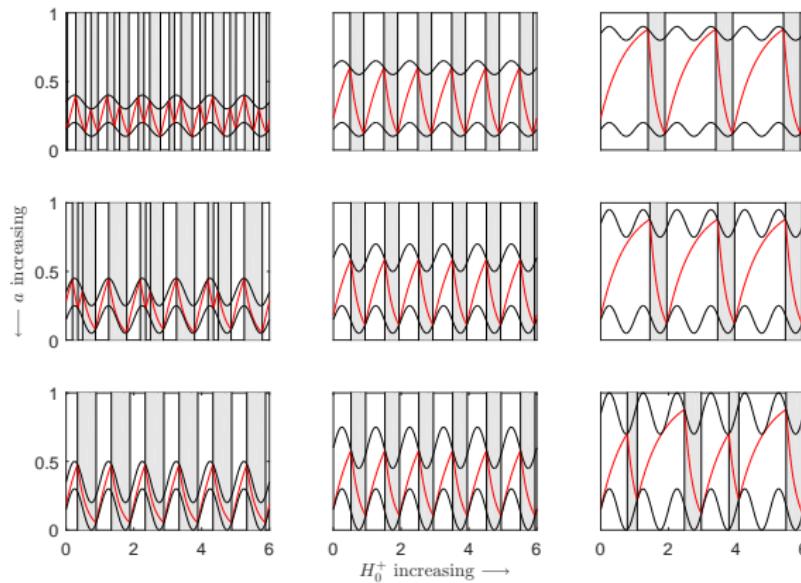
Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

Transitions between monophasic and polyphasic sleep

Model Behaviour as Parameters Vary





Future Work

The Hidden Rhythm

Matthew Bailey

The Two Process Model

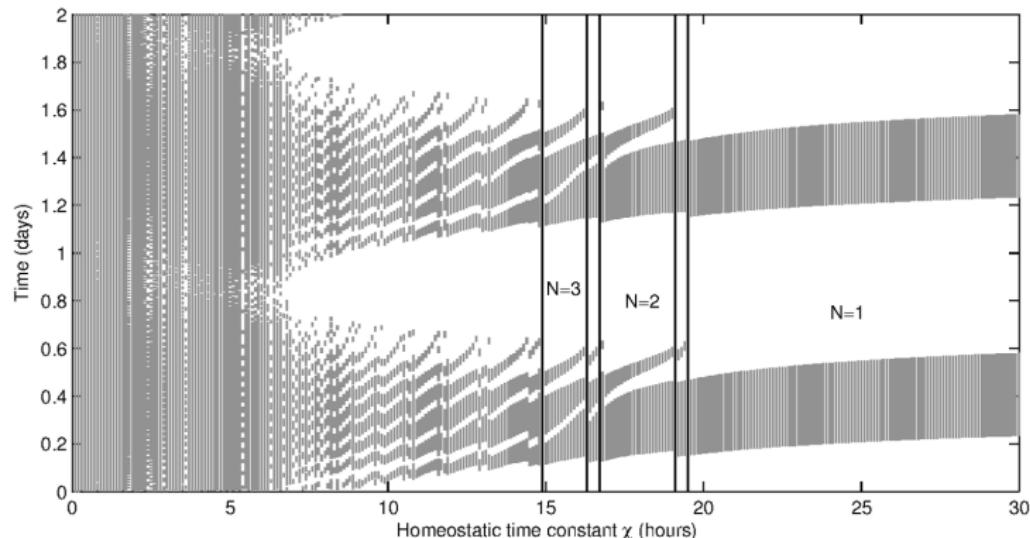
Mammalian Sleep

Sleep Deprivation

Ultradian Rhythms

Conclusion and Future Work

Transitions between monophasic and polyphasic sleep





Acknowledgments

The Hidden
Rhythm

Matthew Bailey

The Two Process
Model

Mammalian Sleep

Sleep Deprivation

Ultradian
Rhythms

Conclusion and
Future Work

Thanks for listening!

Supervisors:

Dr Anne Skeldon

Dr Gianne Derkx

Collaborators:

Dr Daan Van Der Veen

Andreas Psomas (PhD)



UNIVERSITY OF
SURREY

EPSRC

Pioneering research
and skills