Jamming Statistics-Dependent Frequency Hopping

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***Abstract*—This paper studies a jamming statistics-dependent frequency-hopping (FH) pattern generation scheme. Most existing FH patterns are determined by two encryption keys: (a) one for FH in the frequency domain, and (b) the other for time permutation in the time domain. These keys are independent of channel conditions. Hence, an FH signal generated by these two keys occupies the entire spectrum in both frequency and time domains, and the probability of detection by a jammer is low. However, the probability of a hit (or jamming) on the desired user's frequency channels by partial-band noise jamming (PBNJ) can be high because it is inversely proportional to the total number of available frequency positions. Can an FH system with jamming-dependent adaptive FH patterns safeguard the communications systems more effectively? If the answer is yes, then how can an efficient jamming-dependent adaptive FH pattern be found, and can it be implemented cost-effectively for future communications systems against jamming? The aim of this paper is to study answers to the questions posed here.**

***Index Terms*—Frequency hopping, partial-band tone jamming, typical sequence, jamming, multiple-access interference.**

I. INTRODUCTION

THE future protected spread spectrum waveform employs a frequency-hopping (FH) M-ary phase-shift keying (MPSK) scheme in order to provide resistance to jamming and detection by enhancing the existing digital video broadcasting-return channel via satellite (DVB-RCS) and the second-generation DVB satellite (DVB-S2) [1] – [6]. A hopping keystream coming from an end cryptographic unit (ECU) in a terminal determines its FH pattern [1] – [4]. Also, time permutation within a data frame is performed by another key stream different from the frequency-hopping key stream. This is done to ensure that the entire hopping spectrum is uniformly occupied over a large number of hops and to make it difficult for a jammer or an eavesdropper to detect the transmitted information.

Both frequency-hopping and time-permutation keys can reduce the probability of detection by malicious jammers or eavesdropper; however, they cannot reduce the probability of hits in frequency by a jammer. This is because the FH pattern generated by the existing methods is random (i.e., an FH position is uniformly distributed over the entire spectrum) and independent of channel conditions. The probability of a random frequency-hopping position hit by a single-fixed tone jammer is

$$Pr(\text{A random tone hit by a single tone jammer}) = \frac{1}{N_f} \quad (1)$$

where N_f is the total number of frequency-hopping positions in the entire spectrum.

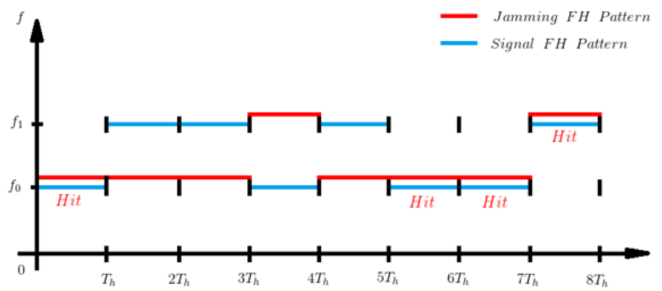

Fig. 1. Example of FH/MPSK patterns with $N_f = 2$ frequency channels, $N_h = 8$ time hops per frame, and $N_s = 1$ symbol per hop.

For illustration, Fig. 1 shows an example of FH/MPSK patterns with $N_f = 2$ frequency channels, $N_h = 8$ time hops per frame, and $N_s = 1$ symbol per hop. Practical parameters are different from these. The blue and red colors show the FH patterns of the desired user and a single-tone jammer, respectively. The frequency channels 0 and 1 are hit by the jammer three times and once, respectively. Hence, Fig. 1 shows that half of $N_h = 8$ time hops are jammed. In addition, a jammer with sufficient jamming power and bandwidth can employ multiple tones instead of a single tone. This jamming is called partial-band tone jamming (PBNJ) [7], which can increase the probability of a hit. Once a signal tone is hit by PBNJ, the transmitted symbol is likely in error, and the bit error rate (BER) would be about $\frac{1}{N_f \cdot \log_2 M}$, where M represents MPSK modulation with Gray encoding. For example, the BER can be 5.2×10^{-3} when $N_f = 64$ and $M = 8$. This is an uncoded BER but still high. Here, for a worst-case scenario, we assume that a jamming tone is centered at the signal tone whenever the signal tone is jammed. Even if the jamming tone is away from the center but within the signal bandwidth, the jamming impact is significant. Therefore, the current strategy of random hopping is not the best option. This is the main motivation of this paper.

An interesting question involving this problem is the following: Can an FH system with channel-dependent adaptive FH patterns safeguard FH systems such as future military satellite communications (MILSATCOM) and other future FH systems more effectively, i.e., yield a lower BER or a lower frame error

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rate (FER) under jamming and MAI than the FH system with channel-independent non-adaptive FH patterns? The answer is yes. Then, can an efficient channel-dependent adaptive FH pattern be found, and can it be implemented cost-effectively for future FH systems against jamming and MAI? The goal of this paper is to study answers to the questions posed here.

Notations: $E[X]$ denotes the expectation of a random variable X; bold lowercase, e.g., $\boldsymbol{x}$, denotes a vector; $\boldsymbol{x}^T$ denotes the transpose of vector $\boldsymbol{x}$; $\|\boldsymbol{x}\|$ denotes the norm of vector $\boldsymbol{x}$; $|x|$ denotes the magnitude of a complex number x; $\lfloor x \rfloor$ denotes the largest integer smaller than or equal to x; $\langle \boldsymbol{x}, \boldsymbol{y} \rangle$ denotes the inner product between two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, and $\begin{pmatrix} n \\ m \end{pmatrix} \triangleq \frac{n!}{m!(n-m)!}$ means "n choose m."

II. System Model

The aim of this paper is to study an FH pattern that requires low computational complexity to generate and simultaneously maximizes the channel capacity (i.e., maximum transmission data rate with arbitrary small BER) and minimizes the SER (or BER) of an FH MPSK under PBNJ and Rician fading environments. This section describes the system model.

In this paper, we adopt the discrete memoryless channel (DMC) model because a pair of interleavers and deinterleavers is employed, and we assume one symbol (of multiple bits) transmission per hop. The results are still applicable for multiple symbols per hop because each transmitted and received symbol is independent of each other due to a memoryless channel. Also, the DMC model assumes a hard decision value at the demodulator, where each received symbol signal is demodulated individually for the decoder instead of using the received soft value. For numerical results, we assume that an MPSK symbol is transmitted at an FH tone frequency f_i through a Rician fading channel with Rician factor K under AWGN and PBNJ environments. Other modulations such as M-ary quadrature amplitude modulation (MQAM) and other fading such as Rayleigh and Nakagami can also be included. Other jamming such as partial-band noise jamming can be also considered, but PBNJ is taken to observe performance under a worse jamming environment. Furthermore, it is assumed that the receiver measures the SNR at every hop and compares it with the required SNR threshold Th. If the received SNR over a hop is smaller than Th, then it is assumed that the signal during the hop is either jammed by PBNJ or interfered with by friendly MA users. The receiver counts the number of jammed hops at each FH tone frequency f_i, $i = 0, \cdots, N_f - 1$, and calculates the ratio of the number of jammed hops over the total number of hops at the end of a certain period, say an epoch or a data frame f_i, of $N_h = 320$ hops (sufficient to represent the jammed probability). Let p_i^J denote the ratio at the FH tone frequency $i = 0, \cdots, N_f - 1$, and the probability be represented as

$$\Pr(tone\ f_i\ is\ jammed\ by\ a\ PBNJ) = p_i^J = \frac{Number\ of\ hops\ jammed\ at\ tone\ f_i}{Total\ number\ of\ hops\ in\ a\ data\ frame}. \tag{2}$$

Let

$$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right) \tag{3}$$

denote the jamming probability vector, and assume it is available at the receiver and used for determining the signal FH pattern for the next frame transmission.

III. Optimum Signal FH Probability Vector

This section presents the optimum signal FH probability vector $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T$ that maximizes the channel capacity and minimizes the SER of the FH/MPSK for a given $\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T$.

Theorem 1: The optimum signal FH probability vector $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T$ that maximizes the channel capacity for a given $\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T$ is

$$\boldsymbol{p}_{opt}^S = \arg min_{\ \boldsymbol{p}^S} \left\langle \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle \tag{4}$$

under probability constraints $p_i^S \geq 0$, $i = 0, \cdots, N_f - 1$, and $\left\langle \mathbf{1}, \boldsymbol{p}^S \right\rangle = \sum_{i=0}^{N_f-1} p_i^S = 1$, where $\mathbf{1} = (1, \cdots, 1)^T$.

Theorem 2: The optimum signal FH probability vector $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T$ that minimizes the symbol error rate for a given $\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T$ is also

$$\boldsymbol{p}_{opt}^S = \arg min_{\ \boldsymbol{p}^S} \left\langle \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle \tag{5}$$

under probability constraints $p_i^S \geq 0$, $i = 0, \cdots, N_f - 1$ and $\left\langle \mathbf{1}, \boldsymbol{p}^S \right\rangle = \sum_{i=0}^{N_f-1} p_i^S = 1$, where $\mathbf{1} = (1, \cdots, 1)^T$.

Theorem 3: An optimum signal FH probability vector $\boldsymbol{p}_{opt}^S = \arg min_{\ \boldsymbol{p}^S} \left\langle \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle$, which simultaneously maximizes the channel capacity and minimizes the symbol error rate for a given $\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T$ and satisfies the probability constraints in (4) or (5), can be found by using a single-tone frequency with probability 1. The location of the single signal tone with probability 1 is the same as the tone location of PBNJ with minimum probability. In other words, it can be written as

$$\boldsymbol{p}_{opt}^S = (0, \cdots,\ 0,\ \underset{\underset{j}{\uparrow}}{1},\ 0,\ \cdots 0)^T$$

$$j = arg\ min_{i \in \{0,1,\cdots,N_f-1\}} p_i^J\ . \tag{6}$$

If there are multiple tones in $\boldsymbol{p}^J$ with the minimum probability, then any one of them can be used for the single-tone location with probability 1.

IV. Typical Sequence-Based Practical Signal FH Probability Vector

Theorem 3 states that the optimum signal FH pattern is just a single tone (i.e., no FH) located at the tone frequency used by PBNJ with a minimum probability. If a transmitter employs this optimum strategy, then both the probability of hits (or jamming) and the SER can be minimized. However, the jammer can detect with high probability which single tone the transmitter is using. This is not desirable in practice because the signal is exposed to the jammer or an eavesdropper with a high probability of detection. Hence, another FH pattern method that can achieve both a low probability of hit and a low probability of detection must be sought. In this section, we propose to use a typical sequence-based FH pattern method.

Prior to discussion of the proposed method, Table I lists the probability of hits and the probability of detection by the jammer for the three FH pattern design methods: (a) existing random FH pattern method used by the random FH system in [1] – [4]; (b) proposed typical sequence-based FH pattern method; and (c) arbitrary FH pattern method. The total number of tones is $N_f = 100$, and the PBNJ jams two tones, say f_3 and f_5, out of 100. Assume that PBNJ can sense which tones have not been used by the transmitter and thus avoids those tones for its detection.

The random FH method transmits a tone signal that is hopping around all 100 tones uniformly and independently of jamming statistics. The probability that PBNJ can detect the presence of a signal or not at a given tone during a hop duration is $\frac{1}{100}$ for the random FH method. But the probability of hits by PBNJ is $\frac{2}{100}$ for the random FH method. On the other hand, the proposed typical sequence-based FH pattern randomly uses one tone out of 98 unjammed tones. Hence, the probability of detection is $\frac{1}{98}$, which is slightly higher than that of the random frequency-hopping method, but the probability of hits is 0 for the proposed method. An arbitrary FH pattern method randomly uses one tone out of five unjammed tones. Hence, the probability of detection is $\frac{1}{5}$, and the probability of hits is zero for the arbitrary FH pattern method. Therefore, the proposed method can achieve both zero jamming probability and a low probability of detection simultaneously. Zero-jamming probability is desirable in the SER sense because if a signal is jammed, then the transmitted symbol will be likely demodulated into another symbol.

Table II lists the proposed typical sequence-based frequency-hopping pattern generation steps. The signal frequency-hopping probability vector $\boldsymbol{p}^S$ is denoted by $\boldsymbol{p}^S_{uniform}$ and $\boldsymbol{p}^S_{inverse}$ for core typical or typical sequence-based FH pattern generations, respectively. The ith component $p^S_{i,\ inverse}$ of $\boldsymbol{p}^S_{inverse}$ represents the ratio (or probability) of the number of hops that the channel frequency f_i is used by the transmitter over the total number of hops in a data frame. The $\boldsymbol{p}^S_{inverse}$ is obtained by taking the inverse method of jamming frequency vector $\boldsymbol{p}^J$, which is explained in Step 3'.

For typical sequence tests in Steps 2 and 3, the definition of a typical sequence is necessary.

TABLE I
Probability of hits and probability of detection by PBNJ for three methods.

FH Pattern Method	$Pr(Hits\ by\ PBNJ)$	$Pr(Detect\ by\ PBNJ)$
Random FH Using 100 Tones Independently of Jamming Conditions	$\frac{2}{100}$	$\frac{1}{100}$
Proposed Typical Sequence-Based FH Pattern Using 98 Unjammed Tones	0	$\frac{1}{98}$
Arbitrary FH Pattern Using 5 Unjammed Tones	0	$\frac{1}{5}$

Definition 1 (*Typical Sequence*) [7, p. 59]: The typical set $A_\epsilon^{(n)}$ with respect to the probability mass function $p(x)$ is the set of sequences $(x_1, \cdots, x_n) \in \chi^n$ having the property

$$2^{-n(H(X)+\epsilon)} \le p(x_1, \cdots, x_n) \le 2^{-n(H(X)-\epsilon)} \qquad (7)$$

where ϵ is a positive small number, χ is the alphabet of random variable X, and $H(X)$ is the entropy of X:

$$H(X) = -\sum_{x\in\chi} p(x)\, log_2\ p(x)\, \text{bits}. \qquad (8)$$

Note 1: Here, the random variable X represents the frequency channel used by the FH system at the current time hop. Also, the alphabet of X is $\chi = \{f_0, \cdots, f_{N_f-1}\}$. The sequence length is $n = N_h$ number of hops in a data frame, which is a sufficiently large number. *Why does this paper consider typical sequences for the FH pattern sequence generations?* The answer is because the typical sequence with the empirical entropy $(-\frac{1}{n} log_2 Pr(x^n))$ is ϵ-close to the true entropy $H(X)$. Entropy means the uncertainty, and it is desirable to enhance the uncertainty of the FH pattern so that a jammer may have a low probability of detection. It would be interesting future work to explore different detection schemes with typical, core, and non-typical hopping sequences.

Note 2: The number of typical sequences is approximately equal to $2^{nH(X)}$ and is bounded between $2^{n(H(X)-\epsilon)}$ and $2^{n(H(X)+\epsilon)}$. Hence, the number of typical sequences is exponentially growing with the FH sequence length n. It is not necessary to search all of these typical sequences for a finite number of MA users. The set of interesting typical sequences can be restricted to a set of "*core typical sequences*."

Definition 2 (*Core Typical Sequence*): Let the number n_i denote the number of frequency f_i being used in a data frame of N_h hops. If $n_i = \left\lfloor p_i^S N_h \right\rfloor$, i.e., $p_i^S \approx \frac{n_i}{N_h}$ for all $i = 0, 1, \cdots, N_f - 1$ and $\sum_{i=0}^{N_f-1} n_i \approx N_h$, then the typical sequence is called a *core typical sequence*.

Theorem 4: All *core typical sequences* have the same properties. For example, they have the same probability of being hit by PBNJ, and they are the ϵ-closet to the true

TABLE II
PROPOSED ALGORITHM FOR TYPICAL OR CORE TYPICAL SEQUENCE-BASED FREQUENCY-HOPPING PATTERN GENERATION.

Step 0	Obtain jamming frequency-hopping statistics $\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T$ for every data frame (or epoch) using SNR threshold test performed at every hop, where p_i^J denotes ratio of number of jammed hops at frequency f_i over total number of hops per frame (or epoch), $i = 0, \cdots, N_f - 1$.
Step 1	Find frequencies f_{π_j} with negligible probabilities $0 \le \boldsymbol{p}_{\pi_j}^J \le \epsilon$ for small positive ϵ. In other words, find unjammed frequencies that jammer has never used or used with very low probability. Let N_{UJ} denote number of unjammed frequencies in entire spectrum. If $N_{UJ} = 0$, then go to Step 3 or Step 3'. Else, go to Step 2 or Step 2'. Here, π_j notation denotes unjammed tone location in frequency domain, $j = 0, \cdots, N_{UJ} - 1$.
Step 2 (w/ Typical Sequence)	Compute entropy $H(X)$ and generate ***typical sequence*** using $\boldsymbol{p}_{uniform}^S = \left(p_{\pi_0}^S, \cdots, p_{\pi_{N_{UJ}-1}}^S\right)^T = \left(\frac{1}{N_{UJ}}, \cdots, \frac{1}{N_{UJ}}\right)^T$, values of which are in range between f_{π_0} and $f_{\pi_{N_{UJ}-1}}$ with equal probabilities: $p_{\pi_j}^S = \frac{1}{N_{UJ}}$. Here, X denotes random variable of FH tone frequency being used, and $j = 0, \cdots, N_{UJ} - 1$.
Step 2' (Alternate: w/ Core Typical Sequence)	Generate ***core typical sequence*** using $\boldsymbol{p}_{uniform}^S = \left(p_{\pi_0}^S, \cdots, p_{\pi_{N_{UJ}-1}}^S\right)^T = \left(\frac{1}{N_{UJ}}, \cdots, \frac{1}{N_{UJ}}\right)^T$. In other words, use tone frequency f_{π_j} with $p_{\pi_j}^S \times N_{frame} = \frac{N_{frame}}{N_{UJ}}$ number of times during frame, where N_{frame} denotes number of hops per frame. Use uniform number generator or hopping keystream coming from end cryptographic unit in the random signal FH system to find time-hop location of f_{π_j} in frame for $j = 0, \cdots, N_{UJ} - 1$. $\boldsymbol{Stop}$.
Step 3 (w/ Typical Sequence)	Compute inverse probabilities, and use them as signal FH probabilities $p_{i,inverse}^S = \frac{\frac{1}{p_i^J}}{\sum_{j=0}^{N_f-1}\frac{1}{p_j^J}}$, for $i = 0, \cdots, N_f - 1$. Then, compute entropy $H(X)$ of X using $\boldsymbol{p}_{inverse}^S$, and generate ***typical sequence*** with values in range between f_0 and $f_{N_f} - 1$ with probabilities $p_{i,inverse}^S$, $i = 0, \cdots, N_f - 1$. $\boldsymbol{Stop}$.
Step 3' (Alternate: w/ Core Typical Sequence)	Compute inverse probabilities, and use them as signal FH probabilities $p_{i,inverse}^S = \frac{\frac{1}{p_i^J}}{\sum_{j=0}^{N_f-1}\frac{1}{p_j^J}}$, for $i = 0, \cdots, N_f - 1$. Then, generate ***core typical sequence*** using $\boldsymbol{p}_{inverse}^S$. In other words, use tone frequency f_i with $p_{i,inverse}^S \cdot N_{frame}$ number of times during frame. Use uniform number generator or hopping keystream coming from end cryptographic unit in random signal FH system to find time hop location of f_i for $i = 0, \cdots, N_f - 1$. $\boldsymbol{Stop}$.
Comment 1	For multiple FH pattern generations for N_u number of MA users, Hamming distances between generated FH patterns for user k and user l should be as maximum as possible in order to minimize or avoid MAI, $k, l = 1, \cdots, N_u$.

entropy $H(X)$, i.e., $|-\frac{1}{N_h}log_2 Pr\left(x^{N_h}\right) - H(X)|$ is the smallest among all typical sequences X of length N_h for a given probability mass function of $p(x)$ of X. When $n_i = \left\lfloor p_i^S N_h \right\rfloor$ is ordered with the largest, the number of core typical sequences $N_{core\ typical\ seq}$ is equal to

$$N_{core\ typical\ seq} = \binom{N_h}{n_0}\binom{N_h - n_0}{n_1}\cdots\binom{N_h - n_0 - n_1 - \cdots - n_{N_f-2}}{n_{N_f-1}}. \tag{9}$$

Recommendation: Obtain an empirical PBNJ FH probability mass vector $\boldsymbol{p}^J$ that can be available in practice with low complexity. Determine the probability mass vector $\boldsymbol{p}^S$ of signal frequency-hopping random variable X using the algorithm in Table II. Use *core typical sequences*. Determine the hop-time locations of frequency f_i in a *core typical sequence* using a uniform random variable or a hopping keystream coming from an end cryptographic unit in a terminal [1] – [4].

There exist many core typical sequences that can be employed by a friendly user as indicated by (9) in Theorem 4. A further interesting question is this: How many multiple-access users (MAUs) can be supported simultaneously with no multiple-access interference during a hop interval when all MAUs want to generate their FH patterns using the same signal FH vector $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T$? Theorem 5 answers this question.

Theorem 5: When all MAUs generate their FH patterns using the same signal FH vector $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T$, the maximum number of multiple-access users that can be supported by the FH system with no MAI (i.e., no hit) is

$$N_{U,\max with\ no\ MAI} = \left\lfloor \frac{1}{\max_i\left\{p_i^S\right\}} \right\rfloor \tag{10}$$

where $i = 0,\ \cdots,\ N_f - 1$.

For example if you have 100 frequency slots and 100 hops in time per frame as well as having a single jammer whose tone jamming bandwidth slighty exceeds the bandwidth of your frequeny hopping slots the jammer will at most partially jam 3 frequency slots. Computing the probability signal vector using the inverse probability rule and applying (10) the system can support 97 simultaneous users. The random hopping and random jamming case will be able to support 100 simultaneous users however 3 of those users will experience degraded channels and increased BERs leading to potentially nonsuccessful transmission of data.

V. Numerical Results

Table III lists parameters used for the simulation results shown in Figure 2.

Figure 2 shows simulation results for five exemplary cases listed in Table III. Case A indicates results for random jamming and the random signal FH pattern that was used in the random FH system [1] - [4]. Case B shows results for nonuniform random jamming and the uniform random

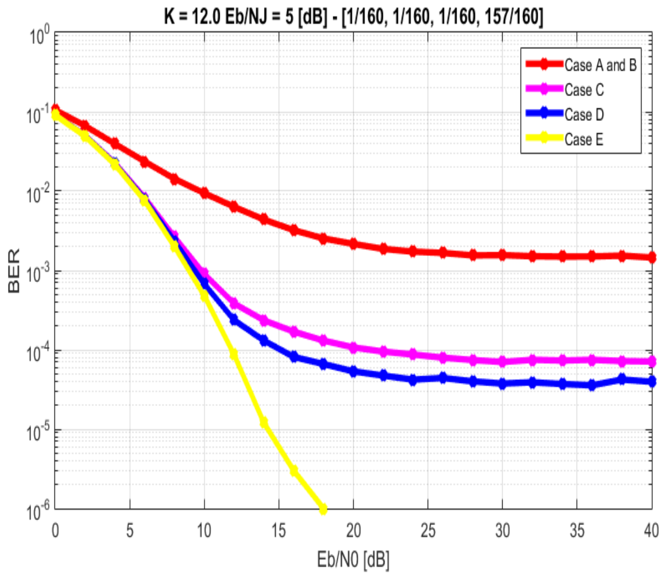

Fig. 2. Simulation BER versus $\frac{E_b}{N_0}$ in dB for five exemplary FH pattern cases when Rician factor $K = 12$, $\frac{E_b}{N_J} = 5\ dB$, $\boldsymbol{p}^J = (\frac{1}{160}, \frac{1}{160}, \frac{1}{160}, \frac{157}{160})^T$, $N_f = 4$ frequency channels, and $N_h = 160$ hops per frame. Hopping pattern is repeated with $N_h = 160$ hop period.

signal FH. Both Cases A and B show almost the same results because both employ a random signal FH pattern. The results for Case B are buried in those of Case A. The slight discrepancy is due to the Rician fading effects on different FH frequencies used.

For example, if there is no fading, then we can compute the BER floor under PBNJ and AWGN assuming a high SNR like $\frac{E_b}{N_0} = 30\ dB$ as

$$P_b\left(E\right)|_{No\ Fading} = p_{cr}^J \left\langle \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle + p_{cr}^{UJ} \left\langle \boldsymbol{1} - \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle$$

$$= Q\left(\sqrt{\frac{2}{\frac{1}{\frac{E_b}{N_0}} + \frac{1}{\beta}\frac{1}{\frac{E_b}{N_J}}}}\right)\Bigg|_{\beta=1} \left\langle \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle + Q\left(\sqrt{2\frac{E_b}{N_0}}\right)\left\langle \boldsymbol{1} - \boldsymbol{p}^J, \boldsymbol{p}^S \right\rangle$$

$$= Q\left(2.397\right)\left(\frac{1}{160}\cdot\frac{1}{4} + \frac{1}{160}\cdot\frac{1}{4} + \frac{1}{160}\cdot\frac{1}{4} + \frac{157}{160}\cdot\frac{1}{4}\right)$$

$$+\ Q\left(44.7\right)\left(\frac{159}{160}\cdot\frac{1}{4} + \frac{159}{160}\cdot\frac{1}{4} + \frac{159}{160}\cdot\frac{1}{4} + \frac{3}{160}\cdot\frac{1}{4}\right)$$

$$= Q\left(2.397\right)\frac{1}{4} + Q\left(44.7\right)\frac{474}{640} \cong 0.002 \qquad (11)$$

where $Q\left(\alpha\right) = \int_\alpha^\infty \frac{1}{\sqrt{2\pi}} \cdot \exp(\frac{-t^2}{2})$ is the tail probability of the normal Gaussian random variable, p_{cr}^J and p_{cr}^{UJ} are the crossover probabilities for BPSK under the DMC for both jammed and unjammed cases, and β denotes the PBNJ fraction ratio. Used here is an assumption that the Rician fading channel of a strong line-of-sight component such as $K = 12$ can be modeled as an AWGN channel for a given jamming state. The reason we used such a high value for K was to simplify validation computation. Simulation results show that $P_b\left(E\right) = 0.0016$, whereas the calculation $P_b\left(E\right) \cong 0.002$,

TABLE III
PARAMETERS USED FOR EXAMPLE SIMULATIONS

Modulation	BPSK
Fading	Rician Fading with Rician Factor $K = 12$
Jamming (with Typical Sequence)	PBNJ
$\frac{E_b}{N_J}$	$5\ dB$
Number of Hops per Frame	$N_h = 160$
Number of Frequencies in Entire Spectrum	$N_f = 4$
Number of Users	$N_U = 1$
Case A (random jamming and random signal FH): Jamming probability vector in spectrum $\boldsymbol{p}^J$ and signal FH probability vector in spectrum $\boldsymbol{p}^S$, and core typical sequence-based signal FH pattern in frame using $\boldsymbol{p}^S$.	$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})^T$ $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})^T$ *Core Typical Sequence-Based Signal FH Pattern* $= (f_0, f_0, f_1, f_1, f_2, f_2, f_3, f_3)$
Case B (random signal FH): Jamming probability vector in spectrum $\boldsymbol{p}^J$ and signal FH probability vector in spectrum $\boldsymbol{p}^S$, and core typical sequence-based signal FH pattern in frame using $\boldsymbol{p}^S$.	$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T = \left(\frac{1}{160}, \frac{1}{160}, \frac{1}{160}, \frac{157}{160}\right)^T$ $p^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})^T$ *Core Typical Sequence-Based Signal FH Pattern* $= (f_0, f_0, f_1, f_1, f_2, f_2, f_3, f_3)$ twenty times repeated
Case C (inverse metric signal FH): Jamming probability vector in spectrum $\boldsymbol{p}^J$ and inverse metric signal FH probability vector in spectrum $\boldsymbol{p}_{inverse}^S$, and core typical sequence-based signal FH pattern in frame using $\boldsymbol{p}_{inverse}^S$.	$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T = \left(\frac{1}{160}, \frac{1}{160}, \frac{1}{160}, \frac{157}{160}\right)^T$ $\boldsymbol{p}_{inverse}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T = \left(\frac{157}{472}, \frac{157}{472}, \frac{157}{472}, \frac{1}{472}\right)^T$ *Core Typical Sequence-Based Signal FH Pattern* $= (f_0, f_0, f_0, f_1, f_1, f_2, f_2, f_2, f_3)$ twenty times repeated
Case D (optimum signal FH): Jamming probability vector in spectrum $\boldsymbol{p}^J$ and optimum signal FH probability vector in spectrum $\boldsymbol{p}_{opt}^S$, and core typical sequence-based signal FH pattern in frame using $\boldsymbol{p}_{opt}^S$.	$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T = \left(\frac{1}{160}, \frac{1}{160}, \frac{1}{160}, \frac{157}{160}\right)^T$ $\boldsymbol{p}_{opt}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T = (1, 0, 0, 0)^T$ *Core Typical Sequence-Based Signal FH Pattern* $= (f_0, f_0, f_0, f_0, f_0, f_0, f_0, f_0)$ twenty times repeated
Case E (PBNJ with $\beta = \frac{1}{2}$): Jamming probability vector in spectrum $\boldsymbol{p}^J$ and signal FH probability vector in spectrum $\boldsymbol{p}^S$, and core typical sequence-based signal FH pattern in frame using $\boldsymbol{p}^S$.	$\boldsymbol{p}^J = \left(p_0^J, \cdots, p_{N_f-1}^J\right)^T = (\frac{1}{2}, \frac{1}{2}, 0, 0)^T$ $\boldsymbol{p}^S = \left(p_0^S, \cdots, p_{N_f-1}^S\right)^T = (0, 0, \frac{1}{2}, \frac{1}{2})^T$ *Core Typical Sequence-Based Signal FH Pattern* $= (f_2, f_2, f_2, f_2, f_3, f_3, f_3, f_3)$ twenty times repeated

which is very close to the calculation for high $\frac{E_b}{N_0}$.

Case C indicates results for the nonuniform random jamming used in Case B and the proposed inverse metric-based signal FH pattern with a core typical sequence method. Simulation results show $P_b(E) = 0.00006$, whereas the calculation $P_b(E) \cong 0.0000665$, which is very close to the calculation for high $\frac{E_b}{N_0}$. This discrepancy is due to Rician fading effects on different FH frequency probabilities, and the quantized value $n_i = \left\lfloor p_i^S N_h \right\rfloor$ is used in the simulation, whereas the true number $n_i = p_i^S N_h$ is used in the calculation. Both simulation and calculation results show that the proposed signal FH pattern shows much better performance than the uniform random signal FH shown in Case B.

Case D shows results for the nonuniform random jamming used in Cases B and C, and the optimum signal FH pattern for which a jammer can have a high probability of detection. Simulation results show $P_b(E) = 0.00004$, whereas the calculation $P_b(E) \cong 0.00005$ for high E_b/N_0. Again, simulation is very close to the calculation for high $\frac{E_b}{N_0}$, the discrepancy being due to quantization and Rician fading effects on different FH frequency probabilities. The calculation result shows that the optimum signal FH pattern is the best among all cases considered except Case E (different PBNJ jamming FH probabilities from those considered in Cases A–D).

Case E shows the most significant gain of the proposed method over th e random signal FH pattern in the random FH system [1] – [4]. Here, PBNJ with $\beta = 1/2$ is assumed. Simulation results show the BER of the proposed FH pattern as being superior to any other signal FH patterns, including the optimum FH pattern. For example, the BER $P_b(E) = 10^{-5}$ at $\frac{E_b}{N_0} = 14\ dB$, whereas all other cases cannot reach $P_b(E) = 10^{-5}$, even if $\frac{E_b}{N_0} = \infty$. This is because the signal FH pattern does not use the jammed frequencies. The BER is independent of the jamming fraction and is dependent on the AWGN and Rician fading environment. The probability of detection by the jammer can be increased. However, as discussed in Table I, the increased probability of detection can be mitigated when the number of frequency channels is high and the PBNJ can jam only a small fraction of the entire spectrum due to its insufficient jamming power, which is a typical case in practice. In conclusion, all proposed methods in Cases C, D, and E show significant dB gains in SNR at the same BER over the existing random FH pattern methods in Cases A and B.

VI. Conclusion

This paper has proposed the following method to obtain a superior future protected spread spectrum waveform: obtain an emperical PBNJ FH probability mass vector $\boldsymbol{p}^J$ with a low generation complexity; convert $\boldsymbol{p}^J$ into a signal FH probability mass vector $\boldsymbol{p}^S$ by employing the algorithm in Table II; use $\boldsymbol{p}^S$ to generate *core typical sequences*; lastely permutate the *core typical sequences* with a seed coming from a uniform random variable or a hopping keystream computed from an end crytographic unit which would output the hop-time locations needed to support multiple users. The numerical and simulated results align closely with each other showing that the proposed method in this paper is superior in both BER as well as channel capacity when compared with the random signal FH pattern method. The simultaneous number of MAUs is slightly lower and the probability of detection by a jammer is slightly higher with the proposed FH pattern generation method when compared with the random hopping random jamming pattern, but both of these inferior metrics are negligible when the number of frequency channels N_f is large and PBNJ can only affect a small fraction of the entire spectrum due to insufficent jamming power. This is a typical case in most practical scenarios so the benefits from the proposed FH generation method described offers a better performing future protected spread spectrum waveform.

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