



Temporal-Spatial Coupling in Gravitational Lensing: A Reinterpretation of Dark Matter Observations

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Abstract

Standard gravitational lensing analysis relies on the Isochrony Axiom—the implicit assumption that the observed image represents a synchronous spatial snapshot of the source. We demonstrate that for evolving sources, this approximation breaks down in the presence of conformal metric couplings, creating a "temporal composite" image. This projects temporal depth onto the spatial plane, generating a Temporal Jacobian contribution that is mathematically indistinguishable from gravitational shear—a phenomenon defined here as Phantom Mass. It is demonstrated that GW170817 does not constrain the conformal component of this coupling; because photons and gravitational waves traverse the same path, conformal time dilation is common-mode and cancels in differential measurements. The widely cited speed-of-gravity constraints apply only to the disformal sector; leaving the conformal "rate of time" unconstrained to explain dark matter phenomenology. The dark sector is thus reinterpreted not as an invisible substance, but as the shadow of temporal transport.

Keywords: gravitational lensing – dark matter – modified gravity – cosmology: theory – galaxies: kinematics and dynamics – temporal equivalence principle

1. Introduction

1.1 The Anomaly of the Dark Sector

The existence of dark matter, inferred from gravitational lensing (Walsh et al. 1979), cluster dynamics (Zwicky 1933), and cosmic microwave background observations (Planck Collaboration 2020), represents a significant challenge in modern physics. Despite decades of increasingly sensitive searches, no dark matter particle has been directly detected (Schumann 2019), and tensions persist between cosmological observations at different scales (Riess et al. 2022; Di Valentino et al. 2021). The prevailing paradigm assumes

that these anomalies indicate the presence of an invisible *substance*. The question arises whether the apparent mass discrepancy can be resolved by relaxing the *Isochrony Axiom*—the assumption that temporal delays across an image are negligible.

Modified Newtonian Dynamics (Milgrom 1983) and its relativistic extensions (Bekenstein 2004; Skordis & Złotnik 2021) have challenged the dark matter hypothesis by modifying the gravitational force law. However, these approaches typically retain the standard metric assumptions regarding light propagation and causality. The present analysis interrogates a deeper, often unstated assumption underlying the interpretation of all astronomical signals: the nature of simultaneity in image formation.

1.2 The Isochrony Axiom

Gravitational lensing is universally treated as the spatial deflection of light rays by mass. This geometric interpretation relies on an implicit but foundational axiom:

The Isochrony Axiom: The observational approximation that the exposure time is much shorter than any differential delay across the source, such that all photons collected can be treated as originating from a single, synchronous epoch.

While this axiom serves as a necessary simplification for standard analysis, it breaks down when the differential lookback time across an image becomes comparable to the evolutionary timescale of the source. Standard analysis corrects for the finite speed of light in the arrival time of signals (lookback time) but has typically neglected the finite *variance* of that speed across the image plane (differential lookback time). In the presence of generalized metric couplings, this approximation fails. Gravitational lensing is fundamentally an *arrival-time* phenomenon. Images form at the stationary points of the Fermat potential, which encodes the total light-travel time (Blandford & Narayan 1986). In standard General Relativity, the difference in arrival times between images is attributed solely to geometric path differences and the Shapiro delay caused by mass. However, if the coupling between matter and gravity involves a second metric—specifically one that affects matter-clock rates or light-cone tilts—the arrival times of photons can be decoupled from the geometric definitions of "mass" used in standard GR.

If the Isochrony Axiom is violated, an observed "image" is not a snapshot of the source at one moment, but a *temporal composite*. Photons arriving at the same detector time may have left the source at significantly different emission times. For an evolving source, this temporal smearing is mathematically indistinguishable from a spatial distortion (convergence and shear) in a static reconstruction. What is interpreted as "dark matter" may be the projection of this temporal depth onto the spatial image plane.

1.3 The Interpretive Bifurcation

Consider two mathematically equivalent interpretations of the same Fermat potential surface, distinguished only by their treatment of simultaneity:

Interpretation A (Standard Framework): Assumes the Isochrony Axiom. An Einstein ring is analyzed with apparent convergence κ_{obs} exceeding what the visible baryonic mass can produce. A dark matter halo with mass $M_{\text{DM}} = M_{\text{obs}} - M_{\text{baryons}}$ is inferred. The "dark matter" is treated as an unseen substance required to explain the lensing geometry.

Interpretation B (TEP Framework): Rejects the Isochrony Axiom. The observed image is recognized as a *temporal composite*: photons arriving simultaneously at the detector left the source at different emission epochs, with differential delays set by the two-metric structure along each ray. For an evolving source, this temporal depth projects onto the image plane as an apparent spatial distortion. The differential proper-time accumulation across the lens is computed, and the "excess convergence" is identified as the signature of temporal-field gradients $\nabla\phi$ in the lens environment. The "dark matter" is reinterpreted not as a substance, but as the shadow of unmodeled time.

The Critical Point: Both frameworks fit the data equally well. The difference is not observational but *interpretive*—it depends on which axiom (Isochrony vs. TEP) is taken as fundamental. The two frameworks become distinguishable only when tested against observables that break the degeneracy: time-domain signatures in rapidly varying sources, achromatic residual timing anomalies across multiple images, and the correlation of inferred "dark" components with source evolution timescales.

This bifurcation illustrates that the existence of "dark matter" is a conclusion contingent on the Isochrony Axiom. If that axiom is false, the conclusion does not necessarily follow.

1.4 The Temporal Equivalence Principle (TEP)

The Temporal Equivalence Principle (TEP), introduced in a companion paper (Smawfield 2025a), represents a shift in fundamental perspective, replacing the standard geometric framework with an operational one:

Temporal Equivalence Principle (TEP): The operative physical observable in any non-local measurement is the accumulated matter proper time along worldlines and signal paths. Under TEP, "gravity" includes the phenomenology of differential proper-time accumulation. The decomposition of this accumulation into "spatial curvature" (mass) and "temporal dilation" (metric coupling) is gauge-dependent; only the total integrated proper time is invariant.

Under TEP, the central question is not "how much mass is bending the light?" but "what is the total proper-time history of the signal?" In the conformal-only limit of a two-metric theory, null cones are preserved, meaning the "speed of light" is unchanged, yet the *rate* of proper time accumulation varies. This creates a disconnect between the "gravitational metric" (which governs orbital mechanics) and the "matter metric" (which governs atomic clocks and photon frequencies).

1.5 Redefining the Dark Sector

This paper develops the TEP framework to show that two-metric temporal coupling can reproduce the phenomenology of dark matter in gravitational lensing without invisible mass. It is demonstrated that:

1. **The "dark" signal is a temporal artifact:** In a spatially varying conformal field, lens-local differential delays create temporal-composite images. When interpreted through standard static lens models (which assume Isochrony), these delays manifest as apparent additional convergence and shear—"phantom mass."
2. **GW170817 is a differential constraint:** The multi-messenger constraint $|c_\gamma - c_g|/c \lesssim 10^{-15}$ is explicitly reanalyzed. It is shown that this bounds only the *disformal* (cone-tilt) component of the coupling. The *conformal* component, which governs clock rates and drives the "dark matter" phenomenology, is unconstrained because photons and gravitational waves share the same null cone and thus experience common-mode dilation.
3. **The Reference Envelope vs. The Reality:** The standard translation of GW170817 timing to propagation-speed bounds is treated as a conservative **Reference Envelope** for the disformal sector. However, for the unconstrained conformal sector, it is demonstrated that the temporal-field gradients required to reproduce lensing anomalies are physically viable. This transforms the "dark matter problem" from a search for particles into a search for unmodeled temporal structure.

By abandoning the Isochrony Axiom, the dark matter problem is transformed from a search for missing particles into a search for unmodeled temporal structure. The parameter space where this structure masquerades as dark matter is defined, offering a falsifiable alternative to the particle paradigm.

The TEP thesis holds that what is conventionally called "dark matter" can be modeled as temporal structure—the accumulated effect of unmodeled proper-time variations across lensing observations. The particle interpretation serves as an effective model under the assumption of synchrony. TEP does not posit an alternative substance; it holds that the phenomenology traditionally attributed to dark matter may be better understood as a metric artifact.

2. Theoretical Framework

2.1 The Two-Metric Postulate

The gravitational interaction and the matter sector are posited to be governed by distinct metrics related by a scalar field ϕ . The *Gravitational Metric* $g_{\mu\nu}$ determines the geodesics of macroscopic test masses and the causal structure of gravitational waves. The *Matter Metric* $\tilde{g}_{\mu\nu}$ determines the behavior of standard model fields, atomic clocks, and electromagnetic propagation. The general two-metric relation (Bekenstein 1993) is adopted:

$$\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu} + B(\phi)\partial_\mu\phi\partial_\nu\phi$$

Here, $\tilde{g}_{\mu\nu}$ is the metric measured by atomic clocks (matter sector) and $g_{\mu\nu}$ is the metric governing gravitational geodesics. The function $A(\phi)$ defines the *Conformal Sector* (isotropic scaling of proper time/length) and $B(\phi)$ defines the *Disformal Sector* (anisotropic stretching along field gradients). This structure represents a modification of measurement rather than a modification of gravity. All physical rulers and clocks are built from matter coupled to $\tilde{g}_{\mu\nu}$, while the "force" of gravity is mediated by $g_{\mu\nu}$.

2.2 The Optical Theory of Gravity

A unified "Optical Theory" of dark matter is developed here, where the phenomenology arises from two distinct optical effects of the scalar field $A(\phi)$: a static refractive index and a dynamic shutter.

1. The Static Refractive Index (Geometric Lensing)

The scalar field acts as a locally variable optical factor $n_{eff} \approx \sqrt{A(\phi)}$ in the travel-time functional governing image formation. The associated excess matter proper-time delay is:

$$\Delta\tilde{\tau}_{\text{static}} = \frac{1}{c} \int (\sqrt{A(\phi)} - 1) dl$$

This *Static Halo* contributes a source-independent term to the arrival-time (Fermat) surface. In multipath configurations, it is therefore operationally degenerate with the "bulk" convergence inferred in standard reconstructions (Einstein rings, major arcs), even though in the conformal limit it preserves null cones and does not generate a differential photon–graviton speed.

2. The Dynamic Shutter (Temporal Lensing)

While the static term shifts the arrival-time surface, the gradient of the differential delay field $\nabla(\Delta\tilde{\tau})$ acts as a "shutter" that modulates the arrival time of photons from different parts of the source. For a source with evolution or motion, this creates a *Temporal Shear*:

$$\mathcal{T}_{\text{shear}} \propto \left(\frac{\partial I}{\partial t} + \vec{v}_s \cdot \nabla I \right) \nabla(\Delta\tilde{\tau})$$

This *Dynamic Shutter* explains the anomalies—the "phantom mass" that appears to fluctuate with source type. It operates as a re-ordering of wavefronts in time rather than a deflection of rays. Crucially, this effect applies even to "static" sources (like elliptical galaxies) due to their proper motion \vec{v}_s across the delay gradient. (See Section 3 for the full derivation).

Together, these two mechanisms constitute the TEP framework: the refractive index replaces the dark matter halo, and the dynamic shutter replaces the "complexity" of substructure.

2.3 Operational Axioms: The TEP Framework

The TEP framework rests on four foundational axioms. These are not approximations or perturbative corrections to General Relativity; they constitute a complete operational framework for interpreting gravitational phenomenology. They are adopted as *primary postulates* from which observational consequences are derived:

Axiom 1 (Causal Universality): In the Conformal Limit ($B = 0$), the null cones of $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$ are identical. Photons and gravitational waves follow the same null geodesics. No "speed of light" difference exists in this limit. *This axiom is exact, not approximate.*

Axiom 2 (Proper Time Primacy): The fundamental observable in any timing measurement is the accumulated matter proper time $\Delta\tilde{\tau}$ along the signal path. Coordinate time differences Δt are not observables; they are inferred quantities dependent on the metric model. *This axiom establishes proper time as the irreducible physical observable; all other timing quantities are derived.*

Axiom 3 (Time-Transport Holonomy): For any signal trajectory γ connecting a fixed emission event to an observation event, define the time-transport functional $\mathcal{T}[\gamma] \equiv \Delta\tilde{\tau}[\gamma]$, the matter proper-time delay registered by an atomic clock. For two alternative trajectories γ_1, γ_2 , the closed-loop holonomy for the loop $C = \gamma_1 \circ \gamma_2^{-1}$ is $\mathcal{H}[C] \equiv \mathcal{T}[\gamma_1] - \mathcal{T}[\gamma_2]$. This holonomy is the invariant discriminator: it is directly observable (multi-image transients, strong-lens time delays) and cannot be reduced to a coordinate "speed" parameter. In the conformal limit, $d\tilde{\tau} = \sqrt{A(\phi)} d\tau_g$ is common-mode for co-propagating messengers; disformal coupling contributes the genuinely path-dependent component that generates differential delays across an image plane. *This axiom formalizes TEP as an operational theory of time transport: the observable content lives in holonomies of $\mathcal{T}[\gamma]$, not in inferred coordinate-time constructs.*

Axiom 4 (Operational Constraint Mapping): The translation from observed arrival-time offsets to inferred propagation-speed bounds is theory-dependent. Existing multi-messenger constraints (e.g., GW170817) are interpreted within the standard framework that assumes a single metric governs all sectors. In the TEP framework, these constraints apply to specific parameter combinations (primarily the disformal sector) and do not constitute blanket exclusions of two-metric effects. *This axiom establishes that constraints derived under Isochrony-assuming frameworks must be re-derived operationally within TEP before being applied as exclusions.*

These axioms are mutually consistent and together define the TEP interpretation of gravitational phenomenology. They replace the implicit Isochrony Axiom of standard lensing analysis with an explicit dynamical-time framework.

The Hidden Closure in Standard Practice: In addition to the Isochrony Axiom, most observational inference quietly assumes that time transport is globally integrable: after correcting for known effects (Sagnac, Shapiro, troposphere, etc.), a single global time coordinate can be assigned such that closed-loop synchronization holonomy vanishes. Operationally, this is the step that licenses the non-local conversion $d = ct$ and the inference "timing residual \Rightarrow mass residual." In TEP, this closure is not assumed; it is replaced by Axiom 3, which treats the loop holonomy $\mathcal{H}[C]$ as the invariant object that can, in principle, be non-zero.

2.4 Conformal vs. Disformal Phenomenology

The distinction between the two sectors is critical for interpreting multi-messenger constraints, and it rests on a fundamental distinction between *Single-Path* and *Multipath* measurements. Furthermore, it necessitates a redefinition of the "speed of light":

- **Conformal Sector ($A(\phi)$):**
 - **Geometry:** Preserves angles and null cones. $\tilde{g}_{\mu\nu}k^\mu k^\nu = Ag_{\mu\nu}k^\mu k^\nu = 0$.
 - **Local Invariance vs. Global Variability:** While c is locally invariant (measured as 299,792,458 m/s by any local clock), the *global effective speed* is variable. Because the rate of proper time accumulation $d\tilde{\tau} = \sqrt{A(\phi)}d\tau_g$ varies with location, the time required to traverse a fixed spatial interval depends on the scalar field value. To an observer assuming a universal clock, light appears to speed up or slow down depending on the path.
 - **Single-Path Physics (GW170817):** Photons and gravitational waves from the exact same source coordinate follow the *same* null geodesic. Any temporal distortion $A(\phi)$ along this path is common-mode. The signals do not diverge because they share the same history.
 - **Multipath Physics (Lensing):** Gravitational lensing involves light rays taking *different* paths around a mass distribution. These paths traverse different regions of the scalar field $\phi(\vec{x})$. The differential time dilation between these paths creates the "dark matter" signature.
- **Disformal Sector ($B(\phi)$):**
 - **Geometry:** Tilts null cones. The effective speed of light differs from the speed of gravity: $c_\gamma \neq c_g$.
 - **Observables:** Differential arrival times between species, constrained by GW170817 to $|c_\gamma - c_g|/c \lesssim 10^{-15}$ for the path-averaged monopole.

2.5 The "Phantom Mass" Mechanism

Standard lensing reconstruction solves for a mass distribution $\Sigma(\vec{\theta})$ that reproduces the observed image distortions. This reconstruction assumes the Isochrony Axiom: $I_{obs}(\vec{\theta}) = I_{src}(\vec{\beta})$. However, in the TEP framework, the image is a temporal integral:

$$I_{obs}(\vec{\theta}) = \int I_{src}(\vec{\beta}, t - \Delta\tilde{\tau}(\vec{\theta})) dt$$

where $\Delta\tilde{\tau}(\vec{\theta})$ is the excess proper time delay along the line of sight at image position $\vec{\theta}$. For a spatially varying conformal coupling, this delay varies across the image plane. If the source has temporal variability (secular evolution, rotation, or fluctuations) on the timescale of $\nabla_\theta(\Delta\tilde{\tau})$, the resulting image is smeared.

The Equivalence: A gradient in arrival time across an image is mathematically equivalent to a shearing of the source frame. To a static observer assuming isochrony, this "temporal shear" is indistinguishable from the "gravitational shear" caused by mass. Thus, *purely temporal structure is misinterpreted as "Phantom Mass" (dark matter)*.

2.6 Two Regimes of TEP-GL

Box 1: The Two Observational Regimes

The TEP-GL framework operates in two distinct regimes, distinguished by the magnitude of the differential proper-time delay $\Delta\tilde{\tau}$ across the lens:

Regime I: The Reference Envelope

- **Assumption (Standard):** The GW170817 multi-messenger timing constraint applies to all metric sectors equally.
- **Constraint basis:** The standard translation of timing to propagation-speed bounds ($\lesssim 10^{-15}$).
- **Delay scale:** $\Delta\tilde{\tau} \sim 10^{-3}$ --1 s (milliseconds to seconds) on halo scales.
- **Primary observables:** Time-domain signatures in rapidly varying sources—lensed FRBs, GRBs.
- **Dark matter status:** TEP is a *precision systematic* in time-delay cosmography, not a wholesale DM replacement.
- **Falsification:** Null detection of achromatic timing residuals at < 0.1 ms excludes this regime.

Regime II: The Extended Regime

- **Assumption (Sector Decoupling):** GW170817 constrains only differential disformal coupling; conformal temporal structure is unconstrained (see Axiom 4).
- **Delay scale:** $\Delta\tilde{\tau} \sim 10^3$ -- 10^5 years, driven by the conformal factor $A(\phi)$ integrated along cosmological paths.
- **Primary observables:** The full phenomenology of "dark matter" in lensing—cluster arcs, cosmic shear.
- **Dark matter status:** Dark matter is modeled as a measurement artifact. The ϕ field constitutes the underlying reality.
- **Falsification:** CMB-galaxy lensing agreement at $< 1\%$; no source-variability correlation in shear.

This work demonstrates that *the Extended Regime (Regime II) is a viable physical alternative*. The Reference Envelope (Regime I) is a useful conservative baseline for calibration, but it represents a scenario where the temporal field is artificially suppressed to match constraints that do not actually apply to the conformal sector.

2.7 Why Lensing May Not Be Purely Spatial

Standard gravitational lensing analysis contains a simplifying temporal assumption that has typically been treated as exact. This assumption is made explicit, and its breakdown under TEP leads directly to the dark matter reinterpretation.

The Hidden Assumption in Image Formation

When a lensed galaxy is observed, photons are collected over an exposure time and an "image" is reconstructed. This reconstruction implicitly assumes:

1. All photons arriving during the exposure left the source at approximately the same epoch
2. Differences in arrival angles correspond to differences in spatial paths, not temporal paths
3. The reconstructed "shape" represents a spatial snapshot of the source at one moment

These assumptions constitute the *Isochrony Axiom* applied to image formation. Standard analysis corrects for the mean lookback time (distance), but assumes that the variance in lookback time across the image is negligible. This overlooked variance is defined as *Differential Lookback Time*. In a two-metric framework, this variance is not negligible; it is the dominant source of the observed distortion.

The Temporal Composite Mechanism

Consider an extended galaxy being lensed. Light from different parts of the galaxy:

- Leaves the source at different times (the source is evolving on Myr timescales)
- Takes different paths through the lens (different impact parameters)
- Accumulates time differently through regions with different $A(\phi)$ values
- Arrives at the detector at the "same" observation time

If $A(\phi)$ varies spatially—forming a halo-like configuration around the lens—then:

$$\Delta t_{\text{path}} = \int_{\text{path}} \frac{\sqrt{A(\phi)}}{c} dl$$

differs between rays at different impact parameters. For cosmological propagation distances $L \sim \text{Gpc}$ and conformal variations $\Delta A/A \sim 10^{-6} \text{--} 10^{-4}$, the differential delay is:

$$\Delta t \sim \frac{1}{2} \frac{\Delta A}{A} \cdot \frac{L}{c} \sim 10^3 \text{--} 10^5 \text{ years}$$

This is *long enough that the source has evolved*. The inner parts of the lensed image show the galaxy from an earlier epoch; the outer parts show it from a later epoch. The observation is not a spatial snapshot—it is a *temporal composite*.

Why This Creates "Phantom Mass"

For a source that evolves on timescales comparable to Δt :

- If the source was more compact in the past \rightarrow inner regions (earlier epoch) appear smaller
- If the source was less compact in the past \rightarrow inner regions appear larger
- The reconstructed "shape" is systematically distorted by temporal mixing

Standard lensing analysis interprets *any* systematic shape distortion as evidence for mass (convergence and shear). The temporal smearing is mathematically indistinguishable from gravitational lensing distortion. What has been interpreted as "dark matter" may be, in whole or in part, *temporal depth projected onto the spatial image plane*.

Why This Effect Is Achromatic

The conformal factor $A(\phi)$ rescales proper time identically for all photon frequencies. Unlike plasma dispersion (which scales as ν^{-2}) or dust extinction (wavelength-dependent), conformal temporal coupling is *perfectly achromatic*. This explains why gravitational lensing appears achromatic—not because there is gravitating mass, but because the temporal field affects all wavelengths equally.

The Central Thesis

Under TEP, the existence of "dark matter" is inferred rather than directly observed. It is a *conclusion contingent on the Isochrony Axiom*. If that axiom fails—if the universe is not temporally synchronous in the way standard analysis assumes—then the inferred "dark mass" can be modeled as unmodeled temporal structure. The dark sector is modeled not as a substance, but as the shadow of time. No claim is made regarding uniqueness of the two-metric realization, only the operational equivalence of temporal gradients to inferred mass under isochrony.

3. The Phantom Mass Mechanism

3.1 Deriving Temporal Shear: Multipath Temporal Divergence

The core insight of TEP is that standard lensing analysis mistakes *Multipath Temporal Divergence* for spatial mass. When light travels from across the universe, it passes through "time-warped regions" of space—areas where the scalar field ϕ modifies the local rate of time flow. Because an extended image is formed by rays taking *different pathways* through these distortions, the arrival times diverge.

We derive the effective shear introduced by these time delays as follows. Let the observed image position be $\vec{\theta}$. The source position $\vec{\beta}$ is a function of $\vec{\theta}$ and the emission time t_{emit} :

$$\vec{\beta} = \vec{\beta}_{geom}(\vec{\theta}) + \Delta\vec{\beta}(t_{emit})$$

where $\vec{\beta}_{geom}$ is the standard gravitational deflection mapping. The emission time is related to the observation time t_{obs} by the total delay $T(\vec{\theta})$:

$$t_{emit} = t_{obs} - T(\vec{\theta}) = t_{obs} - (T_{geom}(\vec{\theta}) + \Delta\tilde{\tau}(\vec{\theta}))$$

For a source with proper motion $\vec{\mu}_s = \partial\vec{\beta}/\partial t$, substituting the time equation yields:

$$\vec{\beta}(\vec{\theta}) \approx \vec{\beta}_{geom}(\vec{\theta}) - \vec{\mu}_s \cdot \Delta\tilde{\tau}(\vec{\theta})$$

The Jacobian matrix of the lens mapping $\mathcal{A}_{ij} = \partial\beta_i/\partial\theta_j$ becomes:

$$\mathcal{A}_{ij} = \frac{\partial\beta_{geom,i}}{\partial\theta_j} - \mu_{s,i} \frac{\partial(\Delta\tilde{\tau})}{\partial\theta_j}$$

The first term is the standard convergence/shear matrix \mathcal{A}_{GR} . The second term is the *Temporal Shear Tensor*:

$$\mathcal{T}_{ij} = -\mu_{s,i} \nabla_j(\Delta\tilde{\tau})$$

The "Static Source" Resolution: Even if a source has a constant luminosity profile $I(\vec{x}, t) = I(\vec{x})$ (zero intrinsic evolution), it is moving across the sky. The term $\vec{\mu}_s \cdot \nabla(\Delta\tilde{\tau})$ represents the source physically moving into a region of different time delay. The "variability" required for the effect is therefore not limited to intrinsic evolution; the proper motion of the source across the delay gradient suffices. Thus, even "static" galaxies are subject to Phantom Mass shearing if they possess transverse velocity relative to the lens.

This tensor introduces anisotropic distortions (shear) and isotropic magnification (convergence) that depend on the source velocity $\vec{\mu}_s$ and the delay gradient $\nabla(\Delta\tilde{\tau})$. Crucially, unlike the gravitational shear tensor which is symmetric and curl-free (derived from a scalar potential ψ), the Temporal Shear Tensor \mathcal{T}_{ij} is constructed from the outer product of a velocity vector and a gradient vector. It generally possesses a non-zero curl if $\nabla \times (\vec{\mu}_s(\nabla\Delta\tilde{\tau})) \neq 0$. This implies that TEP-induced lensing can produce image rotation—a "smoking gun" signature forbidden in standard scalar-potential lensing.

This demonstrates that for an evolving source, the "shape" observed is a mixture of spatial geometry and temporal simultaneity surfaces.

Box 3.1: A Minimal Toy Model Estimate

To demonstrate the order of magnitude, consider a simple spherical conformal halo profile:

$$A(\phi) = 1 + \epsilon \ln(r/r_0)$$

For a coupling strength $\epsilon \approx 10^{-6}$ and a characteristic scale $r_0 = 10$ kpc:

- **Differential Delay:** Across an Einstein radius ($r_E \approx 5$ kpc), the differential proper time accumulation is $\Delta\tilde{\tau} \sim \frac{\epsilon}{2}(L/c) \approx 1.5 \times 10^3$ years (for $L \sim 1$ Gpc).
- **Phantom Convergence:** The resulting effective convergence κ_{eff} mimics an NFW profile core, with magnitude scaling as $\kappa_{eff} \propto \epsilon \cdot (\partial I_s / \partial t)$.

This confirms that even minute conformal variations (10^{-6}) integrated over cosmological distances produce delays sufficient to shear evolving sources.

Box 3.2: Order of Magnitude Estimate for Static Sources

To verify that proper motion alone generates sufficient shear, we calculate the magnitude of the Temporal Shear Tensor $\mathcal{T} \approx \mu_s \nabla(\Delta\tilde{\tau})$.

- **Source Velocity:** Typical cluster transverse velocity $v_s \sim 1000$ km/s at distance $D_A \sim 1$ Gpc yields an angular proper motion $\mu_s \approx 2 \times 10^{-4}$ arcsec/year.
- **Delay Gradient:** From the toy model (Box 3.1), a delay of $\sim 10^3$ years varying over arcsecond scales gives $\nabla(\Delta\tilde{\tau}) \sim 10^3$ years/arcsec.
- **Resulting Shear:** The product is dimensionless shear:

$$\gamma_{eff} \approx (2 \times 10^{-4} \text{ arcsec/yr}) \times (10^3 \text{ yr/arcsec}) \approx 0.2$$

This result ($\gamma \approx 0.2$) is of the order of unity, demonstrating that the proper motion of standard elliptical galaxies is sufficient to generate strong lensing-like distortions without intrinsic luminosity variability.

3.2 Connection to Lens-Model Degeneracies

This mechanism parallels the *Source-Position Transformation (SPT)* (Schneider & Sluse 2013), which identifies a degeneracy class of mass models yielding identical observables but differing time delays. TEP extends this degeneracy into the metric sector itself. Rather than permuting the mass distribution $\kappa(\vec{\theta})$ while holding the metric constant, TEP holds the baryonic mass constant and permutes the spacetime arrival surface $\Delta\tilde{\tau}(\vec{\theta})$. Consequently, the "Phantom Mass" can be understood as a physical realization of the SPT, where the extra freedom resides in the time-transport sector rather than invisible spatial matter.

3.3 The Reference Envelope: Millisecond Delays

Under the conservative *Reference Envelope* (accepting the standard translation of GW170817 timing to propagation-speed bounds), $\Delta\tilde{\tau}$ is small (milliseconds to seconds). In this regime:

- **Static Lensing:** The phantom mass effect is negligible for slowly evolving sources (galaxies). Static mass maps are unaffected.
- **Time-Domain Lensing:** The effect is dominant for fast transients. A millisecond gradient across an image plane is huge for an FRB.

Conclusion: In the *Reference Envelope*, TEP is a precision correction to time-domain astrophysics, not a full dark matter substitute.

3.4 The Critical Discriminator: Source Variability

The magnitude of the Phantom Mass effect depends on $\partial I_s / \partial t$. This leads to a unique falsifiable prediction:

The Variability Bias: The inferred "dark matter" distribution should appear to change depending on the intrinsic variability timescale of the background source population. Static sources see less phantom mass; variable sources see more.

This is a radical prediction that distinguishes TEP from particulate dark matter, which is expected to be passive and source-independent.

4. Reanalysis of GW170817: What is Actually Constrained?

4.1 The Measurement and the Standard Translation

The simultaneous detection of gravitational waves (GW170817) and gamma rays (GRB 170817A) from a binary neutron star merger at approximately 40 Mpc (Abbott et al. 2017) represents one of the most precise measurements in astrophysics. The signals arrived within $\Delta t_{obs} = 1.74 \pm 0.05$ seconds of each other after traveling for approximately 130 million years. This is widely cited as constraining the difference between the speed of gravity c_g and the speed of light c_γ to (e.g., Baker et al. 2017; Creminelli & Vernizzi 2017; Ezquiaga & Zumalacárregui 2017; Sakstein & Jain 2017):

$$\frac{|c_\gamma - c_g|}{c} \lesssim 10^{-15}$$

This standard translation assumes that any delay is due to a uniform difference in propagation speed accumulating linearly over the entire cosmological distance. This section critically re-examines this assumption and analyzes what the measurement strictly constrains in a general two-metric framework.

4.2 The Common-Path Invariant: A Differential Measurement

A crucial but often overlooked geometric fact is that both messengers originated from the *exact same coordinate* in distant space. They passed through the same host halo, the same intergalactic voids, and the same Milky Way halo. In the geometric optics limit, they followed the same spatial trajectory through the scalar field $\phi(\vec{x})$.

The Common-Mode Cancellation:

Because the signals originate from the same coordinate and follow a nearly identical path, they do not diverge significantly. If the metric coupling contains a conformal component $\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$, this factor rescales the proper time along the path for *both* species identically. Any time dilation caused by passing through "time-warped regions" of space is experienced by both messengers. If the universe is "slower" in a specific region due to a scalar potential, it is slower for both the gravitational wave and the photon.

Implication for Conformal Coupling ($A(\phi)$):

As established in Section 2, a purely conformal transformation preserves null cones. *GW170817 imposes no direct constraint on the magnitude of conformal coupling.* The measurement confirms only that light and gravity share the same causal structure along a single path; it does not constrain the *rate* at which they traverse that path relative to other paths in the universe. The constraint applies only to the *difference* in null cone structures, not the absolute rate of time flow.

4.3 The Operational Reality: Decoupling the Sectors

To resolve the conflict between the GW170817 constraint and dark matter phenomenology, the two metric sectors must be explicitly distinguished. A frequent objection concerns the Shapiro delay: if TEP posits significant scalar potentials to mimic dark matter, should these not introduce measurable differential delays? It is crucial to recognize that in the TEP framework, the scalar potential is not an *additional* source of delay superimposed on a gravitational background; rather, it *constitutes* the physical origin of the Shapiro delay itself.

The "mass" inferred from Shapiro delay measurements corresponds, in this framework, to the depth of the conformal potential $A(\phi)$. Since Axiom 1 establishes that photons and gravitational waves traverse the same null geodesics defined by the conformal metric, they experience an identical Shapiro delay as they propagate through these potentials. Consequently, a differential arrival-time measurement like GW170817 cancels this "bulk" common-mode delay entirely, rendering the magnitude of the conformal potential unconstrained by such tests.

Assumption (Sector Decoupling): The 10^{-15} bound on the Disformal sector (the "speed of light" vs "speed of gravity") is accepted. The TEP-GL phenomenology relies entirely on the unconstrained Conformal sector gradients (the "rate of time").

This decoupling makes the TEP framework robust against propagation speed constraints. Consistent with the general disformal relation (Bekenstein 1993), we emphasize that the speed of gravitational waves (c_g) constrains the causal structure (the light cone) governed by the disformal term $B(\phi)$, while the Dark Matter phenomenology in TEP arises from the conformal factor $A(\phi)$ (the clock rate). As long as $c_g = c_\gamma$, the rate at which time accumulates along that path is unconstrained by arrival-time differences. GW170817 strictly constrains cone tilting, but is largely insensitive to the conformal time dilation that mimics mass.

Summary of Constraints:

- **Conformal Sector:** Unconstrained. Can be order unity (subject to other tests).
- **Disformal Mean (Monopole):** Tightly constrained by GW170817 ($\lesssim 10^{-15}$).
- **Disformal Gradient (Multipole):** Constrained only by the requirement that the integral of the gradient not exceed the monopole bound excessively. This permits non-trivial millisecond-scale differential structure across the image plane, sufficient for the time-domain signatures predicted in Section 5.

By distinguishing between the *speed of transmission* (constrained) and the *rate of proper time accumulation* (unconstrained), the viability of TEP as a solution to the dark matter problem is established.

4.4 Environmental Screening and Solar System Constraints

A critical quantitative challenge to the TEP framework is the magnitude of the scalar potential required to explain cluster lensing. To generate differential delays of 10^3 -- 10^5 years over cosmological distances, the conformal factor $A(\phi)$ must have significant depth. If such potentials existed unmodified within the Solar System, they would likely violate high-precision ephemerides constraints (e.g., the Cassini parameter γ).

To reconcile the large potentials required on cluster scales with the strict constraints on local scales, we explicitly invoke a ****Screening Mechanism****. To move beyond qualitative hypotheses, we define the scalar sector via a Galileon-type Lagrangian known to provide Vainshtein screening:

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi + \frac{g}{M_{Pl}}\phi T$$

where Λ is the strong coupling scale and T is the trace of the energy-momentum tensor. In high-density environments (Solar System), the non-linear term $\frac{1}{\Lambda^3}(\partial\phi)^2\Box\phi$ dominates, suppressing the spatial gradients of ϕ and restoring General Relativity to high precision (Vainshtein screening). On cosmological scales (low density), the linear kinetic term dominates, allowing the scalar field to develop the large 10^3 -- 10^5 year potential depths required for TEP phenomenology. This Lagrangian formulation moves the screening argument from "wishful thinking" to standard field theory practice, ensuring mathematical consistency with local tests.

5. Observational Predictions: The Era of Chronometric Mapping

The transition from a geometric to a dynamical-time framework shifts the observational focus from static shapes to dynamic arrival times. The "dark sector" is predicted to reveal its true nature not in deep fields, but in high-cadence time-domain surveys. Explicit, falsifiable predictions are presented, organizing them by scale: from the millisecond "jitter" of fast transients (FRBs), to the statistical bias in variable sources, to the global tension between CMB and galaxy lensing.

5.1 The "Jittering" of Lensed Transients

Prediction: Strongly lensed fast transients (FRBs, GRBs) will exhibit achromatic arrival-time residuals that cannot be explained by geometric time delays (Refsdal 1964) or plasma dispersion.

In standard GR, the time delay Δt_{geom} between images is fixed by the mass distribution. In TEP, there is an additional proper-time delay $\Delta\tilde{\tau}$. Because ϕ fields in halos may have substructure (or "weather"), this delay varies across the image plane. For a millisecond-duration FRB (e.g., Muñoz et al. 2016), even a tiny gradient in $\Delta\tilde{\tau}$ will manifest as a timing anomaly. Unlike plasma dispersion (which scales as ν^{-2}), this delay is *achromatic*. A detection of non-dispersive, millisecond-scale residuals in lensed FRBs would constitute a definitive signature of TEP.

5.2 The Variability Bias Relation

Prediction: The inferred weak-lensing shear should correlate with the variability of the background source population.

As derived in Section 3, the "phantom mass" effect is driven by the *Temporal Shear Tensor* \mathcal{T}_{shear} , which depends on the total source variation (intrinsic evolution + proper motion). A specific test using existing cluster fields is outlined (e.g., Abell 370): construct two independent shear maps of the same cluster. Map A uses only passive elliptical galaxies (dominated by proper motion variability). Map B uses only intrinsically variable sources (quasars, lensed supernovae). TEP predicts a systematic excess convergence in Map B due to the additional intrinsic term $\partial I_s / \partial t$, whereas particle dark matter predicts identical maps. Future surveys (LSST/Rubin) will have the cadence to test this "Variability Bias," potentially falsifying the particle hypothesis.

5.3 The CMB-Galaxy Lensing Tension

Prediction (Extended Regime): Lensing inferred from the Cosmic Microwave Background (CMB) should differ systematically from lensing inferred from galaxy shear.

While the CMB is effectively a static backlight (zero intrinsic evolution), the intervening structures (the lenses) are moving dynamically across the line of sight. Under TEP, the rapid transit of a lens's "time gradient" $\nabla(\Delta\tilde{\tau})$ across a static background induces a temporal shear similar to the *Rees-Sciama effect*, but amplified by the conformal factor. Consequently, TEP predicts that

CMB lensing *exists* and correlates with structure, driven by lens motion. However, galaxy lensing includes an *additional* constructive contribution from the intrinsic evolution of the source galaxies ($\partial I_s / \partial t$). This asymmetry predicts a systematic tension in the inferred amplitude of clustering (S_8) between CMB lensing (lens-motion only) and galaxy weak lensing (lens-motion + source-evolution), offering a physical resolution to the observed S_8 tension.

5.4 Comparison of Predictions: TEP-GL vs. Particle Dark Matter

The following table summarizes the distinguishing predictions of the two frameworks:

Observable	Particle DM Prediction	TEP-GL Prediction
Lensed FRB timing	Achromatic, GR time delays only	Achromatic <i>residual</i> anomaly (ms-scale)
Source-evolution correlation	None (lensing is source-independent)	Strong (phantom mass scales with τ_{var})
CMB lensing	Standard convergence	Identical (CMB is static; no temporal smearing)
Galaxy weak lensing	Identical to CMB lensing	Systematic offset from CMB (source-dependent)
Chromatic dependence	None	None (both achromatic)
Direct detection experiments	Expected signal (WIMP recoil)	No signal (no particle to detect)

The critical discriminant is the *source-dependence* of the inferred dark component. Particle dark matter lenses all sources identically; TEP-GL predicts that the apparent "dark" signal depends on the variability timescale of the background source.

5.5 Falsification Criteria

TEP-GL is a falsifiable hypothesis. The following observations would exclude specific regimes or the framework entirely:

- Reference Envelope Exclusion:** If precision timing of strongly lensed FRBs yields achromatic residuals consistent with zero to better than 0.1 ms across diverse lens environments, the **Reference Envelope** parameter space is excluded.
- Source-Independent Shear:** If weak-lensing shear measurements behind clusters show no correlation with source variability class (static ellipticals vs. variable AGN) after controlling for redshift and morphology, the temporal-composite mechanism is excluded.
- CMB-Galaxy Agreement:** If future surveys (e.g., CMB-S4, Rubin/LSST) demonstrate that CMB lensing and galaxy weak-lensing mass inferences agree to better than 1% after controlling for all modeling systematics, the **Extended Regime** parameter space is excluded.
- Chromatic Anomaly:** If any timing or morphological anomaly shows wavelength dependence after correction for plasma dispersion and dust, the achromatic prediction of two-metric coupling is falsified. This would indicate conventional astrophysical systematics rather than metric effects.
- Direct Detection:** A confirmed detection of dark matter particles in direct-detection experiments would establish the existence of a particulate dark sector. TEP would then be relegated to a sub-dominant correction rather than a primary explanation.

The framework stands or falls on empirical grounds. It does not claim immunity from observation; the claim is that the observations required to test TEP have not yet been performed with sufficient precision.

6. Discussion: A Paradigm Shift in the Dark Sector

6.1 Limitations of the "Stack of Corrections"

Gravitational physics has historically advanced by refining the fundamental geometric framework rather than adding ad-hoc corrections. TEP represents a similar upgrade to the fundamental object, following the natural historical progression of physical theory:

- Newton:** Gravity is a Force; Time is Absolute.
- Einstein:** Gravity is Geometry; Time is Relative (Coordinate-Dependent).
- TEP:** Gravity is Geometry; Time is a *Dynamical Field*.

By treating the rate of proper time accumulation as a physical field with its own degrees of freedom (rather than a fixed function of the metric), TEP unifies the "stack" into a single framework. The "dark matter" anomaly is simply the observation that this field has spatial gradients.

6.2 Interpretational Challenges

TEP effects mimic standard lensing signatures, making them difficult to distinguish without time-domain analysis.

1. **Static Degeneracy:** Most lensing data are analyzed under static assumptions. In this limit, temporal shear is mathematically indistinguishable from spatial mass (Section 3).
2. **Differential vs. Common-Mode Effects:** The GW170817 constraint is often interpreted as excluding modified metric couplings. However, as shown in Section 4, this constraint applies to the differential disformal sector. Conformal proper-time dilation is a *common-mode effect* for co-propagating signals, leaving the "speed of gravity" constraint satisfied while permitting significant time-domain phenomenology.

6.3 Reinterpreting the Evidence: Multipath vs. Single-Path

When viewed through the lens of TEP, the disparate constraints on the dark sector resolve into a consistent picture based on signal topology:

- **Lensing (Multipath):** Observations that require "dark matter" (e.g., lensing) are fundamentally *multipath* measurements. These different pathways sample different values of the time field ϕ , creating differential delays that manifest as "phantom mass."
- **GW170817 (Single-Path):** Observations that constrain "modified gravity" (GW170817) are fundamentally *single-path* measurements. The signals originate from the same coordinate and traverse the same time-warped regions. The temporal distortions are common-mode and cancel out.

The apparent contradiction—"how can gravity be modified enough to create dark matter but unmodified enough to pass GW170817?"—is resolved. Gravity may not be modified; time is. And time manifests as a differential observable only when comparing divergent paths.

6.4 Global Variation of Effective Light Speed

If TEP is correct, then c is locally invariant but globally variable. This challenges the foundational assumption used to interpret all astrophysical data.

- **Standard Assumption:** c is constant; therefore, any anomaly in t must be due to extra path length (spatial curvature) or extra mass (Shapiro delay). Conclusion: *dark matter* exists.
- **TEP Reality:** t is modulated by the field $\sqrt{A(\phi)}$; c is effectively variable along the path. The anomaly in t is due to the scalar field gradient. Conclusion: *dark matter* admits the interpretation of variable light speed.

TEP removes the need for an invisible substance by correcting the assumption about the constancy of the global speed of light.

6.5 TEP as a Bridge Between Paradigms

TEP offers a potential unification of competing frameworks:

- **CDM:** In the limit where the scalar field ϕ relaxes into stable halos and source variability is ignored, TEP reproduces the phenomenology of Cold Dark Matter (via the Phantom Mass mechanism).
- **MOND:** In the regime of low acceleration (large gradients in ϕ), TEP reproduces the scaling relations of Modified Newtonian Dynamics without modifying Newton's laws, but by modifying the clock metric.

Thus, TEP is not merely a competitor to these paradigms; it is a framework that explains *why* CDM works on large scales (where static assumptions hold) and *why* MOND works on galactic scales (where the field gradients are dominant).

6.6 The Path Forward

The analysis suggests that *dark matter phenomenology can be reproduced without a substance*. What exists is a temporal field with spatial gradients, and what has been measured as "dark matter" is the projection of those gradients onto observations that assume temporal synchrony. To maintain the particle interpretation, it is necessary to demonstrate that the universe is synchronous to a precision that excludes TEP effects. TEP offers a geometric identification of the phenomenology, explaining the universality of dark matter profiles and the success of MOND on galactic scales through a single parsimonious framework.

"Dark Time" is not merely an alternative to dark matter; it offers a geometric identification of the phenomenology that admits this interpretation.

7. Conclusions

7.1 Limitations of the Isochrony Axiom

For nearly a century, the dark matter paradigm has rested on a single, simplifying assumption: that the observed universe is synchronous. By treating the photons in telescopes as representing a single spatial slice of the source, standard models have been forced to interpret all observed distortions as spatial deflections caused by mass. It has been shown that this *Isochrony Axiom* is an effective approximation that may break down in the presence of generalized metric couplings.

The *Temporal Equivalence Principle (TEP)*, introduced in a companion paper (Smawfield 2025a), provides a corrected framework. Under TEP, "gravity" includes the phenomenology of differential proper-time accumulation. When the assumption of isochrony is relaxed, the "dark" signals in lensing maps can be modeled as *temporal-composite artifacts*—the projection of unmodeled time dilation onto the spatial image plane.

7.2 Summary of Findings

1. **Phantom Mass from Dark Time:** It was demonstrated that a gradient in proper-time accumulation across an image plane acts as a "temporal shear" on evolving sources. In a static reconstruction, this is mathematically indistinguishable from the gravitational shear of a dark matter halo. The "dark sector" may simply be the difference between the gravitational metric (which guides orbits) and the matter metric (which guides clocks).
2. **The GW170817 "Speed Limit" is Nuanced:** The standard multi-messenger constraint was deconstructed. Because the conformal component of the metric coupling preserves null cones, it is *invisible* to differential speed-of-gravity tests. Photons and gravitational waves share the common-mode dilation. The widely cited 10^{-15} bound constrains only the disformal (cone-tilt) sector, accepted here as the "speed of transmission" limit, while the unconstrained conformal sector governs the "rate of proper time accumulation" responsible for dark matter phenomenology.
3. **A New Observational Era:** Two regimes for testing TEP are defined. In the conservative *Reference Envelope*, the effects are millisecond-scale and detectable only in high-time-resolution astrophysics (FRBs, pulsars). In the *Extended Regime*, where environmental screening and/or a revised operational mapping allow larger effective delays, TEP becomes a full dynamical alternative to particle dark matter.

7.3 Future Outlook: Time, Not Mass

The persistence of the dark matter problem despite decades of particle searches suggests a potential error in the underlying premises. The error may lie in the treatment of time. By treating time as a passive parameter rather than an active dynamical field, there is a risk of blinding analysis to the true nature of the "dark" universe.

The path forward lies in *Chronometric Lensing*. The field must move beyond static mass-mapping and towards *chronometric mapping*—measuring the detailed arrival-time structure of the universe. If "dark matter" halos are actually "time dilation" halos, the definitive indicator will not be a particle recoil in a xenon tank, but a millisecond residual in a Fast Radio Burst. The focus of inquiry must shift from searching for missing mass to characterizing the dynamical structure of time.

7.4 The Paradigm Shift

Within the operational axioms adopted here, the phenomenology traditionally attributed to a particulate dark sector can be reinterpreted as the projection of two-metric time transport onto inference pipelines that assume isochrony. The anomalies are real; the claim is that their standard "missing mass" interpretation is not unique once the Isochrony Axiom is relaxed.

The observational program outlined herein—time-domain lensing of fast transients, precision strong-lens residual timing, and source-variability-dependent shear consistency tests—provides a viable route to distinguish particle dark matter from a temporal-composite channel while maintaining a conservative Reference Envelope baseline anchored to existing multi-messenger constraints, thereby offering a promising avenue for resolving the dark matter problem.

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Appendix A: Mathematical Derivations

A.1 Proof of Null Cone Invariance in the Conformal Limit

Proposition: If two metrics are conformally related by $\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$, they share the same null geodesics as unparameterized curves.

Proof:

Let $k^\mu = dx^\mu/d\lambda$ be a null vector in $g_{\mu\nu}$, satisfying $g_{\mu\nu}k^\mu k^\nu = 0$ and the geodesic equation $k^\nu \nabla_\nu k^\mu = 0$ (where ∇ is the Levi-Civita connection of g).

In the metric $\tilde{g}_{\mu\nu}$, the null condition holds immediately:

$$\tilde{g}_{\mu\nu}k^\mu k^\nu = A g_{\mu\nu}k^\mu k^\nu = 0$$

The connection coefficients $\tilde{\Gamma}_{\mu\nu}^\lambda$ for \tilde{g} are related to $\Gamma_{\mu\nu}^\lambda$ by:

$$\tilde{\Gamma}_{\mu\nu}^\lambda = \Gamma_{\mu\nu}^\lambda + \frac{1}{2}(\delta_\mu^\lambda \partial_\nu \ln A + \delta_\nu^\lambda \partial_\mu \ln A - g_{\mu\nu} g^{\lambda\sigma} \partial_\sigma \ln A)$$

Substituting this into the geodesic equation for \tilde{g} :

$$k^\nu \tilde{\nabla}_\nu k^\mu = k^\nu \nabla_\nu k^\mu + \frac{1}{2}(2k^\mu k^\nu \partial_\nu \ln A - g_{\nu\sigma} k^\nu k^\sigma \dots)$$

Since k is null ($g_{\nu\sigma} k^\nu k^\sigma = 0$) and $k^\nu \nabla_\nu k^\mu = 0$, this yields:

$$k^\nu \tilde{\nabla}_\nu k^\mu = (k^\nu \partial_\nu \ln A) k^\mu$$

This is the geodesic equation with a non-affine parameterization ($Dk/d\lambda \propto k$). Thus, the curve is a geodesic of \tilde{g} , differing only by the parameterization (the clock rate). \square